Effective Field Theory – Zuoz 2014

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Note Title

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Lecture 1 – Introduction to EFT in general

- General philosophy
- Explicit example
- Matching, power counting etc

Lecture 2 – Varieties of EFT and advanced techniques

- Survey and limits of EFT
- Techniques outside of textbooks

Lecture 3 - Gravity as an EFT

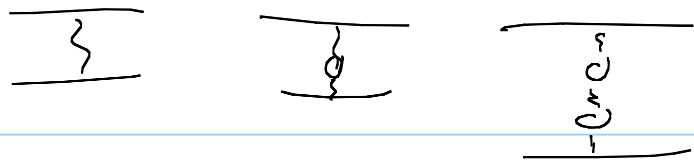
- GR as a gauge theory
- How to think about quantum gravity
- Reliable aspects of gravity and QM

Why does QM work?

$$\langle f | V | i \rangle + \sum_I \frac{\langle f | V | I \rangle \langle I | V | i \rangle}{E - E_I}$$

Uncertainty Princ. HE $\Rightarrow$ LOCAL $\Rightarrow$ some term in $\mathcal{L}$

$\Rightarrow$ measure coeff. $\leftarrow$ H.E..

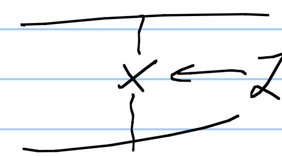


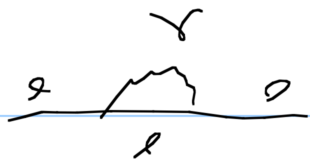
$$\Longrightarrow \frac{e_0^2}{1-\Pi(q^2)}$$

$$e^2 = \frac{e_0^2}{1-\Pi(0)}$$

$$\begin{aligned}
\Pi(q) = \frac{e_0^2}{12\pi^2} \Bigg[& \frac{1}{\epsilon} + \ln(4\pi) - \gamma \\
& -6\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2 - q^2 x(1-x)}{\mu^2}\right) + \mathcal{O}(\epsilon)\Bigg] \\
= \frac{e_0^2}{12\pi^2} & \begin{cases} \dfrac{1}{\epsilon} + \ln(4\pi) - \gamma + \dfrac{5}{3} - \ln\dfrac{-q^2}{\mu^2} + \dots & (|q^2| \gg m^2), \\[2ex] \dfrac{1}{\epsilon} + \ln(4\pi) - \gamma - \ln\dfrac{m^2}{\mu^2} + \dfrac{q^2}{5m^2} + \dots & (m^2 \gg |q^2|). \end{cases}
\end{aligned}$$

$$\frac{q^2}{m^2} \rightarrow \quad \mathcal{L}_{eff} = \frac{\alpha}{60\pi} F_{\mu\nu} \frac{\Box}{m_H^2} F^{\mu\nu}$$





$\rightarrow \underline{\delta m^2}, \underline{Z}$

$= \underline{\underline{Z_\mu \gamma_\mu}} + \Gamma_\mu(p,p')$

Appelquist Carrazonne Thm — renorm of couplings

suppressed $\left(\frac{1}{M}\right)^{km}$

Effective Lagrangians

New Lagrangians ("non renorm.")

$$\mathcal{L} = \frac{\alpha}{60\pi} F_{\mu\nu} \frac{\Box}{m^2} F^{\mu\nu} + \frac{\alpha}{90 m^4}\left[\left(F_{\mu\nu} F^{\mu\nu}\right)^2 - \frac{7}{4}\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right)^2 \right]$$

$$+ \; c \, F \frac{\Box^2}{m^4} F + \cdots$$

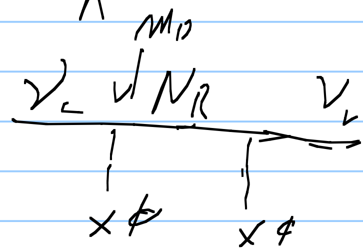
EFT 1.0 Probes of new physics

$$\mathcal{L}_{New} = \sum_{n} \frac{c_i}{\Lambda^n} \mathcal{L}_i$$

local

Example – Dim 5

$$\mathcal{L} = \frac{1}{\Lambda} (\phi \ell)^T (\phi \ell_L) \to \frac{v^2}{\Lambda} \nu_L^T \nu_L \Longrightarrow \text{Majorana mass term}$$



$$\Longrightarrow \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \to m_\nu = \frac{m_D^2}{M_R} = \frac{y^2 v^2}{M_R}$$

EFT = QFT

"Effective" → useful
→ able to ~~have~~ have effects

⋮

EFT → QFT of <u>active</u> DOF
QFT at low energy
QFT with focus on energy scales

Key

1) HE is local, LE not local

2) Energy expansion $\Rightarrow \mathcal{L} = \sum_n \frac{1}{\Lambda^n} \mathcal{L}_n$

3) "Matching" or "Measuring"

Example Linear σ model (Higgs)

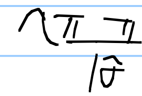
$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_0 + i\phi_3\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi_1 + i\pi_2 \\ \sigma + \pi_3\end{pmatrix} \leftarrow \pi^+$$

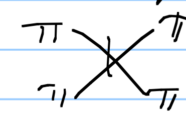
$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial_\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}, \quad v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi_1 + i\pi_2 \\ v + \tilde{\sigma} + \pi_3\end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \tilde{\sigma})^2 - 2\mu^2 \tilde{\sigma}^2\right] + \frac{1}{2}(\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi}) - \lambda v \tilde{\sigma}(\tilde{\sigma}^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\tilde{\sigma}^2 + \vec{\pi}^2)^2$$



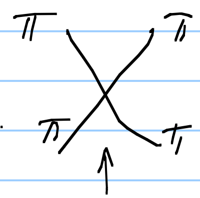


$$\mathcal{L}_{eff} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U\, \partial^\mu U^\dagger) \quad ! \qquad U = \exp\left[i \frac{\vec{\tau} \cdot \vec{\pi}}{v}\right]$$

$$= \frac{1}{2}(\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi}) + \frac{1}{6v^2}\left[(\vec{\pi}\, \partial_\mu \vec{\pi})^2 - \vec{\pi}^2 (\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi})\right]$$

Example: $\pi^- \pi^0 \to \pi^- \pi^0$

1) Full theory



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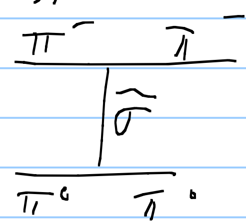
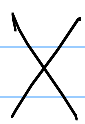


$$-i\mathcal{M} = -2i\lambda + (-2i\lambda v)^2 \frac{i}{q^2 - m_{\tilde\sigma}^2} = -2i\lambda\left[1 + \frac{2\lambda v^2}{q^2 - 2\lambda v^2}\right]$$

$$= i\left(\frac{q^2}{v^2} + \frac{q^4}{v^2 m_{\tilde\sigma}^2} + \cdots\right)$$

2) EFT



$$-i\mathcal{M} = i\,\frac{q^2}{v^2}$$

$$\left(\frac{q^2}{m_{\tilde\sigma}^2}\right)^n$$

Constructing $\mathcal{L}_{eff}$ ϕ

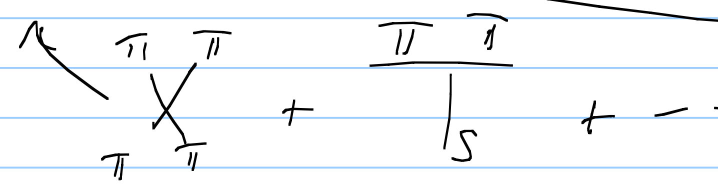
$$\Sigma = \sigma + i\vec{\tau}\cdot\vec{\pi} \implies \frac{1}{2}\mathrm{Tr}(\Sigma^\dagger \Sigma) = (\sigma^2 + \pi^2) = 2\phi^\dagger\phi$$

$$\mathcal{L}_\sigma = \frac{1}{4}\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \frac{\mu^2}{4}\mathrm{Tr}(\Sigma^\dagger\Sigma) - \frac{\lambda}{16}\left[\mathrm{Tr}(\Sigma^\dagger\Sigma)\right]^2$$

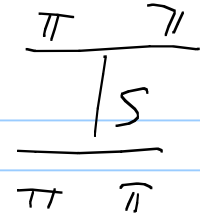
<u>Rename</u>: $\Sigma = (v + s)\,U$, $U = \exp\left[i\,\frac{\vec{\tau}\cdot\vec{\pi}}{v}\right]$

Note $\Sigma = v + \sigma + i\vec{\tau}\cdot\vec{\pi}$
$= v + s + i\vec{\tau}\cdot\vec{\pi}' + \dots$
↑ <u>nonlinear</u>

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu s)^2 - 2\mu^2 s^2\right] + \frac{(v+s)^2}{4}\mathrm{Tr}(\partial_\mu U\, \partial^\mu U^\dagger)$$
$$- \lambda v s^3 - \frac{\lambda}{4}s^4$$



Exchanges of S are suppressed



$\leftarrow g^2$

$\leftarrow \frac{1}{q^2 - m_s^2} \sim \frac{g^4}{m_s^2} + \cdots$

g^2

To get $\mathcal{L}_{eff} \Rightarrow$ <u>drop S</u>

<u>Haag's Thm:</u> Names don't matter

$\phi_i(\psi_i)$ – $\phi_i = \psi_i + \cdots$

Next term

$$\times \leftarrow \mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger$$

$$| S$$

$$\times$$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{v^2}{8 M_\sigma^2}\left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)\right]^2 + \cdots$$

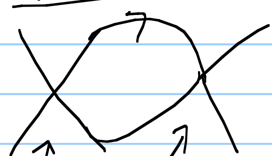
$$-i\mathcal{M} = i\left[\frac{q^2}{v^2} + \frac{q^4}{M_\sigma^2 v^2}\right]$$

"MATCHING" at Tree Level

HW $U(1)$ $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi)$

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial \phi + \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

Full QFT



$\frac{g^2}{N^2}$

$$= \frac{1}{N^4} I(p_i) = \frac{1}{N^2} p_i^4 \left[\frac{1}{\epsilon} + \ldots \underbrace{\text{finite}}_{\ln g^2/\mu^2} \right]$$

Divergences are local $\Longrightarrow$ look like $\mathcal{L}_{eff}(\partial_\mu^4)$

$$\frac{\partial^4}{N^4} \to \mathcal{L} = \left[\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)\right]^2$$

Symmetry of original theory $\mathrm{Tr}(\Sigma^\dagger \Sigma)$ $\boxed{\Sigma \to \overset{SU(2)}{L} \Sigma R^\dagger}$

$$U \to L U R^\dagger$$

Most general $\mathcal{L} = \frac{v^2}{4} \operatorname{Tr} \partial_\mu U \partial^\mu U^\dagger + \ell_1 \left[\operatorname{Tr}(\partial_\mu U \partial^\mu U^\dagger)\right]^2$

$$+ \ell_2 \operatorname{Tr}(\partial_\mu U \partial_\nu U^\dagger) \operatorname{Tr}(\partial^\mu U \partial^\nu U^\dagger)$$

Amplitude at 1 loop (Gasser Leutwyler)

$s = E_{cm}^2$

$t = q^2$

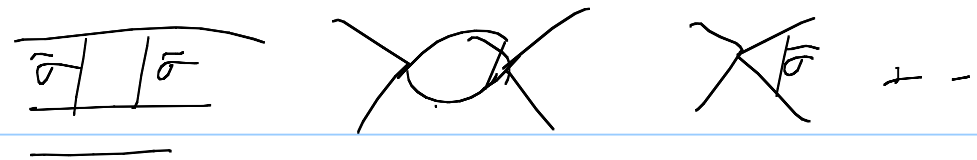
$u = (p_1 - p_4)^2$

$$\begin{aligned}\mathcal{M}_{eff} = \frac{t}{v^2} &+ \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4} \\ &+ \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u) + u(u-s)]/v^4 \\ &- \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right]\end{aligned}$$

$$\ell_1^r = \ell_1 + \frac{1}{384\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

$$\ell_2^r = \ell_2 + \frac{1}{192\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

Full Theory



Taylor expanded:

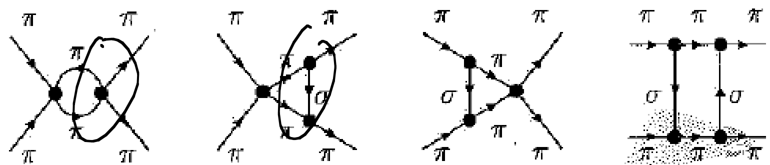
$$\mathcal{M}_{full} = \frac{t}{v^2} + \left[\frac{1}{m_\sigma^2 v^2} - \frac{11}{96\pi^2 v^4}\right] t^2 - \frac{1}{144\pi^2 v^4}[s(s-u) + u(u-s)] - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{m_\sigma^2} + s(s-u)\ln\frac{-s}{m_\sigma^2} + u(u-s)\ln\frac{-u}{m_\sigma^2}\right]$$

Match at 1 loop

$$\ell_1^r = \frac{v^2}{8m_\sigma^2} + \frac{1}{384\pi^2}\left[\ln\frac{m_\sigma^2}{\mu^2} - \frac{35}{6}\right]$$

$$\ell_2^r = \frac{1}{192\pi^2}\left[\ln\frac{m_\sigma^2}{\mu^2} - \frac{11}{6}\right]$$

Why this works



(a) (b) (c) (d)

← full theory

at low E

$$i\mathcal{M}_{\text{full}} = \int \frac{d^4k}{(2\pi)^4}\left[-2i\lambda + (-2i\lambda v)^2 \frac{i}{(k+p_+)^2 - m_\sigma^2}\right]\frac{i}{(k+p_+ + p_0)^2}\,\frac{i}{k^2}$$
$$\times\left[-2i\lambda + (-2i\lambda v)^2 \frac{i}{(k+p'_+)^2 - m_\sigma^2}\right]. \qquad (3.4)$$

$\mathcal{L}_{eff}$.

$$i\mathcal{M}_{\text{eff}} = \int \frac{d^4k}{(2\pi)^4}\,\frac{i(k+p_+)^2}{v^2}\,\frac{i}{(k+p_+ + p_0)^2}\,\frac{i}{k^2}\,\frac{i(k+p'_+)^2}{v^2}$$

diff at High E ⇒ shift in Local $\mathcal{L}$

same at low E

⇒ same LE behavior