Air-core transformer electrical modeling

Two ideal coupled resonant circuits

The differential system has no general analytic solution except for $LC=L_2C_2$. The oscillations of the system are not necessarily periodic.

Differential system

Solution

$$2\omega_1 = \omega_3$$

$$2\sqrt{\frac{1}{LC(1+K_x)}} = \sqrt{\frac{1}{LC(1-K_x)}}$$

Critical coupling

$$K_x = 0.6$$

Wave forms

For critical coupling ($K_c=0.6$) the oscillation consist of the fundamental and the second harmonic. The signal is not symmetric with respect to zero. The ratio between pos. and neg. maximum is 1.78.
Air-core transformer electrical modeling

Numeric simulations of lossy air-core resonant transformer

System sensitivity due to components value variation was examined using numeric simulations.

Small coupling factor deviations from \( K_c = 0.6 \) can be compensated by changing the tuning of the resonators.

Design and construction of a transformer with a precise coupling factor is a challenge.
Air-core transformer electrical modeling

Parametric study of lossy air-core resonant transformer

The study used two criteria:
- Maximum output amplitude
- Maximum positive to negative peak voltage ratio

The varied parameters were coupling factor and primary resonator tuning (varying the primary capacitance).

The value of the resistor in primary side was iteratively determined to ensure that the loss factor is kept constant (~30% per cycle)

The determination of negative to positive peak voltage ratio was difficult because, if there is no significant loss in the system, after many cycles the negative peak could appear as positive one.
Air-core transformer magnetic modeling

Numerical and physical modeling of the transformer

The full 3D numerical simulation is limited by:

- enormous aspect ratio;
- excitation pulse length limitations

Simplified geometry was a necessity.

Measurements on scaled physical models:
- confirmed 3D numeric simulations
- confirmed scalability of the air-core coils / transformers
- defined the best geometry of the air-core transformer
- showed the sensitivity of the transformer to surrounding space and close metal objects

Used physical models
Air-core transformer magnetic modeling

Physical models vs. 3D numeric simulations

An additional conductor is placed in 3D model in order to shorten the secondary.

Coupling factor $K$ is given by the formula:

$$K = \sqrt{\frac{L - L_{sh}}{L}}$$

where $L$ and $L_{sh}$ are the values of primary inductance when the secondary is open and shorted. The coils are used in autotransformer mode and one turn primary.

The relative error between the simulated and measured inductance values was <6% and coupling values <1%. Simulation conditions were kept strictly unchanged for $L$ and $L_{sh}$ (meshing, coil geometry, etc.).

Insensitivity to coupling errors because meshing errors tend to cancel.

<table>
<thead>
<tr>
<th>Number of turns</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$, nH</td>
<td>1090</td>
<td>1090</td>
<td>1090</td>
<td>1090</td>
<td>1090</td>
</tr>
<tr>
<td>$L_{sh}$, nH</td>
<td>276</td>
<td>451</td>
<td>553</td>
<td>619</td>
<td>666</td>
</tr>
<tr>
<td>coupling, %</td>
<td>0.864</td>
<td>0.766</td>
<td>0.702</td>
<td>0.657</td>
<td>0.624</td>
</tr>
<tr>
<td>Simulated</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$, nH</td>
<td>1101</td>
<td>1101</td>
<td>1101</td>
<td>1101</td>
<td>1101</td>
</tr>
<tr>
<td>$L_{sh}$, nH</td>
<td>290</td>
<td>436</td>
<td>563</td>
<td>625</td>
<td>678</td>
</tr>
<tr>
<td>coupling, %</td>
<td>0.858</td>
<td>0.761</td>
<td>0.699</td>
<td>0.658</td>
<td>0.620</td>
</tr>
<tr>
<td>L relative error, %</td>
<td>1.01%</td>
<td>1.01%</td>
<td>1.01%</td>
<td>1.01%</td>
<td>1.01%</td>
</tr>
<tr>
<td>$L_{sh}$ relative error, %</td>
<td>5.07%</td>
<td>2.66%</td>
<td>1.81%</td>
<td>0.97%</td>
<td>1.80%</td>
</tr>
<tr>
<td>Coupling factor error, %</td>
<td>0.68%</td>
<td>0.58%</td>
<td>0.41%</td>
<td>0.03%</td>
<td>0.62%</td>
</tr>
</tbody>
</table>

Measured values vs. simulated values and the relative errors between them.
Air-core transformer magnetic modeling

Air-core transformers scalability

Mutual inductance of air-core transformers depends only on geometry.

Using the definitions of magnetic flux density $B$, magnetic flux $\Phi$, self inductance $L$, mutual inductance $M$ and coupling factor $K$, the inductances scale linearly with the geometrical dimensions and the coupling factor stays constant.

Basic relations

\[ B = \frac{\mu_0 i}{4\pi} \int \frac{d\vec{L} \times \vec{r}}{r^3} \quad B \sim \frac{1}{r} \]

\[ \Phi = \int_{S} \vec{B} \cdot d\vec{s} \quad \Phi \sim r \]

\[ L = \frac{\Phi}{i} \quad L \sim r \]

\[ M = \frac{\Phi_{21}}{i_1} \quad M \sim r \]

\[ K = \frac{M}{\sqrt{L_p L_S}} \quad K = const \]

Measured results
Air-core transformer magnetic modeling

To optimize the coupling factor of air-core transformer the influence of the coils shape was studied. The model coils were used in autotransformer mode.

Three basic configurations were compared:

- Spiral shape with peripheral excitation
- Spiral shape with central excitation
- Helical shape

Highest coupling factor is obtained for peripherally excited spiral secondary coil.

The higher the number of turns (higher step up ratio) the lower the coupling.
Air-core transformer magnetic modeling

Optimal air-core transformer geometry – coil dimensions and conductor cross-section

Another factor that influences the coupling is the transformer diameter and the conductor cross-section. The larger the transformer diameter and the wider the strip the higher the coupling.

Conductors span is kept 15mm for all coils.

Influence of coil diameter

Influence of conductor cross section dimensions

Conductor Shape

- Strip 14x1 mm
- Strip 50x1 mm
- Helix D220 mm
- Helix D380 mm
- Spiral Dm380 mm
Air-core transformer magnetic modeling

Optimal air-core transformer geometry – conductor cross section

The conductor cross-section shape was studied. The two extreme cases was investigated. Conductors span was kept constant.

**Round conductor**

**Strip conductor (width >> thickness)**

Coupling factor depends on the perimeter of the conductor cross-section and it is relatively insensitive to the conductor cross-section shape.
Air-core transformer magnetic modeling

Optimal air-core transformer geometry – proximity effects

There was a concern about the deterioration of the transformer performance due to limited surrounding space and near large metal objects.

<table>
<thead>
<tr>
<th>Primary Loop type</th>
<th>Free space</th>
<th>In the tank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L, nH</td>
<td>Lsh, nH</td>
</tr>
<tr>
<td>D165/14/0.5mm (1MHz)</td>
<td>330</td>
<td>210</td>
</tr>
<tr>
<td>D165/28/0.5mm (1MHz)</td>
<td>270</td>
<td>170</td>
</tr>
</tbody>
</table>

Coupling factor deteriorated by 15% due to limited surrounding space. The metal stalk in the center of the transformer did not show any significant further deterioration.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Model</th>
<th>Estimation</th>
<th>Measured</th>
<th>Error</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary (Oct 2004)</td>
<td>248 nH</td>
<td>903 nH</td>
<td>1078 nH</td>
<td>19%</td>
<td>Since the final shape of the coil was not known the model was not exact</td>
</tr>
<tr>
<td>Primary (Dec 2006)</td>
<td>150 nH</td>
<td>1088 nH</td>
<td>1078 nH</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>Secondary (Oct 2004)</td>
<td>290 uH</td>
<td>141 uH</td>
<td>133 uH</td>
<td>6%</td>
<td>Since the final shape of the coil was not known the model was not exact</td>
</tr>
</tbody>
</table>

The study gave an input for the electrical simulations.
Later on, the comparison between the estimated values based, on the scaled models, and the real transformer values showed good agreement.
The shortest possible secondary length gives the shortest pulse length. A planar spiral secondary coil with peripheral single-turn primary was chosen.

The secondary consist of 16.5 turns of 4mm diameter copper tube (compatible with 3.6 mm semi-rigid coax cable)

To prevent voltage breakdown the transformer is operated in sulfur hexafluoride ($\text{SF}_6$) gas at up to 5 bar.

The transformer base is made out of Acrylglas (Plexiglas). OD 600 mm, ID 240 mm. On one face is a spiral channel for the conductor and on the other corrugations to prevent surface discharges.