String Theory

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More than Higgs – Effective Theories for Particle Physics

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Introduction

You've heard a lot about particle physics in the past few days:

- particles of the standard model;
- particles that have long been predicted,
  and that have recently been observed;
- particles that have long been predicted,
  and that have recently not been observed;
- further conjectured particles,
  that have not been observed yet,
  and probably never will be observed,
  because they probably do not exist.

This talk:

String Theory

Effective Theory

Why talk about String Theory? Here?!

- string theory is a model of particle physics ...
- ... at low energies (compared to Planck scale).
- it predicts new particles and features, many of them, ...
- ... at least at excessively high energies.
- so we can only dream about them. After this talk, please!

Many results in string theory use effective theory concepts.

- In fact, if you believe string theory describes nature,
  all that you've heard about here
  is nothing but an effective theory of string theory.

... but still a long way to go.

Overview

Therefore let me introduce string theory
and some of its basic concepts:

- What is String Theory?
- Problem of Quantum Gravity.
- String Theory Basics.
- String Theory and Gravity.
- D-branes.

Material from a one-semester lecture course . . .

I. What is String Theory?

Einstein with a violin?

Nothing to do with violins!

What is String Theory?

String Theory: [Wikipedia]

- In physics, string theory is a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional objects called strings.
- String theory aims to explain all types of observed elementary particles using quantum states of these strings.
- In addition to the particles postulated by the standard model of particle physics, string theory naturally incorporates gravity, and so is a candidate for a theory of everything, a self-contained mathematical model that describes all fundamental forces and forms of matter.
- Besides this hypothesized role in particle physics, string theory is now widely used as a theoretical tool in physics, and has shed light on many aspects of quantum field theory and quantum gravity.

What else is String Theory?

- incredibly exciting (for some),
- rich in hidden, exceptional and beautiful structures,
- popular:
  - Brian Greene,
    The Elegant Universe
    (book, TV)
  - Woody Allen,
    Whatever Works
    (movie)
  - ...

Cartoon: Witten's Dog
from Futurama, Episode 11: Mars University

String Theory vs. Loop Quantum Gravity
Except from “The Big Bang Theory”
Episode 2.2 – “The Codpiece Topology”

So What is String Theory?

Cartoon on String Theory
xkcd webcomic: 171

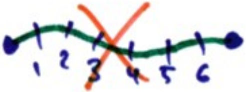
Strings

String theory is a physical model whose fundamental objects are strings. Strings have the following properties:

- spatially 1D extended

- with tension

- without inner structure

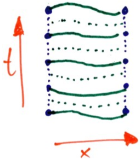


- open or closed

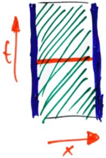
Spacetime Diagrams

When painting the path of a string one gets 2D surfaces in spacetime:

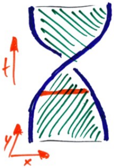
compare:
particle



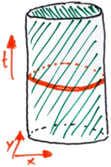
string
motion



open
string



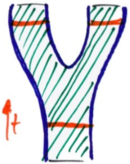
rotating
string



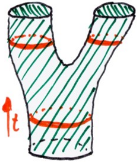
closed
string

String Interactions

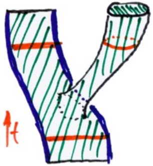
fundamental interactions between strings:



open strings



closed strings



mixed strings

compare: point-particle

Benefits of string theory (precitive power):
There are merely two constants of nature, namely

- string tension (within a string),
- string coupling (coupling between strings).

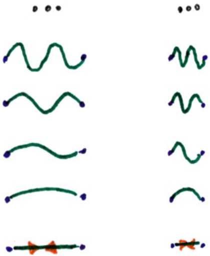
Cartoon: Witten's Dog
from Futurama, Episode 11: Mars University

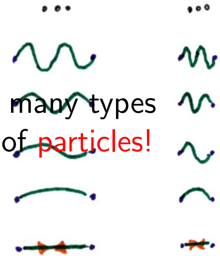
Why String Theorie?

Cartoon on String Theory
xkcd webcomic: 171

String Spectrum

How does this relate to particle physics?
Limit of large string tension:





linear spectrum $m_n^2 = m_0^2 + n \cdot \Delta m^2$.
spectrum of hadronic excitations similar!
String theory was a promising candidate.
Later better explanation: quark model, QCD, SM.
Instead string theory incorporates a consistent quantum gravity!

II. Problem of Quantum Gravity

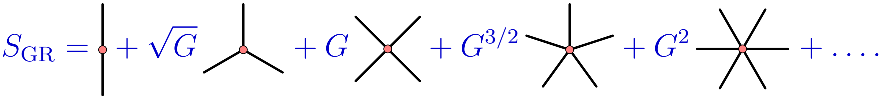
Einstein Gravity

Einstein's general relativity described by Hilbert action

$$S_{\mathrm{GR}} = \frac{1}{16\pi G} \int d^4x \, \sqrt{-\det g} \, R[g].$$

- fundamental field is metric $g_{\mu\nu}$ measuring distances.
- $R = g^{\mu\nu} R_{\mu\nu}[g] = g^{\mu\nu} R^{\rho}{}_{\mu\rho\nu}[g]$ is curvature scalar.
- coupling to matter via diffeo.-covariant derivative: $\partial_\mu \to D_\mu[g]$.

Gravity has a non-linear action translating to infinitely many vertices



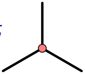
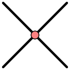
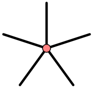
Not a fundamental problem; moreover G is small (gravity is weak).

Non-Renormalisability

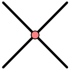
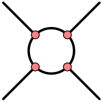
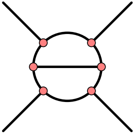
Additional terms in gravity action conceivable

$$S \sim \int d^4x \big[R + *R^2 + *R^3 + *D^2R^2 + \ldots\big].$$

Translates to additional interaction vertices

$S \sim \sqrt{G}$  $+ (G + c_4)$  $+ (G^{3/2} + *)$  $+ \ldots.$

Additional constants required to absorb divergences in loops, e.g.

$(G + c_4)$  $+ G^2$  $+ G^3$  $+ \ldots.$

Problem: Infinitely many constants needed to absorb all divergences!

String Theory and Quantum Gravity

String theory promises to solve the problem of quantum gravity:

- string excitation spectrum contains massless spin-2 particles;
- they behave as Einstein gravitons to first approximation;
- string theory is finite, no divergences (of the above kind);
- string theory has just two fundamental coupling constants.

Is all well now!? Beware of the vacuum...

Other Features.

- String spectrum contains many other particles (gauge bosons, ...); but still unclear how to extract the standard model.
- GUT-like groups $\mathrm{SU}(5)$, $\mathrm{SO}(10)$, ..., E_8 appear.
- String theory is supersymmetric; need to break at low energies.
- Exciting geometrical / mathematical concepts arise.

III. String Theory Basics

Particle Worldline Action

Start with an analogous system: **Relativistic Particle**
Embedding of worldline

$$X^\mu(\tau) : \mathbb{R} \to \mathbb{R}^{D-1,1}.$$

Proper time action

$$S \sim m \int ds := m \int d\tau\, |\dot{X}| := m \int d\tau \sqrt{-\dot{X}^\mu \dot{X}_\mu}\,, \qquad \dot{X}^\mu = \frac{dX^\mu}{d\tau}\,.$$

Equations of motion

$$\dot{P}^\mu = 0 \quad \text{with} \quad P^\mu = \frac{m}{|\dot{X}|}\, \dot{X}^\mu.$$

Note: momentum P^μ and equation of motion covariant under reparametrisation $\tau \to \tau'(\tau)$.

New Particle Worldline Action

Can use an alternative action with additional field $e(\tau) : \mathbb{R} \to \mathbb{R}$

$$S \sim \int d\tau \left(-\tfrac{1}{2}e^{-1}\,\dot{X}^\mu \dot{X}_\mu + \tfrac{1}{2}em^2\right).$$

Equations of motions

$$\dot{P}^\mu = 0 \quad \text{with} \quad P^\mu = \frac{\dot{X}^\mu}{e} \qquad \text{and} \qquad |\dot{X}| = em.$$

First equation linear in $\dot{X}^\mu$ and equivalent to above (with second).

Second equation is algebraic in e!

- e does not carry additional (quantum) degrees of freedom;
- recover the original action upon substituting $e = |\dot{X}|/m$.

$$S \sim \int d\tau \left[-\frac{m}{2|\dot{X}|}\,\dot{X}^\mu \dot{X}_\mu + \frac{|\dot{X}|}{2m}\,m^2\right] = \int d\tau\, m|\dot{X}|.$$

Nambu–Goto Action

Repeat steps for a string embedding

$$X^\mu(\tau,\sigma) : \mathbb{R}^{1,1} \to \mathbb{R}^{D-1,1}$$

Proper area action (Nambu–Goto)

$$S \sim T \int d^2\xi \sqrt{-\det\gamma} \qquad \text{with} \qquad \gamma_{\alpha\beta} := \partial_\alpha X^\mu \, \partial_\beta X_\mu .$$

Remarks:

- $\gamma_{\alpha\beta}[X]$ is induced metric on worldsheet;
- $d^2\sigma\sqrt{-\det\gamma}$ is worldsheet "area" element;
- T is string tension.

Equation of motion extremises worldsheet area

$$\partial_\alpha\left[\sqrt{-\det\gamma}\,\gamma^{\alpha\beta}\partial_\beta X^\mu\right] = 0.$$

Equation is invariant under worldsheet reparametrisation $\xi^\alpha \mapsto \xi'^\alpha(\xi)$.

Polyakov Action

Nambu–Goto equations of motion non-linear; difficult!!
Introduce worldsheet metric $g_{\alpha\beta}$

$$S \sim \frac{T}{2}\int d^2\xi\sqrt{-\det g}\,g^{\alpha\beta}\,\partial_\alpha X^\mu\,\partial_\beta X_\mu.$$

Again, two equations of motion

$$\partial_\alpha\Big[\sqrt{-\det g}\,g^{\alpha\beta}\partial_\beta X^\mu\Big]=0,\qquad \gamma_{\alpha\beta}[X]-\tfrac{1}{2}g_{\alpha\beta}g^{\gamma\delta}\gamma_{\gamma\delta}[X]=0.$$

Remarks:

- first equation linear in X;
- second equation algebraic in g;
  worldsheet metric proportional to induced metric: $g_{\alpha\beta}\sim\gamma_{\alpha\beta}[X]$;
- equations equivalent to original ones.

Symmetries and Conformal Gauge

Action and equations of motion invariant under:

- target space Poincaré transformations $X^\mu(\xi) \to M^\mu{}_\nu X^\nu(\xi) + B^\mu$.
- worldsheet diffeomorphisms $\xi^\alpha \mapsto \xi'^\alpha(\xi)$;
- worldsheet metric rescaling $g_{\alpha\beta}(\xi) \to \alpha(\xi) g_{\alpha\beta}(\xi)$;

$2 + 1$ fields carry redundant information (gauge).

Conformal Gauge: Exploit redundancy to set $g_{\alpha\beta} = \eta_{\alpha\beta}$ (3 eq.). Reduced action and equations of motion:

$$S \sim \frac{T}{2} \int d^2\xi \, \eta^{\alpha\beta} \, \partial_\alpha X^\mu \, \partial_\beta X_\mu, \qquad \eta^{\alpha\beta} \partial_\alpha \partial_\beta X^\mu = 0.$$

Almost simple linear system, X^μ are massless waves on worldsheet.

Do not forget other equation! Virasoro constraint (non-linear):

$$T_{\alpha\beta}[X] := \gamma_{\alpha\beta}[X] - \tfrac{1}{2} \eta_{\alpha\beta} \eta^{\gamma\delta} \gamma_{\gamma\delta}[X] = 0.$$

String Modes

General solution for X by massless plane waves

$$X^\mu(\xi) = \int \frac{dk}{2|k|}\,\left[a^\mu(k)\,e^{ik\sigma+i|k|\tau} + a^\mu(k)^*\,e^{-ik\sigma-i|k|\tau}\right].$$

Need to impose boundary conditions. E.g. closed string:

$$X^\mu(\tau,\sigma+2\pi) = X^\mu(\tau,\sigma).$$

Periodicity forces k to be integer: string Fourier modes

$$X^\mu(\xi) = x^\mu + \frac{p^\mu\tau}{2\pi T} + \operatorname{Im}\sum_{n=1}^{\infty}\left[\frac{\alpha^\mu_{\mathrm{L},n}}{\sqrt{\pi T}\,n}\,e^{-in(\tau+\sigma)} + \frac{\alpha^\mu_{\mathrm{R},n}}{\sqrt{\pi T}\,n}\,e^{-in(\tau-\sigma)}\right].$$

Solutions parametrised by:

- centre of gravity position and momentum x^μ, p^μ;
- string mode excitation amplitudes $\alpha^\mu_{\mathrm{L/R},n}$, $n = 1, 2, \ldots$.

Quantisation

We want to quantise the system

- a free particle (x^μ, p^μ)
- a set of harmonic oscillators ($\alpha^\mu_{\mathrm{R/L},n}$, $\alpha^{\mu,*}_{\mathrm{R/L},n}$)

Canonical quantisation:

$$\left[x^\mu, p^\nu\right] = i\eta^{\mu\nu}, \qquad \left[\alpha^\mu_{\mathrm{R/L},m}, \alpha^{\nu,*}_{\mathrm{R/L},n}\right] = m\delta_{m,n}\eta^{\mu\nu}.$$

States are tensor products of

- momentum eigenstate $|P\rangle$;
- excitation eigenstate $|N\rangle = (\alpha^\dagger)^N|0\rangle$; for each string mode.

String ground state $|0;P\rangle = |P\rangle \otimes |0\rangle \otimes |0\rangle \otimes \ldots$

- Almost point particle $\alpha \approx 0$: $X^\mu \approx x^\mu + p^\mu\tau/2\pi T$.
- HO ground state localisation: $\mathit{\Delta} X \sim 1/\sqrt{T}$, typical string length.

String Spectrum

Furthermore need to consider constraints $T_{\alpha\beta} = 0$:

- removes negative norm states;
- balances left/right total excitation numbers

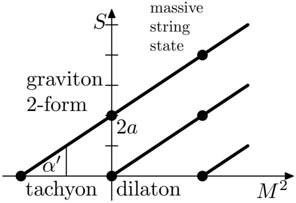
$$N_{\mathrm{L}} - a_{\mathrm{L}} = N_{\mathrm{R}} - a_{\mathrm{R}}, \qquad N_{\mathrm{L/R}} = \sum_{n=1}^{\infty} \eta_{\mu\nu} \alpha^{\mu,\dagger}_{\mathrm{L/R},n} \alpha^{\nu}_{\mathrm{L/R},n};$$

  $a_{\mathrm{L/R}}$ are normal ordering constants of the quantum theory;
- determines mass of string states

$$P^2 = -M^2, \qquad M^2 = 8\pi T(N - a).$$

Furthermore, spin is bounded by $S \leq 2N$.
Leading Regge trajectory

$$M^2 = 4\pi T(S - 2a) = \frac{S - 2a}{\alpha'/2}.$$



Anomalies and Consistency

To obtain a consistent spectrum without negative norm states:

- need $D = 26$ dimensions of spacetime;
- need $a_{\mathrm{L}} = a_{\mathrm{R}} = 1$;

Underlying reason is quantum anomaly of symmetries:

- cannot have worldsheet reparametrisation *and* Lorentz symmetry
- unless $D = 26$, $a_{\mathrm{L}} = a_{\mathrm{R}} = 1$.

Interesting:

- particle at level $N = 1$ with spin $S = 2$ is massless: graviton?!

Strange:

- particle at level $N = 0$ with $S = 0$ is tachyonic;
- need 22 extra dimensions (visible at least at string scale).

Why $D = 26$? $26 = 2 + 24$; "$\sum_{n=1}^{\infty} \frac{1}{2} n = \frac{1}{2}\zeta(-1) = -1/24$".

Superstrings

Can we do something about the problems?

- compactify additional dimensions (many options . . .);
- find true ground state of quantum system (in practice?!);
- . . . but will never have half-integer spin / fermionic states.

Need additional fermionic fields on worldsheet. Long story:

- various bosonic and fermionic particles;
- tachyon is absent (gladly);
- critical dimension is reduced to $D = 10$ (somewhat better);
- supersymmetry is inevitable (at least at string scale).

Stick to bosonic strings here:

- conceptually simpler system;
- qualitatively the same physics;
- can largely ignore the tachyonic mode.

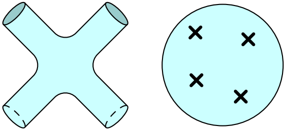
IV. Strings and Gravity

Virasoro–Shapiro Amplitude

Want to compare massless spin-2 state with graviton.

Can compute scattering of string states:

- create external states on string worldsheet;
- integrate over insertions.



Scattering of 4 tachyons (simpler than gravitons)

$$A_4 \sim g_{\mathrm{s}}^2 \delta^D(P) \int d^2 z \, |z|^{p_2 \cdot p_4 / 2\pi T} |1 - z|^{p_3 \cdot p_4 / 2\pi T}$$

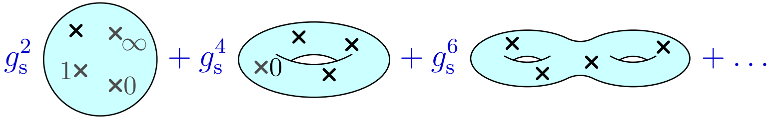
$$\sim g_{\mathrm{s}}^2 \delta^D(P) \, \frac{\Gamma(-1 - s/8\pi T)\,\Gamma(-1 - t/8\pi T)\,\Gamma(-1 - u/8\pi T)}{\Gamma(+2 + s/8\pi T)\,\Gamma(+2 + t/8\pi T)\,\Gamma(+2 + u/8\pi T)} \,.$$

- very soft in the UV;
- manifestly crossing symmetric;
- poles of Γ correspond to virtual particles exchanged in s, t and u channels: string modes!

Graviton Scattering

Corresponding scattering amplitude for spin-2 excitations:

- conceptually similar;
- correct polarisation dependence;
- agrees with leading Einstein gravity at leading large T;
- Planck length equals string length;
- correction terms at higher orders: $\alpha' \sim 1/T$ corrections;
- higher orders in g_s: genus corrections



  integration over moduli of higher-genus surfaces;
- corrections equivalent to gravity correction terms $D^{2k} R^n$; effective low-energy theory summarising string modes;

Strings and Gravity

String theory contains gravity!

- gravitons behave like in special relativity at low energy;
- there are stringy corrections to gravity at higher orders;

So far:

- started with strings in flat Minkowski space;
- quantisation led to the emergence of gravitons;
- gravitons are the quanta of general relativity;
- general relativity is a theory of dynamical geometry.

What if we started from a curved background spacetime?

Strings in Curved Backgrounds

Put strings into a curved background. Introduce metric field

$$\eta_{\mu\nu} \to G_{\mu\nu}(x).$$

Simple replacement for Nambu–Goto action (induced metric γ)

$$S \sim T \int d^2\xi \sqrt{-\det\gamma} \qquad \text{with} \qquad \gamma_{\alpha\beta} := G_{\mu\nu}(X)\,\partial_\alpha X^\mu\,\partial_\beta X^\nu.$$

Polyakov action becomes a non-linear sigma model

$$S \sim \frac{T}{2} \int d^2\xi \sqrt{-\det g}\, g^{\alpha\beta}\, G_{\mu\nu}(X)\,\partial_\alpha X^\mu\,\partial_\beta X^\nu.$$

Equations of motion now non-linear; cannot be solved in general.

Gravitons and Geometric Deformations

Can show equivalence of stringy gravitons and geometric deformation.
Compare for graviton with momentum p and polarisation $\epsilon_{\mu\nu}$:

- massless spin-2 excitation at level 1:

$$|\epsilon; p\rangle = \epsilon_{\mu\nu}(\alpha^\mu_{\mathrm{L},1})^\dagger(\alpha^\nu_{\mathrm{R},1})^\dagger|0;p\rangle;$$

- translation to string fields: $(\alpha^\mu_n)^\dagger \to \partial^n X^\mu$; $|0;p\rangle \to e^{ip\cdot X}$;
- graviton excited by worldsheet operator

$$V[\epsilon,p] = \int d^2\xi\sqrt{-\det g}\,\epsilon_{\mu\nu}\,g^{\alpha\beta}\,\partial_\alpha X^\mu\,\partial_\beta X^\nu\,e^{ip\cdot X};$$

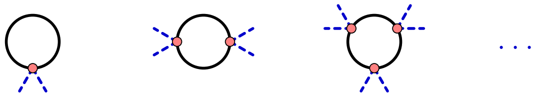
- graviton is plane wave deformation

$$G_{\mu\nu}(x) = \eta_{\mu\nu} + \epsilon_{\mu\nu}e^{ip\cdot x} + \ldots.$$

Variation of Polyakov action is excitation operator $\delta S = V[p;\epsilon]$.

Divergences of Worldsheet Theory

Curved background changes quantum theory. UV divergences?



Metric field $G(x)$ serves as collection of coupling constants for QFT

$$S \sim \frac{T}{2} \int d^2\xi \sqrt{-\det g}\, g^{\alpha\beta}\, G_{\mu\nu}(X)\, \partial_\alpha X^\mu\, \partial_\beta X^\nu .$$

Divergences at loop level to be absorbed into G (and similar fields).
Running coupling constants

$$\frac{\mu\partial}{\partial\mu} G_{\mu\nu} = \beta_{\mu\nu} = \frac{1}{2\pi T} R_{\mu\nu}[G].$$

Anomalies

Beta-function introduces a new scale on worldsheet.
Scale would violate reparametrisation invariance!

$$\beta_{\mu\nu} = 0 \quad \Longleftrightarrow \quad R_{\mu\nu}[G] = 0.$$

Einstein equation! Strings like to propagate in general relativity.

In fact, for general dimension D:

$$\beta_{\mu\nu} = \tfrac{1}{3}(D - 26)\, G_{\mu\nu} + \frac{1}{2\pi T}\left(R_{\mu\nu}[G] - \tfrac{1}{2} G_{\mu\nu} R[G]\right).$$

In other words $D - 26$ serves as a cosmological constant.
For $D \neq 26$ space curved at the order of the Planck scale.

Low-Energy Effective Action

Gravitons are low-energy excitations (massless).
Can summarise their dynamics by effective action

$$S \sim \int d^D x \sqrt{-\det G}\, \left[-\frac{2}{3}(D-26) + \frac{1}{2\pi T}\, R + \ldots\right].$$

Remarks:

- Hilbert action with cosmological term;
- corrections from higher orders in $1/T$ and g_s;
- resulting equations of motion are equivalent to $\beta_{\mu\nu} = 0$: string excitations will not cause an anomaly.

Summary

- One of the massless excitations is the graviton;
- string theory clearly contains quantum gravity; finite!
- general relativity plus stringy corrections;
- string scale is the Planck scale;
- may start with flat or curved background.

Background dependence in string theory?

- Quantum theory depends on background;
- similar example: QED; clearly depends on background; e.g. vacuum vs. constant magnetic field; motion of electrons;
- also ordinary quantum gravity depends on background;
- other backgrounds are highly excited, coherent quantum states; not accessible in perturbation theory;
- asymptotical behaviour of background may make a difference.

V. D-Branes

Open Strings

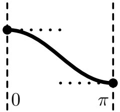
For closed strings we simply assume periodicity $X^\mu(\sigma + 2\pi) = X^\mu(\sigma)$.

For open strings, we vary the action

$$S \sim \int d^2\xi\, \eta^{\alpha\beta}\, \partial_\alpha X^\mu\, \partial_\beta X_\mu$$

and pay attention to boundary terms (integration by parts) at $\sigma = 0, \pi$

$$\delta S \sim \int d\tau \left[\delta X^\mu\, X'_\mu\right]_{\sigma=0}^{\sigma=\pi} + \ldots .$$



Remarks:

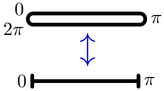
- For general variations one finds Neumann condition $X'_\mu = 0$.
- Virasoro constraints $T_{\alpha\beta} = 0$ furthermore imply $\dot{X}^2 = 0$.
- Ends of string must move at the speed of light.

Open String Spectrum

General solution with open boundary conditions $X' = 0$ at $\sigma = 0, \pi$:

- similar to closed strings
- boundary conditions couple left- and right-moving modes

$$\alpha^{\mu}_{\mathrm{L},n} = \alpha^{\mu}_{\mathrm{R},n}.$$



Mass formula for open strings:

$$M^2 = 2\pi T(N - a).$$

Features:

- vacuum $|0; p\rangle_{\mathrm{o}}$ is tachyonic;
  ignore; can be avoided in superstrings;
- massless vector state at level 1: $(\alpha^{\mu}_1)^{\dagger}|0; p\rangle_{\mathrm{o}}$: photon!

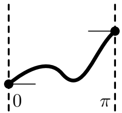
D-Branes

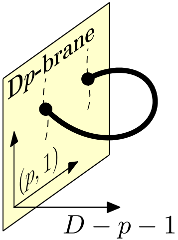
Alternative option: constrain variation at boundary, $\delta X^\mu = 0$:

- Dirichlet boundary condition: fix ends $X^\mu = \text{const.}$;
- mix boundary conditions: $p+1$ Neumann, $D-p-1$ Dirichlet;
- free motion in $p+1$ directions; fixed in orthogonal directions;
- string ends confined to $(p+1)$-dimensional surface: Dp-brane.

Why?

- why should there be such objects?
- where should they be located?
- breaks Poincaré symmetry of background!





Why not?!

- add more features to background;
- go further: allow strings to end on curved hyperplane.
- string dualities; two different models may yield same physics; here: T-duality introduces/changes D-branes.

Spectrum with D-Branes

What changes about the spectrum?

- no transversal momentum; motion confined to D-brane;
- identification $\alpha_{\mathrm{L}}^{\mu} = \pm\alpha_{\mathrm{R}}^{\mu}$ depends on μ;
- same mass formula.

level-1 massless states $(\alpha_1^{\mu})^{\dagger}|0;p\rangle$ split up:

- longitudinal μ: photon along D-brane;
- transversal μ: $D-p-1$ scalar particles on D-brane;

What do massless open string modes mean?

Brane Dynamics

Recall massless closed string modes:

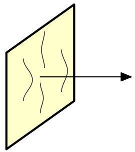
- spin-2 particles are gravitons;
- have the same effect as deforming geometry.

Now consider longitudinal massless open string modes:

- spin-1 particles are photons;
- have the same effect as coupling of gauge field to string ends

$$\int d\tau\, \dot{X}^\mu A_\mu(X);$$

- gauge field naturally confined to D-brane.



How about transversal massless open string modes?

- have the same effect as deforming the D-brane itself;
- D-branes become dynamical upon string quantisation!

Effective Theories

Can do same steps as for closed strings.

Avoiding UV divergences and anomalies:

- for gravitons: Einstein equations;
- for photons: Maxwell equations;
- for deformations: D-brane equations of motion.

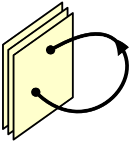
Corresponding low-energy effective actions:

- gravitons: Hilbert action;
- photons: action of electromagnetism;
- deformations: Dirac action for brane motion (induced volume);
- combinations with gravity and higher orders: Dirac–Born–Infeld

$$S \sim \int d^{p+1}x \sqrt{-\det(G_{ab} + 2\pi\kappa^2 F_{ab})}\,.$$

Multiple Branes and Gauge Groups

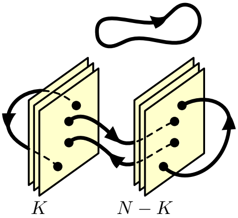
Can now combine branes to construct interesting physics.



Simplest case: N coincident branes

- strings will start on brane $j = 1, \ldots, N$;
- strings will end on brane $k = 1, \ldots, N$;
- N^2 massless photon-like modes.

Yang–Mills theory with $\mathrm{U}(N)$ gauge group.



Splitting a stack of branes: $N \to K + (N - K)$

- $2NK$ strings stretch between stacks;
  minimum length leads to mass shift.
  corresponding photons become massive spin-1 particles.
- K^2 plus $(N - K)^2$ strings end on same stack;
  corresponding photons remain massless;

Spontaneous breaking of gauge symmetry to $\mathrm{U}(K) \times \mathrm{U}(N - K)$.

String Phenomenology

Finally, combine various features of backgrounds:

- non-compact dimensions and compact manifolds;
- different types of branes;
- stacks of branes, intersections of branes, parallel branes;

in order to obtain the desired

- spectrum of light particles (w.r.t. Planck scale);
- set of global symmetries and supersymmetries;
- set of gauge symmetries;
- symmetry breaking patterns;

Pay attention to:

- stability;
- anomalies;
- number of continuous moduli for the configuration.

VI. Conclusions

Conclusions

Introduced and sketched some basic concepts of string theory:

- Formulation(s) of classical string theory.
- Quantisation and the string spectrum.
- Scattering in string theory.
- How gravity emerges from string theory.
- Low-energy effective theory.
- Open strings and emergent extended objects.
- Construction of suitable particle physics.

Many further results and insights from string theory:

- AdS/CFT correspondence
- mathematics and geometry
- ...