

Radius of the proton

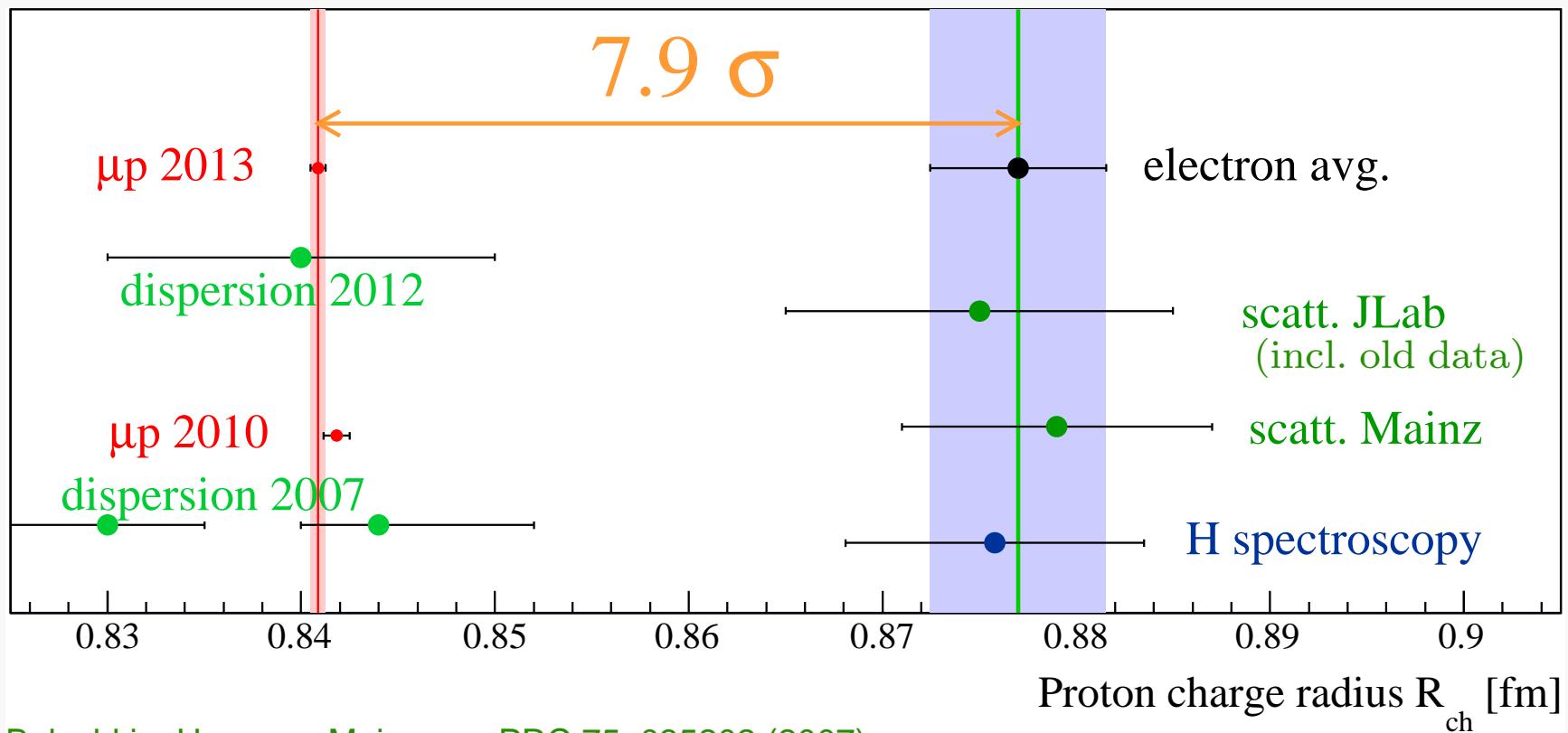
from the Lamb shift in muonic hydrogen

F. Kottmann, ETH Zürich, Switzerland

- Puzzle, media hype, some history
- μp levels, proton finite size effect
- Principle of experiment, apparatus
- Results, proton radius puzzle
- What may be wrong ?
 - (1) μp experiment
 - (2) μp theory
 - (3) H spectroscopy
 - (4) H theory
 - (5) electron-proton scattering
- New physics ?
- muonic deuterium μd
- μHe^+ – Conclusions & outlook

The proton radius puzzle

The proton rms charge radius measured with
electrons: 0.8770 ± 0.0045 fm
muons: 0.8409 ± 0.0004 fm



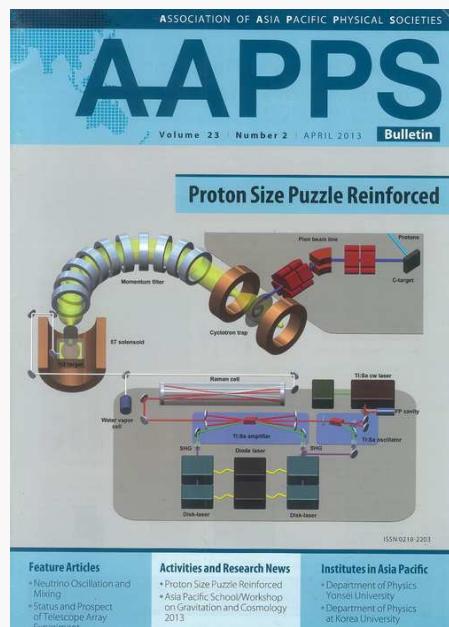
Belushkin, Hammer, Meissner PRC 75, 035202 (2007).

Lorenz, Hammer, Meissner EPJ A 48, 151 (2012).

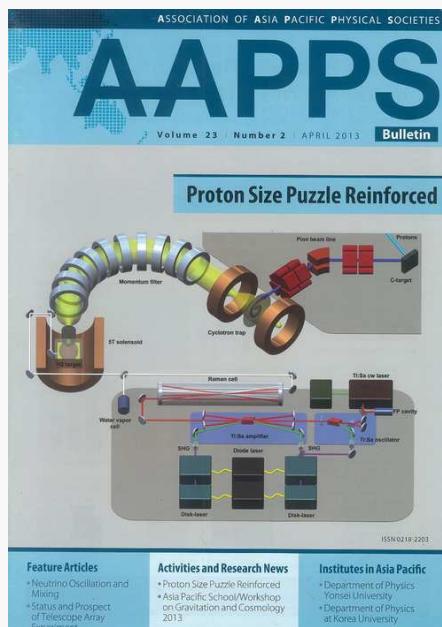
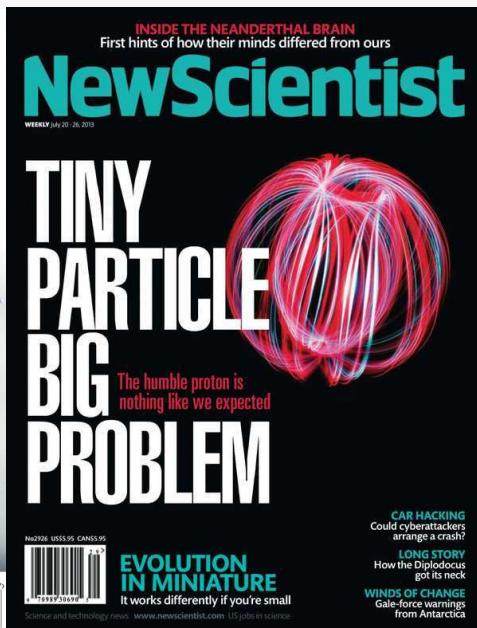
‘Inflation’ in the news ...



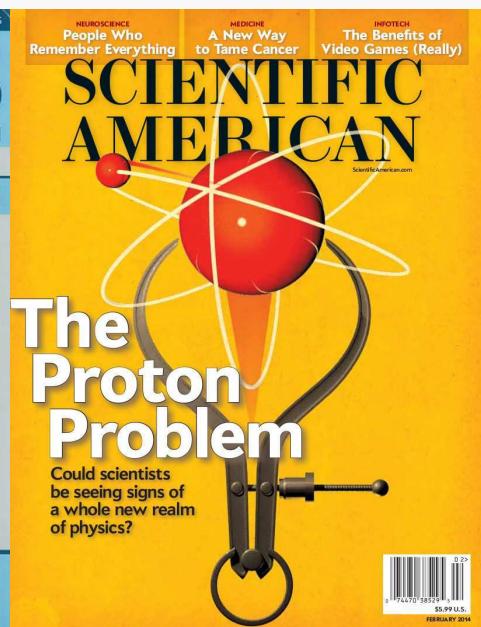
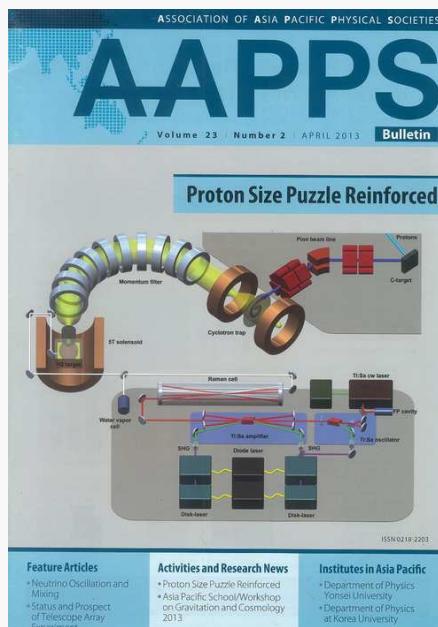
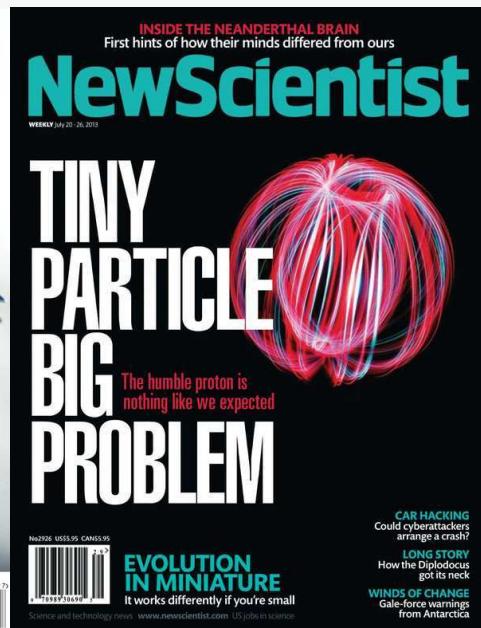
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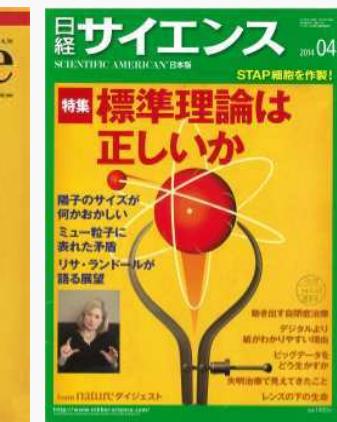
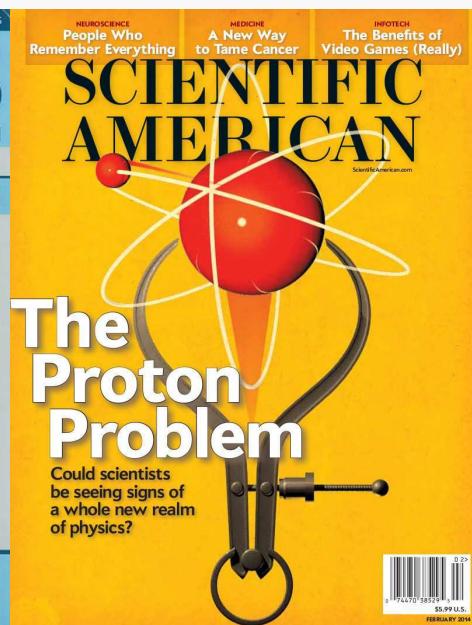
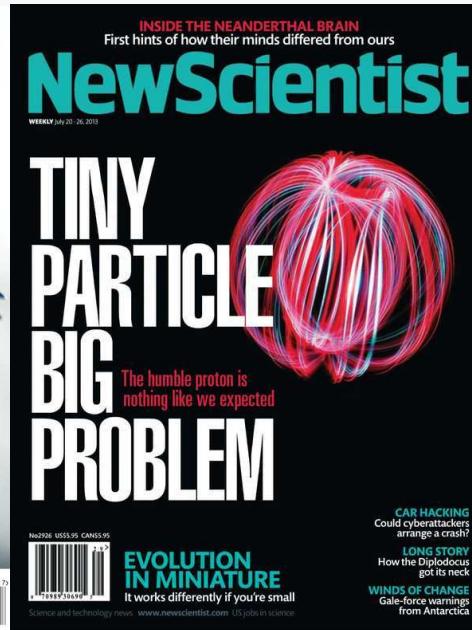
'Inflation' in the news ...



'Inflation' in the news ...



'Inflation' in the news ...



... our own journal:



CREMA:

Charge Radius Experiments
with Muonic Atoms

History of μp Lamb shift

- 1947 Lamb shift = $\Delta E(2S_{1/2}-2P_{1/2})$ in H (~ 1 GHz) → QED
- ~1948 muonic atoms $\mu^- Z$, muonic cascade [Fermi-Teller 1947, Chang 1949]
- 1949 Wightman discusses formation of μp (small neutral system → interacts!)
- 1953 μZ spectroscopy with NaI(Tl) → nuclear radii
- ~1966 μZ spectroscopy with Ge(Li) [Backenstoss et al.]
- 1969 Di Giacomo calculates $\Delta E(2S-2P) = -0.2$ eV in μp → $\lambda = 6 \mu\text{m}!$
V. Hughes, V. Telegdi, E. Zavattini consider $\mu p(2S-2P)$ → $\tau_{2S} = ?$
- 1970 2-keV x-rays from μp measured at 4 bar [Zavattini et al.]
- 1971 Proposal at NEVIS, Columbia: search for long-lived $\mu p(2S)$ [V. Hughes et al.]
- 1973 $\mu \text{He}^+(2S-2P)$ measured by Zavattini et al. (at 40 bar !)
- ~1975 SIN, LAMPF, TRIUMF: meson factories

1960 first lasers

History of μ p Lamb shift

1975 Propaganda slide (when μ p(2S-2P) was first considered at SIN):

“pure-QED tests”

e:	$g_e - 2$	0.1 ppm	H(2S-2P)	~ 30 ppm
μ :	$g_\mu - 2$	8 ppm	μ p(2S-2P)	~ 50 ppm (ideas...)

$\mu = e$? (Discrepancies found in μ -atoms !)

History of μ p Lamb shift

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(status 2014:)

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μ :	$g_\mu - 2$ 8 ppm <i>0.5 ppm (Brookh.)</i>	μ p(2S-2P) ~50 ppm (ideas...) <i>12 ppm (PSI, 2013)</i>

$\mu = e ?$ (Discrepancies found in μ -atoms!) ... resolved

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$\mu = e ?$ (Discrepancies found in μ -atoms!) ... resolved

1979 Proposal for $\mu p(2S-2P)$ at SIN [H. Hofer et al.] (0.3 mbar)

1981 SIN: no long-lived $\mu p(2S)$ at ~mbar; problems with laser development

~1985 no motivation for a “test of vac.pol.” at 50 ppm-level!

“THE END”

History of μp Lamb shift

1975 Propaganda slide (when $\mu p(2S-2P)$ was first considered at SIN):

“pure-QED tests”

(status 2014:)

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... Intermezzo:

- 1989 at SIN: $\mu He^+(2S-2P)$ measured at “ λ (Zavattini)”, **40 mbar**
- D. Taqqu continues to think ...
- L. Simons: new Cyclotron Trap (delivered 1996) for πp , πd
- PSI-Proposal R-93-06: $\mu p(3D-3P)$ with FEL (Zavattini et al.)

History of μp Lamb shift

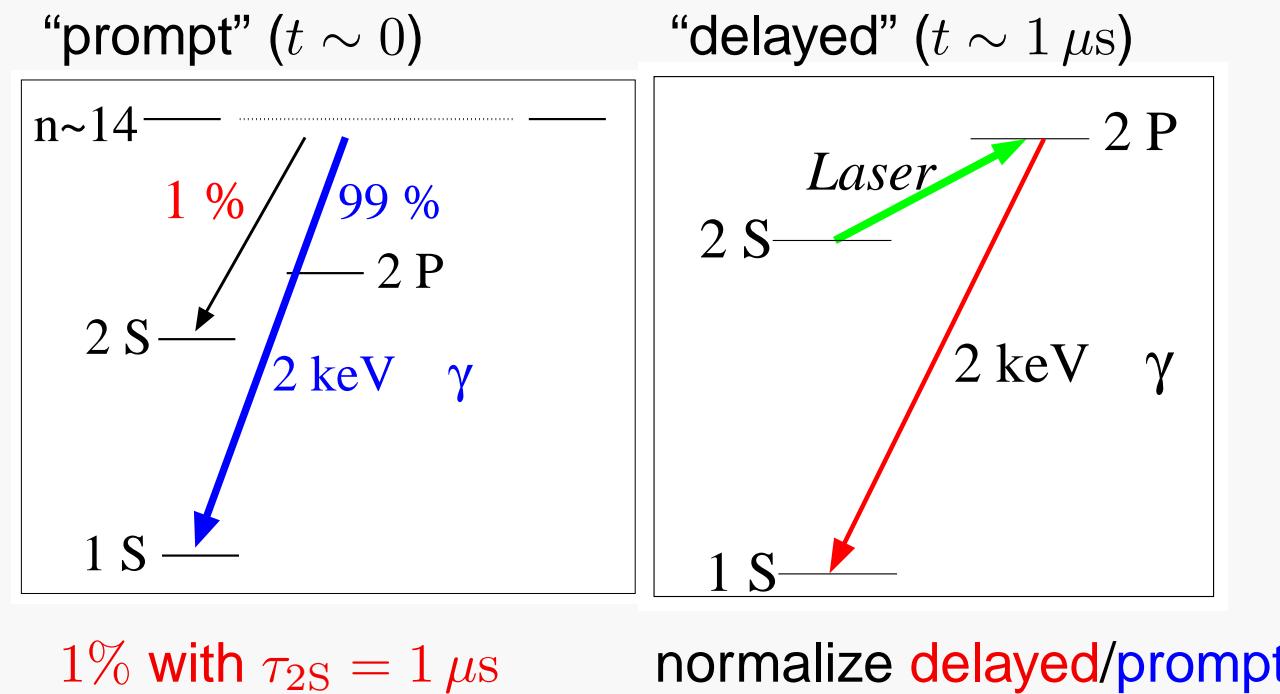
- ~1995
 - big progress in H-spectroscopy [Haensch et al.]
 - new motivation: determine r_p precisely (2 % → 0.1 %)
 - new μ^- -beams, new ideas for $\mu p(2S-2P)$ [L. Simons, D. Taqqu, F.K.]
- 1998 new Proposal for $\mu p(2S-2P)$ at PSI [new collaboration: MPQ, Paris, Coimbra, FR...]
- 2000 long-lived $\mu p(2S)$ measured (non-radiative $2S \rightarrow 1S$ “quenching”) [R. Pohl et al.]
- 2009 2S-2P resonance found, 5σ off! (nothing found in 2003, 2007)
 - unexpected new situation, new motivation: *solve puzzle!*

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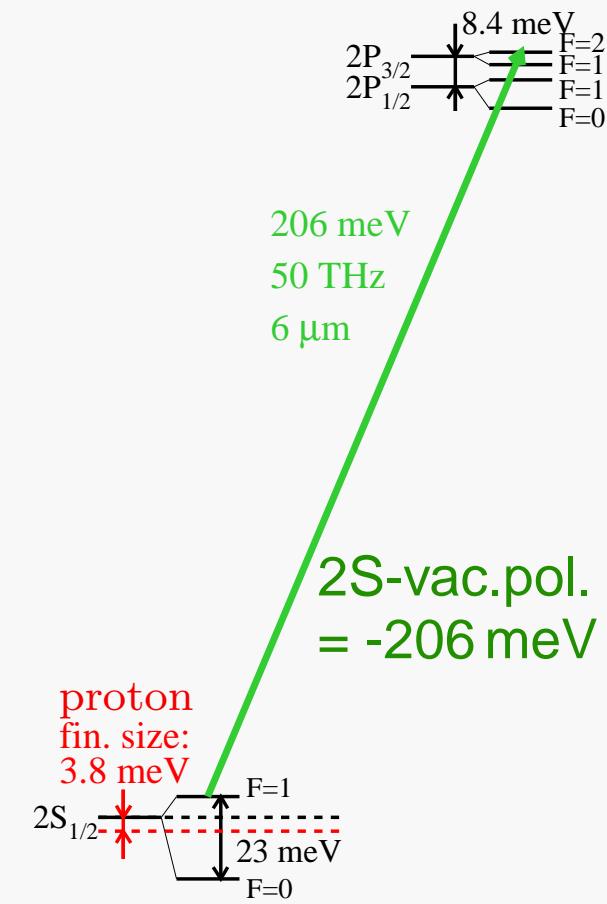
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- 2010
 - First $\mu p(2S-2P)$ resonance published in Nature
 - New Proposal for $\mu He^+(2S-2P)$ at PSI
 - New e-p scattering data from Mainz [PRL 105, 242001]
- 2011 New \vec{e} -p scattering data from JLab [Phys. Lett. B 705, 59]
- 2013/4 Five 2S-2P resonances measured in μ^4He^+ and μ^3He^+

Principle of μ^- (2S-2P) experiment

- special low-energy μ^- beam-line at PSI (unpulsed!)
- μ^- detected in-flight \rightarrow trigger of laser system
- μ^- p atoms formed in 1 mbar H₂ gas
- laser pulse excites the 2S-2P transition ($\lambda \approx 6 \mu\text{m}$)
- delayed 2P-1S X-ray detected: **signature**

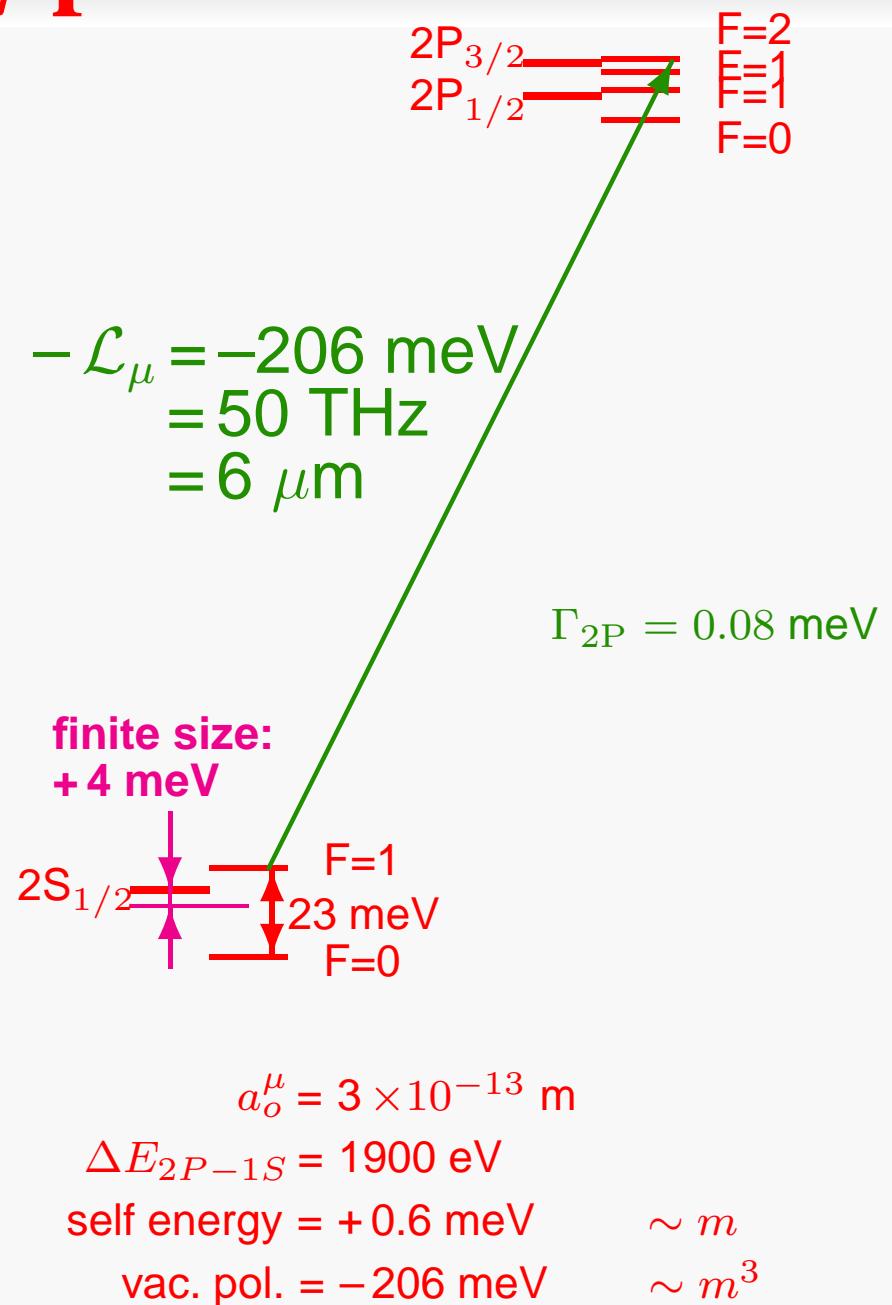
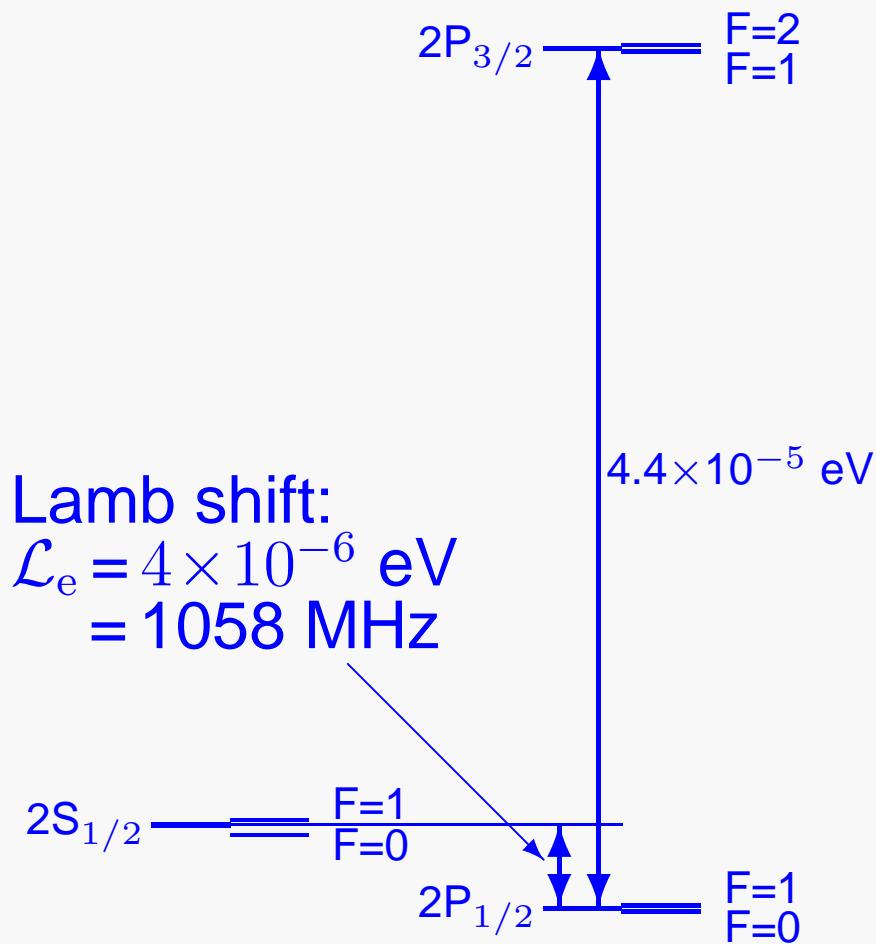


[R. Pohl ..., PRL 97,193402 (2006)]

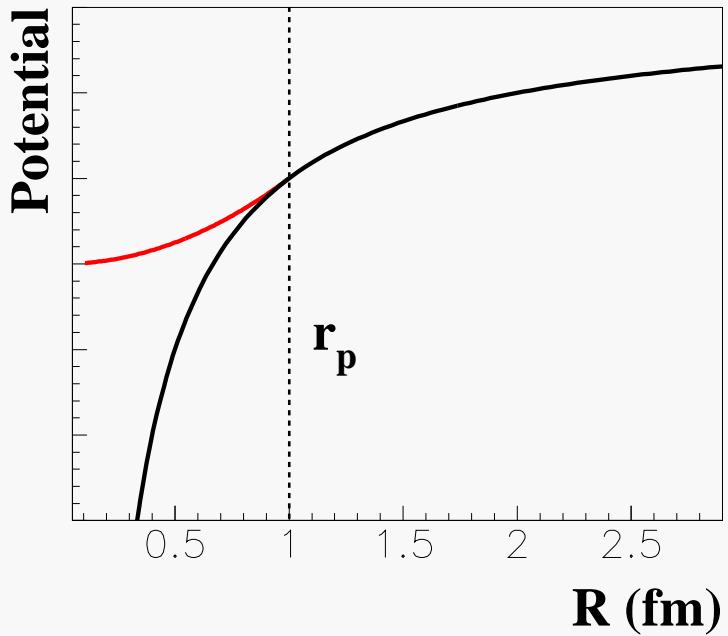


linewidth = $\Gamma_{2P} = 18.6 \text{ GHz}$
 \rightarrow 6 transitions separated!

(n=2) - states of ep and μ p



Finite size effect (in leading order)



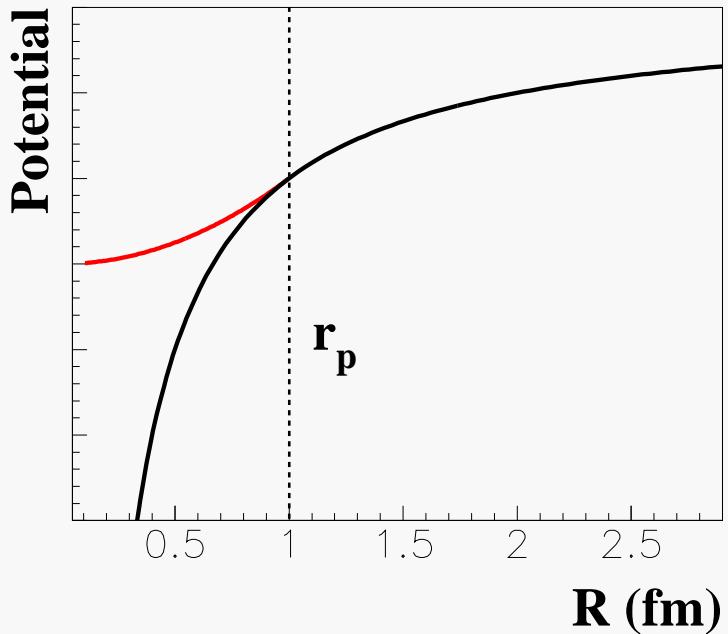
Maxwell equation: $\nabla E = 4\pi\rho$

$$V = \begin{cases} -\frac{Z\alpha}{2r_p} \left(3 - \left(\frac{r}{r_p}\right)^2\right) & (r < r_p) \\ -\frac{Z\alpha}{r} & (r > r_p) \end{cases}$$

$$\Delta V = \begin{cases} -\frac{Ze^2}{2r_p} \left(3 - \left(\frac{r}{r_p}\right)^2 - \frac{2r_p}{r}\right) \\ 0 \end{cases}$$

$$\Delta E^{FS} = \langle \bar{\Psi} | \Delta V | \Psi \rangle$$

Finite size effect (in leading order)

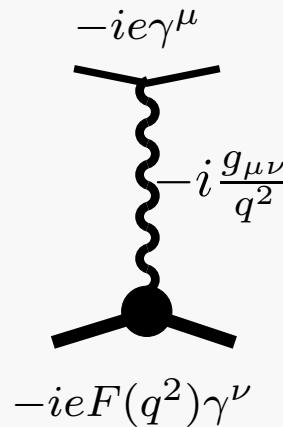


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$$\Delta E^{FS} = \langle \bar{\Psi} | \Delta V | \Psi \rangle$$



$$\frac{1}{q^2} \rightarrow \frac{F(q^2)}{q^2}$$

$$r_p^2 \equiv \int d^3r \rho(\mathbf{r})r^2$$

$$F(\mathbf{q}^2) = \int d^3r \rho(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} \simeq Z \left(1 - \frac{\mathbf{q}^2}{6} r_p^2 + \dots\right)$$

$$\Delta V(r) = V(r) - \left(-\frac{Z\alpha}{r}\right)$$

$$\Delta V(\mathbf{q}) = \frac{4\pi Z\alpha}{\mathbf{q}^2} (1 - F(\mathbf{q})) \simeq \frac{2\pi(Z\alpha)}{3} r_p^2$$

$$\Delta V(r) = \frac{2\pi(Z\alpha)}{3} r_p^2 \delta(r)$$

$$\begin{aligned} \Delta E^{FS} &= \frac{2\pi(Z\alpha)}{3} r_p^2 |\Psi_n(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3 r_p^2 \delta_{l0} \end{aligned}$$

... there are several “proton radii” :

- rms charge radius r_p^2 : $r_p^2 \equiv \langle r_p^2 \rangle = \int d^3r \rho_E(r) r^2 = 0.774(8) \text{ fm}^2$ ($r_p \approx 0.88 \text{ fm}$)
↔ Lamb shift
- rms magnetic radius: $r_{\text{mag}}^2 \equiv \langle r_{\text{mag}}^2 \rangle = \int d^3r \rho_M(r) r^2 = 0.604(20) \text{ fm}^2$
- “Zemach radius”: $R_Z \equiv \langle r_p \rangle_{(2)} = \int d^3r \int d^3r' \rho_E(\vec{r} - \vec{r}') \rho_M(\vec{r}') r = 1.045(4) \text{ fm}$
↔ HFS
- “Third Zemach moment”: $\langle r_p^3 \rangle_{(2)} = \int d^3r \int d^3r' \rho_E(\vec{r} - \vec{r}') \rho_E(\vec{r}') r^3 = 2.85(8) \text{ fm}^3$
↔ Lamb shift, “NLO”

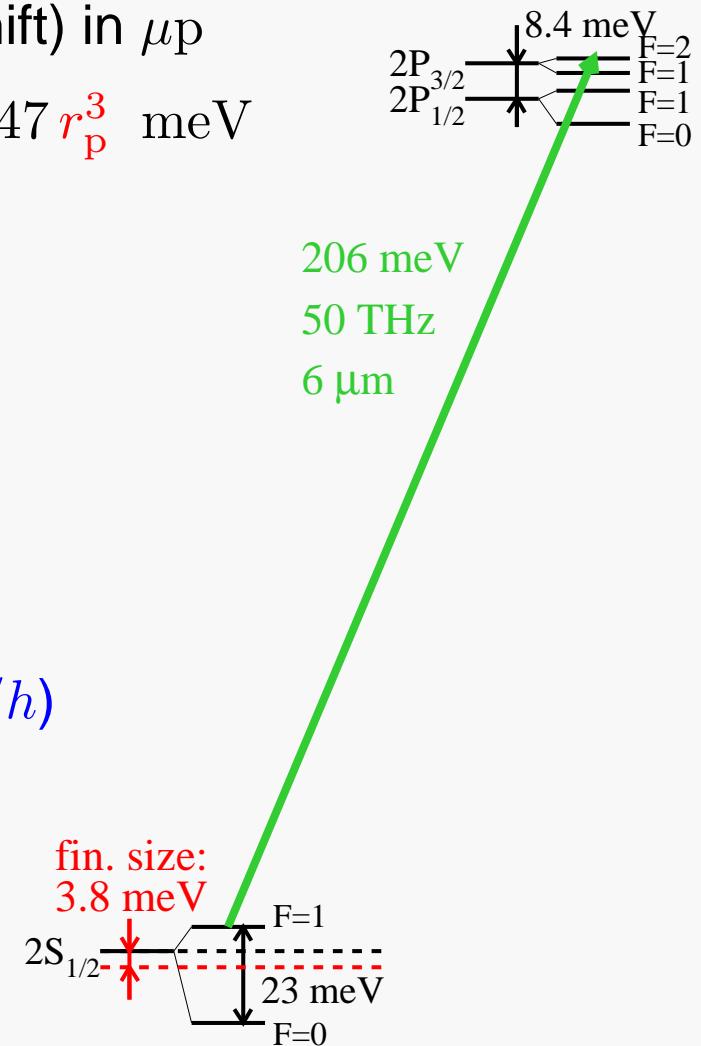
values from e-scattering [Distler, Bernauer, Walcher, arxiv:1011.1861]

Aim of the μp Lamb shift experiment

(before we dit it !)

- Measure the $2S - 2P$ energy difference (Lamb shift) in μp

$$\Delta E(2S - 2P) = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV}$$
with 30 ppm precision.
- Extract $r_p \equiv \sqrt{r_p^2}$ with $u_r \approx 10^{-3}$ (rel. accuracy)
 - bound-state QED test in hydrogen to a level of $u_r \approx 3 \times 10^{-7}$ ($10\times$ better)
 - improve Rydberg constant ($cR_\infty = \frac{1}{2}\alpha^2 m_e c^2 / h$) to a level of $u_r \approx 1 \times 10^{-12}$ ($6\times$ better)
 - benchmark for lattice QCD calculations
 - confront with electron scattering results



Apparatus

Apparatus

(why realized only after 2000 ?)

- Low energy muon beam line at PSI

$\tau_{2S} \sim 1 \mu\text{s}$  stop μ^- in 1 mbar H_2 ($\geq 100/\text{s}$ in small volume, $\sim 10^{-6} \text{ g}$)
detect keV- μ^- (sub- μm range) \rightarrow trigger for DAQ and laser
 \rightarrow “trigger quality” is crucial !

- Laser system

tunable around $\lambda = 6 \mu\text{m}$

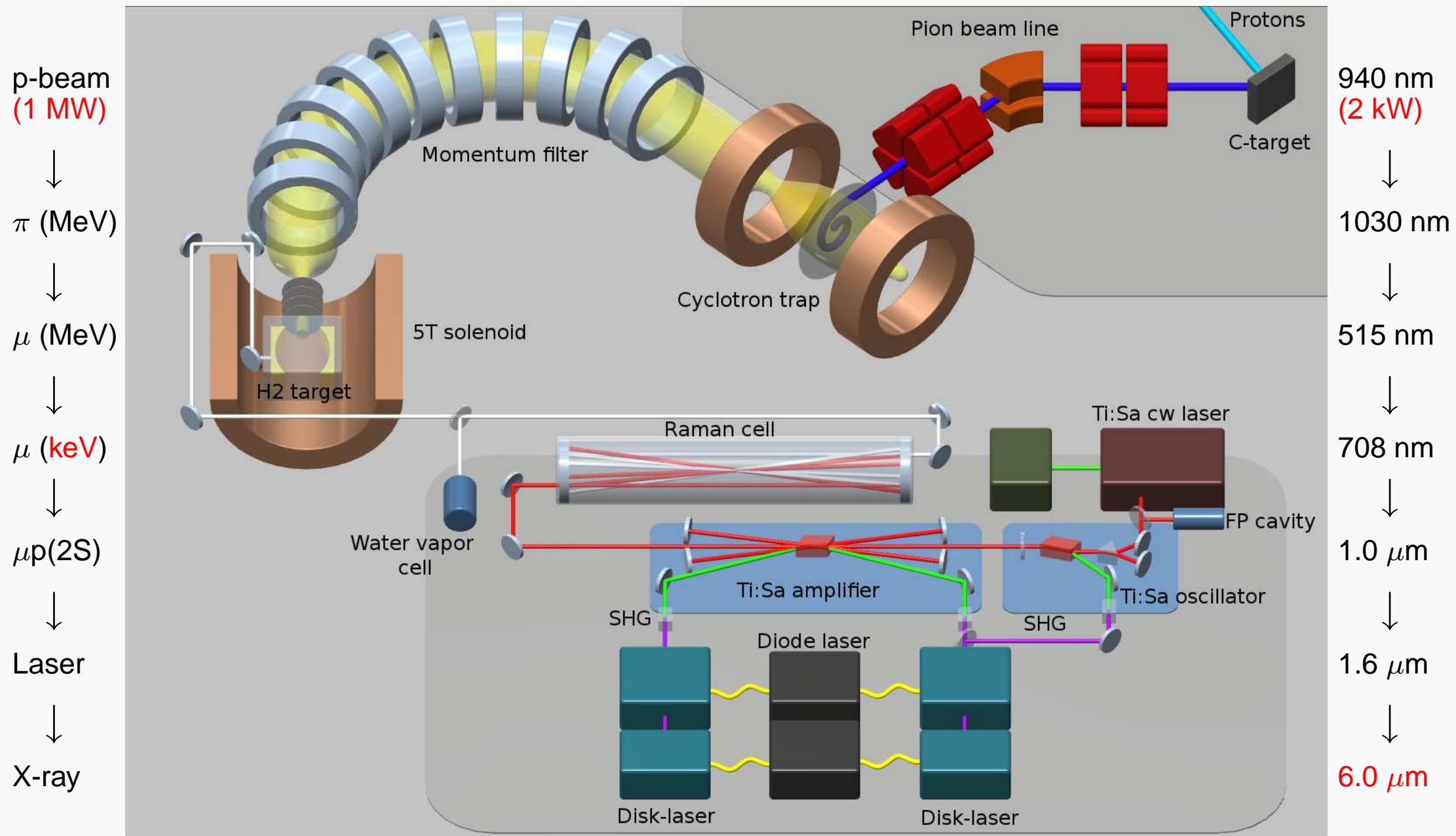
triggerable within $\sim 1 \mu\text{s}$ on stochastic muon-trigger (PSI !?)

$< 1 \text{ mJ}$ pulse energy (1979: $\sim 100 \text{ mJ}$)

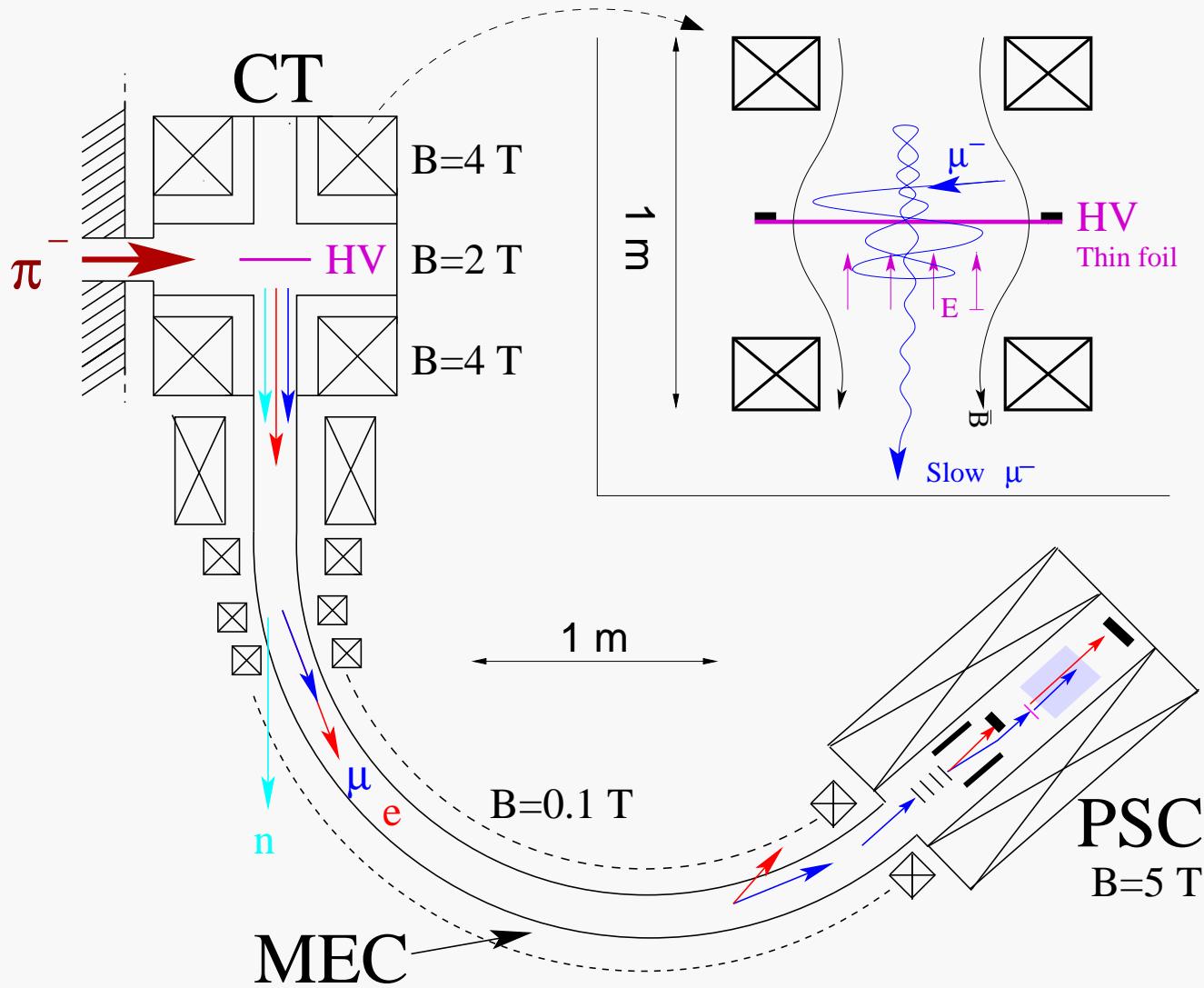
- Detectors and DAQ

2 keV photons: soft X-rays; t - and E -resolution; high B -fields

The μp Lamb shift setup



5 keV energy muon beam line



- Production of 20-50 keV μ^-
 - $10^8 \pi^-/\text{s}$ injected in CT
 - π^- decay in MeV μ^-
 - μ^- decel. to 20-50 keV by crossing thin foil
- Extraction of μ^- from CT:
$$\frac{T_{\parallel}(0)}{T_{\perp}(0)} > \left(\frac{B_{\max}}{B_0} - 1 \right) - \frac{qV}{T_{\perp}(0)}$$

$$\sim 0.5 \quad \approx 1 \quad 0.01 \dots 1$$
- Momentum selection
 - toroidal magnetic field \rightarrow vertical drift
 - eliminate e^- and n bg.
- μ^- detection
- μp formation and laser exp.

How to stop μ^- in a low-density H₂ target

1979: “muon bottle”

$$V_{\text{stop}} \approx 8 \times 8 \times 35 \text{ cm}^3 \approx 2200 \text{ cm}^3$$

$$(1 \text{ mbar:}) \quad \sim 150 \frac{\mu^- \text{ stop}}{\text{s}}$$

$$\Rightarrow \quad \sim 0.07 \frac{\mu^- \text{ stop}}{\text{cm}^3 \text{ s}}$$

2001: “MEC beam”

$$V_{\text{stop}} \approx 0.5 \times 1.5 \times 20 \text{ cm}^3 \approx 15 \text{ cm}^3$$

$\sim 100 \frac{\mu^- \text{ stop}}{\text{s}}$ pulsed accelerators
still excluded

$$\Rightarrow \quad \sim 7 \frac{\mu^- \text{ stop}}{\text{cm}^3 \text{ s}}$$

PSI proton accelerator: 10×
dedicated μ^- beam: 10×

mirrors for laser experiment:

~100 reflexions (proposed)

\Rightarrow 6 μm laser: ~100 mJ needed



impossible !

~1000 reflexions (measured) new design!

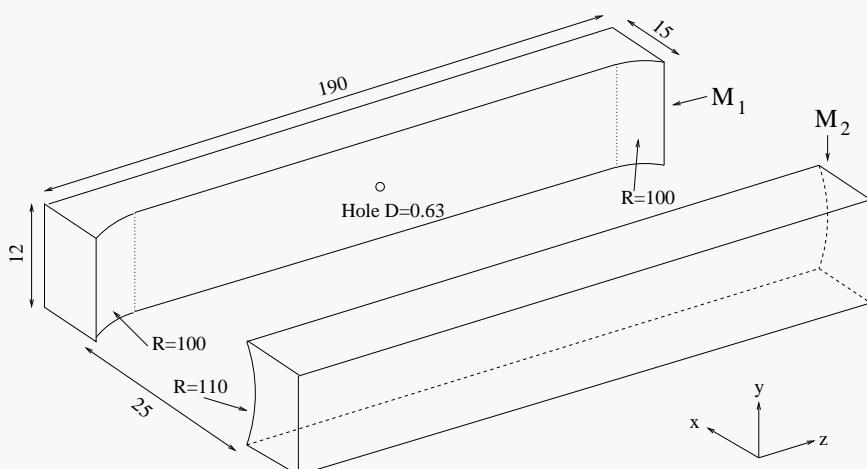
\Rightarrow ~0.2 mJ needed



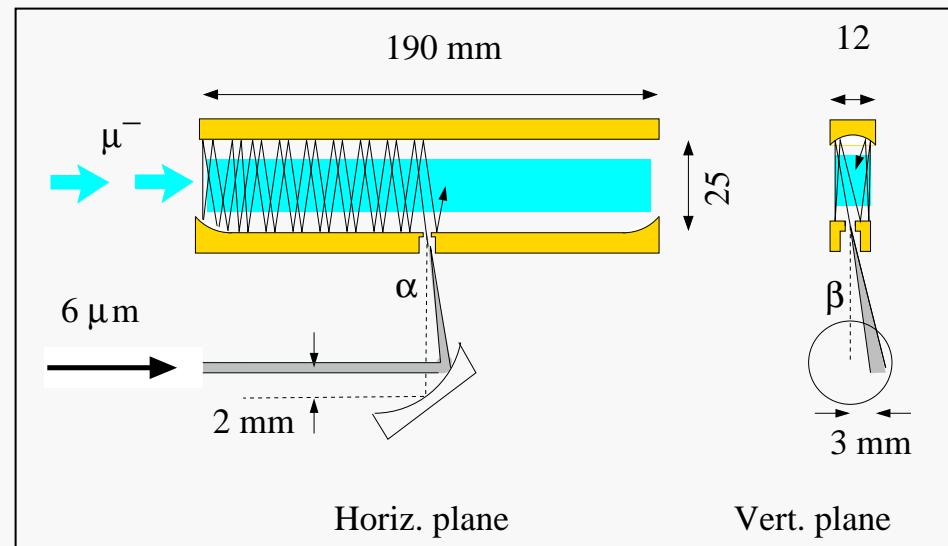
possible, we have 0.3 mJ

\longleftrightarrow Progress in muon beam technologies !

Setup: $6 \mu\text{m}$ multipass mirror cavity



Multipass cavity (curvatures exaggerated)

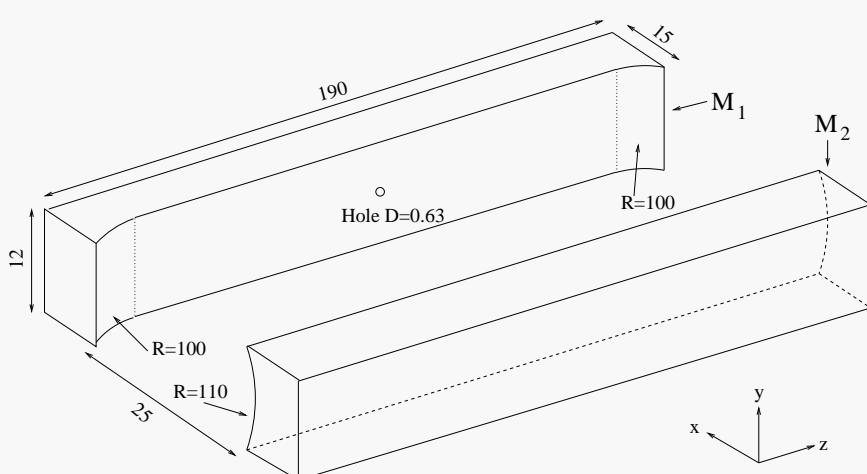


Off-axis coupling into cavity

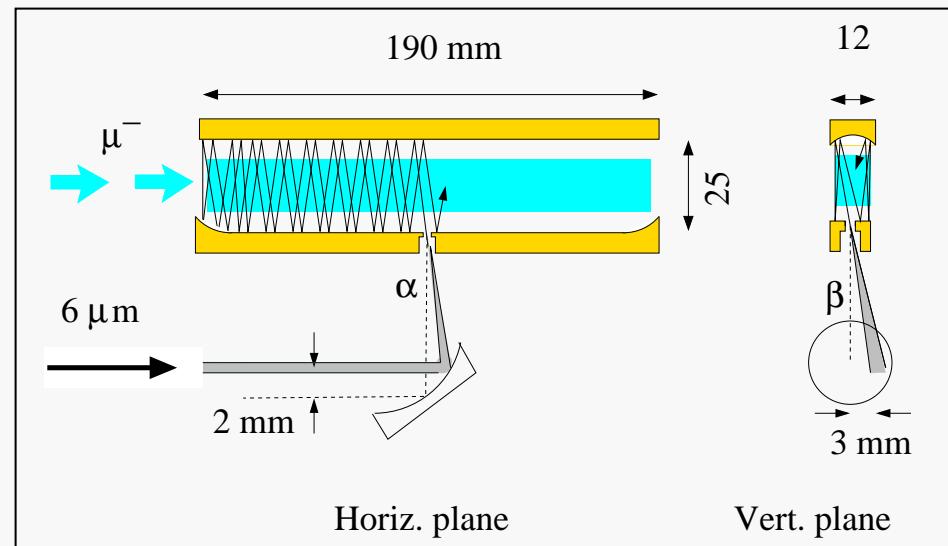
- fused silica mirrors, dielectric coating of ZnSe and ThF₄ with 26 layers [Lohnstar Optics]
- $R = 99.97\%$ at $6 \mu\text{m}$, small additional losses \rightarrow 1700 reflexions in cavity
- non-resonant cavity with curved mirrors: quite stable against misalignment
 \rightarrow no active adjustment devices needed!

Jan Vogelsang *et al.*, Opt. Express 22, 13050 (2014)

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Multipass cavity (curvatures exaggerated)



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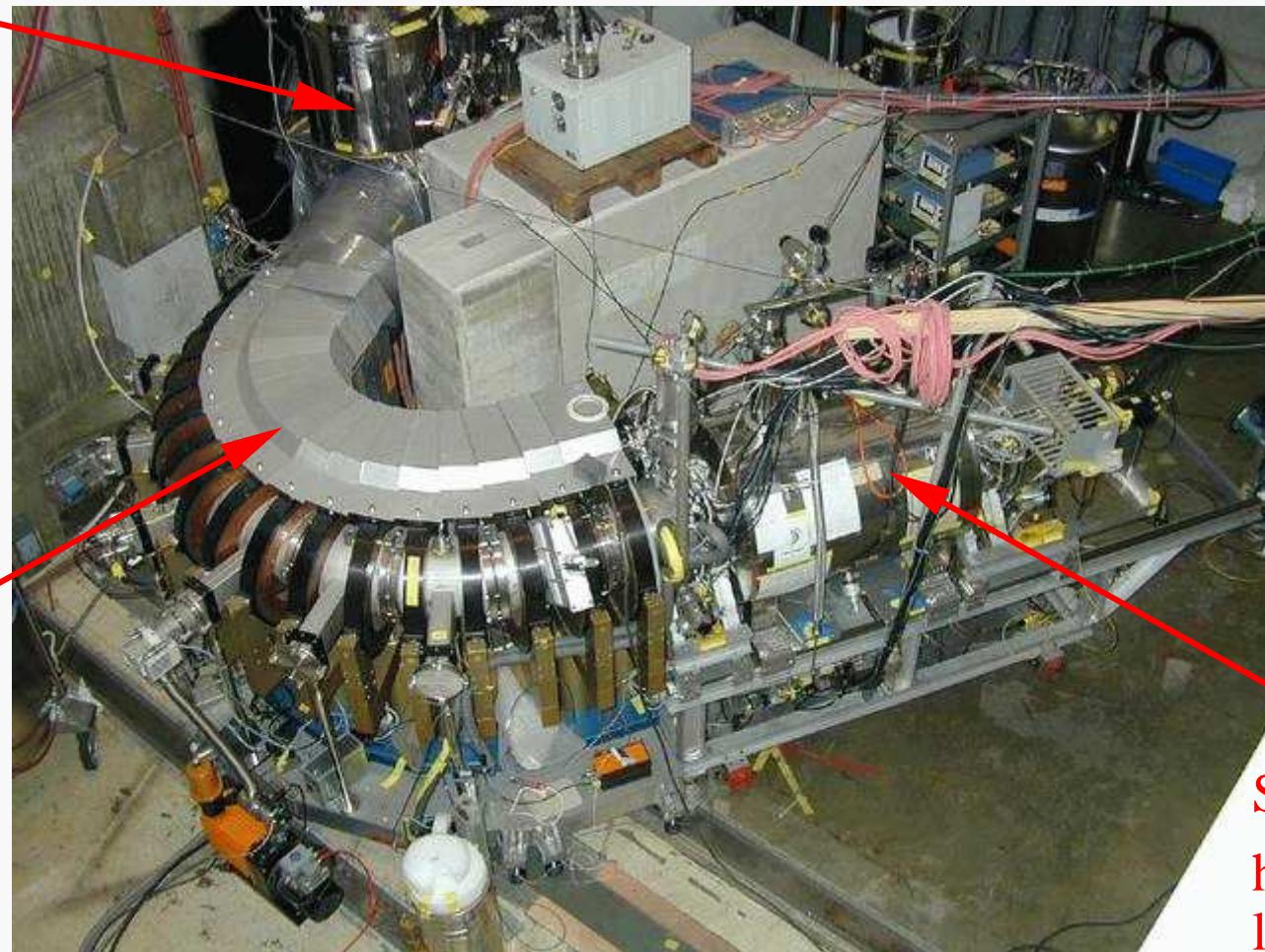
Ge

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Setup: Beam line for keV-muons in $\pi E5$ area

"Cyclotron trap"



"MEC"
Muon
extraction
channel

Measured 2009:

$400 \mu^-/\text{s}$ ($3 \dots 6 \text{ keV}$, 0.75 cm^2)

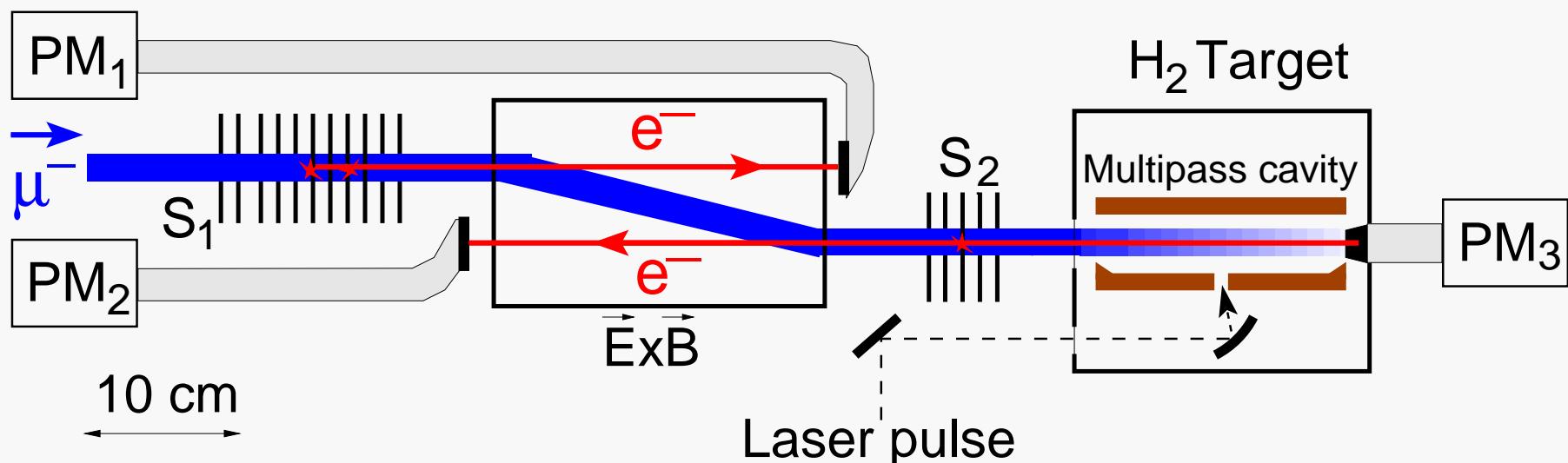
1 m

Solenoid with
hydrogen target
laser cavity
x-ray detectors
($B = 5 \text{ Tesla!}$)

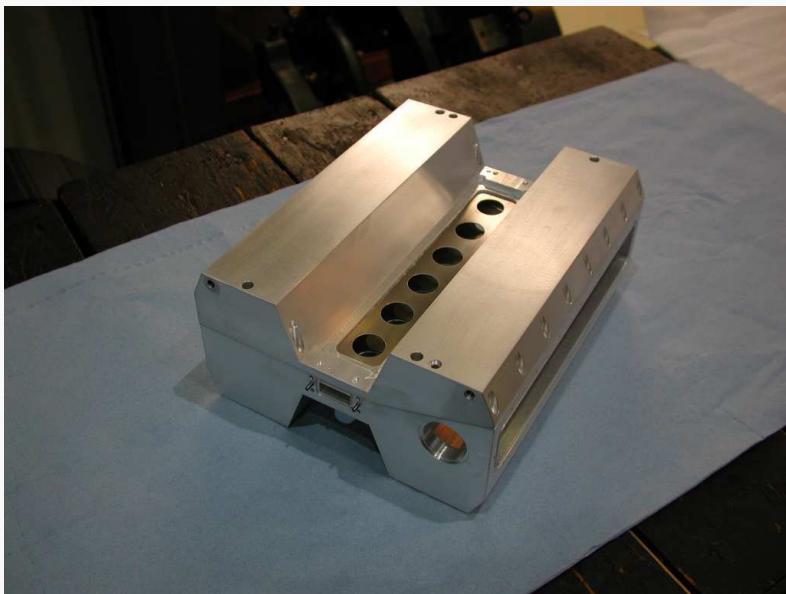
$\dots \mu^-$ inside the 5 Tesla solenoid

At target entrance: 5 keV μ^- , 400 s⁻¹ (detected)

- From the muon extraction channel (MEC): 20-50 keV μ^-
slowing down + frictional cooling + e^- emission + $E \times B$ + TOF + trigger
(laser, DAQ)
- Stacks of C-foils $\rightarrow \mu^-$ -detectors: $\epsilon_{S_1} = 85\%$, $\epsilon_{S_2}^{\text{up}} = 35\%$, $\epsilon_{S_2}^{\text{down}} = 55\%$
- Stopping volume in 1 hPa H₂: $5 \times 15 \times 190 \text{ mm}^3$



Setup: Gas target

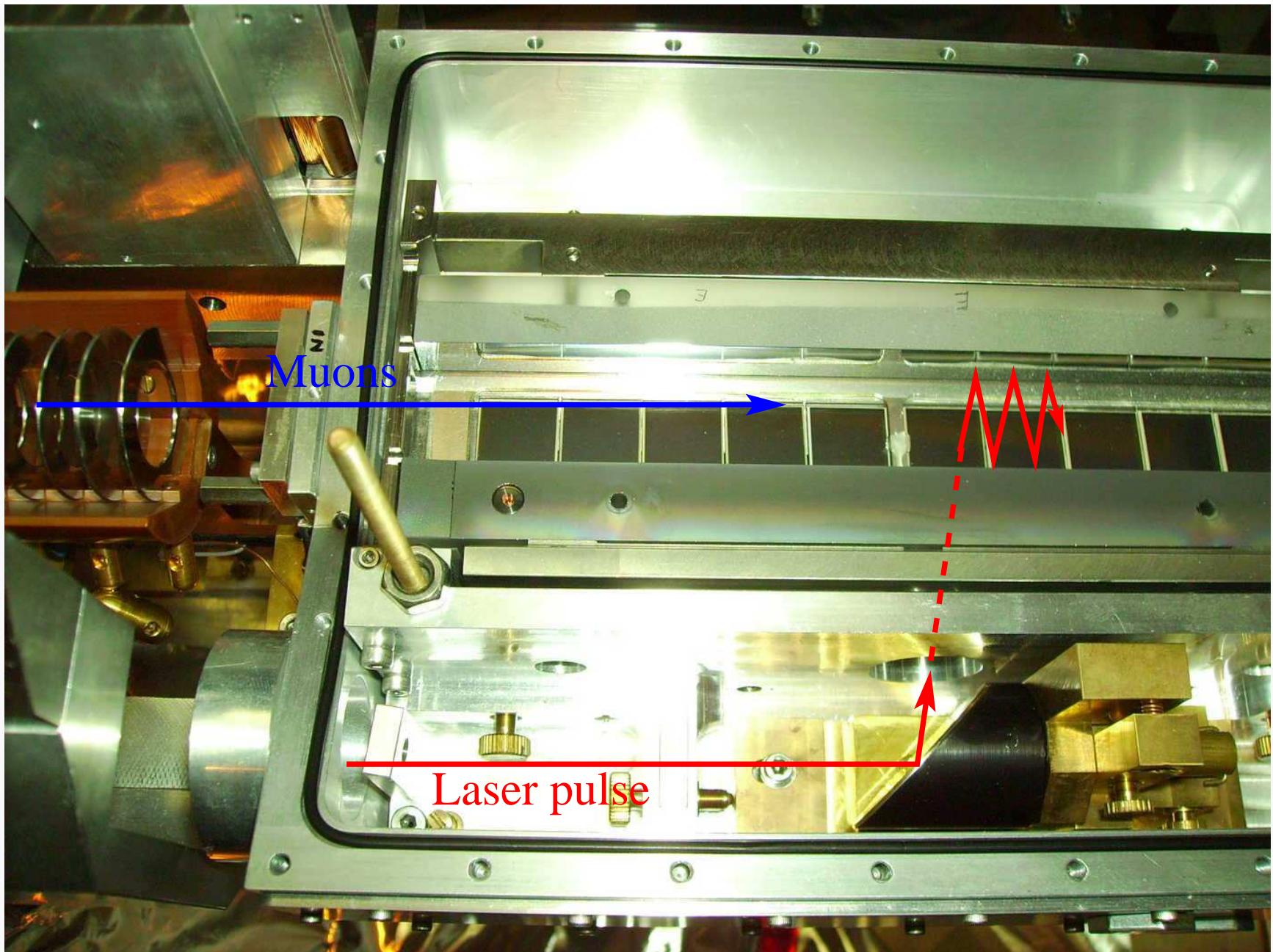


- $\Delta p \sim 1 \text{ hPa H}_2$
- window for μ^- beam entrance:
30 nm Formvar
- windows for APDs (2 keV det.):
1 μm polypropylene
- space for laser mirrors inside
target vessel



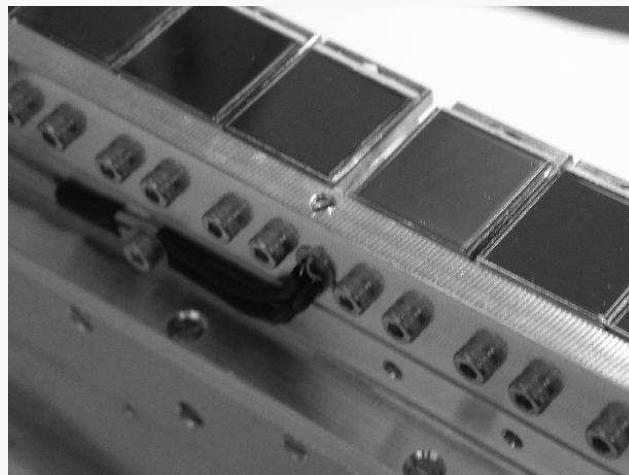
Mr Gross from PSI workshop

Open target

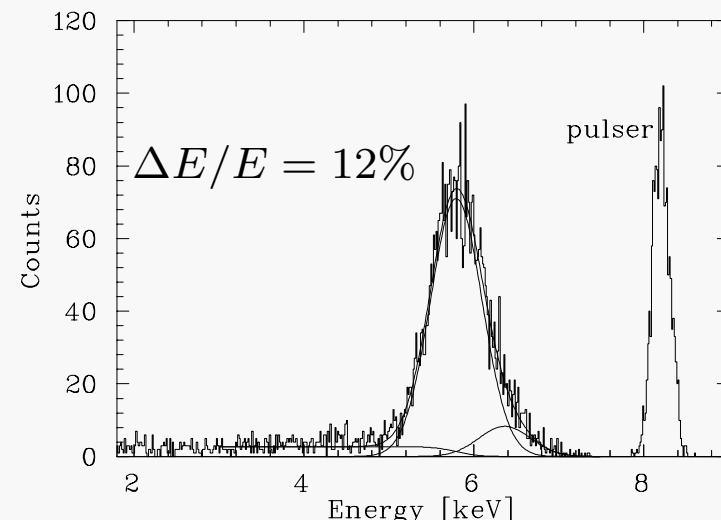


Setup: APD as 2 keV x-ray detector

- 20 avalanche photo diodes (APD), mounted in two rows at top and bottom of target vessel (at ± 8 mm) $\rightarrow \sim 30\%$ solid angle
- RMD company: APD with $14 \times 14 \text{ mm}^2$ sensitive area, square shaped
- cooled to -30°C $\rightarrow \sim 15 \text{ nA}$ leakage current
- $\Delta E/E \approx 30\%$ (FWHM), $\Delta t \approx 35 \text{ ns}$ (FWHM) for 2 keV x-rays
- operated at $B = 5$ Tesla without problems

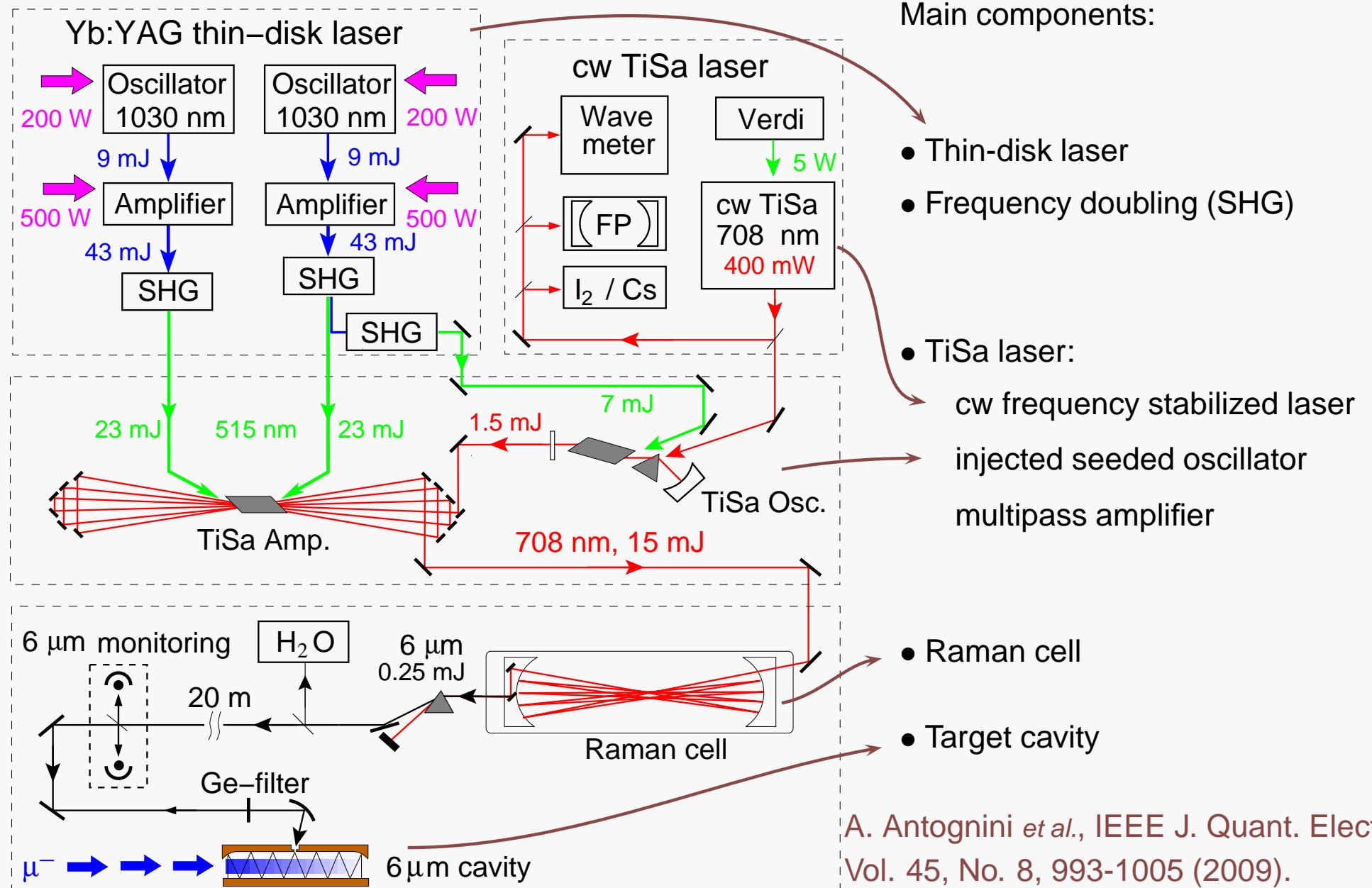


Central part of one detector array



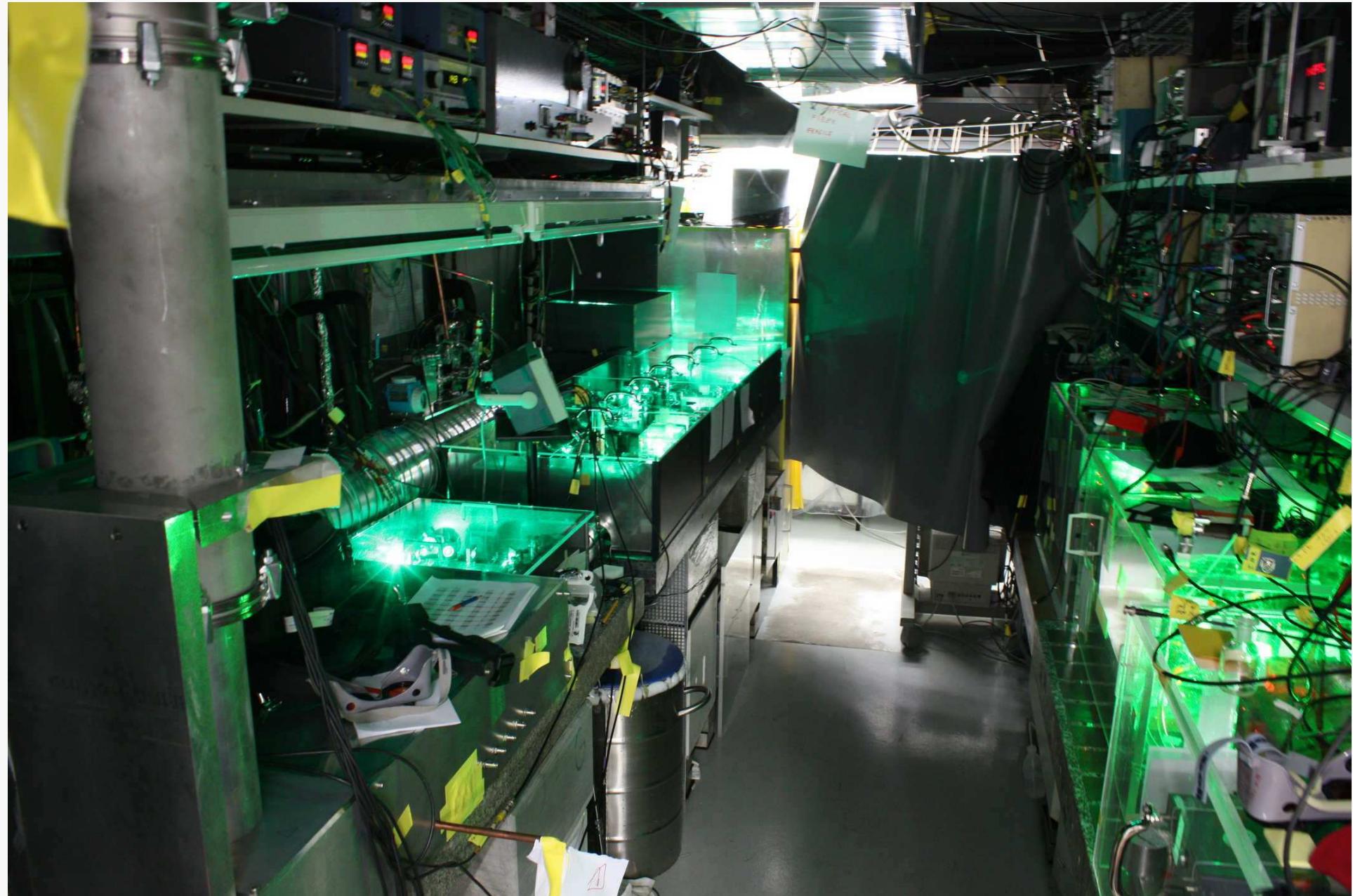
Energy spectrum of ^{55}Fe source

The laser system (2009)

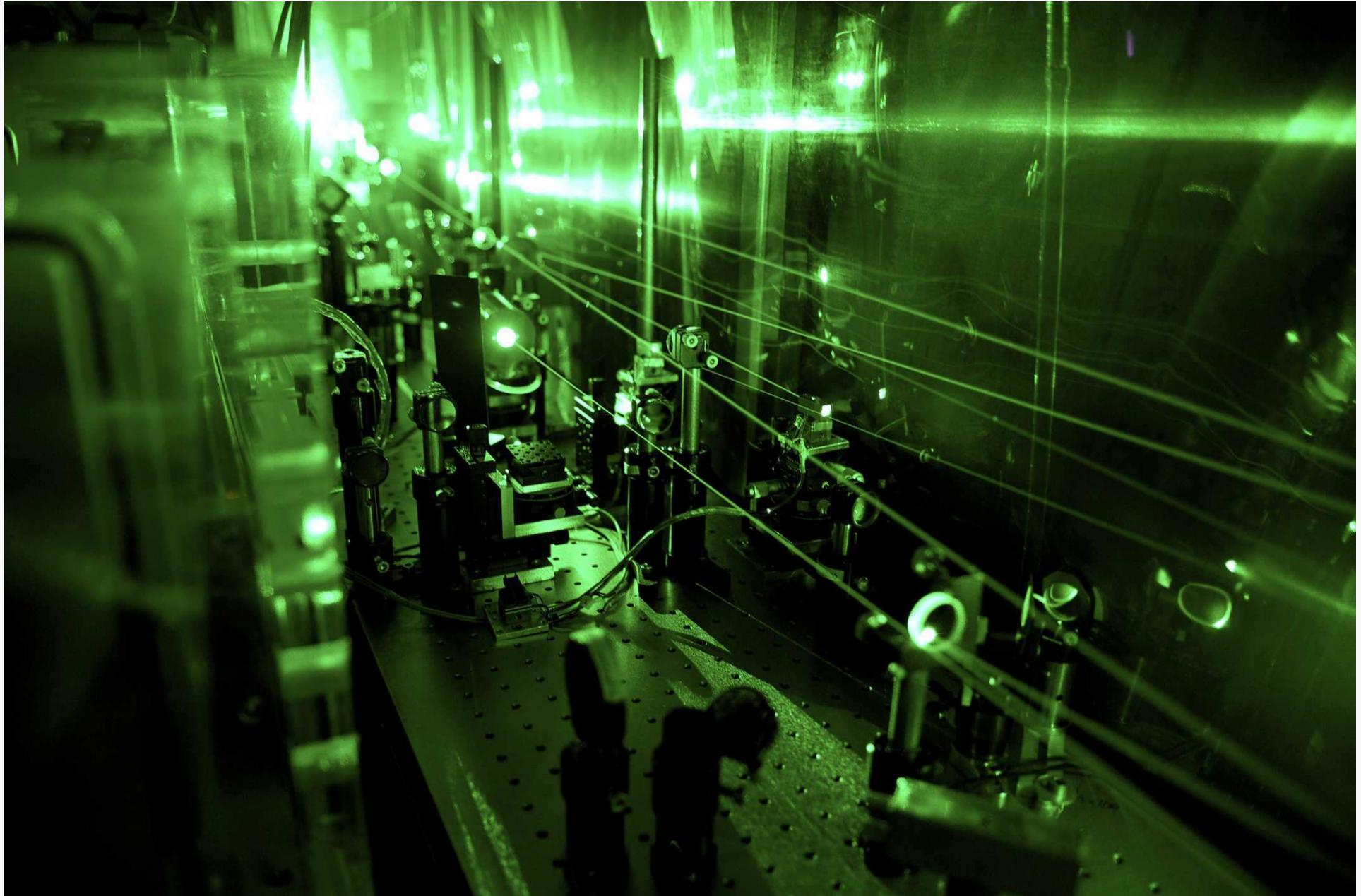


A. Antognini *et al.*, IEEE J. Quant. Electr.
Vol. 45, No. 8, 993-1005 (2009).

Impressions from the laser hut

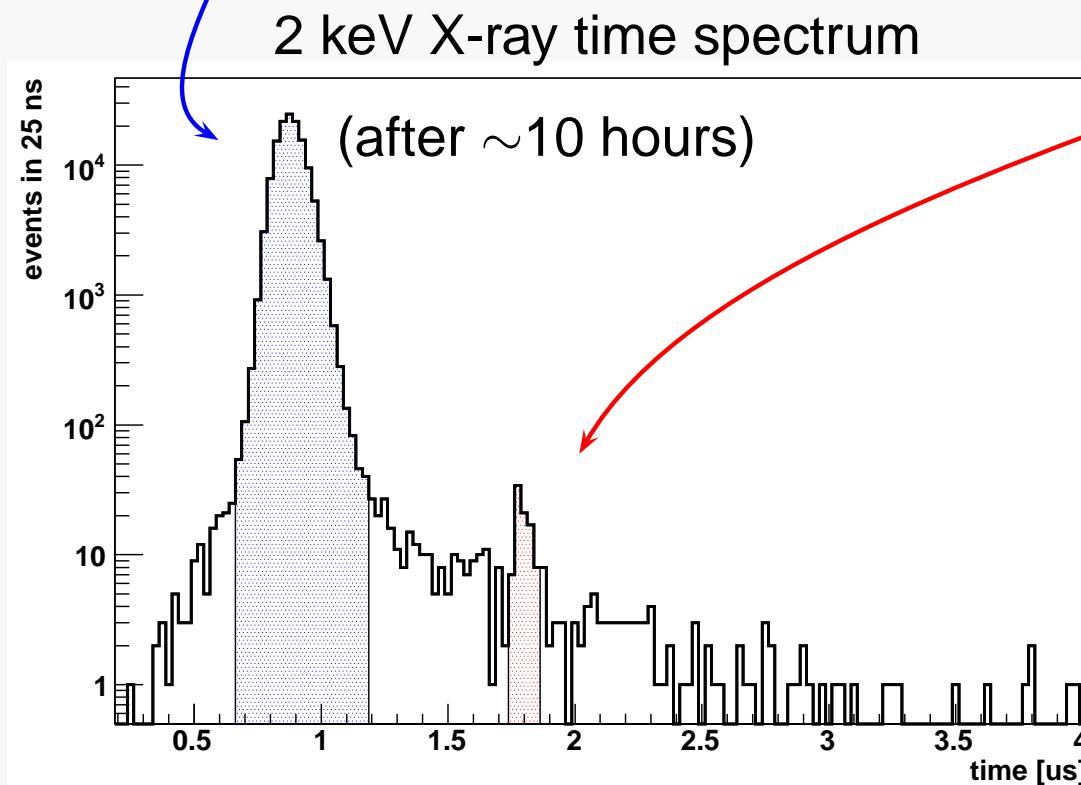
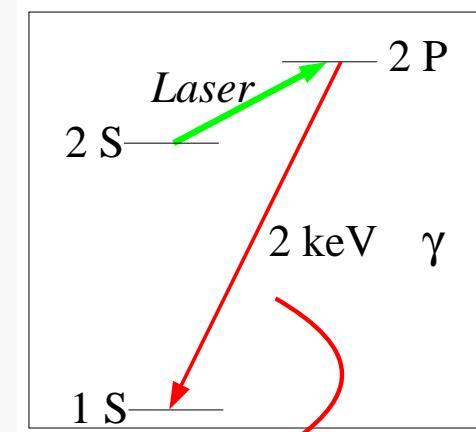
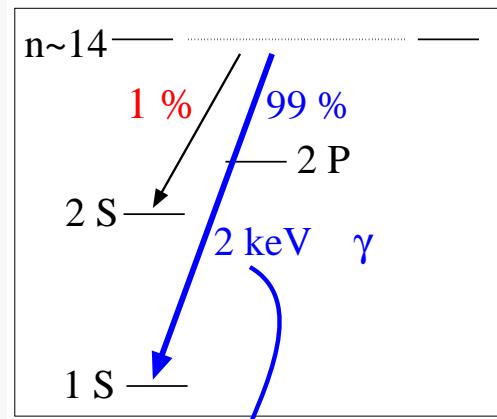


Disk laser doubling stages



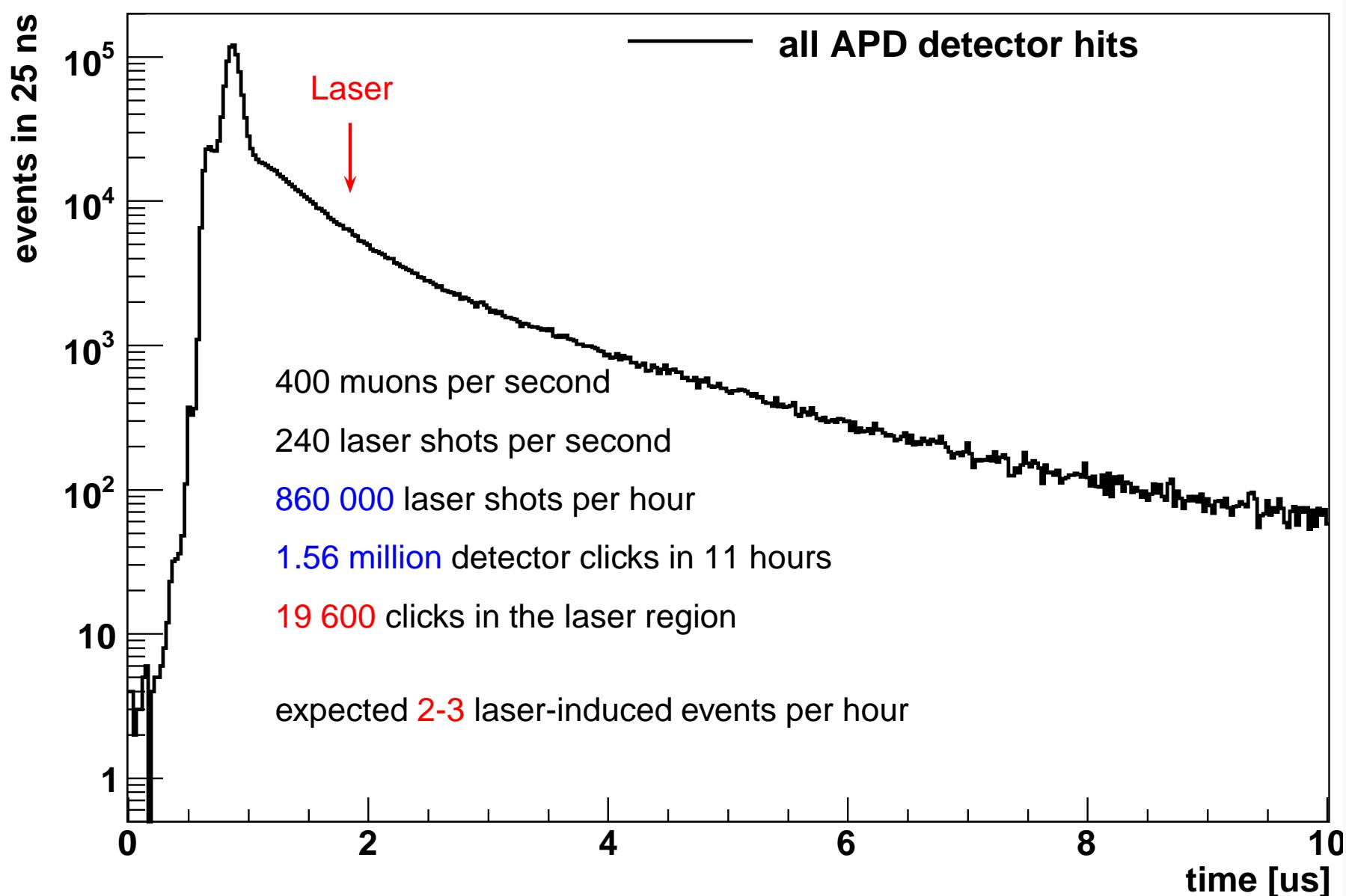
Results

Principle of the experiment . . .



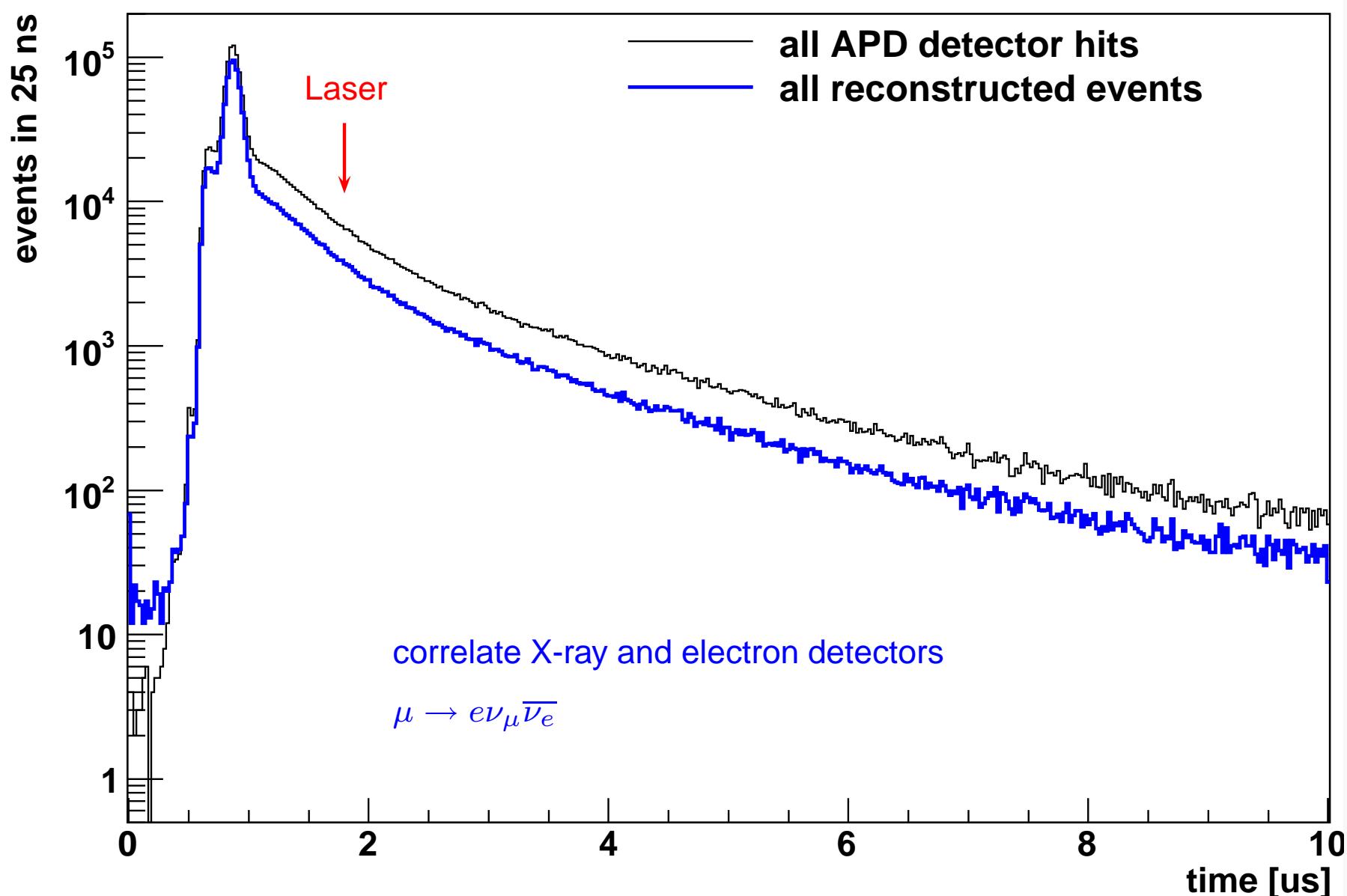
Data analysis: time spectra

FP 900, 11 hours measurement



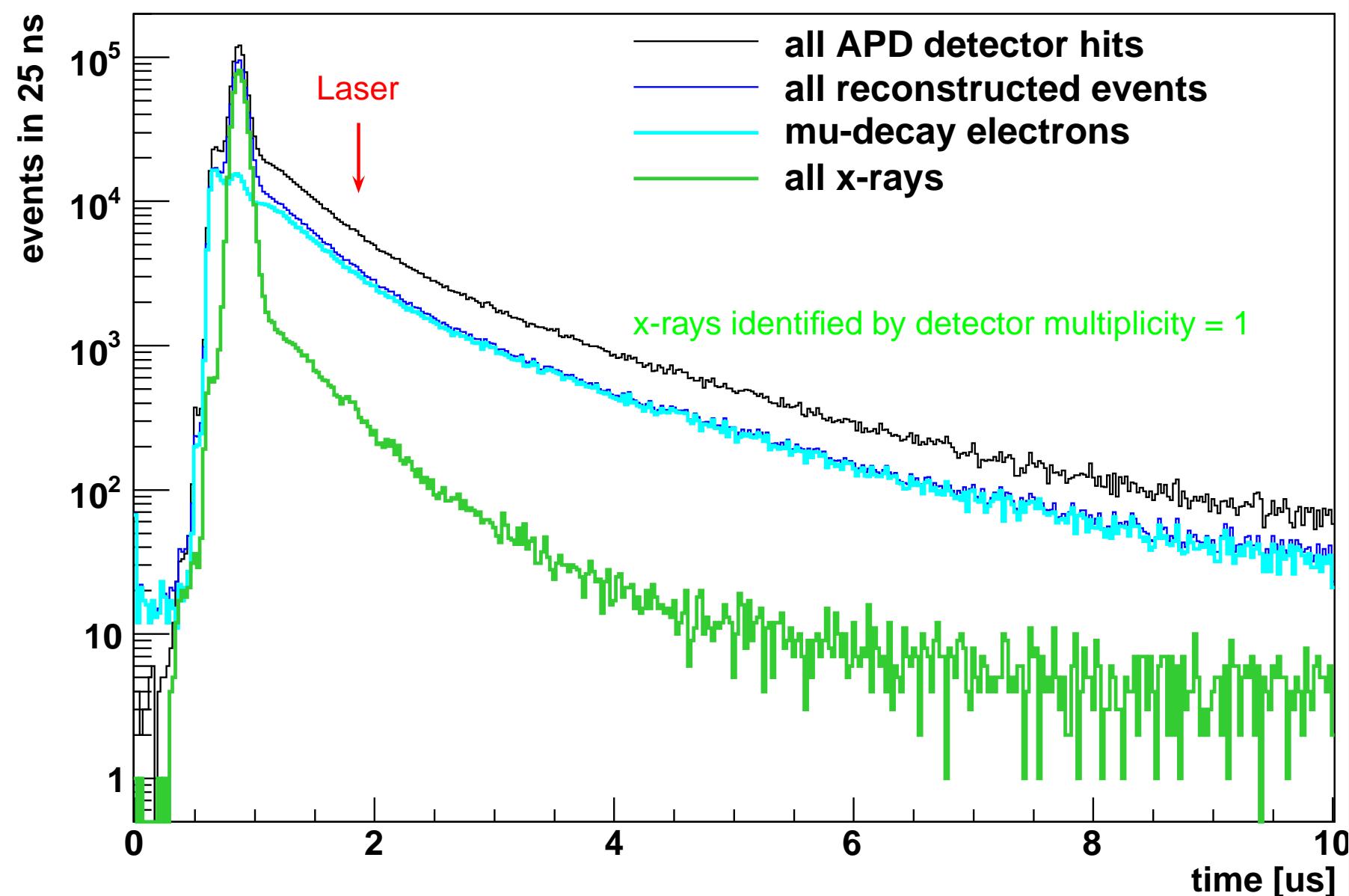
Data analysis: time spectra

FP 900, 11 hours measurement



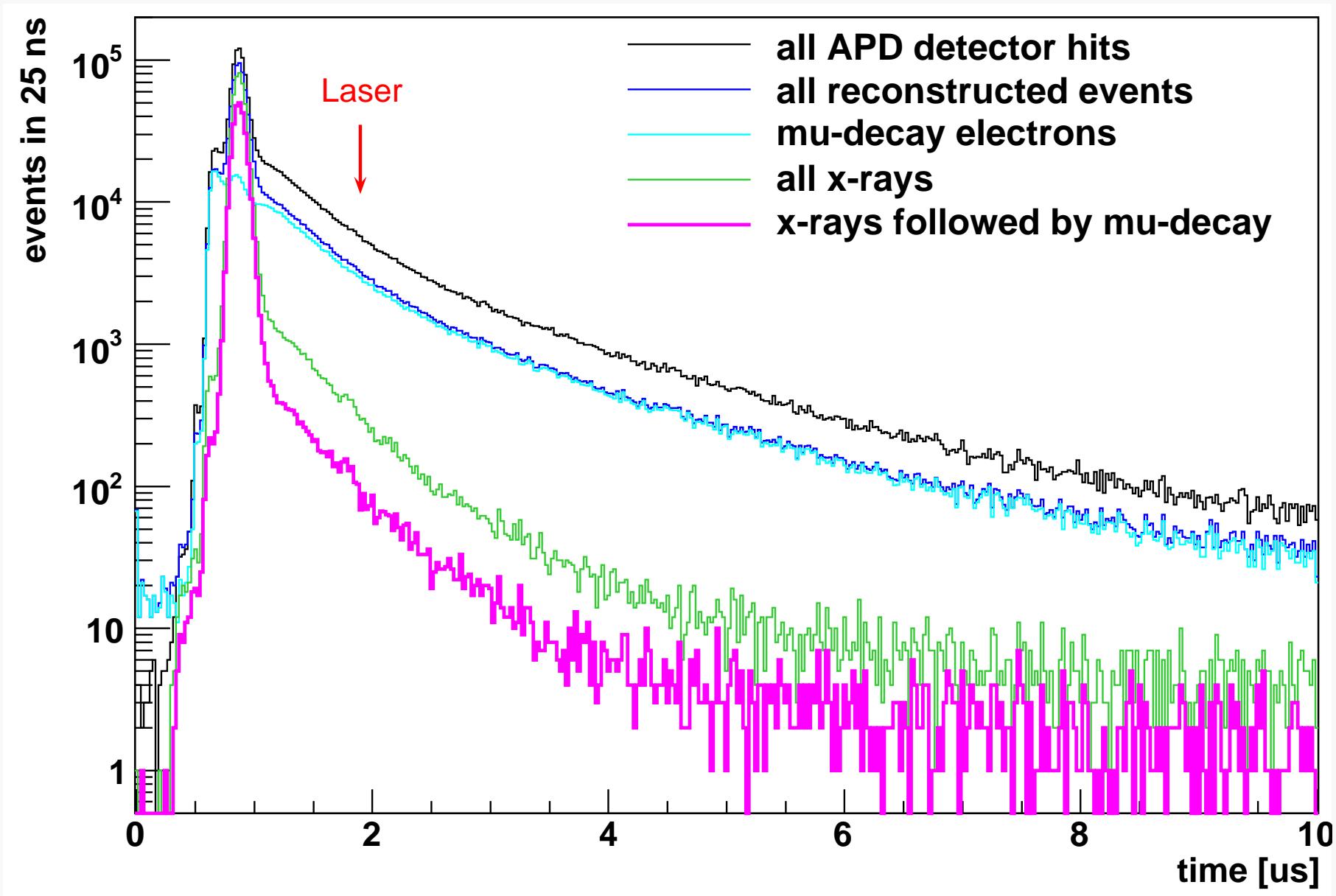
Data analysis: time spectra

FP 900, 11 hours measurement



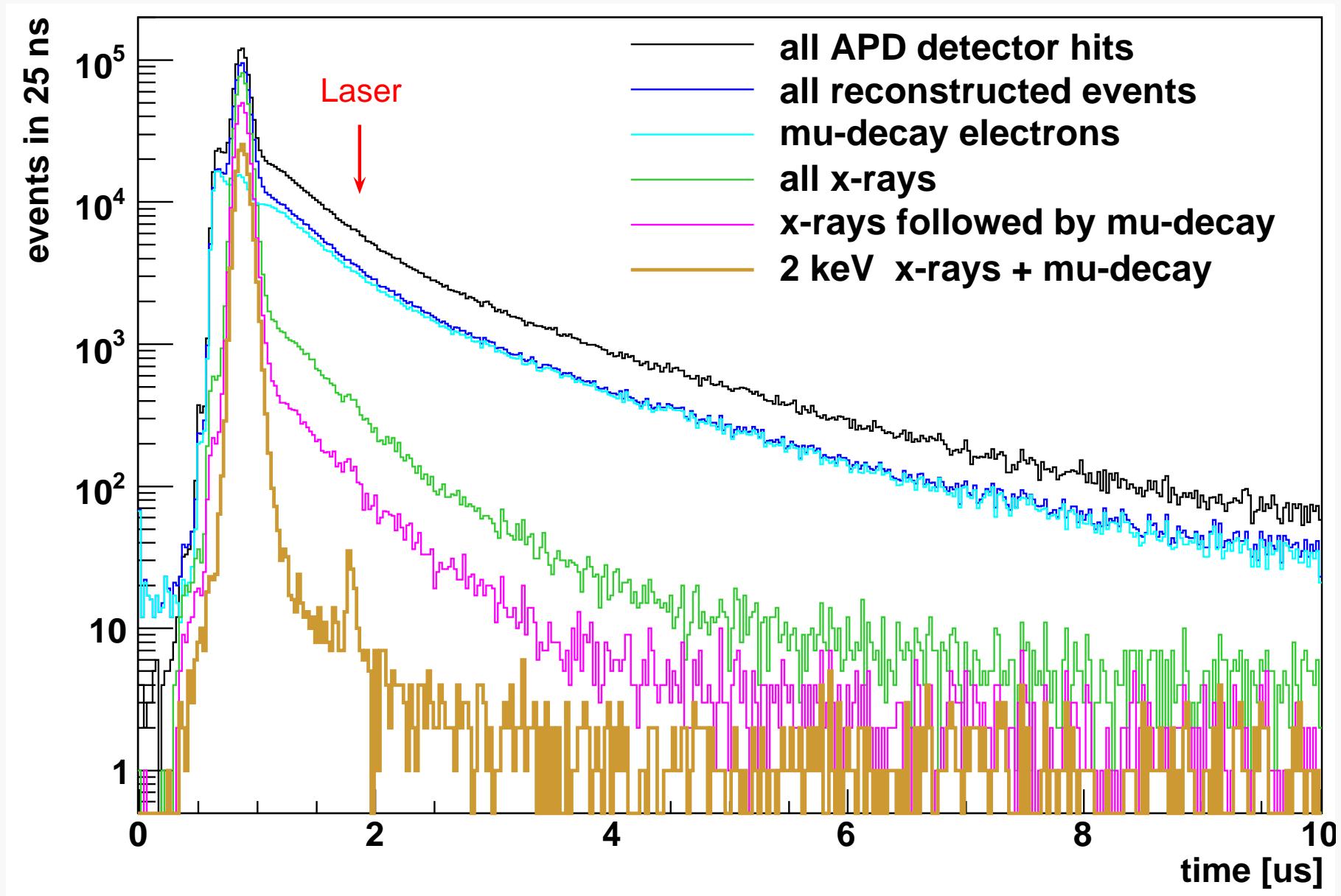
Data analysis: time spectra

FP 900, 11 hours measurement

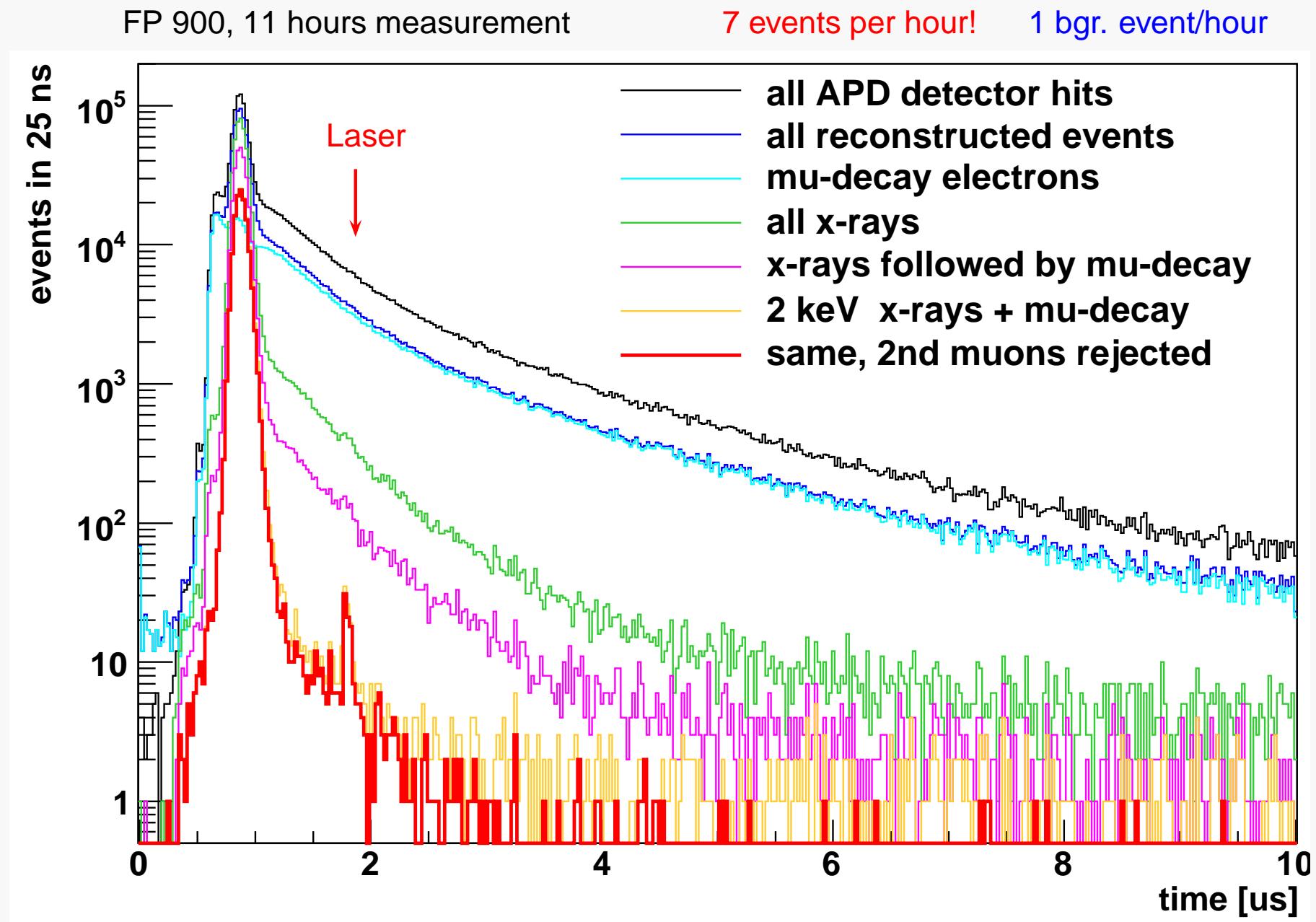


Data analysis: time spectra

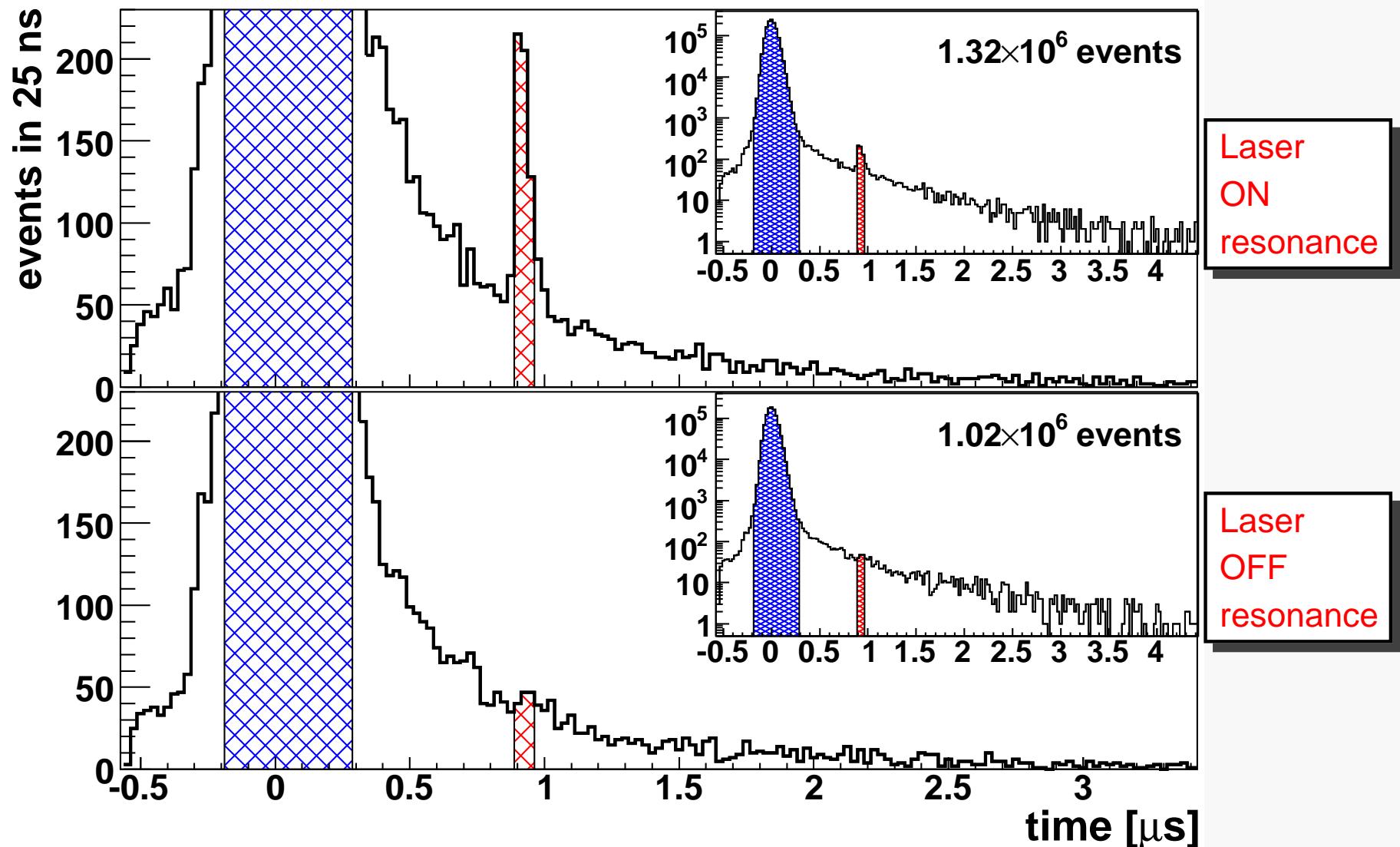
FP 900, 11 hours measurement



Data analysis: time spectra



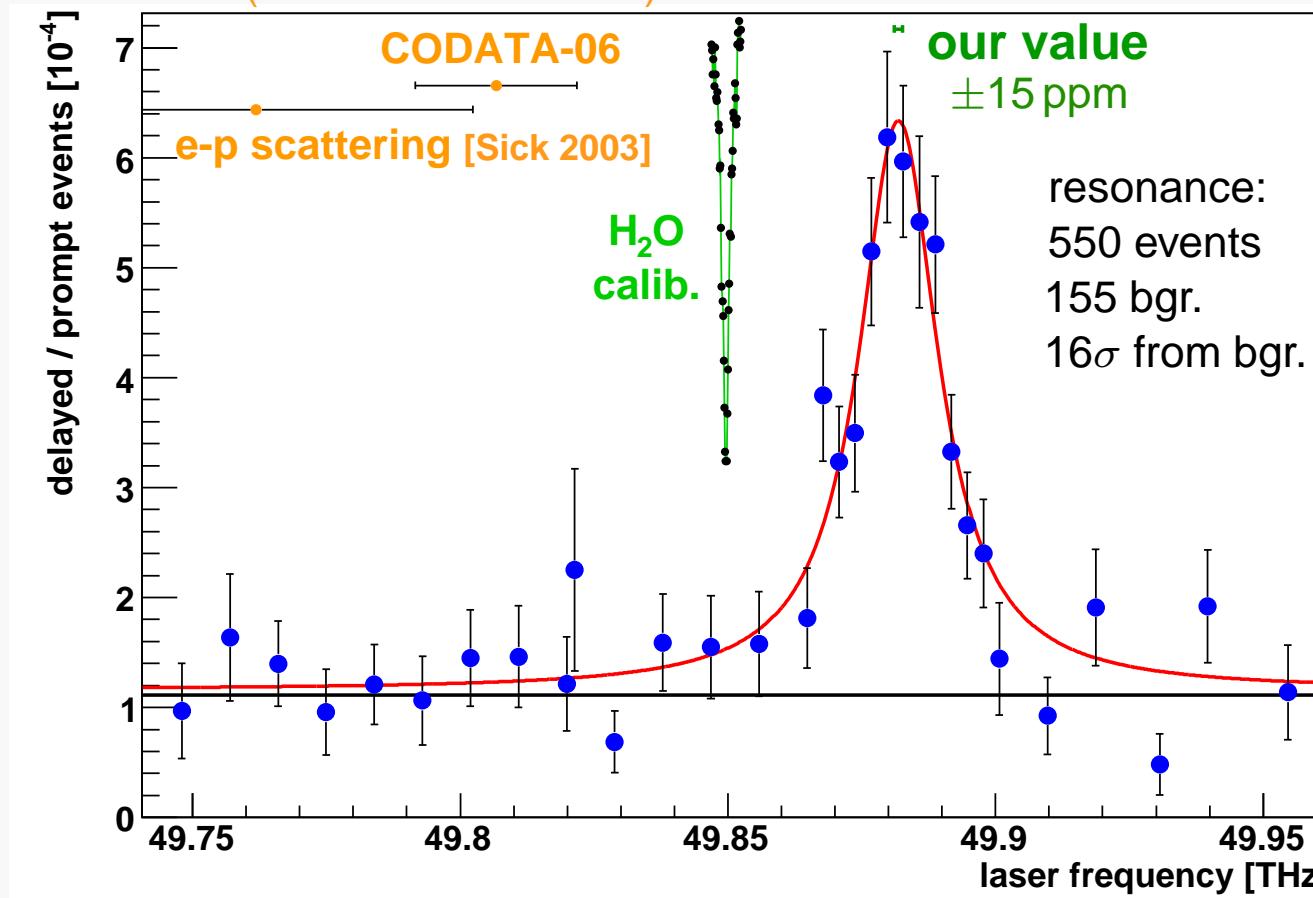
... and measured time spectra



Time-spectrum fit around laser time \Rightarrow Extract precise background level

Measured resonance $\mu p(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2})$

(known before 2009:)



Reference:

R. Pohl, A. Antognini,
F. Nez, D. Taqqu, et al.,
Nature **466**, 213 (2010)

Collaboration:

- MPQ Garching
- LKB Paris
- Coimbra and Aveiro
- Stuttgart
- Fribourg
- Yale
- PSI - ETHZ - ...

Statistics: ± 0.70 GHz

Systematics: ± 0.30 GHz (laser calibration)

Discrepancy (to CODATA-06):

~ 75 GHz $\leftrightarrow 5.0\sigma \leftrightarrow \delta\nu/\nu = 1.5 \times 10^{-3}$

Collaboration (μp and μHe^+)

F. Biraben, P. Indelicato, E.-O. LeBigot, L. Julien, F. Nez, C. Szabo

Lab. Kastler Brossel, Paris

M. Diepold, B. Franke, J. Götzfried, T.W. Hänsch,
J. Krauth, T. Nebel, R. Pohl

MPQ, Garching, Germany

F.D. Amaro, J.M.R. Cardoso, L.M.P. Fernandes,
A. L. Gouvea, J.A.M. Lopes, C.M.B. Monteiro, J.M.F. dos Santos

Uni Coimbra, Portugal

D.S. Covita, J.F.C.A. Veloso
P. Amaro, J. Machado, J. P. Santos

Uni Aveiro, Portugal

Uni Nova, Lisboa, Portugal

A. Voss, T. Graf
K. Schuhmann, A. Giesen

IFSW, Uni Stuttgart
D&G GmbH, Stuttgart

A. Antognini, K. Kirch, F. Kottmann, D. Taqqu
M. Hildebrandt, A. Knecht

ETH Zürich
PSI, Switzerland

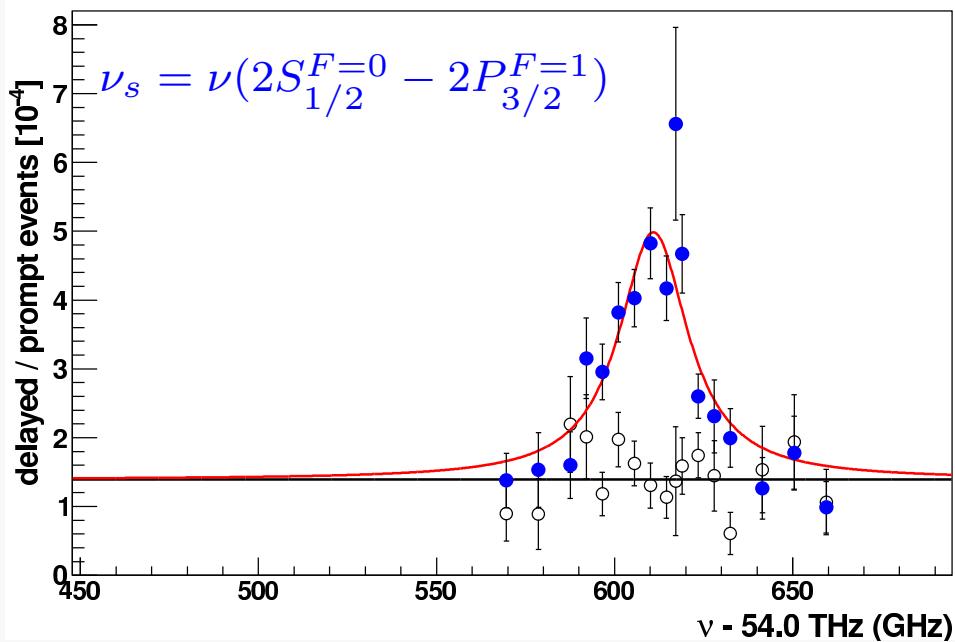
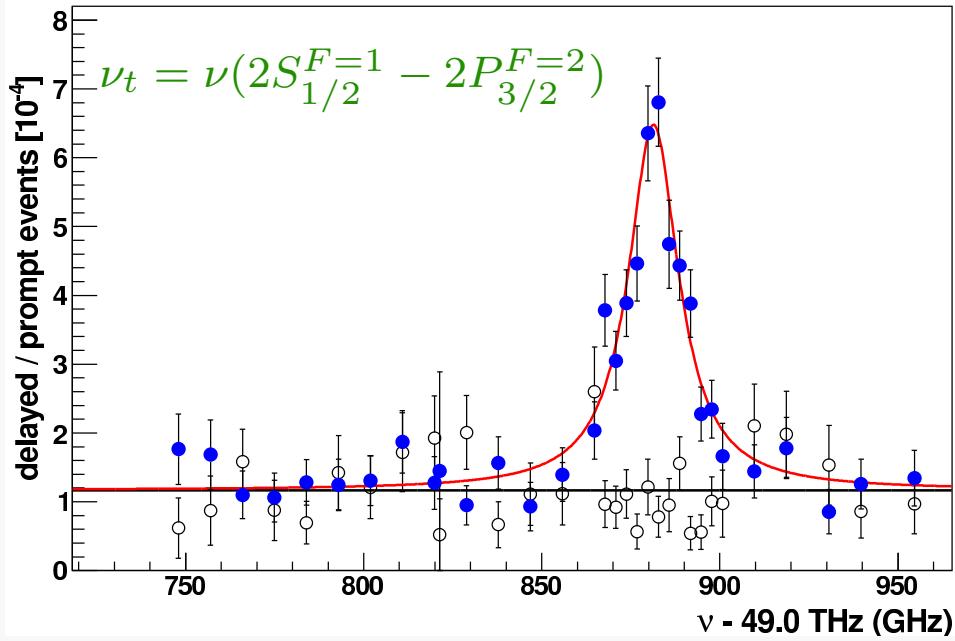
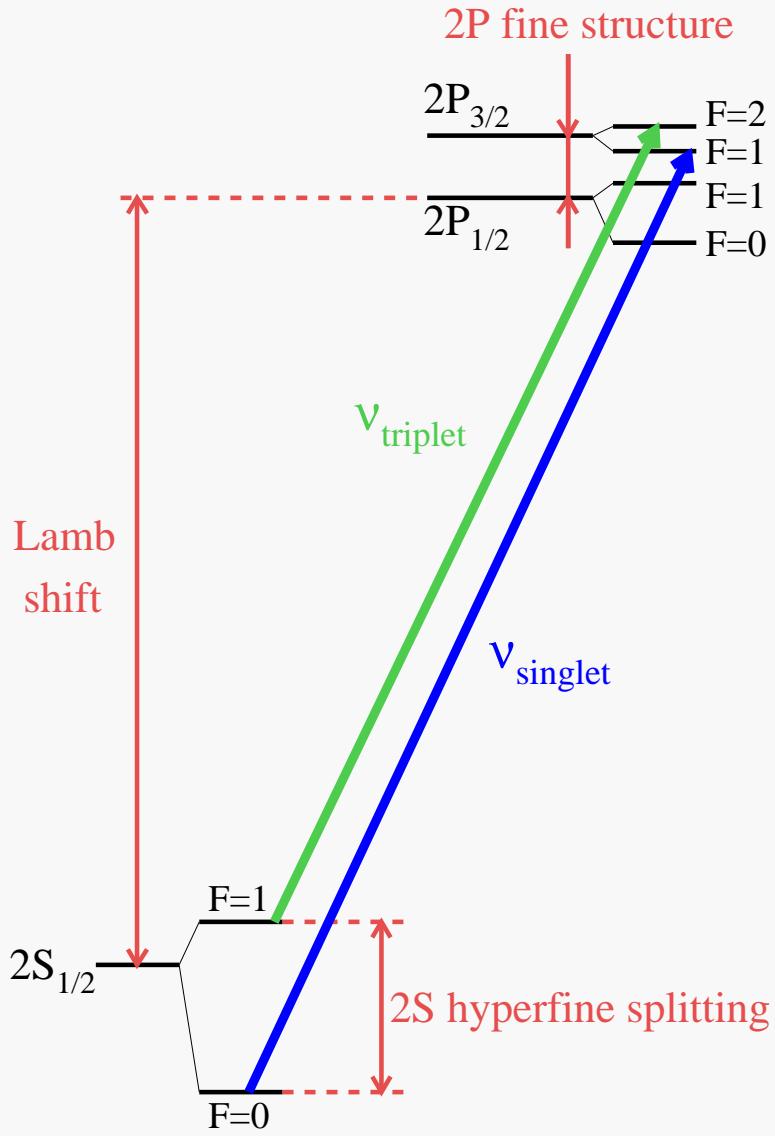
P.E. Knowles, L. Ludhova, F. Mulhauser, L.A. Schaller

Uni Fribourg, Switzerland

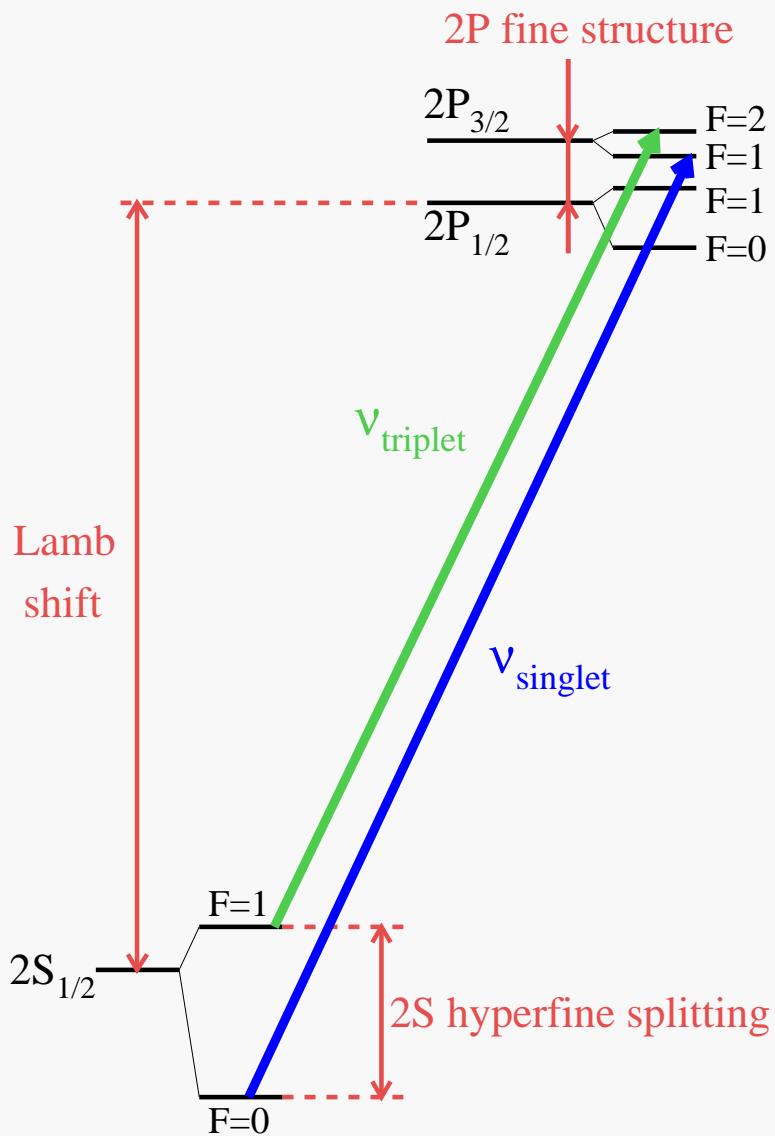
P. Rabinowitz
A. Dax, S. Dhawan, (V.W. Hughes)
T. L. Chen, C.-Y. Kao, Y.-W. Liu

University of Princeton, USA
Yale University, USA
N.T.H. Uni, Hsinchu, Taiwan

We have measured two transitions in μ p



We have measured two transitions in μ p



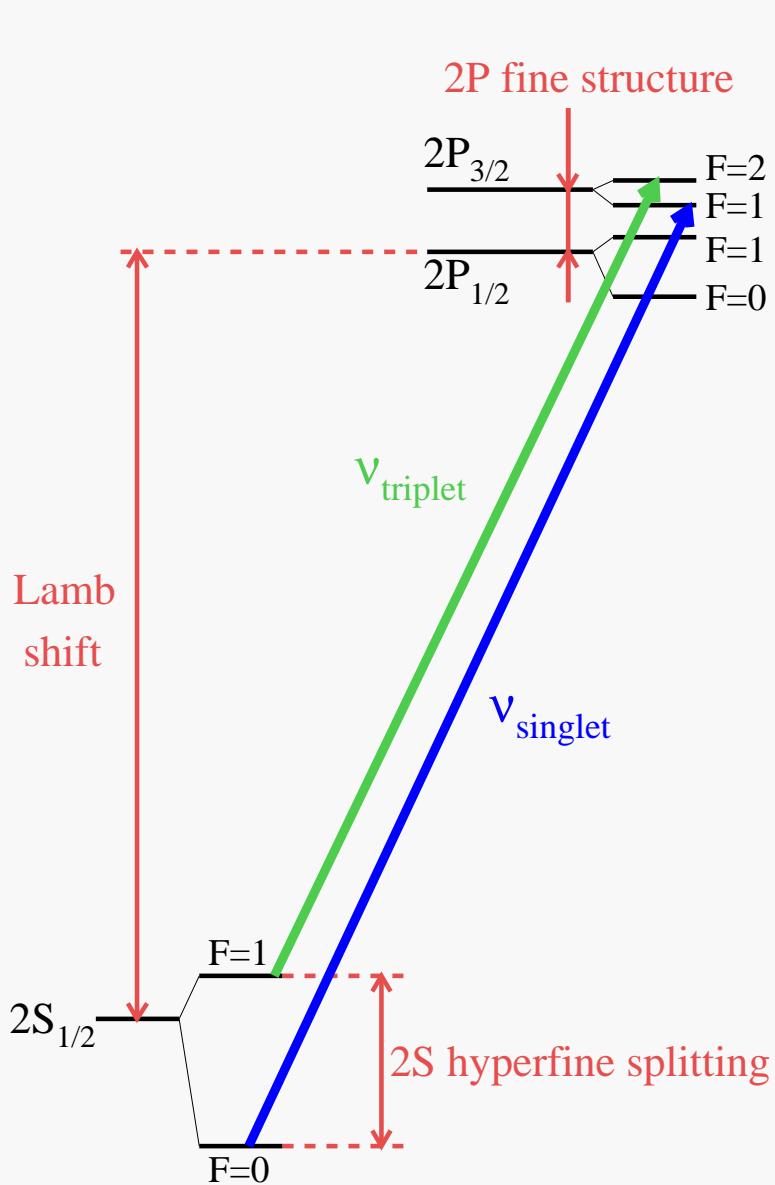
- Consider the two measurements separately

Two independent determinations of r_p

($\nu_t \rightarrow r_p$, $\nu_s \rightarrow r_p$)

Consistent results!

We have measured two transitions in μ p



- Consider the two measurements separately

Two independent determinations of r_p

($\nu_t \rightarrow r_p$, $\nu_s \rightarrow r_p$)

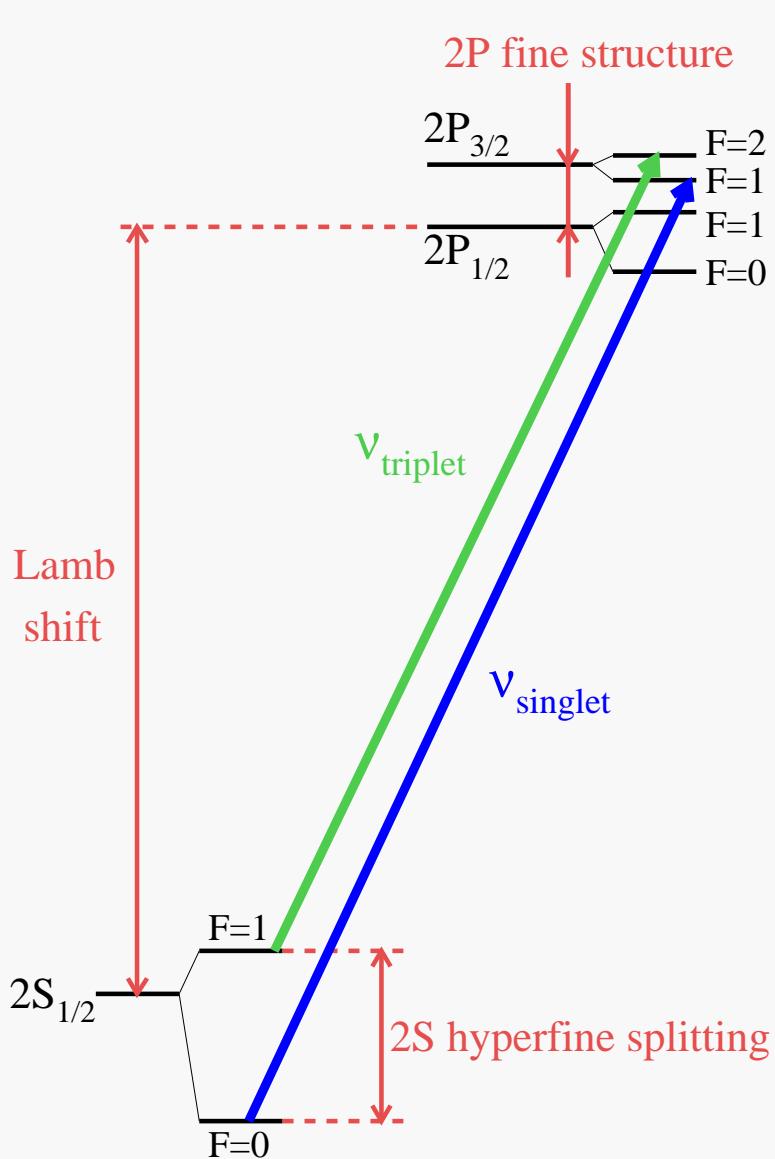
Consistent results!

- Combine the two measurements

Two measurements \rightarrow determine two parameters

$\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{\text{HFS}} \rightarrow [r_p, r_Z]$

We have measured two transitions in μ p



- Consider the two measurements separately

Two independent determinations of r_p

($\nu_t \rightarrow r_p$, $\nu_s \rightarrow r_p$)

Consistent results!

Using the 2S-HFS prediction

- Combine the two measurements

Two measurements \rightarrow determine two parameters

$\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{\text{HFS}} \rightarrow r_p, r_Z$

r_p does NOT require 2S-HFS prediction

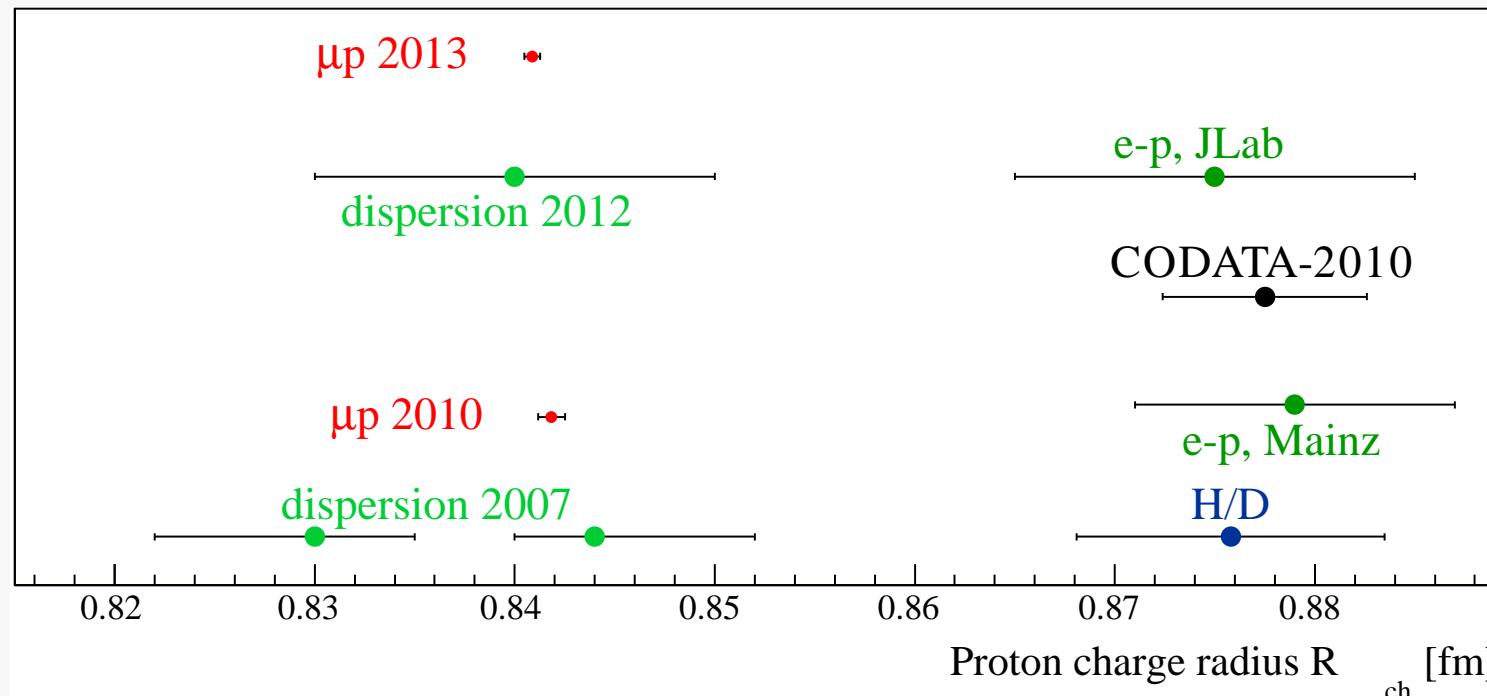
Proton charge radius

$$\nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2}) = 49881.88(76) \text{ GHz} \quad \text{R. Pohl } \textit{et al.}, \text{ Nature 466, 213 (2010)}$$

$$\begin{aligned} \nu(2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1}) &= 49881.35(65) \text{ GHz} \\ &= 54611.16(1.05) \text{ GHz} \end{aligned} \quad \left. \right\} \text{A. Antognini } \textit{et al.}, \text{ Science 339, 417 (2013)}$$

Proton charge radius: $r_p = 0.84087(26)_{\text{exp}}(29)_{\text{th}} = 0.84087(39) \text{ fm}$

μp theory summary: A. Antognini *et al.*, Ann. Phys. 331, 127 (2013) [arXiv:1208.2637]



Proton Zemach radius

2S hyperfine splitting in μp is: $\Delta E_{\text{HFS}} = 22.9843(30) - 0.1621(10) r_Z$ [fm] meV

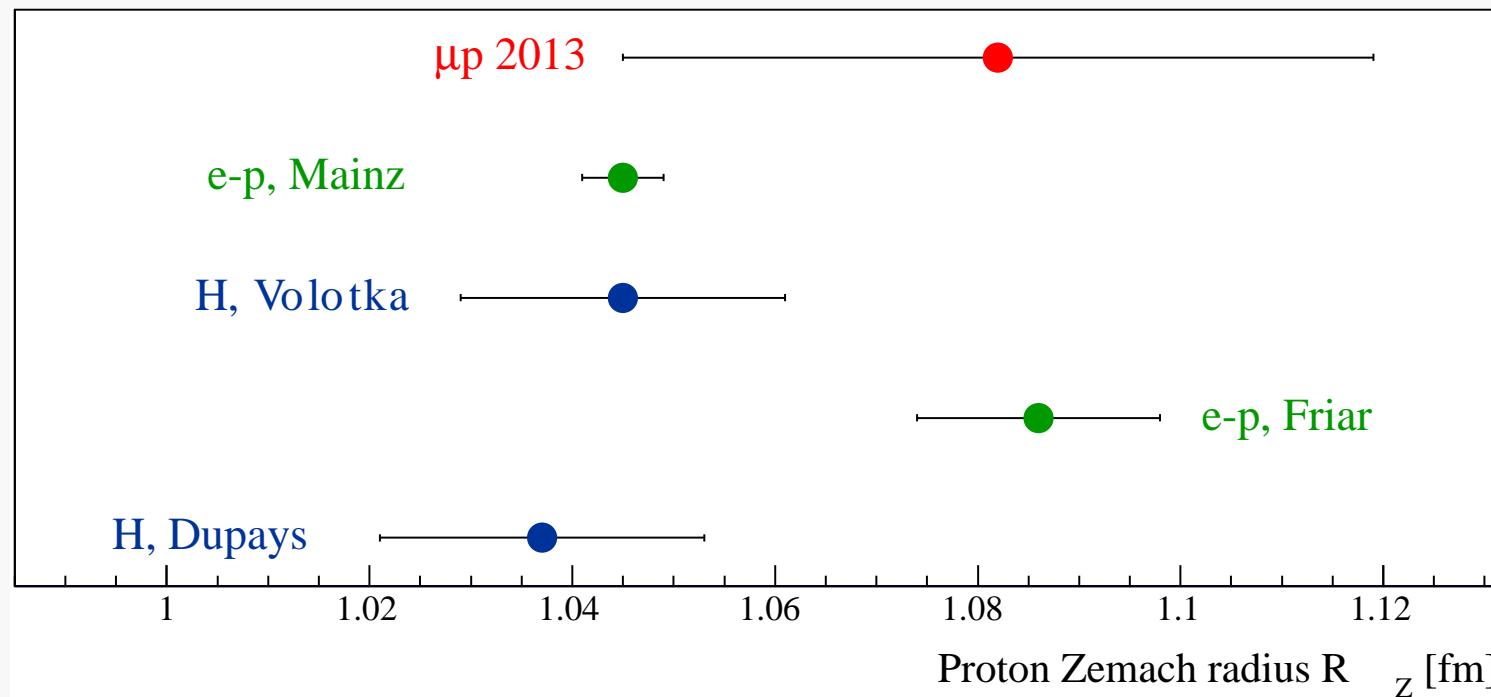
$$\text{with } r_Z = \int d^3r \int d^3r' r \rho_E(r) \rho_M(r - r')$$

We measured

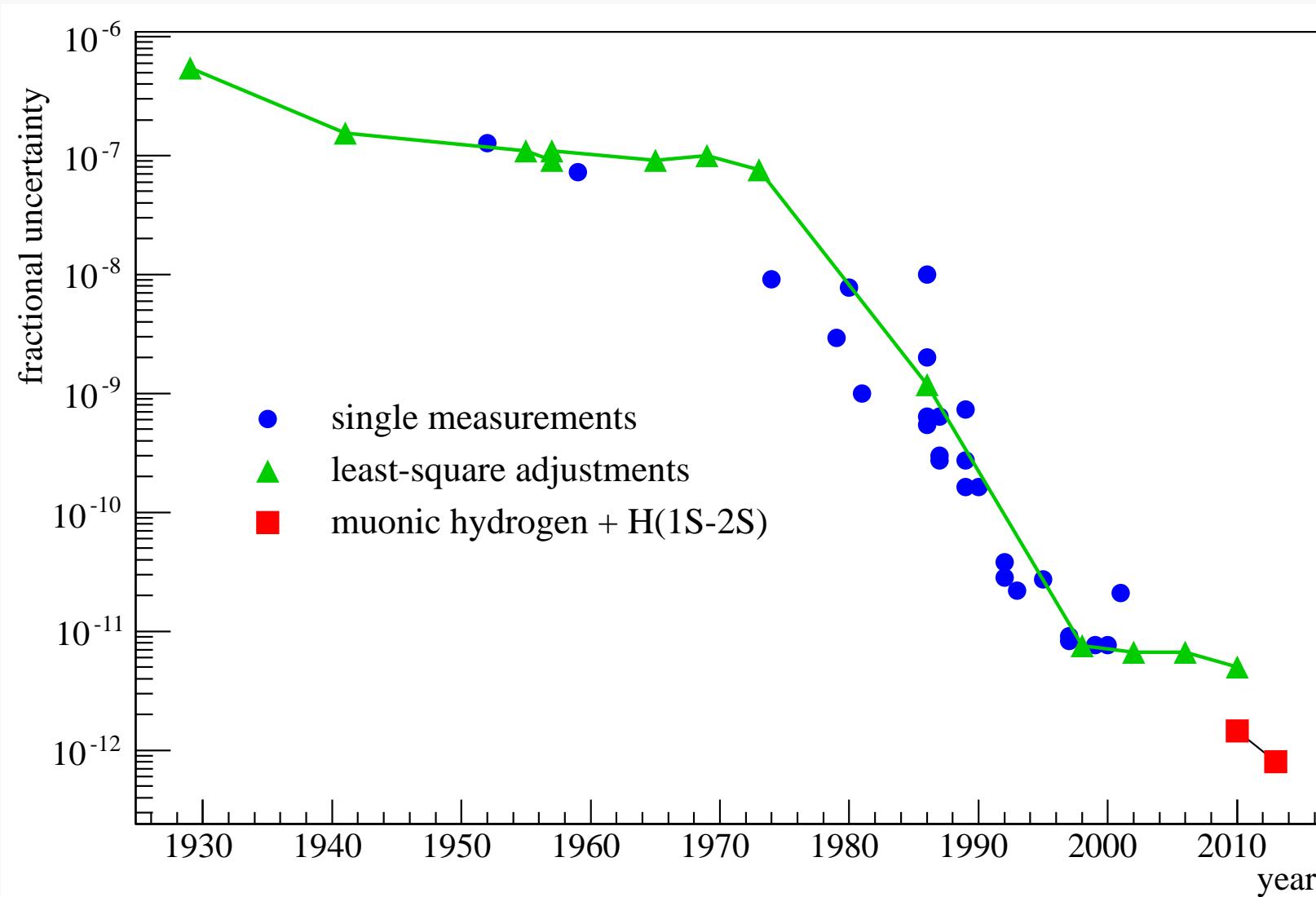
$$\Delta E_{\text{HFS}} = 22.8089(51) \text{ meV}$$

This gives a proton Zemach radius $r_Z = 1.082(31)_{\text{exp}}(20)_{\text{th}} = 1.082(37) \text{ fm}$

A. Antognini, et al., Science 339, 417 (2013)



Rydberg constant



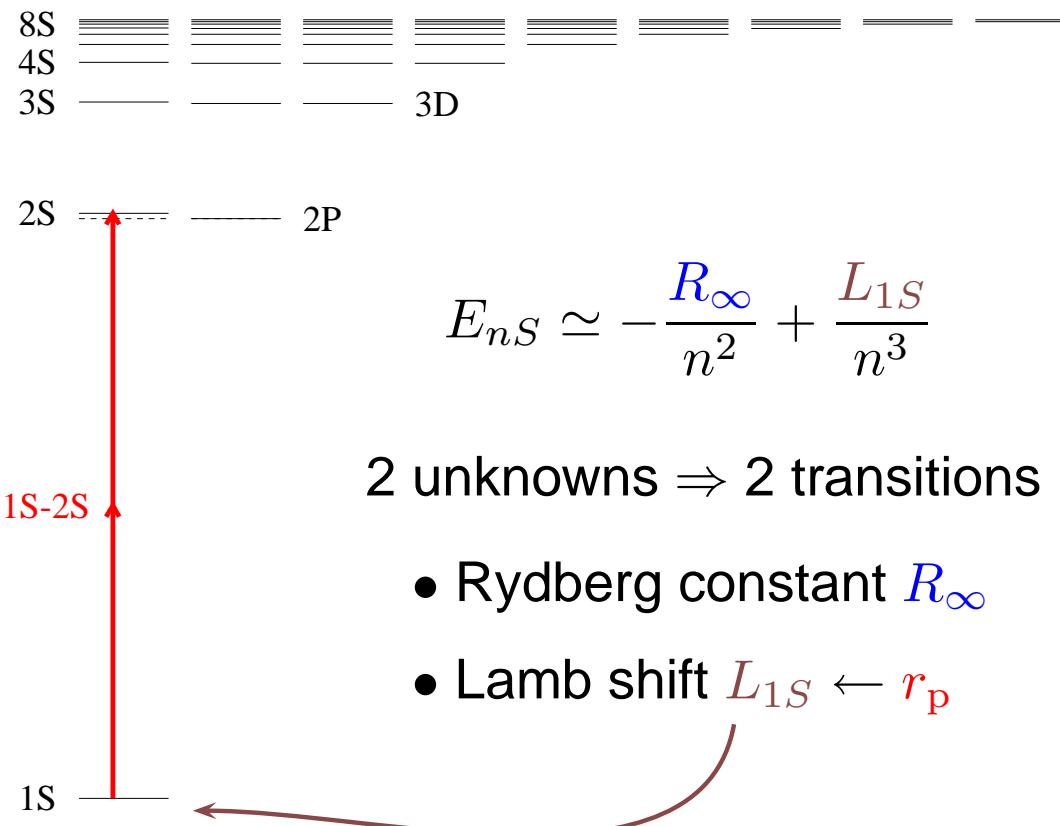
H(1S-2S): C.G. Parthey *et al.*, PRL 107, 203001 (2011).

r_p : A. Antognini *et al.*, Science 339, 417 (2013).

Rydberg constant

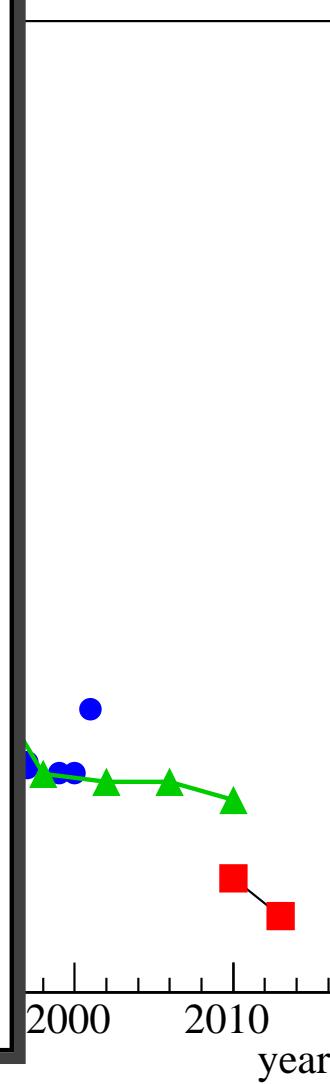
Hydrogen spectroscopy (Lamb shift):

$$L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz}$$



2 unknowns \Rightarrow 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_p$

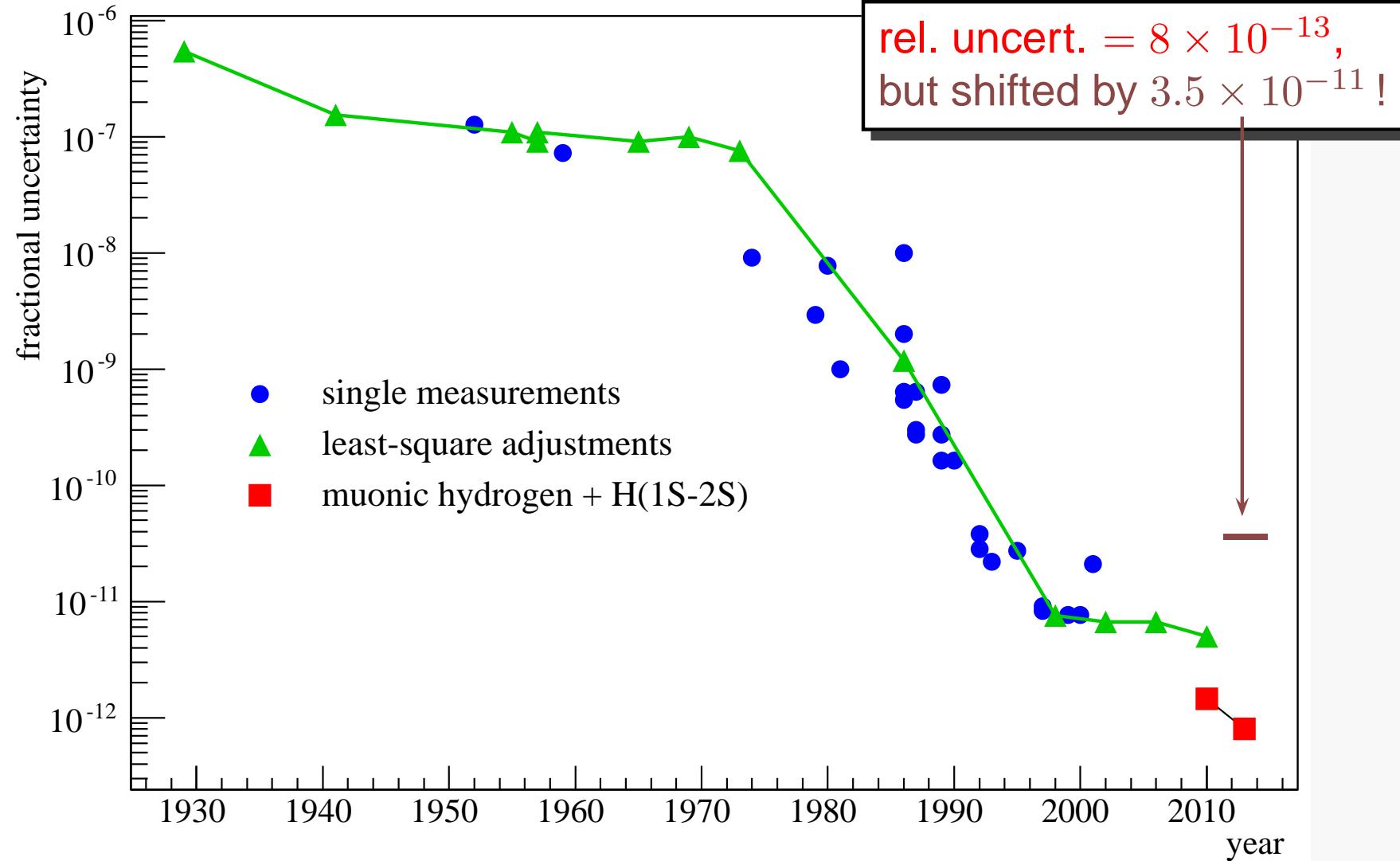


H(1S-2S): C.G. Parthey *et al.*, PRL 107, 203001 (2011).

r_p : A. Antognini *et al.*, Science 339, 417 (2013).

Rydberg constant

$$= 3.289\ 841\ 960\ 249\ 5 (10)^{r_p} (25)^{\text{QED}} \times 10^{15} \text{ Hz/c}$$



H(1S-2S): C.G. Parthey *et al.*, PRL 107, 203001 (2011).

r_p : A. Antognini *et al.*, Science 339, 417 (2013).

What may be wrong ?

Proton radius puzzle: What may be wrong?

Discrepancy:

$$\Delta E_{\mu p}^{\text{th.}}(r_p^{\text{CODATA}}) - \Delta E_{\mu p}^{\text{exp.}}$$

$$= \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases}$$

(2) μp theory wrong? but

- mainly pure QED (vac.pol., etc.)
- 'huge' relative discrepancy
- hadronic terms small
- weak interaction: only HFS, small
- proton *shape*?
- proton *polarizability*?

(1) μp exp. wrong? but

- good statistics ($\sigma = 0.65 \text{ GHz} \ll \text{discrepancy}$)
- two $\mu p(2S-2P)$ transitions measured
- linewidth $\sim 19 \text{ GHz} \ll \text{discrepancy}$
- systematics, *molecular effects*?

(3) H spectroscopy wrong? but

- 2S-8S, 2S-8D, 2S-12S, etc. all consistent ...

(4) H theory wrong? but

- uncertainties 10 \times smaller than discrepancy ...

(5) e-p scattering wrong? but

- new Mainz and JLab results ...

both?

r_p puzzle (1): Is the μp experiment wrong?

$$\Delta E\text{-discrepancy} = 75 \text{ GHz} \leftrightarrow u_r = 1.5\% \leftrightarrow 4\Gamma \quad \text{and} \quad \Gamma^{\text{th}} = \Gamma^{\text{exp}}$$

- Pressure shift ?

- pressure shift of H(1S-2S) in H₂ gas: ~10 MHz/mbar
- μp is m_e/m_μ smaller (stronger E-fields):
 - less disturbed by external fields
 - smaller mixing of states

Detailed calculations give a pressure shift of ~2 MHz at 1 mbar

- Spectroscopy of (pp μ)^{*}-molecules, or (μp_{2S}) e^- -ions, instead of μp ?

- (a) $\mu p(2S) + H_2 \rightarrow \{[(pp\mu)^+]*pee\}^* \rightarrow \mu p(1S) + \dots$ (muon-cat.-fusion)
- (b) $\mu p^* + H_2 \rightarrow (\mu p_{2S})e^- + \dots$?? [Jentschura, Ann. Phys. 326, 516 (2011)]

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(a)	$\{(pp\mu)^+*pee\}^*$ formation/deexcitation	exp:	[PRL 97, 193402 (2006)]
		th:	[PRA 68, 032502 (2003)]
			[PRA 70, 042506 (2004)]

$\tau_{pp\mu} \lesssim 1 \text{ ps}$ caused by strong Auger/Coulomb/radiative deexcitations

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- (b) $\mu p^* + H_2 \rightarrow (\mu p_{2S})e + \dots$?? [Jentschura, Ann. Phys. 326, 516 (2011)]

- (b) Idea: H⁻ ion is stable ! $\rightarrow (\mu p_{2S})e = p\mu^-e^-$ also stable ?
 - The e^- in (μp_{2S}) e leads to $\Delta E \sim 0.4 \text{ meV}$ if $r_e = a_0$ [Jentschura]
 - What is the probability of (μp_{2S}) e formation ?
 - Lifetime of this ion ? Internal and external Auger emission rate ?
 - Loosly bound system: “each” collision ionizes it. No population left.

r_p puzzle (1): Is the μp experiment wrong?

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- (b) $\mu p^* + H_2 \rightarrow (\mu p_{2S})e^- + \dots$?? [Jentschura, Ann. Phys. 326, 516 (2011)]

(a+b) *More detailed theoretical investigation:*

Karr and Hilico [PRL 109, 103401 (2012)] exclude both $p\mu^-e^-$ ions and (pp μ^-)⁺ molecular ions as explanation of the proton radius puzzle.

r_p puzzle (1): Is the μp experiment wrong?

$$\Delta E\text{-discrepancy} = 75 \text{ GHz} \leftrightarrow u_r = 1.5\% \leftrightarrow 4\Gamma \quad \text{and} \quad \Gamma^{\text{th}} = \Gamma^{\text{exp}}$$

- Pressure shift ?

- pressure shift of H(1S-2S) in H₂ gas: ~10 MHz/mbar
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- Spectroscopy of (pp μ)^{*}-molecules, or (μp_{2S}) e^- -ions, instead of μp ?

- (a) $\mu p(2S) + H_2 \rightarrow \{[(pp\mu)^+]*pee\}^* \rightarrow \mu p(1S) + \dots$ (muon-cat.-fusion)
- (b) $\mu p^* + H_2 \rightarrow (\mu p_{2S})e^- + \dots$?? [Jentschura, Ann. Phys. 326, 516 (2011)]

(a+b) *Experimental argument:*

no broadening or double line has been measured

→ “All” μp_{2S} have to be in such a molecular or ionic state
during the laser excitation: **impossible !**

r_p puzzle (1): Is the μp experiment wrong?

$$\Delta E\text{-discrepancy} = 75 \text{ GHz} \leftrightarrow u_r = 1.5\% \leftrightarrow 4\Gamma \quad \text{and} \quad \Gamma^{\text{th}} = \Gamma^{\text{exp}}$$

- Pressure shift? → **NO**
- Spectroscopy of $(pp\mu)^*$ -molecules, or $(\mu p)e^-$ -ions, instead of μp ? → **NO**
- Laser frequency calibration
 - (i) at $6 \mu\text{m}$ with H_2O lines (20 measurements of 5 different lines)
 - (ii) at 708 nm with λ -meter, wavemeter, and FP (calibrated to I_2 , Rb , Cs lines)

Raman cell: $\nu(6\mu\text{m}) = \nu(708\text{nm}) - 3\hbar\omega_{\text{vib}}$. Fluctuations → $\sigma = 0.3 \text{ GHz}$
- Systematic uncertainties:

- laser frequency calibration	0.300 GHz
- Zeeman effect ($B = 5$ Tesla)	0.060 GHz
- AC-Stark, DC-Stark shift	< 0.001 GHz
- Doppler shift	< 0.001 GHz
- collisional shift (1 mbar)	0.002 GHz
- black body radiation shift	$\ll 0.001$ GHz

r_p puzzle (1): Is the μp experiment wrong?

$$\Delta E\text{-discrepancy} = 75 \text{ GHz} \leftrightarrow u_r = 1.5\% \leftrightarrow 4\Gamma \quad \text{and} \quad \Gamma^{\text{th}} = \Gamma^{\text{exp}}$$

- Pressure shift? → **NO**
- Spectroscopy of $(pp\mu)^*$ -molecules, or $(\mu p)e^-$ -ions, instead of μp ? → **NO**
- Laser frequency calibration → **ok**
- Systematic uncertainties → **ok**
- 0.5% air in 1 mbar H_2 → $p_{N_2} = 0.005 \text{ mbar}$
→ $\ll 1\%$ of all $\mu p(2S)$ see any N_2 → **ok**
- Second measured $\mu p(2S-2P)$ resonance ($\sigma_{\text{stat}} = 1.0 \text{ GHz}$, $\sigma_{\text{syst}} = 0.3 \text{ GHz}$):
in agreement with first resonance → **ok**
(calculated 2S-HFS uncertainty: $\sim 2 \text{ GHz}$, assuming a conservative value
for the Zemach radius of $r_Z = 1.05 \pm 0.05 \text{ fm}$)

r_p puzzle (1): Is the μp experiment wrong?

NO!

r_p puzzle (2): Is $\mu p(2S-2P)$ theory wrong?

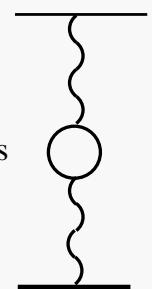
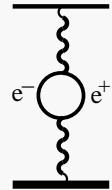
#	Contribution	Value	Unc.	
3	Relativistic one loop VP	205.0282		
4	NR two-loop electron VP	1.5081		
5	Polarization insertion in two Coulomb lines	0.1509		
6	NR three-loop electron VP	0.00529		
7	Polarisation insertion in two and three Coulomb lines (corrected)	0.00223		
8	Three-loop VP (total, uncorrected)			
9	Wichmann-Kroll	-0.00103		
10	Light by light electron loop ((Virtual Delbrück))	0.00135	0.00135	
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	-0.00500	0.0010	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	-0.00150		
13	Mixed electron and muon loops	0.00007		
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	0.01077	0.00038	
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	0.000047		
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	-0.000015		
17	Recoil contribution	0.05750		
18	Recoil finite size	0.01300	0.001	
19	Recoil correction to VP	-0.00410		
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	-0.66770		
21	Muon Lamb shift 4th order	-0.00169		
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	-0.04497		
23	Recoil of order α^6	0.00030		
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	-0.00960		
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability)	0.015	0.004	
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	0.00019		
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	-0.00001		
	Sum	206.0573	0.0045	

Vac.pol.

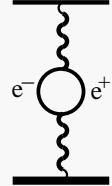
Hadron

Recoil

Status of
2010



r_p puzzle (2): Is $\mu p(2S-2P)$ theory wrong?

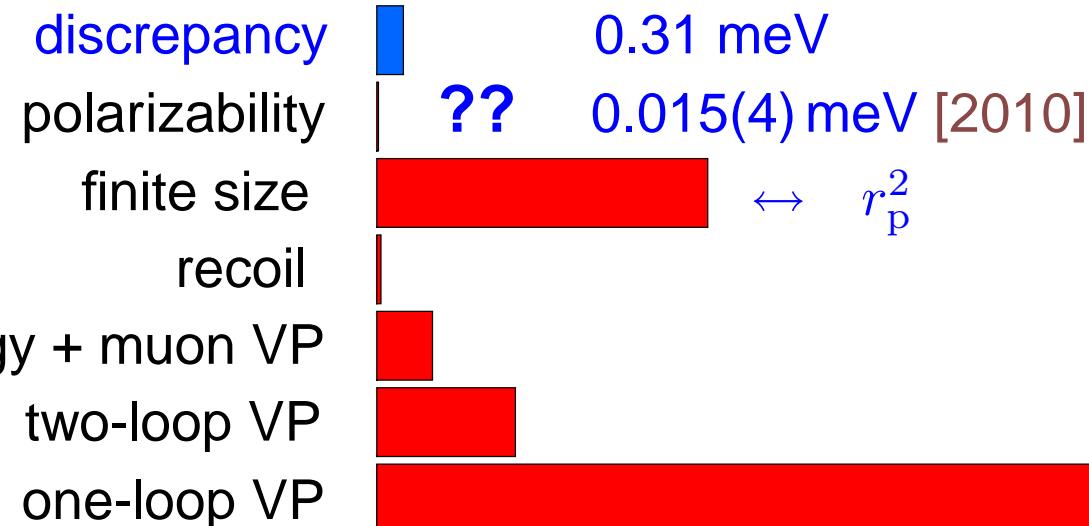
#	Contribution		Value	Unc.	
3	Relativistic one loop VP	Vac.pol.	205.0282		
4	NR two-loop electron VP		1.5081		
5	Polarization insertion in two Coulomb lines		0.1509		
6	NR three-loop electron VP		0.00529		
7	Polarisation insertion in two and three Coulomb lines (co)		0.00223		
8	Three-loop VP (total, uncorrected)				
9	Wichmann-Kroll		-0.00103		
10	Light by light electron loop ((Virtual Delbrück)		0.00135	0.00135	
11	Radiative photon and electron polarization in the Coulom		-0.00500	0.0010	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$		-0.00150		
13	Mixed electron and muon loops		0.00007		
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	Hadron	0.01077	0.00038	Hadrons
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$		0.000047		
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4$		-0.000015		
17	Recoil contribution	Recoil	0.05750		
18	Recoil finite size		0.01300	0.001	
19	Recoil correction to VP		-0.00410		
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$		-0.66770		
21	Muon Lamb shift 4th order		-0.00169		
22	Recoil correction to nuclear polarizability				
23	Recoil of order $\alpha(Z\alpha)^5 m_r$				
24	Radiative recoil induced correction to nuclear polarizability	A. Antognini et al., Ann. Phys. 331, 127 (2013) [arXiv:1208.2637]			
25	Nuclear struc				
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$		0.00019		
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$		-0.00001		
	Sum		206.0573	0.0045	

r_p puzzle (2): Is $\mu p(2S-2P)$ theory wrong?

#	Contribution	Value	Unc.
3	Relativistic one loop VP	205.0282	
4	NR two-loop electron VP	1.5081	
5	Polarization insertion in two Coulomb lines	0.1509	
6	NR th.		
7	Polariz.		
8	Three		
9	Wichr.		
10	Light l.		
11	Radia		
12	Electr		
13	Mixed		
14	Hadro		
15	Hadro		
16	Hadro		
17	Recoi		
18	Recoi		
19	Recoi		
20	Radia		
21	Muon		
22	Recoi		
23	Recoi		
24	Radia		
25	Nucle		
26	Polari		
27	Radia		
	Sum	206.0573	0.0045

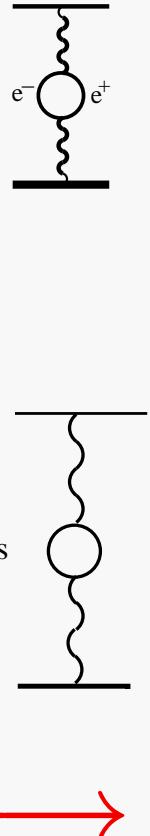
Vac.pol.

Main contributions to the μp Lamb shift:



If there is a problem in μp theory, it is probably related to

- proton shape \rightarrow higher moments in $\langle r_p^n \rangle$
- proton polarizability



r_p puzzle (2): Is $\mu_{\text{p}}(2\text{S}-2\text{P})$ theory wrong?

- Finite size contributions [Friar, Ann.Phys. 1979]

$$E_{\text{FS}} = -\frac{2Z\alpha}{3} \left(\frac{Z\alpha m_r}{n} \right)^3 \left[r_p^2 - \frac{Z\alpha m_r}{2} \langle r_p^3 \rangle_{(2)} + \dots \right]$$

$\downarrow \quad \quad \quad \downarrow$
3.7 meV 0.02 meV

Third Zemach moment:

$$\langle r_p^3 \rangle_{(2)} = \int d^3r \int d^3r' \rho(\vec{r})\rho(\vec{r}')|\vec{r}-\vec{r}'|^3$$

(our discrepancy = 0.31 meV)

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- Can we find a proton shape so that the discrepancy is solved?

In principle yes $\Leftrightarrow \langle r_p^3 \rangle_{(2)} = 37(7) \text{ fm}^3$ [De Rújula, PLB 693, 555 (2010)]

"QED is not endangered by the proton's size"

But measured is $\langle r_p^3 \rangle_{(2)} = 2.71(13) \text{ fm}^3$ [Friar and Sick 2005, Cloët and Miller 2011]

and $\langle r_p^3 \rangle_{(2)} = 2.85(8) \text{ fm}^3$ [New Mainz data: Distler et al., PLB 696, 343 (2011)]

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Cloët and Miller give even "a rigorous upper bound":

$$\langle r_p^3 \rangle_{(2)} \leq 4.5 \text{ fm}^3 \quad [\text{PR C 83, 012201 (2011)}]$$

Solving the puzzle with a large Third Zemach moment
is in contradiction with e-p scattering data!

r_p puzzle (2): Is $\mu p(2S-2P)$ theory wrong?

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What about higher order $(Z\alpha)^6$ finite-size terms?

- M. Distler (Mainz): these terms are not negligible, but small compared to our "discrepancy" (for "non-crazy" form-factors).
- The higher momenta $\langle r_p^n \rangle$ are approx. measured by e-scattering.

r_p puzzle (2): Is μp (2S-2P) theory wrong?

- Most contributions to $\Delta E(2S-2P)$ recalculated 2010-2012 by several groups
⇒ only minor corrections found!

[Karshenboim, Indelicato+Mohr, Jentschura+Pachucki, Eides, Borie, Martynenko, Pineda, ...]

- The rms proton radius r_p is defined consistently for all three experiments (μp , H-spectroscopy, e-p scattering)!

e.g. Darwin-Foldy term, radiative corrections, hfs-structure effects, ...

[Jentschura, EPJD 61, 7 (2011)]; CODATA-2010: P. Mohr *et al.*, Rev. Mod. Phys. 84, 1527 (2012).

Preliminary conclusion:

If μp -experiment (1) and μp -theory (2) are both correct, then $r_p \approx 0.84$ fm

⇒ H experiment (3) or theory (4), and e-p scattering (5)
are both wrong ($r_p \approx 0.87 \dots 0.88$ fm) !?

r_p puzzle (2): Is $\mu p(2S-2P)$ theory wrong?

..... 2011, new players come into the (theory) game:

- Hill & Paz [PRL 107, 160402 (2011)]: “Proton structure effects ... are analyzed using NR QED effective field theory”
→ uncertainty of proton **polarizability** term “underestimated by at least an order of magnitude”

$$\Delta E_{\text{pol}} = 0.015 \pm 0.004 \text{ meV}; \text{discrepancy} = 0.310 \text{ meV} \rightarrow \text{"not enough" ?}$$

Background: Third-Zemach contribution \approx modification of the wave function caused by finite-size. In a quantum field framework, it is part of the **two-photon exchange (TPE)** diagrams which include also an inelastic part (ΔE_{pol}).

- unified treatment of TPE (elastic + inelastic), using
 - ↔ doubly virtual Compton amplitude, (+ dispersion relations)
 - ↔ measured form-factors and structure functions
- unknown “subtraction term” calculated with **heavy baryon χ PT**
by Birse + McGovern [2012] ... , but Hill+Paz and Miller et al. still have doubts:
Gerry Miller at Mainz: “ χ PT is for low Q^2 , but integral goes over all Q^2 ”

r_p puzzle (2): Is $\mu p(2S-2P)$ theory wrong?

TPE (two-photon exchange), continued:

- Pascalutsa et al. [EPJC 74, 2852 (2014)] summarize 7 different calculations of the proton polarizability term, from ~ 0.005 to ~ 0.021 meV
→ large values unlikely!
- Correct treatment of TPE-subterms “elastic”, “non-pole”, “inelastic”, “subtraction” still under discussion ... [Birse+McGovern, Hill+Paz, ...]

→ Conclusions:

- It is unlikely that “ μp theory” can explain our discrepancy
- The new $\mu^\pm - p / e^\pm - p$ scattering experiment (“MUSE” at PSI, ~ 2016) will restrict TPE effects!

[Pohl, Gilman, Miller: Pachucki, Annu.Rev.Nucl.Part.Sci. 63, 175 (2013)]

r_p puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

TPE (two-photon exchange), continued:

In our NATURE-2010 paper, we treated the Third-Zemach moment “classically”,
in the SCIENCE-2013 paper, we preferred to quote the more modern TPE approach.

Savely K. analyzed this at the Mainz-workshop, and in his Summary he said:

“... this result is from SCIENCE, not from NATURE ...”

and was irritated that the audience started to laugh, because people understood

“This result is from science, not from nature.”

r_p puzzle (3): Is H-spectroscopy wrong?

- 1S Lamb shift and R_∞ can be deduced from two measurements in H

$$\left. \begin{array}{ll} \nu_{1S-2S} & (u_r = 10^{-14}) \\ \nu_{2S-8S/D} & (u_r = 10^{-11}) \\ & \vdots \end{array} \right\} \Rightarrow L_{1S}^{\text{exp}} = 8172.840(19) \text{ MHz}$$

$E_{nS} \simeq \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$

- 1S Lamb shift, theoretical prediction in H

$$\left. \begin{array}{l} \text{QED} \\ r_p \\ \alpha, m_e, m_p, \dots \end{array} \right\} \Rightarrow L_{1S}^{\text{th}}(r_p) = 8171.636(4) + 1.5645 r_p^2 \text{ MHz}$$

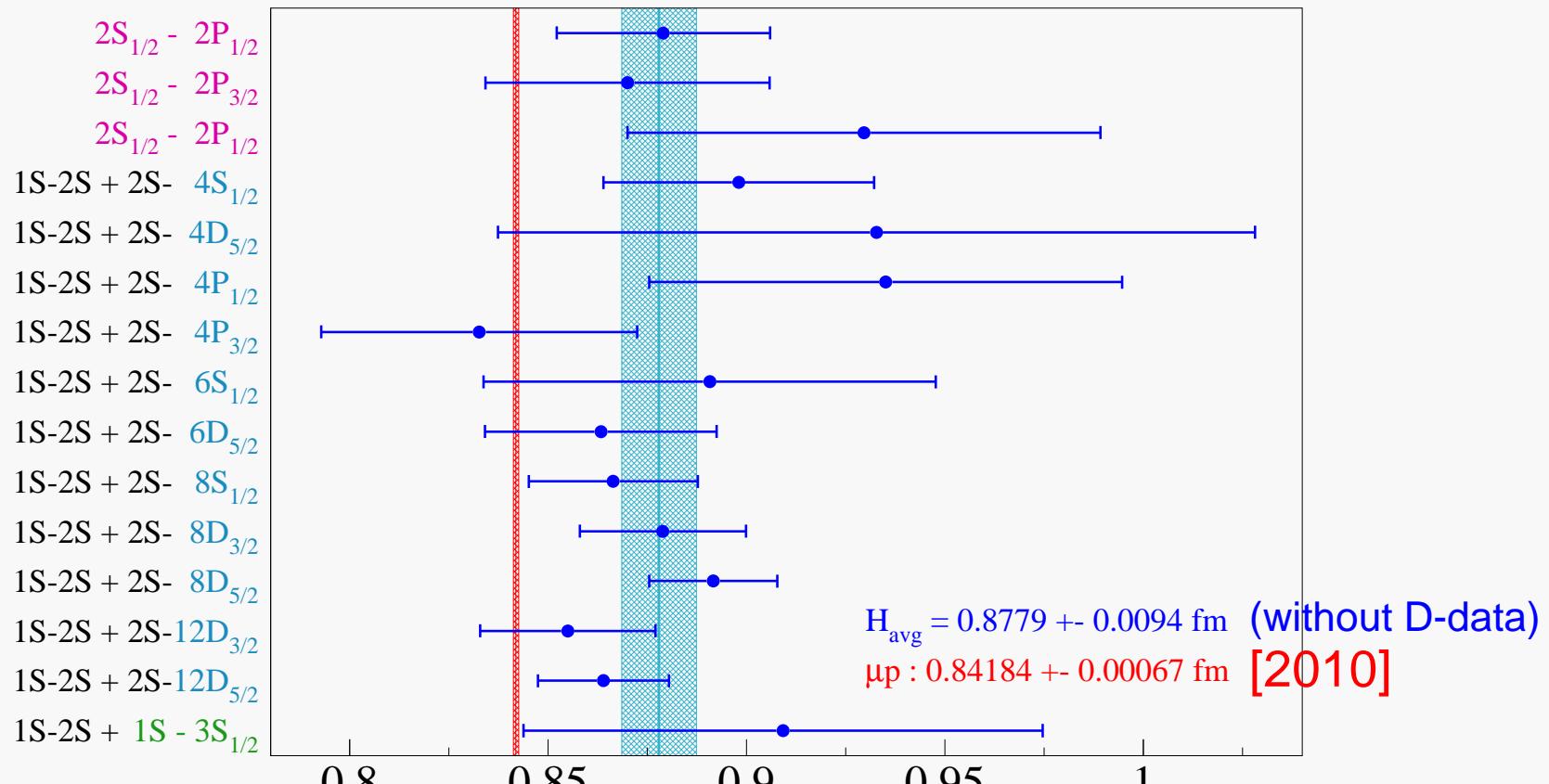
- Proton radius from H and D spectroscopy

$L_{1S}^{\text{th}}(r_p) = L_{1S}^{\text{exp}} \implies r_p = 0.876(8) \text{ fm, with } u_r = 1\%$

r_p puzzle (3): Is H-spectroscopy wrong?

r_p from H spectroscopy:

- 2S-2P transition in H (independent on R_∞)
- two transitions $n \rightarrow n'$ in H ($\Rightarrow r_p$ and R_∞)



CODATA says: " $\mu_p - H$ " is 4.4σ , but:
The maximal deviation from our result is $\sim 3\sigma$

r_p puzzle (3): Is H-spectroscopy wrong?

Is Rydberg R_∞ , the best measured physical constant ($u_r \sim 10^{-11}$), wrong?

- H(1S-2S) measured ultra-precisely ($\sim 10^{-14}$) at MPQ
 - ⇒ strong corr. $R_\infty \leftrightarrow L_{1S}^{\text{exp}}$, because $\nu_{1S-2S} \simeq \frac{3}{4}R_\infty + \frac{7}{8}L_{1S}^{\text{exp}}$
 - ⇒ strong corr. $R_\infty \leftrightarrow r_p$, using QED calculation $L_{1S}^{\text{th}}(r_p) = 8171.636(4) + 1.5645 r_p^2$ MHz
 - ⇒ our $r_p(\mu p)$ shifts R_∞ by -115 kHz, or 6.6σ away from the CODATA value
- New measurements of R_∞ (or r_p) are thus needed:
 - H(1S-3S) Paris, in progress; MPQ, in progress
 - H(2S-4P) MPQ, in progress
 - H(2S-2P) $\rightarrow r_p$ York Uni, Toronto, in progress
 - H-like atoms at medium- Z NIST, planned
 - He $^+$ combined with μ He $^+$ MPQ, Mainz (proposed), PSI (compl.)
 - Myonium μ^+e^- (1S-2S) PSI, planned
- new R_∞ , together with QED(H-atom)
puzzle (4) → independent r_p

r_p puzzle (4): Is H-theory wrong ?

- Free QED

[Hanneke et al., PRL 100, 120801 (2008)]

electron anomaly: $a_e = \frac{1}{2}(g_e - 2)$ → determination of α ($\approx 2\pi a_e$)

$$a_e = C_1 \left(\frac{\alpha}{\pi}\right) + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + C_5 \left(\frac{\alpha}{\pi}\right)^5 + \Delta(\text{had.}, \dots)$$

$$u[a_e^{\text{exp}}] = 2.4 \times 10^{-10}, \quad u[a_e^{\text{th}}] = 2.8 \times 10^{-10}, \quad u[\text{QED test}] = 7.7 \times 10^{-10}$$

[new h/M → α measurement: PRL 106, 080801 (2011)]

- Bound-state QED in Hydrogen now: $u[\text{test}] \approx 7 \times 10^{-6}$!

- Binding effects ($Z\alpha$) bad convergence, all-order approach/expansion
- Radiative corrections (α and $Z\alpha$)
- Recoil corrections (m/M and $Z\alpha$) relativity \nLeftrightarrow two-body system
- Radiative–recoil corrections (α , m/M and $Z\alpha$)
- Proton structure corrections (r_p , r_{Zemach} and $Z\alpha$)

r_p puzzle (4): Is H-theory wrong ?

Bound state QED:

- All corrections are mixed up: $\alpha^x \cdot (Z\alpha)^y \cdot (m/M)^z \rightarrow$ “book-keeping”?
- Cannot develop the calculation in a systematic way, like in $g - 2$
- Relativistic QED is not suitable for precision calculation of bound-states

⇒ NRQED: “A field theory describing the interactions of photons and non-relativistic matter. The Lagrangian is constructed to yield predictions valid to any fixed order in small parameters $\alpha \dots$ etc.” [Hill & Paz, PRL 107, 160402 (2011)]

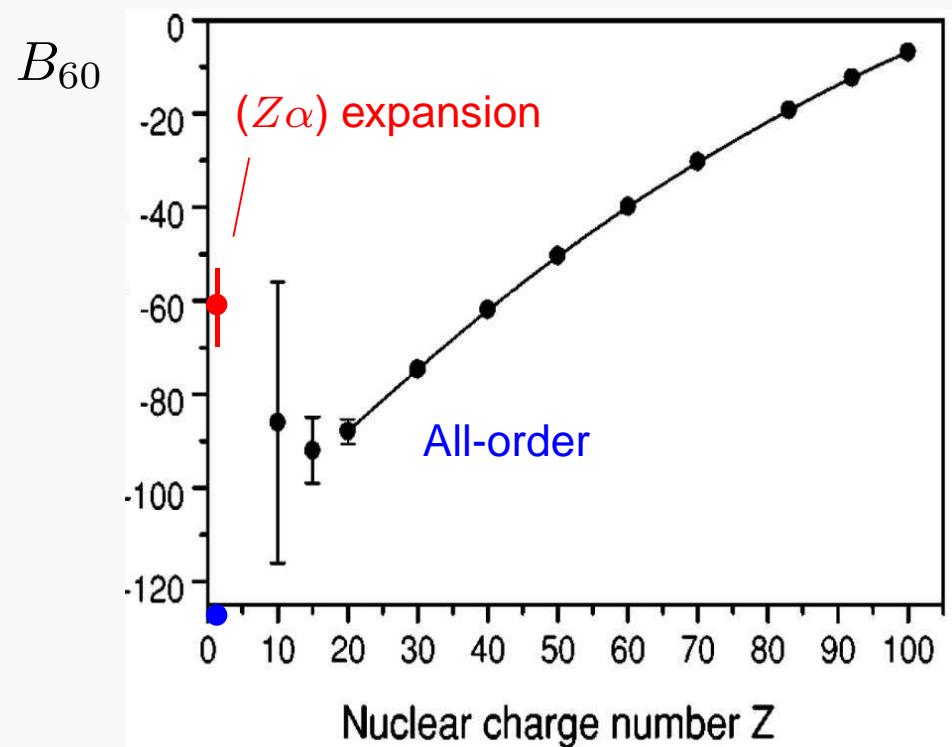
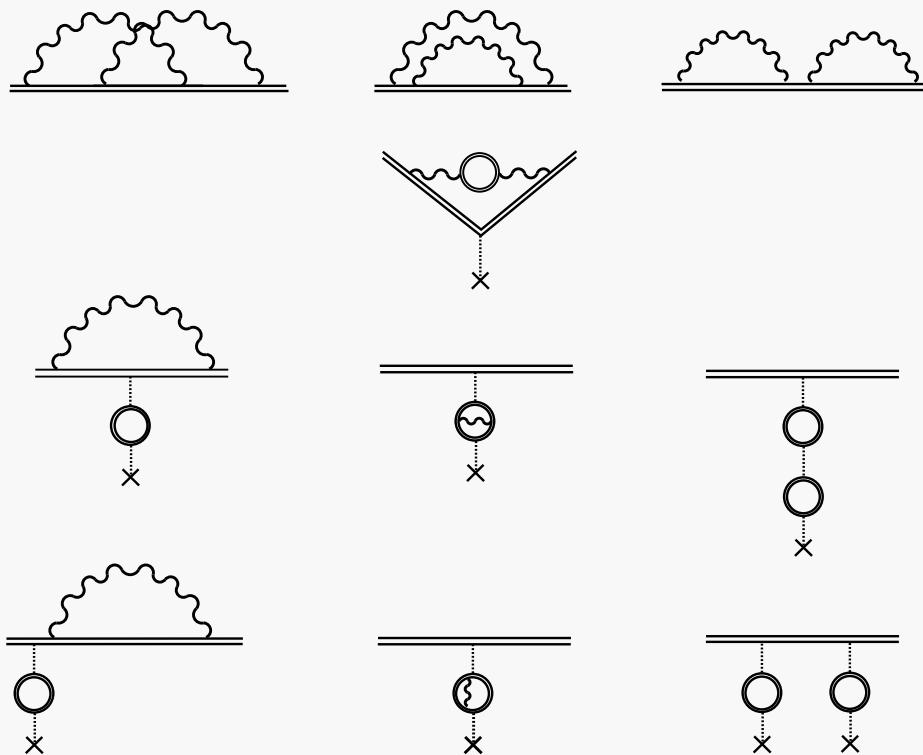
Pineda: “Potential Non-Relativistic QED” describes the (muonic) hydrogen dynamics and profits from the hierarchy $m_\mu \gg m_\mu \alpha \gg m_\mu \alpha^2$

- **HBEFT** → (QED) → NRQED → **pNRQED**: compute QED and hadronic effects
heavy baryon effective field theory [Pineda, PR C 77, 035202 (2008), and previous]

→ χ PT can predict the leading order of third-Zemach and polarizability terms:

$\langle r_p^3 \rangle (\chi\text{PT}) \sim \langle r_p^3 \rangle (\text{e} - \text{scattering})$, but in disagreement with $\langle r_p^3 \rangle (\text{DeRujula})$

r_p puzzle (4): Is H-theory wrong ?



$$\Delta E_{SE}^{(2)} = m \left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^4}{n^3} G_n(Z\alpha)$$

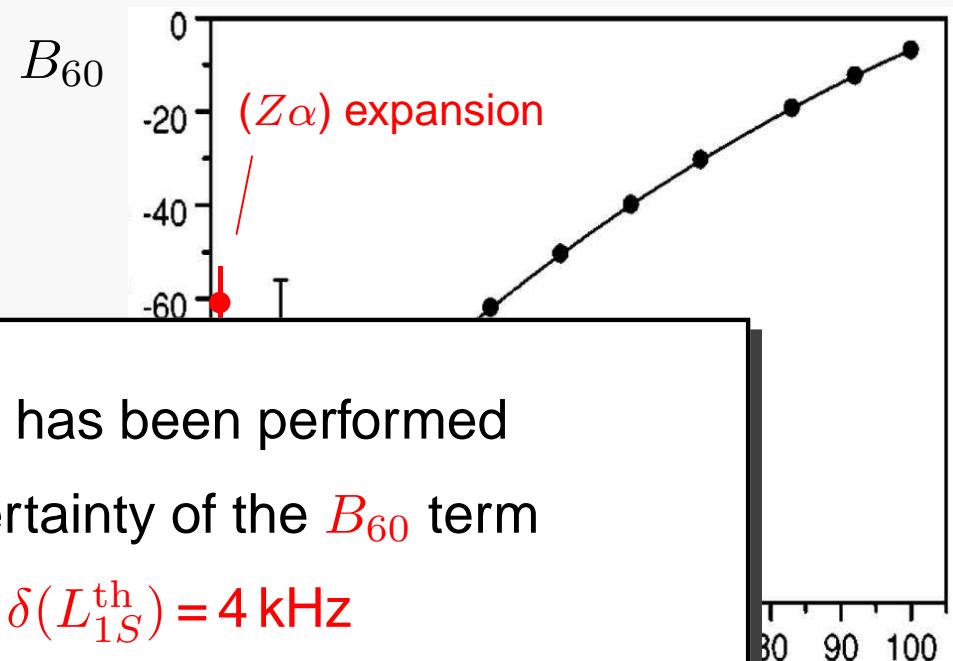
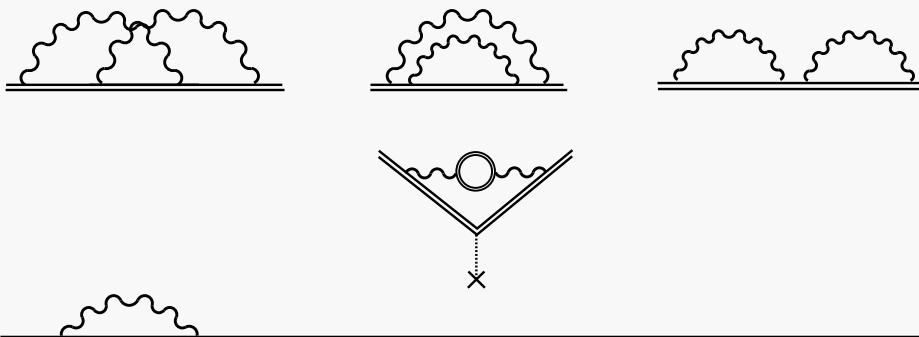
$B_{60} = -86(15)$, $G_{60}^{h.o.} = -101(15)$ Yerokin (2009)

$$G_n = B_{40} + (Z\alpha)B_{50} + (Z\alpha)^2 [B_{63} \ln^3 (Z\alpha)^{-2} + B_{62} \ln^2 (Z\alpha)^{-2} + B_{61} \ln (Z\alpha)^{-2} + B_{60}] + \dots$$

$$G_n = 1.409 - 0.177 + [-0.015 - 0.003 + 0.026 - 0.003 + \dots] + \dots$$

Bad convergence of the $(Z\alpha)$ expansion

r_p puzzle (4): Is H-theory wrong?



- Calculation of all two-loop terms in H has been performed in the last decade. The present uncertainty of the B_{60} term (and other higher-order terms) gives $\delta(L_{1S}^{\text{th}}) = 4 \text{ kHz}$
- But to bring $r_p(H)$ in agreement with $r_p(\mu p)$, L_{1S}^{th} has to be shifted by $\sim 100 \text{ kHz}$!
- Future: combining He^+ (MPQ, Mainz) and μHe^+ spectroscopy → determine B_{60} term and Rydberg constant much better

okin (2009)

$[60] + \dots$

$+ \dots$

r_p puzzle (5): Is e-p scattering wrong?

- Rosenbluth cross section → Sachs form factor → r_p

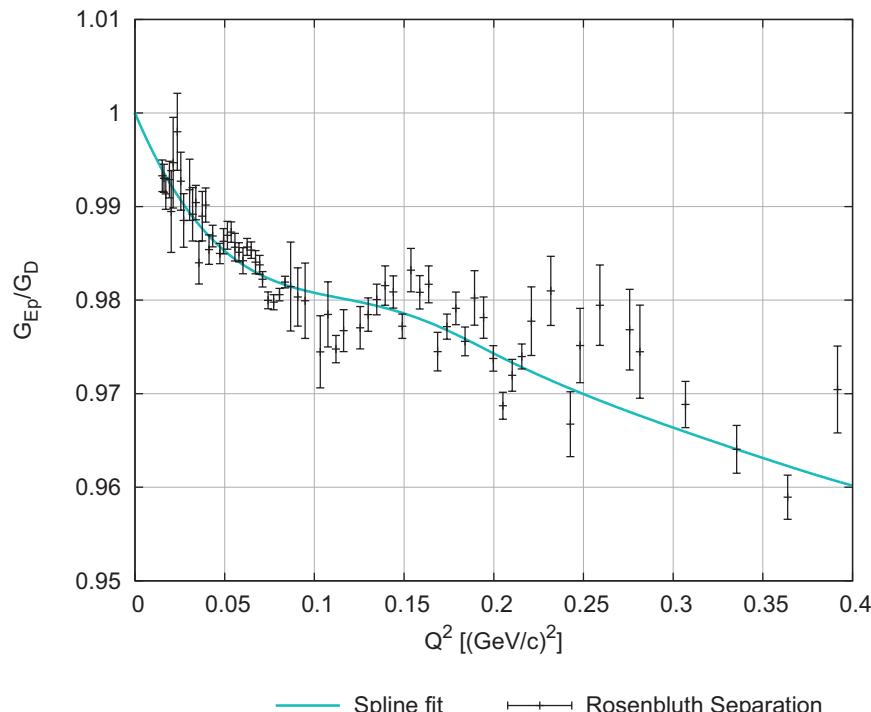
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon(1 + \tau)}$$

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

$$Q^2 [(\text{GeV}/c)^2] = \begin{cases} \sim 10^{-6} & (\mu p) \\ > 10^{-3} (10^{-4} ?) & (\text{e-p scatt.}) \end{cases}$$

extrapolation to $Q^2 \rightarrow 0$ required

Example: part of new Mainz data, G_E/G_{dipole} vs. Q^2



Systematic studies about extrapolation have been performed

$$r_p = (0.879 \pm 0.005_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.005_{\text{model}}) \text{ fm}$$

→ old r_p values from e-p scatt. confirmed!

[Bernauer *et al.*, PRL 105, 242001 (2010);
PRC 90, 015206 (2014)]

r_p puzzle (5): Is e-p scattering wrong?

- Rosenbluth cross section → Sachs form factor → r_p

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- Open questions regarding the extrapolation

- We can not totally exclude the presence of unexpected “bump/dip” at lower Q^2 .
- Model assumption of the functional behavior of the form factor?
- Normalization problems. Fitting with $G_E(Q^2 = 0) = 1 \rightarrow$ underestimation of uncertainty.

r_p from new scattering data with 1% accuracy. Is that realistic?

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r_p from new scattering data with 1% accuracy. Is that realistic?

- Old data, mainly from Mainz ~1985, reanalyzed by Rosenfelder, Sick, ... [2000-2011]

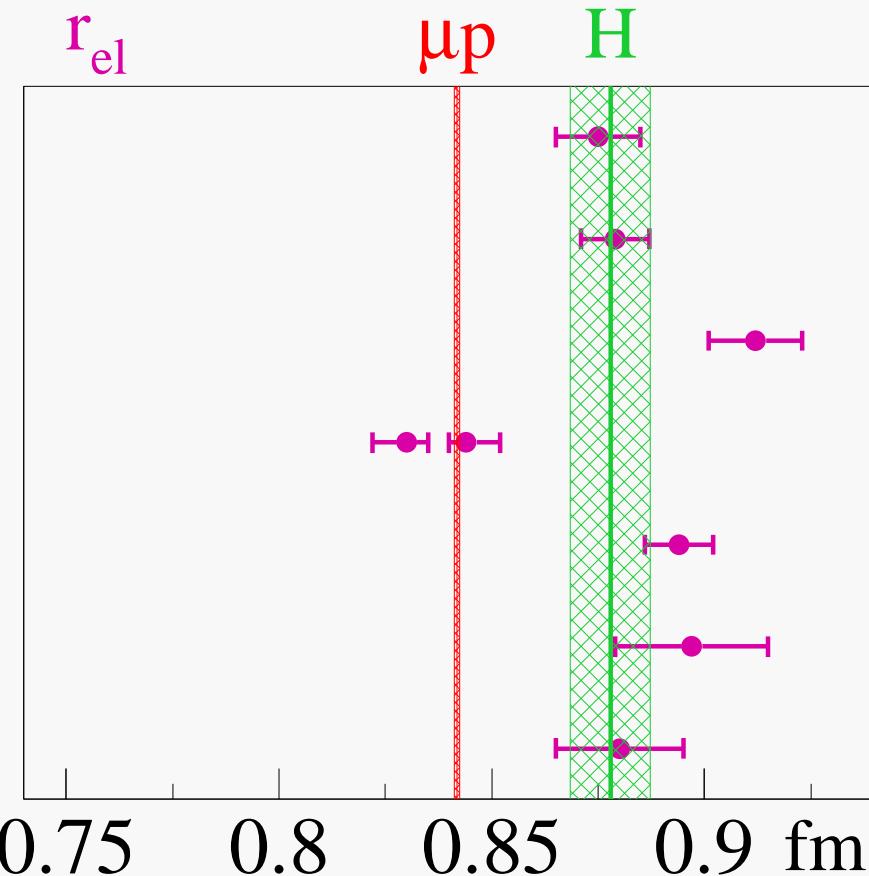
- New data: more statistics

- Mainz “MAMI A1” [Bernauer et al., PRL 105, 242001 (2010); PRC 90, 015206 (2014)]
[Vanderhaegen & Walcher, Nucl. Physics News 21, 14 (2011)]
[Distler et al., Phys Lett B 696, 343 (2011)]

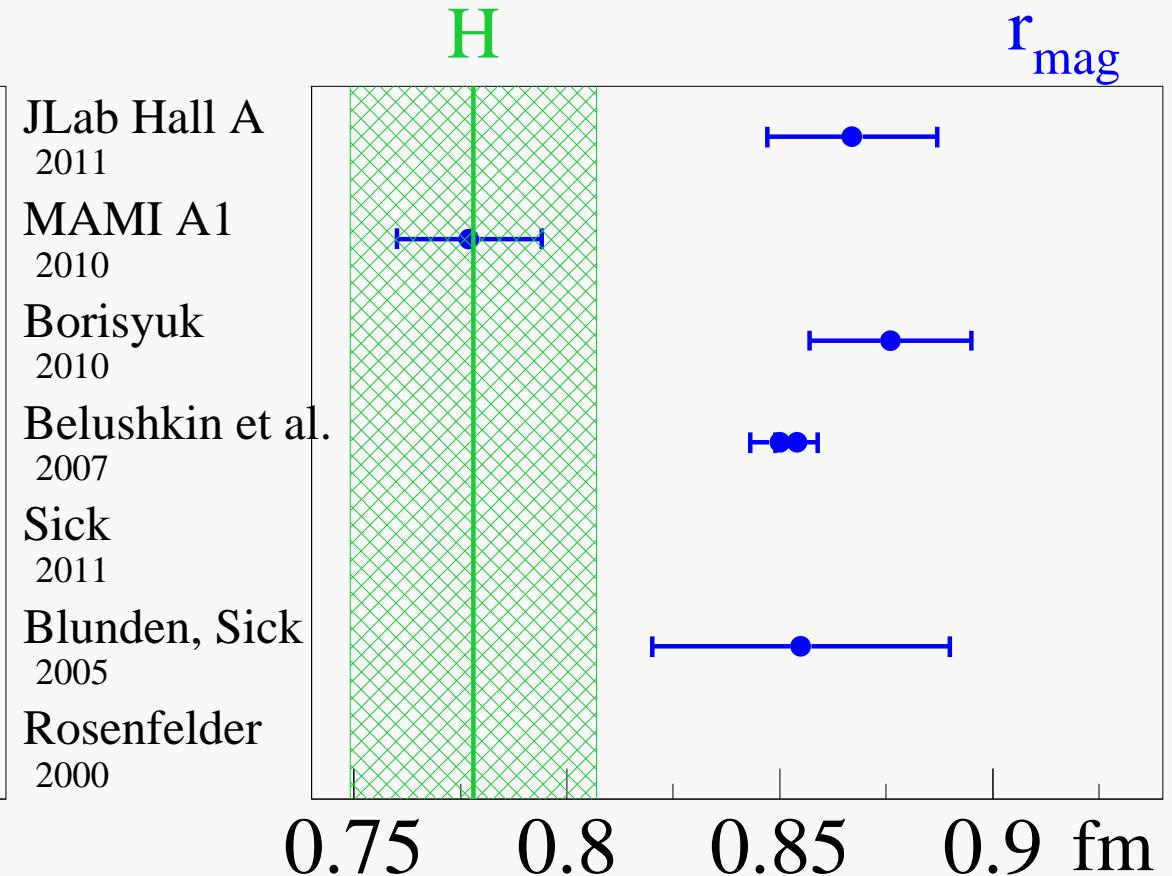
- Jefferson Lab “Hall A” [Zhan et al., arXiv:1102.0318; Ron et al., 1103.5784]

r_p puzzle (5): Is e-p scattering wrong?

Charge and magnetic rms-radii of the proton:



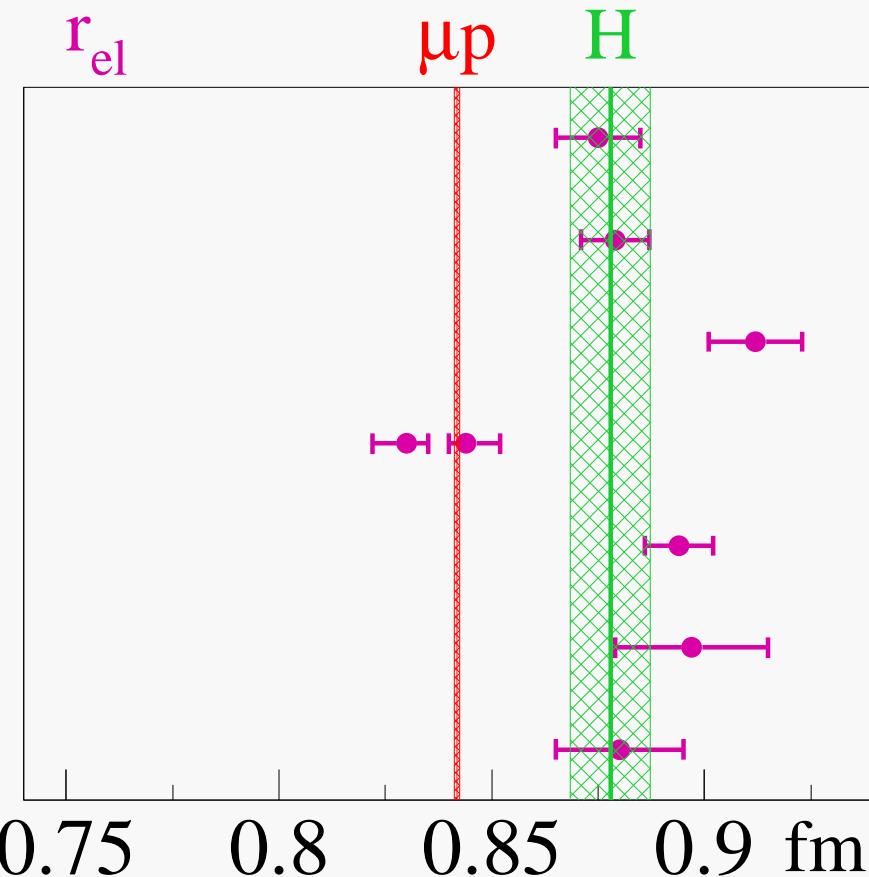
Rosenfelder , Phys Lett B 479, 381 (2000)
Blunden, Sick , PRC 72, 057601 (2005)
Sick , Few Body Syst. (2011)
Belushkin et al. , PRC 75, 035202 (2007)



Borisuk , Nucl Phys A 843, 59 (2010)
MAMI A1 Bernauer et al., PRL 105, 242001 (2010)
JLab Hall A Zhan et al., 1102.0318 (nucl-ex) (2011)

r_p puzzle (5): Is e-p scattering wrong?

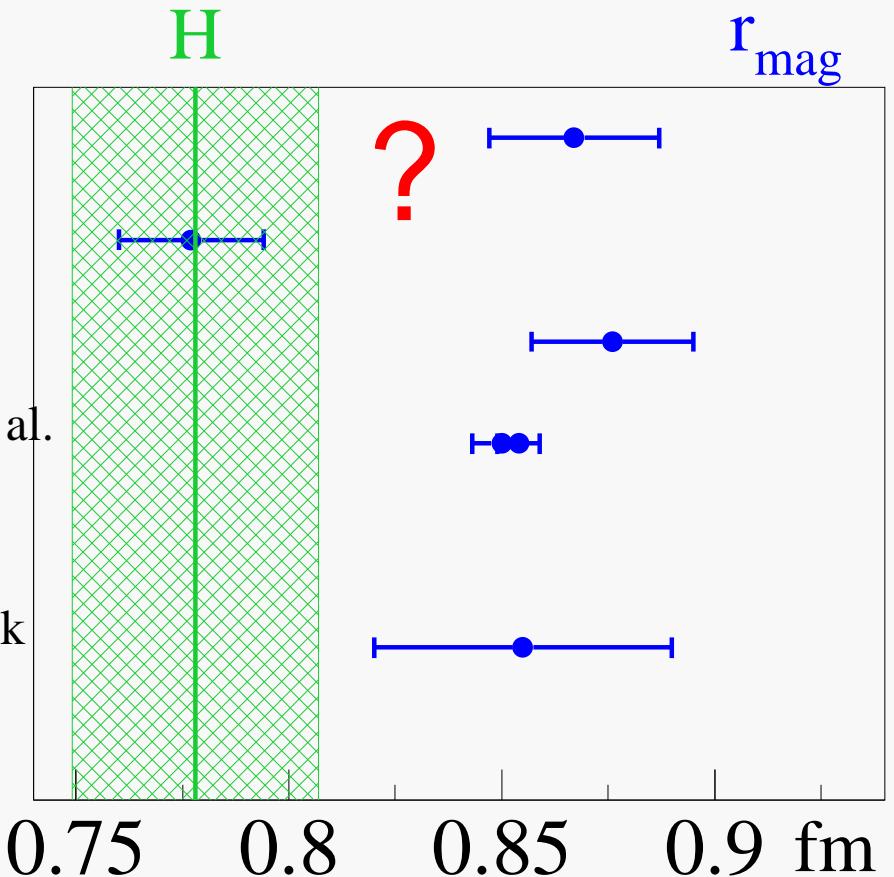
Charge and magnetic rms-radii of the proton:



JLab Hall A
2011
MAMI A1
2010
Borisyuk
2010
Belushkin et al.
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r_p puzzle (5): Is e-p scattering wrong?

Charge and magnetic rms-radii of the proton:



Discrepancies between fits of e-p data, using:

- “sum of Gaussians” (e.g. Sick et al.) → $r_p \approx 0.88 \text{ fm}$
 - functions based on “dispersion relations” with “analyticity and unitarity” ... (e.g. Meissner et al.) → $r_p \approx 0.84 \text{ fm}$
- more DATA needed!

New experiments at

- JLab ($Q^2 \sim 10^{-4} \text{ GeV}^2$, in progress)
- Mainz (e-d, analysis in progress), and more...
- MUSE at PSI: μ^\pm, e^\pm

Puzzle (1) - (5): Present status

- (1) μp experiment:
 - no doubt about statistics, position, width
 - molecular or “ionic” effects: **excluded**
- (2) μp theory:
 - pure QED checked, using different methods: **ok** (only minor effects found)
 - proton shape (e.g. third-Zemach) ? **excluded** (all momenta of $\rho_E(r)$ measured at Mainz)
 - proton **polarizability** ? in discussion (but unlikely to explain discrepancy)
 - modern “effective theories” have been introduced to treat nuclear effects
- (3)+(4) H spectroscopy:
 - $R_\infty \leftrightarrow r_p$ individually $\leq 3\sigma$ → new experiments in progress
 - theory: now at 4 kHz uncertainty → discrepancy of ~ 100 kHz: unlikely
- (5) e-p scattering:
 - new data from Mainz and JLab **confirm old values!** Analysis, systematics ?
(There are inconsistencies!)
- New physics ?? : dark photons, new couplings, mini-charged ...

New physics ?

New physics ? Some ideas :

- Jentschura [arxiv:1011.5453; Annals of Physics 326, 516 (2011)]:
Modification of vacuum polarization due to a **millicharged particle** or an unstable **intermediate vector boson**: **excluded** by g_μ -2, g_e -2, H-spectroscopy
- Barger et al. [arxiv:1011.3519; PRL 106, 153001 (2011)]:
... new scalar, pseudoscalar, vector, and tensor flavor-conserving non-universal interactions may be responsible for the discrepancy. We consider exotic particles that among leptons, couple preferentially to muons, and mediate an attractive nucleon-muon interaction. We find that many constraints from low energy data disfavor new spin-0, spin-1 and spin-2 particles as an explanation.

New physics ? Some ideas :

- Jentschura [arxiv:1011.5453; Annals of Physics 326, 516 (2011)]:
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*In response to Rabi's gibe about the existence of the muon,
"Who ordered that?" we declare, "The proton!"*

New physics ? Some ideas :

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- Batell, McKeen, Pospelov [PRL 107, 011803 (2011), “New parity-violating muonic forces”]:
We identify a class of models with gauged right-handed muon number, which contains new vector and scalar force carriers at the 100 MeV scale or lighter, that is consistent with observations. Such forces would lead to an enhancement by several orders-of-magnitude of the parity-violating asymmetries in the scattering of low-energy muons on nuclei.
This model predicts a shift of the effective charge radius from $\mu \text{He}^+(2S-2P)$ by $\Delta r_{\text{He}}/r_{\text{He}} = -1.0\%$ (to be compared with $\sigma_{r_{\text{He}}}/r_{\text{He}} = 0.25\%$ from e-scattering [Sick]).
For “photon” masses $< 4 \text{ MeV}$, $\Delta r_{\text{He}}/r_{\text{He}} \leq -0.5\%$.

New physics ? Some ideas :

- Jaeckel & Roy [arxiv:1008.3536 and 1011.0692; PR D 82, 125020 (2010)]:

High precision spectroscopy can provide a sensitive tool to test Coulomb's law on atomic length scales. This can then be used to constrain particles such as extra "hidden" photons or minicharged particles that are predicted in many extensions of the standard model, and which cause small deviations from Coulomb's law.

H-spectroscopy **rules out** hidden photons and restricts deviations from Coulomb's law.

$$V(r) = -\frac{Z\alpha}{r}(1 + \alpha'e^{-mr}) \quad \text{or} \quad V(r) = -\frac{Z\alpha}{r}(1 + \alpha''(\mathbf{s}_1 \cdot \mathbf{s}_2)e^{-mr})$$

From simple atoms, there are constraints on light bosons with ultra-weak coupling:

$m \in [1\text{eV}, \text{MeV}]$ and $\alpha' < 10^{-13}$, $\alpha'' < 10^{-17}$ [Karshenboim, PRL 104,220406 (2010)]

- Tucker-Smith & Yavin [PRD 83, 101702 (2011), "Muonic hydrogen and MeV forces"]:
 - new interaction between muons and protons
 - new force carrier with $\sim\text{MeV}$ mass, can account for discr. in μp and g_μ^{-2}
 - predicts effects on μd , μHe^+ (comparable to Pospelov's).
- Brax & Burrage [PR D 83, 035020 (2011)]:
 - **negligible** contribution of a scalar field which couples to matter and photons

New physics ? Some ideas :

- Jentschura [PR A 88, 062514 (2013)] :

*“...Speculative presence of light sea fermions as a **nonperturbative** physical property of the hadron. ... Due to the highly nonlinear nonperturbative nature of QCD, this reshaping can be much larger than the electromagnetic perturbation itself, and therefore there is room for ... electron-positron pairs inside the proton, which cannot be accounted for by perturbative QED considerations alone. ... not excluded by any known experiments.*

A fraction of $\sim 10^{-7}$ sea fermion pairs (positrons!) per valence quark would be enough to explain the proton radius puzzle.

For μHe^+ , the radius is expected to shrink by $\Delta r_{\text{He}}/r_{\text{He}} \sim -2\%$

- Pachucki & Meissner [arXiv:1405.6582, “Proton charge radius and the perturbative QED”] :

“... perturbative picture of quantum electrodynamics within the proton may fail ... The proton charge radius difference can be attributed to the existence of additional forms of the lepton-proton interaction ... If there are nonperturbative terms beyond the proton formfactors, the proton charge radius as seen by positron can be different from that seen by the electron.”

New physics ? Some ideas :

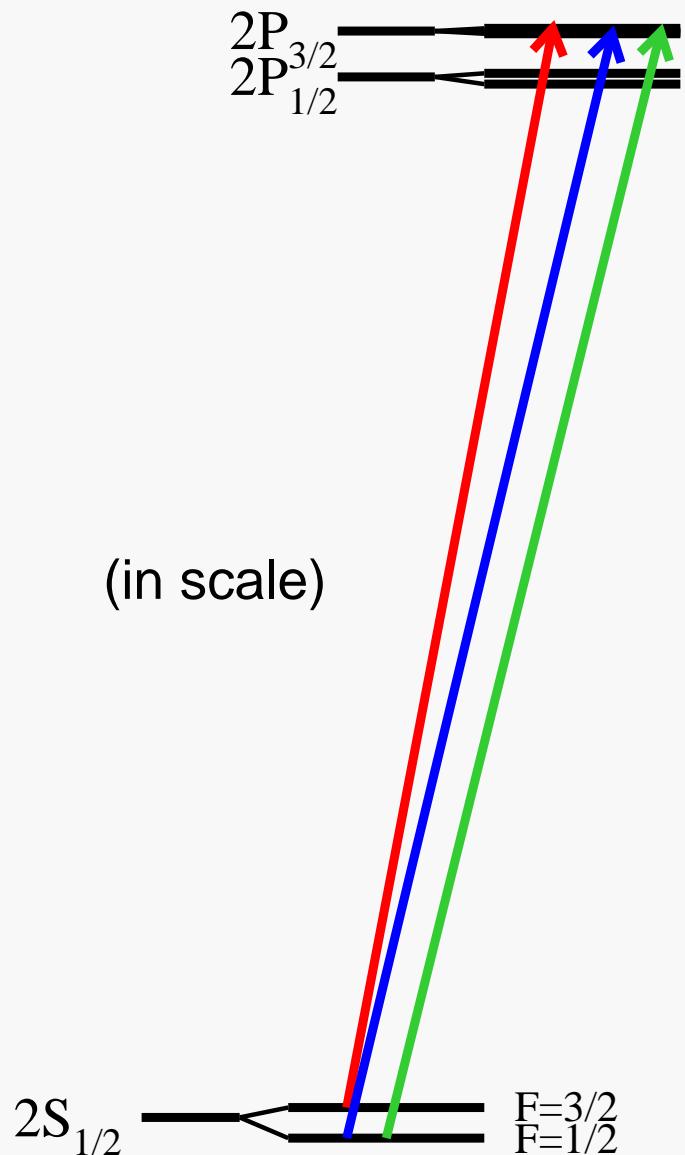
... this is all fine ...

... but, as Roland Rosenfelder said:

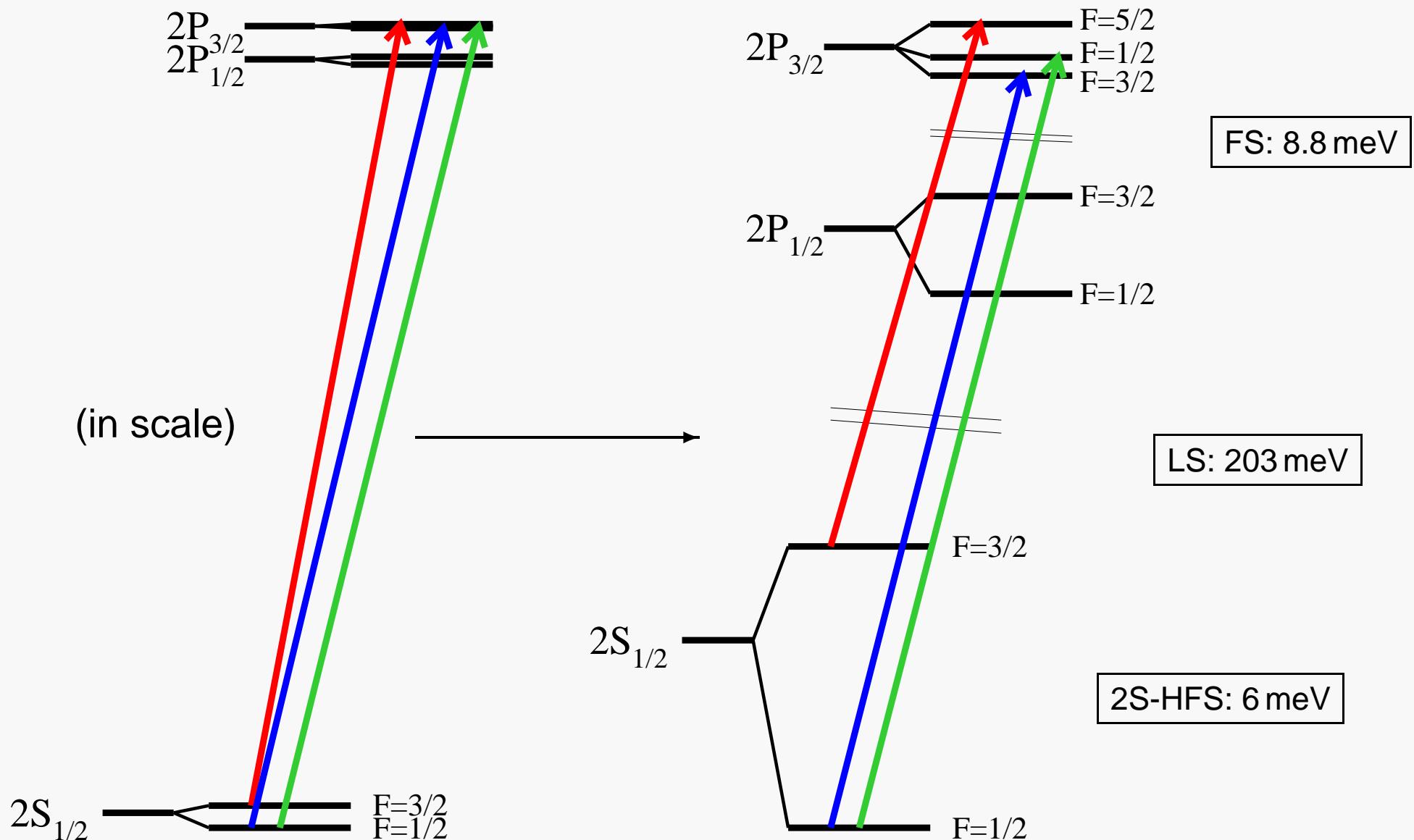
Over many years I have witnessed how all alleged "anomalies" in the low-energy sector of the Standard Model have disappeared after careful examination of all effects.

Muonic deuterium

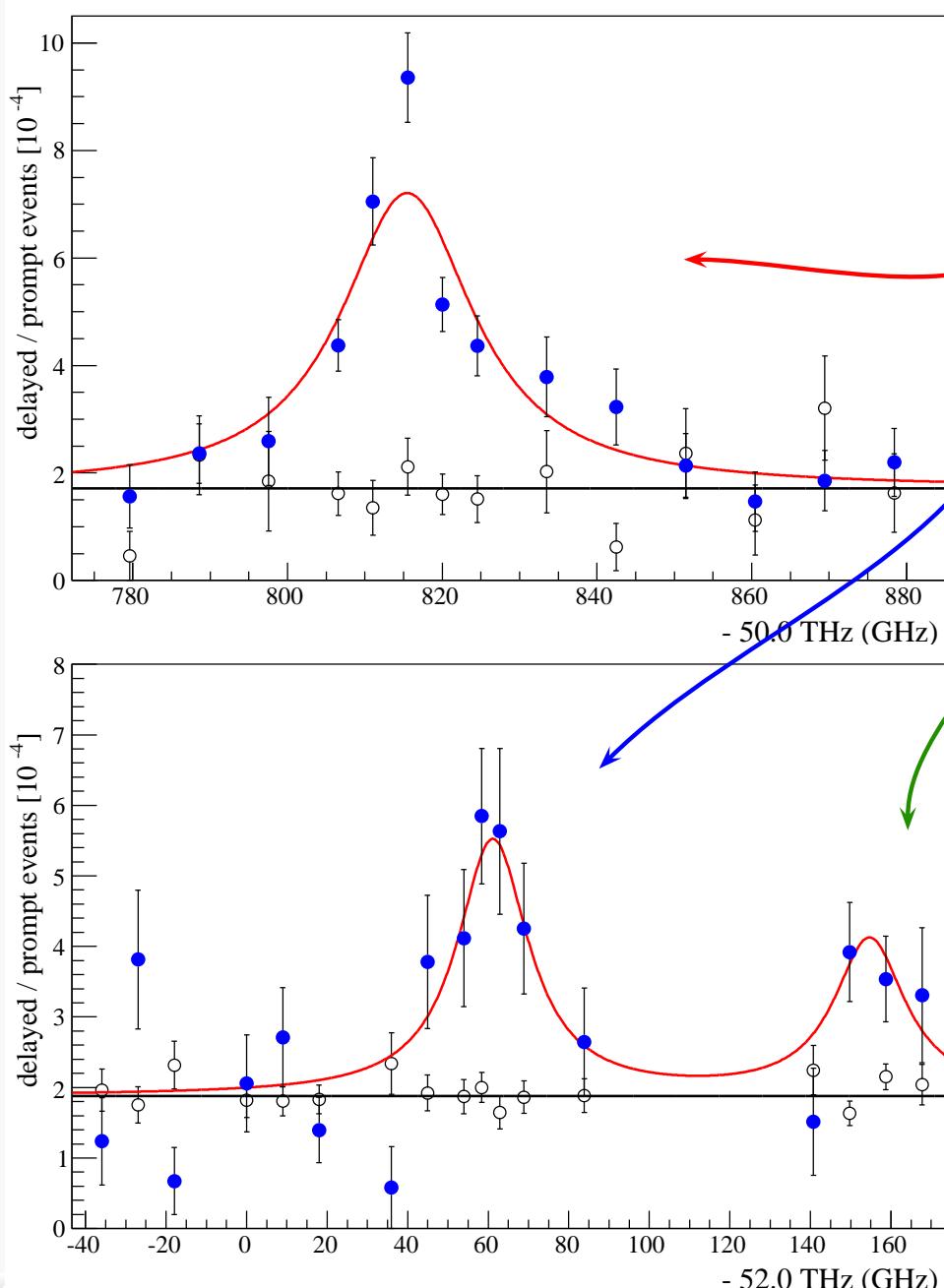
Muonic deuterium Lamb shift



Muonic deuterium Lamb shift

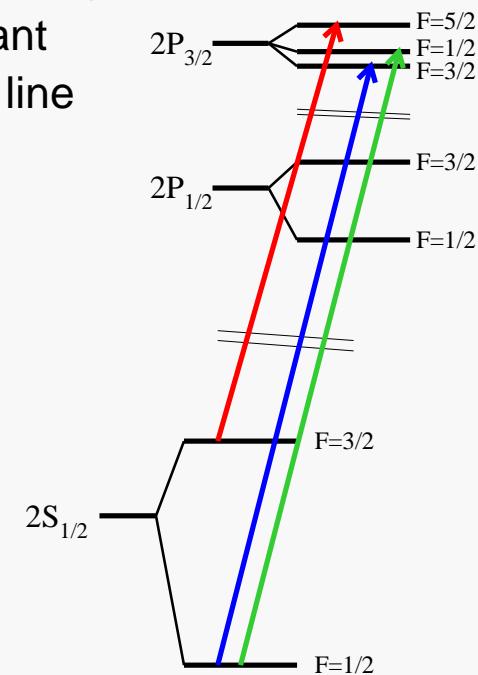


Muonic deuterium



2.5 resonances in muonic **deuterium**

- μd [$2S_{1/2}(F=3/2) \rightarrow 2P_{3/2}(F=5/2)$]
20 ppm (stat., online)
- μd [$2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=3/2)$]
45 ppm (stat., online)
- μd [$2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=1/2)$]
70 ppm (stat., online)
only 5σ significant
identifies $F=3/2$ line



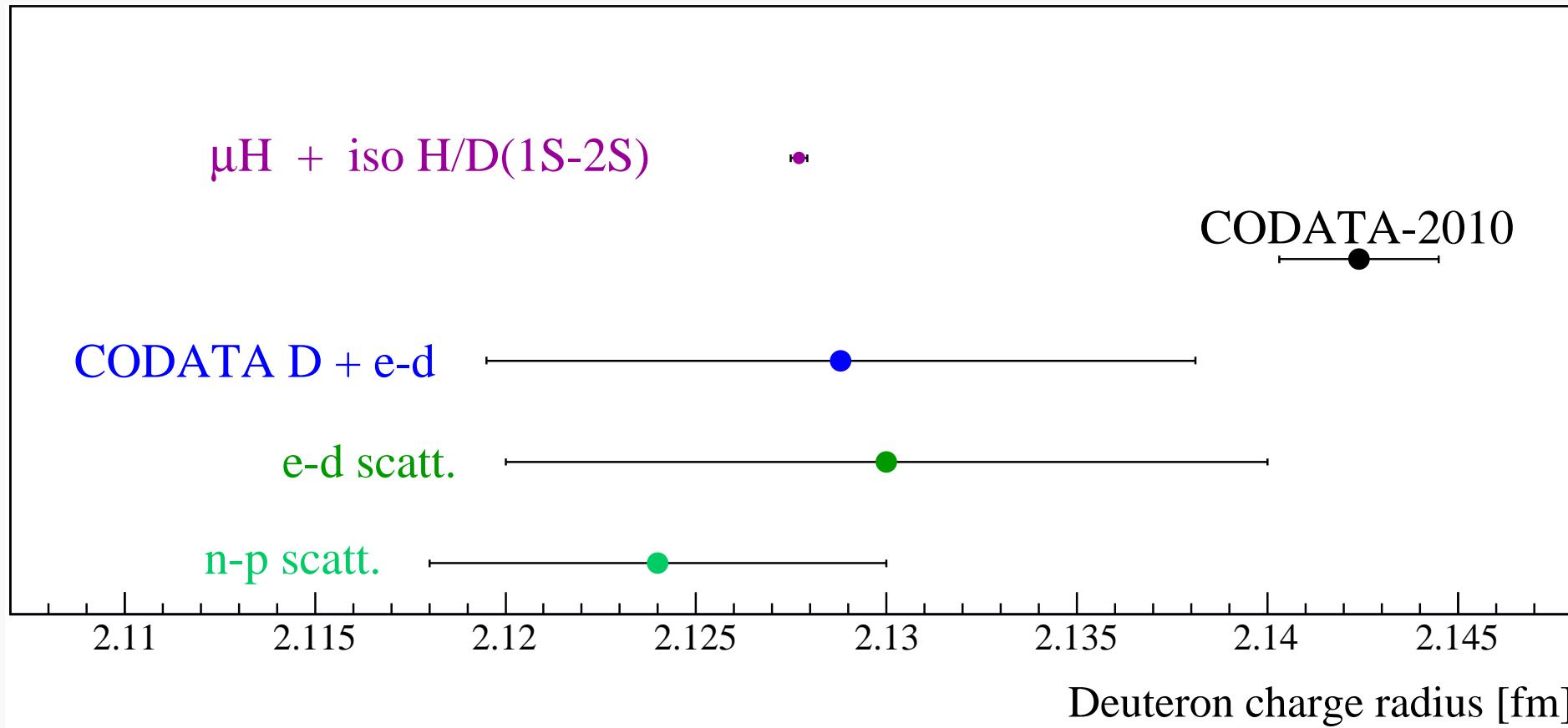
Deuteron charge radius

H/D isotope shift: $r_d^2 - r_p^2 = 3.82007(65) \text{ fm}^2$

C.G. Parthey, RP *et al.*, PRL 104, 233001 (2010)

CODATA 2010 $r_d = 2.1424(21) \text{ fm}$

$r_p = 0.84087(39) \text{ fm}$ from μH gives $r_d = 2.1277(2) \text{ fm}$



Deuteron charge radius

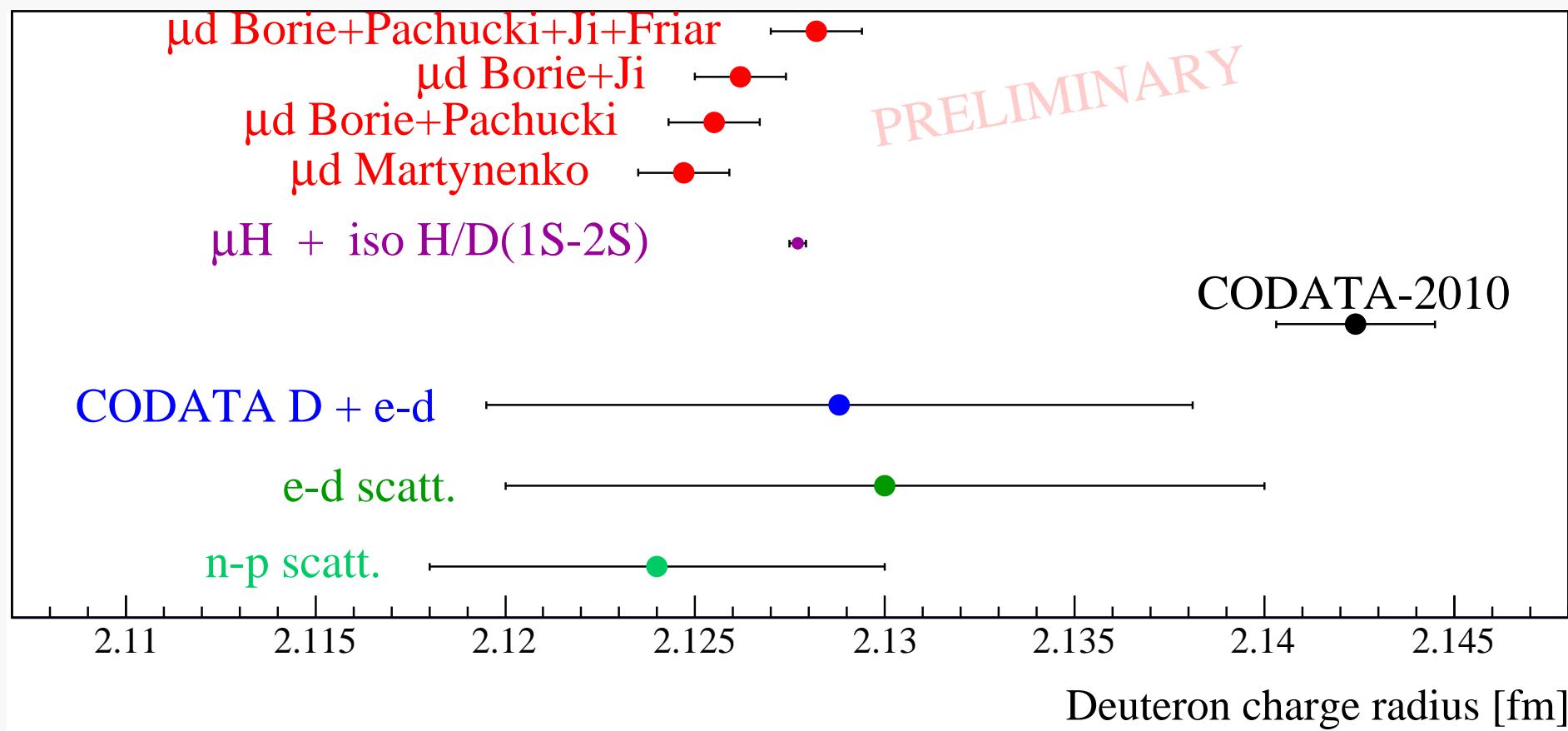
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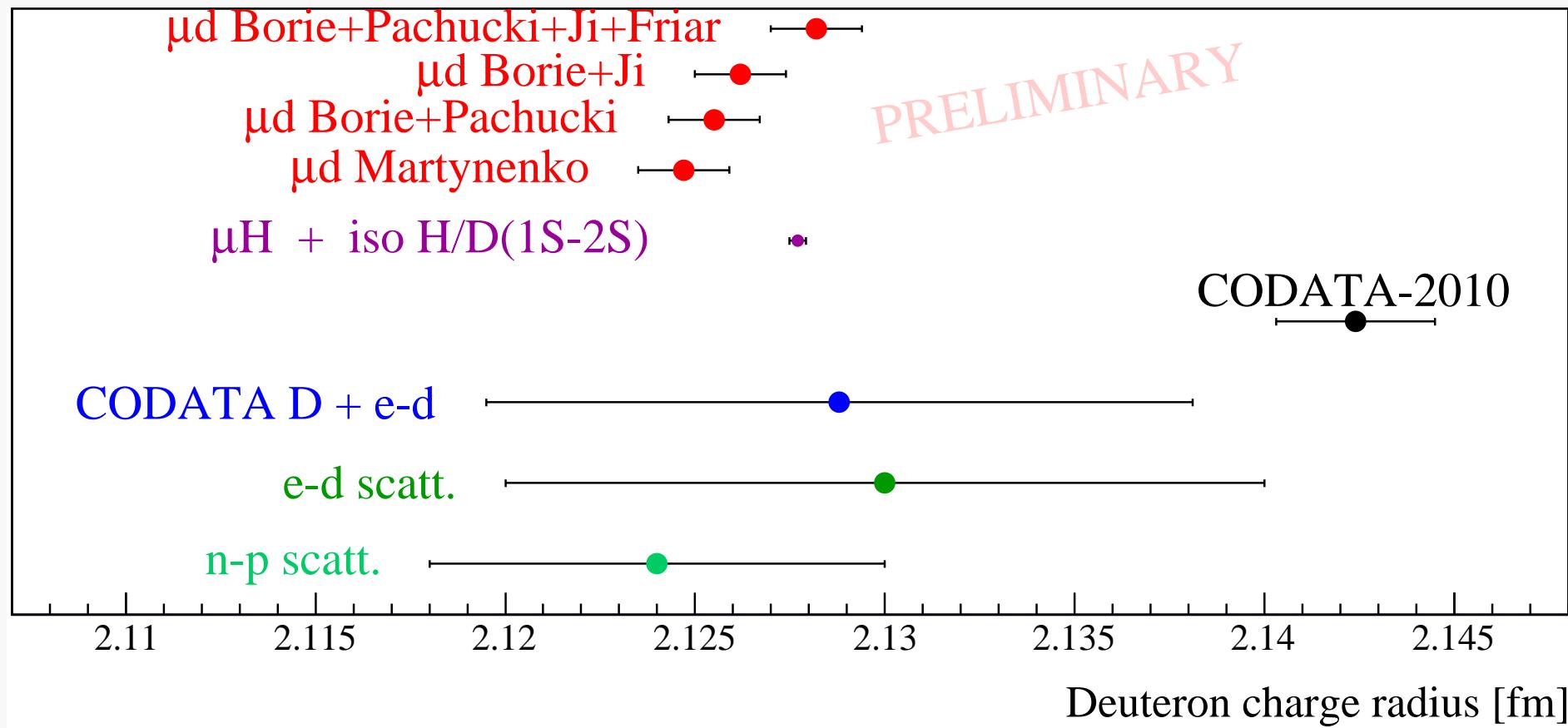
$r_p = 0.84087(39) \text{ fm}$ from μH gives $r_d = 2.1277(2) \text{ fm}$

Lamb shift in muonic deuterium $r_d = 2.1282(12) \text{ fm}$ PRELIMINARY!



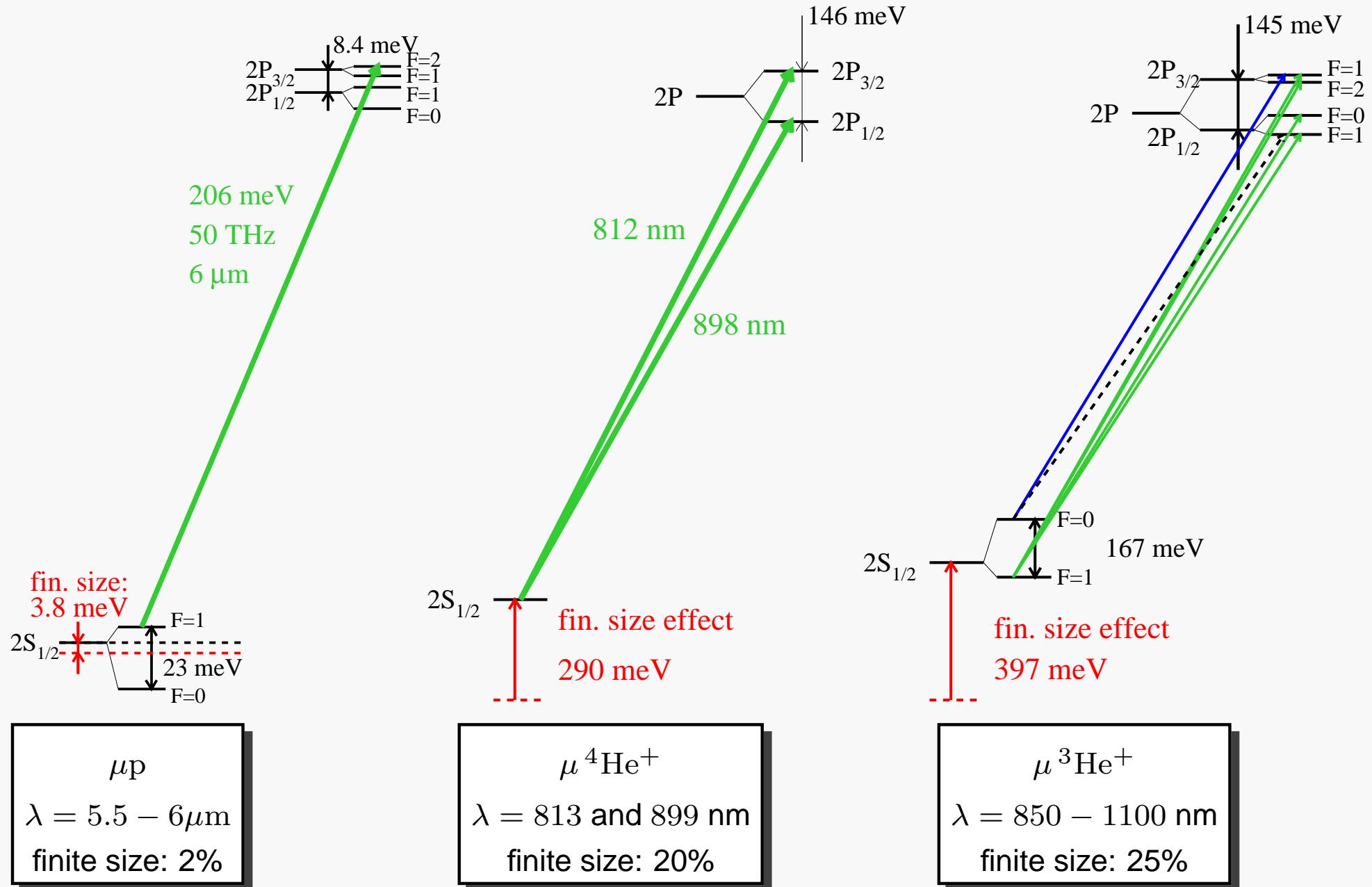
Deuteron charge radius

- μH and μD are **consistent!**
- a new “dark-photon” interaction would probably have no coupling to neutrons
- deuteron polarizability: theory complete? double-counting?



Muonic helium $\mu^4\text{He}^+$, $\mu^3\text{He}^+$

Muonic helium transitions



Aim and motivation of μ He Lamb shift

Measure the 2S-2P Lamb shift in $\mu^3\text{He}^+$ and $\mu^4\text{He}^+$ with **50 ppm**



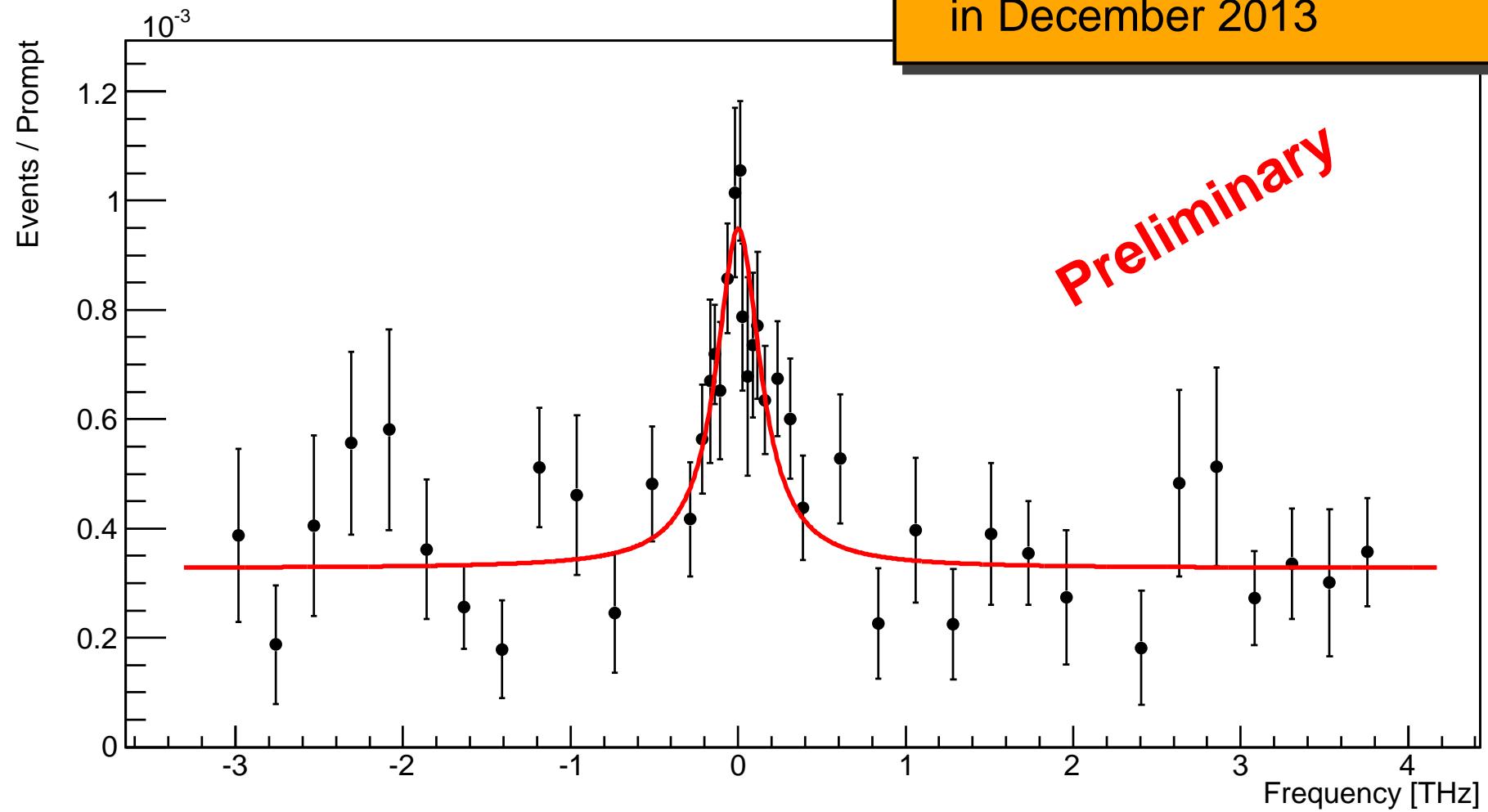
$r_{^3\text{He}}$ and $r_{^4\text{He}}$ with $u_r = 3 \times 10^{-4} \iff 0.0005 \text{ fm}$

- May help to solve the discrepancy observed in H - μ p! Sensitive to new physics
- Nuclear physics:
 - Significant test of few-nucleon theories
 - **Absolute** radii for ${}^3\text{He}$, ${}^4\text{He}$, ${}^6\text{He}$, ${}^8\text{He}$ when combined with isotopic shifts
 - Comparison between isotopic shift (${}^3\text{He}$ - ${}^4\text{He}$) in muonic and electronic sector
- Enhanced bound-state QED test when combined with $\text{He}^+(1\text{S}-2\text{S})$ [MPQ, Mainz]
 - Check interesting/problematic QED terms in He^+

The $2S_{1/2} - 2P_{3/2}$ resonance in $\mu^+ {}^4\text{He}^+$

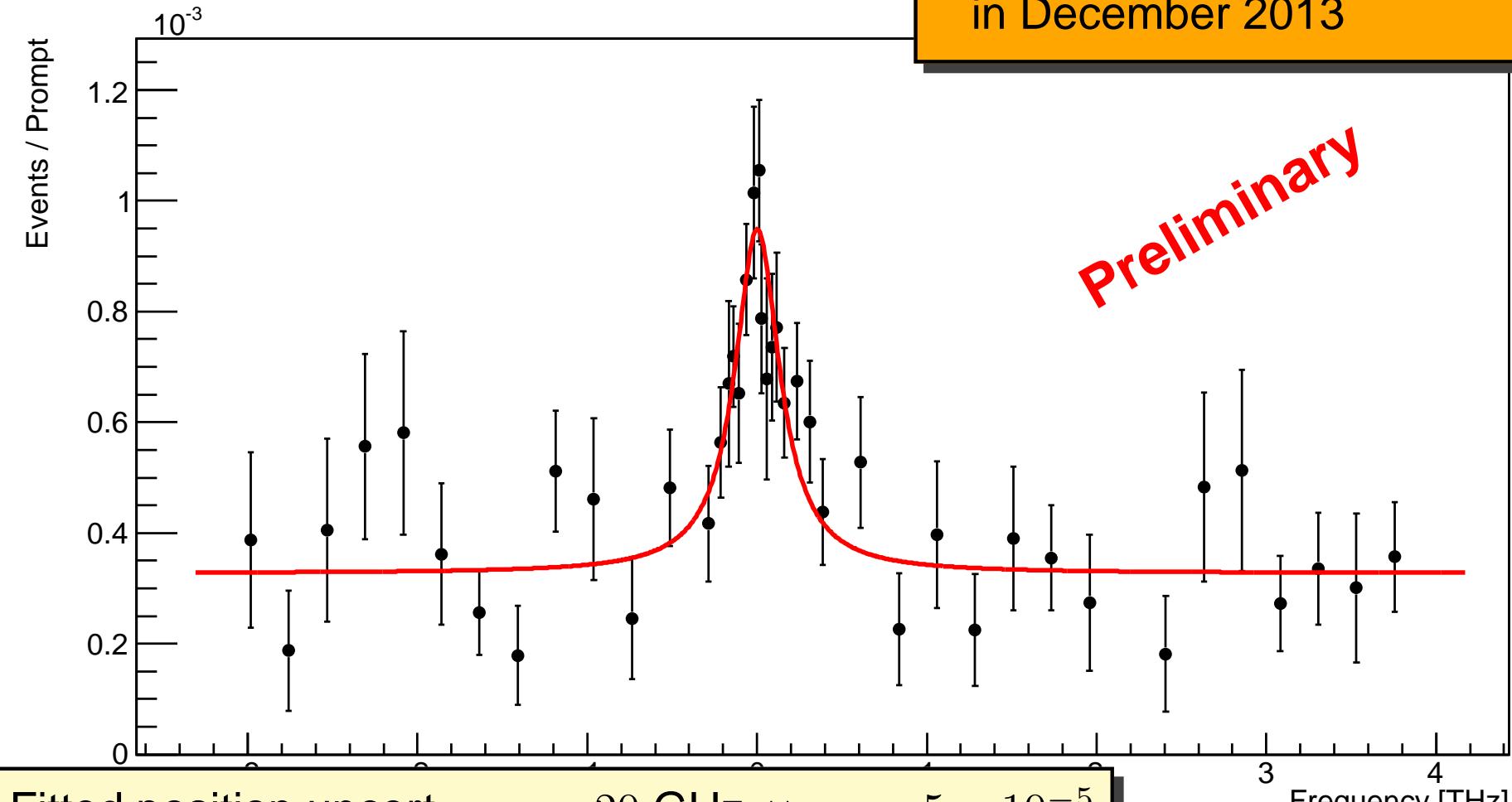
Two weeks of measurements
in December 2013

Preliminary



The $2S_{1/2} - 2P_{3/2}$ resonance in $\mu^+ {}^4\text{He}^+$

Two weeks of measurements
in December 2013



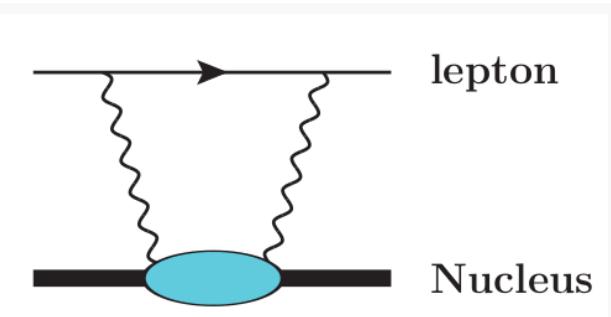
Fitted position uncert. $= 20 \text{ GHz} \Leftrightarrow u_r = 5 \times 10^{-5}$

Laser frequency uncert. $< 100 \text{ MHz}$

Systematics $< 10 \text{ MHz}$

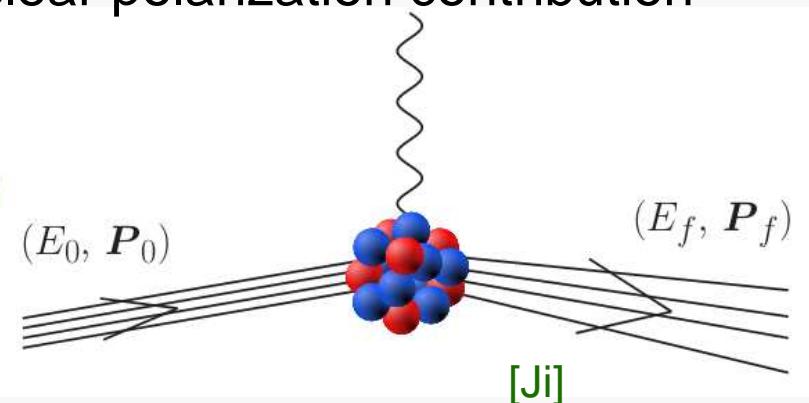
Nuclear polarization contribution in μ He⁺

$$\Delta E_{\text{LS}}^{\text{th}} = \Delta E_{\text{QED}} - \frac{m_r^3}{12} (Z\alpha)^4 \langle r^2 \rangle + \frac{m_r^4}{24} (Z\alpha)^5 \langle r^3 \rangle_{(2)} + \delta_{\text{pol}}$$



- From nuclear response function $S_0(\omega)$ → nuclear polarization contribution

$$S_O(\omega) = \sum_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



- Two ways to get the response function:

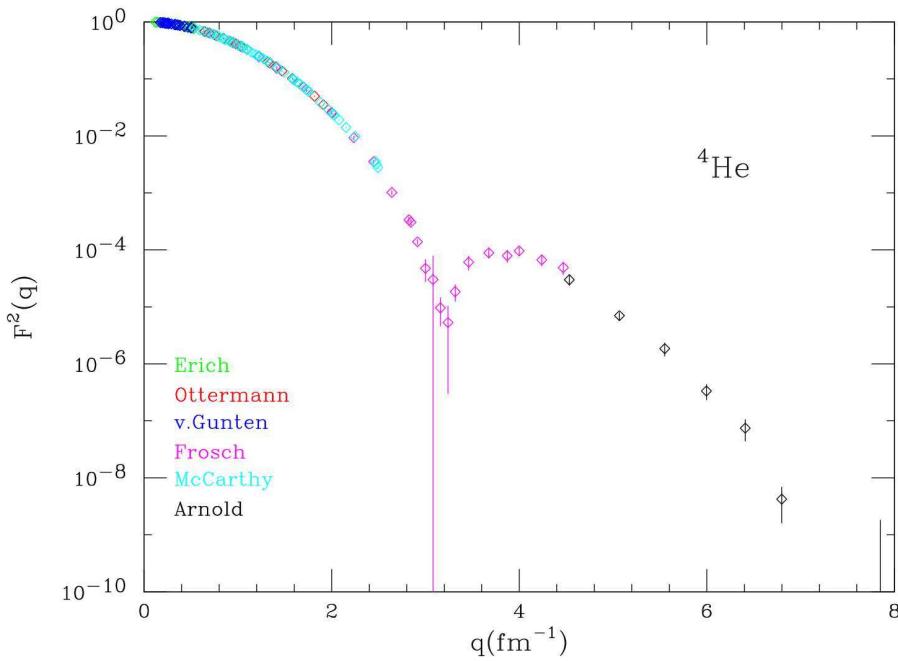
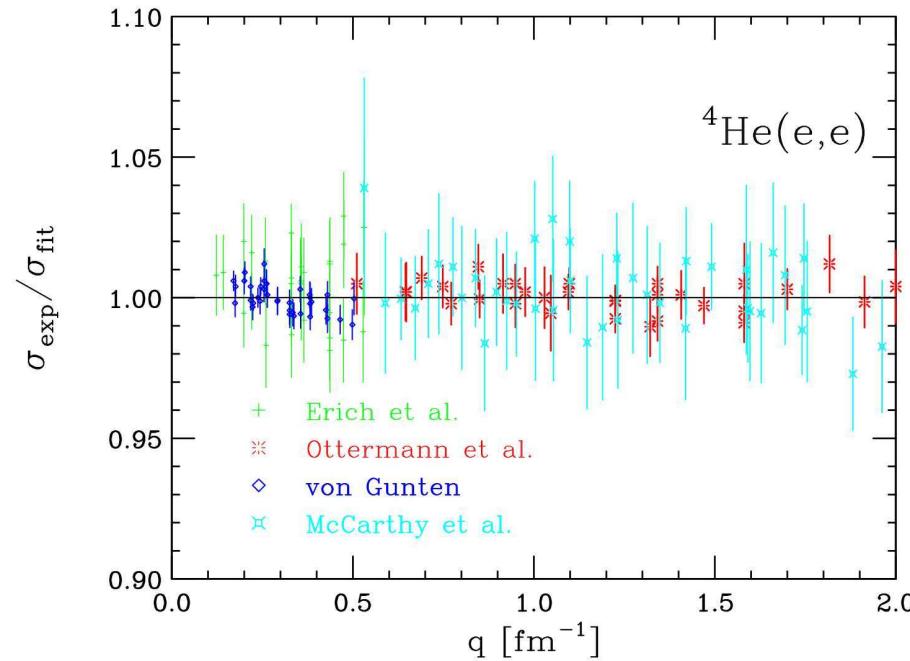
- From photo-absorption [Bernabeau & Jarlskog, Rinker, Friar]

$$\delta_{\text{pol}} = 3.1 \text{ meV} \pm 20\%$$

- From state-of-the-art potentials (chiral EFT, AV18/UIX)

$$\delta_{\text{pol}} = 2.47 \text{ meV} \pm 6\% \quad [\text{Ji et al., PRL 111, 143402 (2013)}]$$

He radius from e-scattering



- world data of e-scattering.
- constraints of density at large r :
 - shape: from p-wavefunction \sim Whittaker.
 - absolute density: from p-He scattering + FDR.
- point density from potential + GFMC (small r) + FDR (large r).
- fold point density with charge density distribution of p and n.
- include Coulomb distortions.

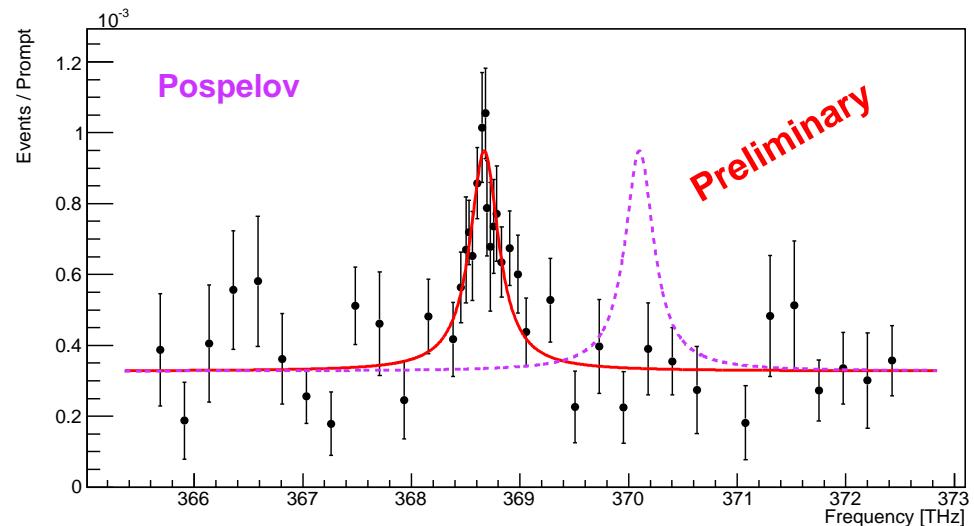
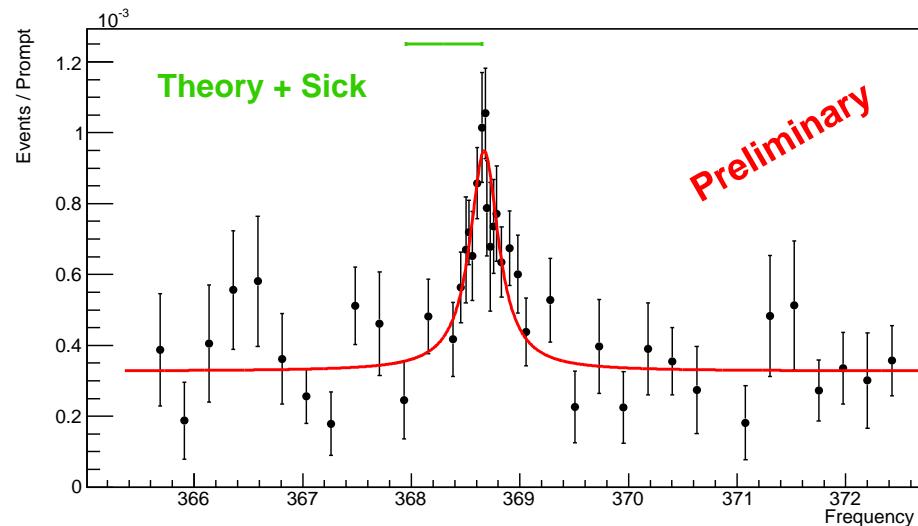
Fit with sum-of-Gaussians

$$\rightarrow R = 1.681(4) \text{ fm}$$

(best known radius from e-scattering)

[Sick, PRC 77, 941392(R) (2008)]

First resonance in $\mu^4\text{He}^+$

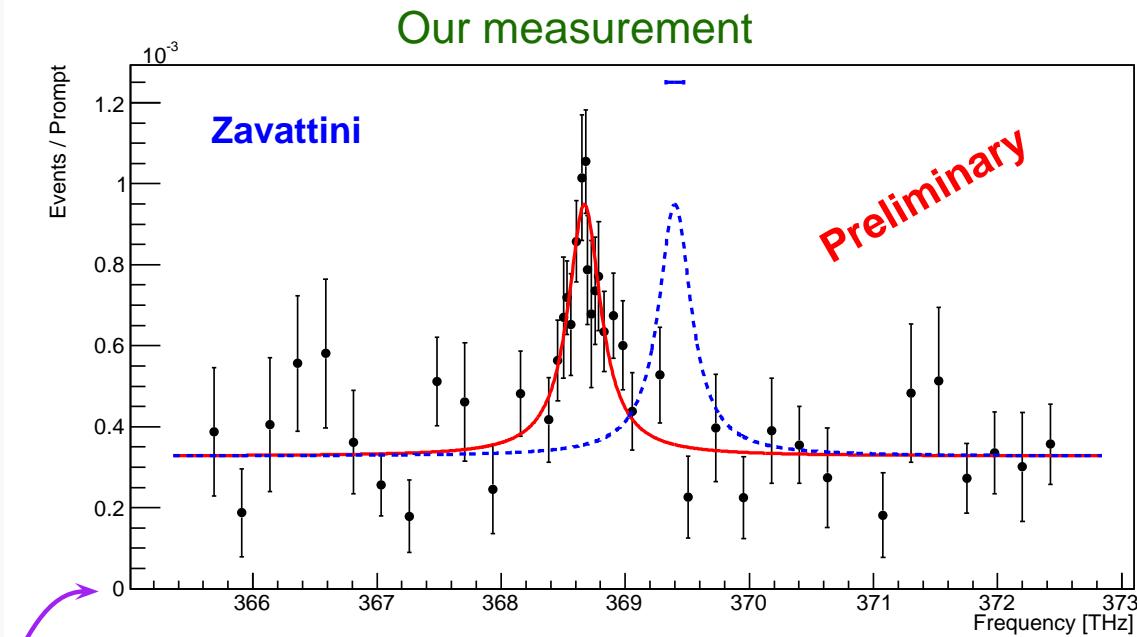
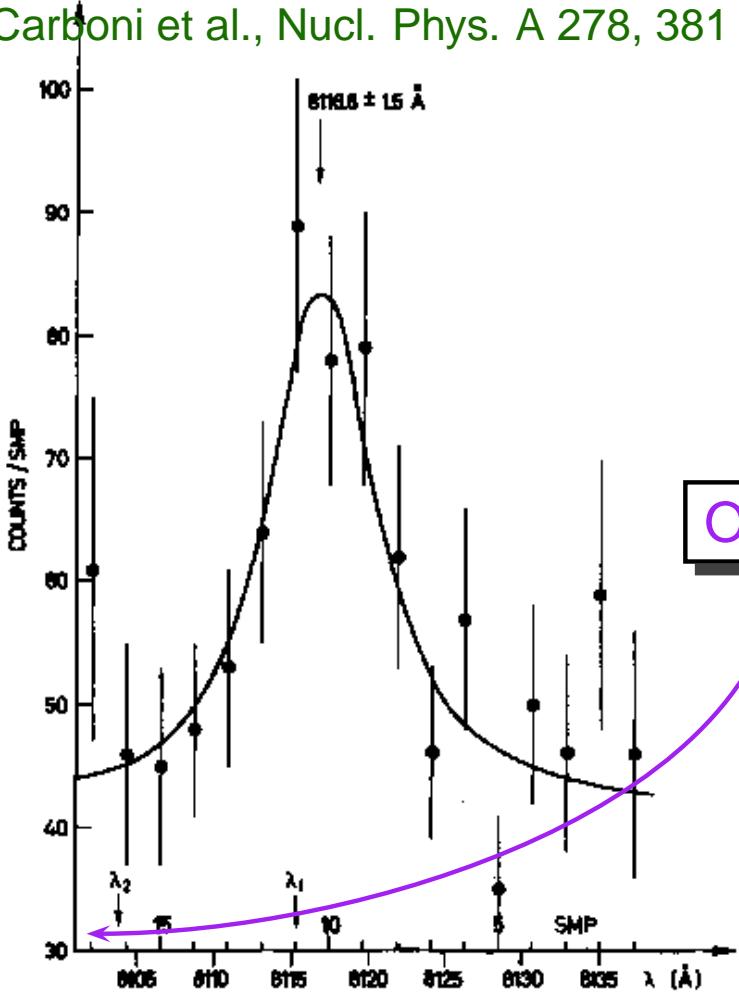


- The transition has been found at the expected position i.e., within the uncert. given by r_{He} from $e-\text{He}$ scattering.
- New physics model of Pospelov excluded
- Zavattini value from old μHe^+ experiment excluded

Zavattini “resonance”

→ Zavattini wrong

[Carboni et al., Nucl. Phys. A 278, 381 (1977)]



Zavattini experiment was performed at **50 bar** pressure:
⇒ 2S-population is collisionally quenched.
⇒ No population left for a laser experiment.
(for comparison: we are measuring μ He⁺ at **3 mbar**)

[Hauser et al., PRA 46, 2363 (1992)]

More resonances measured in $\mu^4\text{He}^+$, $\mu^3\text{He}^+$

In 2014, we successfully measured

- $\mu^4\text{He}^+(2S_{1/2} - 2P_{1/2})$ at $\lambda \approx 899 \text{ nm}$: 2nd transition in $\mu^4\text{He}^+$
- $\mu^3\text{He}^+(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2})$ at $\lambda \approx 863 \text{ nm}$: 1st transition in $\mu^3\text{He}^+$
- $\mu^3\text{He}^+(2S_{1/2}^{F=0} - 2P_{3/2}^{F=1})$ at $\lambda \approx 958 \text{ nm}$: 2nd transition in $\mu^3\text{He}^+$
- $\mu^3\text{He}^+(2S_{1/2}^{F=1} - 2P_{1/2}^{F=1})$ at $\lambda \approx 965 \text{ nm}$: 3rd transition in $\mu^3\text{He}^+$

→ in agreement with expectations !

Conclusions & Outlook

- Original motivation: test theory of H energy levels (limited by uncertainty of r_p)
- Conclusion from μp (2S-2P): r_p -discrepancy
 - μp experiment correct
 - $\Rightarrow \mu p$ theory wrong ? H-spectroscopy wrong ? + e-scatt. wrong ?
- when puzzle solved:
 - best test of bound-state QED (combine μp and H)
 - fundamental constants (R_∞)
 - test of lattice QCD for p, few-nucleon theory for d
- 2009: 3 resonances measured in μd (2S-2P) \rightarrow r_d , d-polarizabilities
- 2013/14: μHe^+ (2S-2P)
 - sensitive to (some) “new physics”
 - more sensitivity to “QED”-effects than μp -H
 - less sensitive to R_∞ (in He^+)
 - test of few-nucleon theory for 3He , 4He

Conclusions & Outlook

- Future (when r_p puzzle solved!):

- $\mu p(2S-2P)$ more precisely → r_p and R_{Zemach} : better understanding of p
- $\mu p(1S-HFS)$ precise R_{Zemach} → magnetic radius, polarizabilities
- $\mu p(3D-3P)$? “pure QED”, but large linewidth (0.6 %)
- μp Rydberg-states → muon mass
- $\mu Li(2S-2P)$ etc.: *ab initio* nuclear structure calc., few-electron QED-calc.

- More generally,

there is a revival of precision spectroscopy of simple atomic systems like

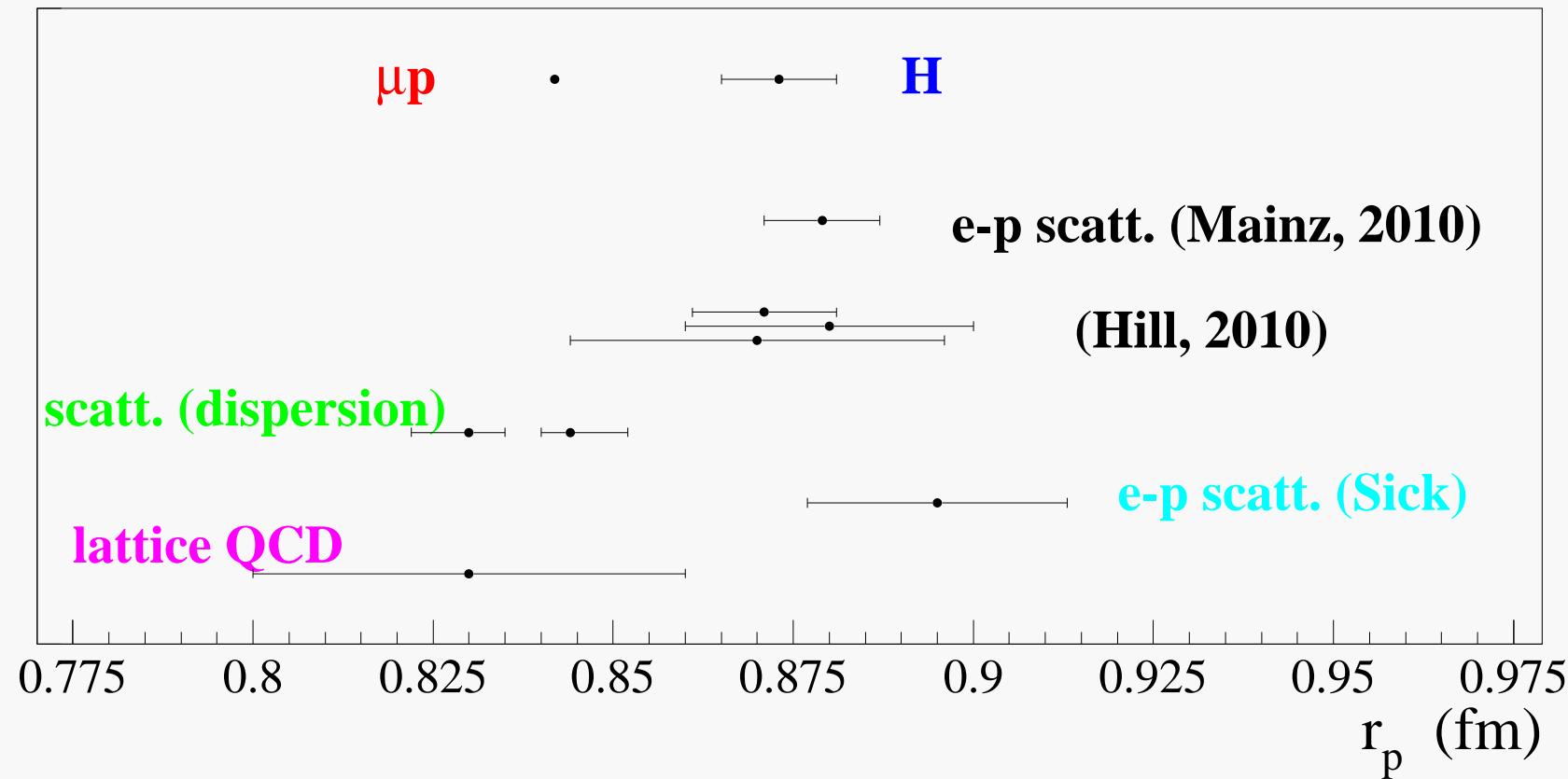
- antihydrogen \bar{H} at CERN
- Muonium μ^+e^- at PSI!
- Positronium e^+e^- at ETHZ, ...
- H-like med- Z ions at GSI, Paris, ...

Personal conclusion:

There are surprises in physics.

Back up slides

r_p puzzle (5): Is e-p scattering wrong?

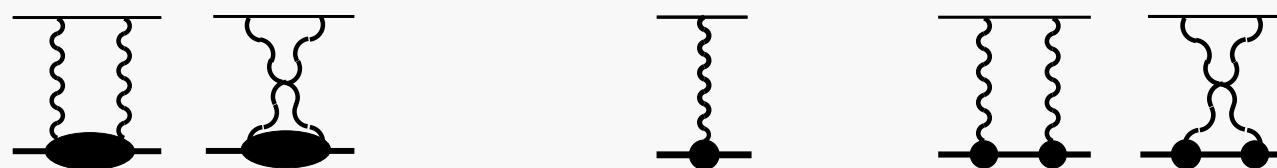


- Pohl et al., Nature 466, 213 (2010)
- Mohr et al., Rev. Mod. Phys. 80, 633 (2008)
- Bernauer et al., PRL 105, 242001 (2010)
- Hill and Paz, PRD 82, 113005 (2010)
- Belushkin et al., Phys. Rev. C 75, 035202 (2007)
- Sick, Phys. Lett. B 576, 62 (2003)
- Wang et al., Phys. Rev. D 79, 094001 (2009)

r_p puzzle (2): Is $\mu_{\text{p}}(2S-2P)$ theory wrong?

Radius (structure) dependent contributions:

Contribution	Value [meV]	$r_p = 0.84 \text{ fm}$
Leading nuclear size contribution	-5.19745	$\langle r_p^2 \rangle$
Radiative corrections to nuclear finite size effect	-0.0275	$\langle r_p^2 \rangle$
Nuclear size correction of order $(Z\alpha)^6 \langle r_p^2 \rangle$	-0.001243	$\langle r_p^2 \rangle$
Total $\langle r_p^2 \rangle$ contribution	-5.22619	$\langle r_p^2 \rangle$
Nuclear size correction of order $(Z\alpha)^5$	0.0347	$\langle r_p^3 \rangle$ (\leftrightarrow Third Zemach moment)
Nuclear size correction of order $(Z\alpha)^6 \langle r_p^4 \rangle$	-0.000043	$\langle r_p^2 \rangle^2$
Proton polarizability	0.015(4)	



$$E(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}) = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV} \quad (\text{HFS+FS included})$$

Uncertainty??

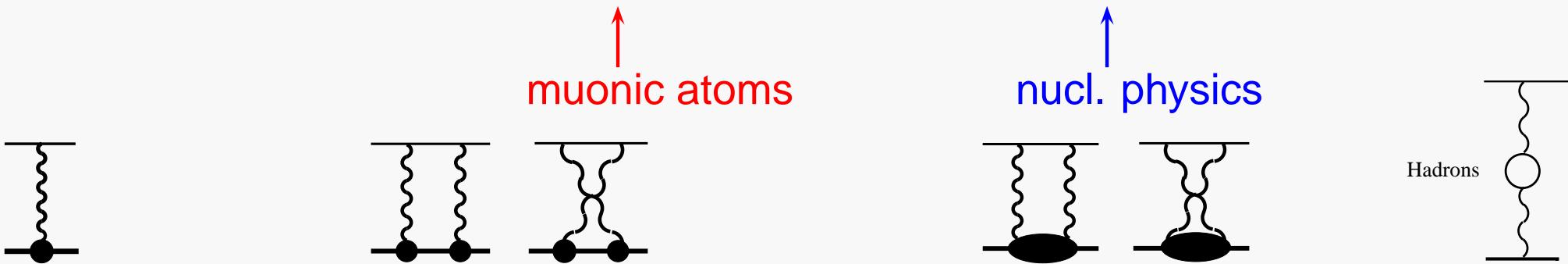
The role of nuclear physics in atomic physics

Atomic physics means high-precision measurements.

However their interpretations are usually limited by nuclear-physics effects

Interpretation of H, D, $^{3,4}\text{He}^+$, μp , μd , $\mu^{3,4}\text{He}^+$:

- Lamb shifts limited by: r^2 , shape, nucl. pol., hadronic VP pol.
- HFS limited by: Zemach radius, shape, nucl. pol., hadronic VP pol.



$$\text{Zemach radius} \sim \int_0^\infty \frac{dQ}{Q^2} \left[G_E(-Q^2) \frac{G_M(-Q^2)}{1 + \kappa_p} - 1 \right]$$

$$\text{Nucl. Pol.} \sim T_{\mu\nu}^A = \frac{i}{m_p} \epsilon_{\nu\mu\alpha\beta} \left[(G_1(\nu, q^2) + G_2(\nu, q^2)) S^\beta - G_2(\nu, q^2) \frac{S \cdot q^2 p^\beta}{p \cdot q} \right] \quad \text{Im}G_1 = \frac{1}{\nu} g_1(\nu, q^2) \dots$$

ab-initio calculation for d and He very promising