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Searching for New Physics beyond the Standard Model in Electric Dipole Moment

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Abstract

This is the theoretical review of exploration of new physics beyond the Standard Model (SM) in electric dipole moment (EDM) in elementary particles, atoms, and molecule. EDM is very important CP violating phenomenon and sensitive to new physics.

Starting with the estimations of EDM of quarks-leptons in the SM, we explore the new signals beyond the SM. However, these works drive us to more wide fronteers where we serach fundamental physics using atoms and molecules and vice versa.

Paramagnetic atoms and molecules have great enhancement factor on electron EDM. Diamagnetic atoms and molecules are very sensitive to nuclear P and T odd processes.

Thus EDM becomes the key word not only of New Physics but also of unprecedented fruitful collaboration among particle, atomic. molecular physics.

This review intends to help such collaboration over the wide range of physicists.

I. INTRODUCTION

This article is a review of the search of new physics beyond the Standard Model (SM) concentrating on electric dipole moments (EDM) of elementary particles like neutron, proton, leptons, quarks as well as atoms and molecules. The presence of EDM implies T-odd and P-odd interactions. So if it exists, it indicates the direct T noninvariance as well as CP violation if CPT invariance is assumed.

It is very important that these fundamental EDMs are enhanced in paramagnetic atoms (d_{atom}) and molecules ($d_{molecule}$) which have an unpaired electron. Also in diamagnetic atoms and molecules, proton and neutron EDMs appear via Schiff moment due to CP-violating hadron interactions.

The discovery of CP violation in $K_L^0 \to \pi^+\pi^-$ decay [1] in 1964 was an amazing event for the majority of theorists since the model at that time could not produce CP violation. The introduction of CP phase in the mixing matrix by Kobayashi-Maskawa [2] was 7 years after that, which becomes the unique origin of CP violation in the SM [3]. This CP phase opened the door to new frontiers in a vast range of physics, especially in B factories: Belle at KEKB (KEK) and BaBar at PEP-II (SLAC).

CP violation in B mesons is measured by observing the asymmetry

$$\begin{aligned} a_f(\tau) &\equiv \frac{\Gamma(B^0(\tau)\to f)-\Gamma(\overline{B}^0(\tau)\to f)}{\Gamma(B^0(\tau)\to f)+\Gamma(\overline{B}^0(\tau)\to f)} \\ &= C_f\cos(\Delta m\tau)-S_f\sin(\Delta m\tau), \end{aligned} \tag{1}$$

where

$$C_f\equiv\frac{1-|\lambda|^2}{1+|\lambda_f|^2},\quad S_f\equiv\frac{2\Im\lambda_f}{1+|\lambda_f|^2}. \tag{2}$$

Here $B=B_d^0=|d\bar{b}>$ or $B=B_s^0=|s\bar{b}>$, and f is a CP eigenstate such as $J/\psi K_S,\ \pi^+\pi^-,\ \rho K_S$. $\Im$ means an imaginary part. For $B_d^0\to J/\psi K_s$ [4] [5],

$$\Im\lambda_{\psi K}=0.734\pm0.054, \tag{3}$$

with world average. It may be more advantageous in searching for new physics to consider the SM loop suppressed process like $B^0\to\phi K^0$ etc. However, these results seem to be consistent with the CKM mechanism [6].

As we will show, the EDM values predicted by the SM are very tiny because they appear first in three loops (quarks) and four loops (leptons) and are far smaller than the upper limit of the present and near future experiments. [1] On the other hand, there are some physical phenomena which suggest new physics beyond the SM other than neutrino oscillation experiments. The anomalous muon magnetic moment, $a_\mu\equiv(g_\mu-2)/2$, is one such example [8]

$$a_\mu^{EXP}-a_\mu^{SM}=(26.1\pm8.0)\times10^{-10}, \tag{4}$$

corresponding to a 3.3σ discrepancy from the SM. There are also other indirect problems of the SM like the observed baryon asymmetry, $n_B/n_\gamma=1\times10^{-10}$. Indeed, in the SM, CP violation is parametrized by the Jarlskog invariant, which is too tiny to produce this amount of asymmetry; we need other CP violating terms. The other implicit deficiencies of the SM are Dark Matter candidates and the hierarchy problem etc.

Under these situations, the EDM is very important since some models beyond the SM give rather marginal predictions on the electron and neutron EDMs on the upper bounds of ongoing experiments.

[1] In the broad sense, there is another CP phase called the θ term [7] in the SM, playing an essential role in especially the EDM of diamagnetic atoms (see section **5.3**).

So new models are required to recover all such discrepancies. Furthermore, it must reproduce much larger phenomena which the SM predicts beautifully like, for instance, flavour changing neutral currents (FCNCs) and other vast low energy physics phenomena.

Here we point out a peculiar property of the EDM:

As is well known, EDMs of elementary particles are enhanced in atoms and molecules. In this sense, the EDM provides an unprecedented strategy of using atoms and molecules for the search of fundamental properties of elementary particles.

Several review works on this subjects have been already published [9][10][11][12]. The new features of this review is that it is written by the author who is studying new physics beyond the SM, and, therefore, emphasis is on this point. However, EDM studies drive us necessarily to a wide range of physics (and chemistry), particle physics, atomic and molecular physics. The great achievements are possible only by the collaboration of theoretical and experimental scientists over this wide range of fields. Under these situations, we try in this review to make a small but significant bridge between these wide communities of scientists.

Accordingly, we endeavor to give a self-complete concept of EDMs as far as possible, sometimes sacrificing the exhaustive citation of important references.

II. BASICS OF EDM

In this section we give the definitions and conventions used in this review, and basic formulae useful for the EDM.

A. Definitions and Conventions

Metric:

$$g^{\mu\nu} \equiv \begin{pmatrix} 1 & 0 \\ 0 & -\mathbf{1}_{3\times 3} \end{pmatrix} . \tag{5}$$

Pauli matrices and spin matrices:

$$\sigma^1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

$$S^i \equiv \frac{1}{2}\sigma^i . \tag{7}$$

Gamma matrices:

$$\gamma^0 \equiv \begin{pmatrix} \mathbf{1}_{2\times2} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1}_{2\times2} \end{pmatrix}, \quad \gamma^i \equiv \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}, \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbf{0} & \mathbf{1}_{2\times2} \\ \mathbf{1}_{2\times2} & \mathbf{0} \end{pmatrix}. \tag{8}$$

Chirality projection:

$$P_L \equiv \frac{1}{2}(1-\gamma^5), \quad P_R \equiv \frac{1}{2}(1+\gamma^5). \tag{9}$$

Antisymmetric tensor:

$$\sigma^{\mu\nu} \equiv \frac{1}{2}[\gamma^\mu, \gamma^\nu] \equiv \frac{1}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right). \tag{10}$$

The electromagnetic field tensor is

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu, \quad F^{0i} = -E^i, \quad F^{ij} = -\epsilon^{ijk}B^k, \text{ so } F^{12} = -B^3 \text{ cyclic.} \tag{11}$$

The Cabbibo-Kobayashi-Maskawa matrix [2] and Jarlskog invariant [13]:

$$V \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{12}$$

$$j^\mu = (\bar{u}, \bar{c}, \bar{t}\,)\gamma^\mu P_L V \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{13}$$

$$J_{CP} \equiv \left|\Im(V_{\alpha j}V^*_{\beta j}V^*_{\alpha k}V_{\beta k})\right| = s_{12}s_{23}s_{13}c_{12}c_{23}c^2_{13}\sin\delta. \tag{14}$$

Here J_{CP} is the base independent CP phase called the Jarlskog parameter, appearing in CP violation processes via the Kobayashi-Maskawa mechanism. Hereafter, we denote the imaginary part (real part) of O by $\Im(O)$ ($\Re(O)$). Apart from the EDM process discussed later, we also mention on neutrino oscillation processes,

$$\begin{aligned} P(\nu_\beta \to \nu_\alpha) = \delta_{\alpha\beta} \; - \; & 4\sum_{j<k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\sin^2\left(\frac{\Delta p_{jk}L}{2}\right) \\ & + \; 4i\sum_{j<k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\sin\left(\Delta p_{jk}L\right). \end{aligned} \tag{15}$$

Thus we can determine the CP odd term (the third term) by measuring both $P(\nu_\beta \to \nu_\alpha)$ and $P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$. T2K [14] found evidence of a nonzero θ_{13} and recently the Daya Bay Collaboration [15] fixed it as

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}). \tag{16}$$

Therefore the above mentioned CP phase experiments have become crucial.

B. Effective Dipole Operator

A permanent EDM of the electron must lie along its spin, namely $\mathbf{d} = d_e \boldsymbol{\sigma}$ [16].

At tree level in the SM, a fermion ψ (of mass m_ψ and electric charge e, so electron's charge is $e = -|e|$) in the presence of electromagnetic field satisfies

$$(\gamma(p - eA) - m)\, \psi = 0, \quad \overline{\psi}\, (\gamma(p + eA) + m) = 0. \tag{17}$$

However if we include loop corrections, the effective electromagnetic interaction is given in general by

$$V = -e\overline{u}_2(p_2)\Gamma^\mu u_1(p_1) \equiv J^\mu A_\mu(k) \tag{18}$$

with

$$P \equiv p_1 + p_2, \quad k = p_2 - p_1. \tag{19}$$

Here

$$A^\mu = (\phi, \mathbf{A}) \tag{20}$$

is a true vector and transfoms as

$$A^\mu \to (\phi, \; -\mathbf{A}) \quad \text{under P, T transformation.} \tag{21}$$

Whereas J^μ can be either a true or a pseudo vector.

First we consider the case where J^μ is a true vector. In this case, J^μ takes the general form

$$J^\mu = F_1(k^2)(\overline{u}_2 u_1)P^\mu + F_2(k^2)\overline{u}_2\gamma^\mu u_1 + F_3(k^2)(\overline{u}_2 u_1)k^\mu. \tag{22}$$

However, from gauge invariance, the current is conserved

$$k_\mu J^\mu = 0 \tag{23}$$

and

$$F_3(k^2) = 0. \tag{24}$$

Using Gordon's decomposition for the bilinear form of a spinor of mass m,

$$(\overline{u}_2\sigma^{\mu\nu}u_1)\, k_\nu = -2m\overline{u}_2\gamma^\mu u_1 + \overline{u}_2 u_1 P^\mu, \tag{25}$$

the interaction term is then given by

$$- e_\psi\overline{\psi}\gamma^\mu\psi A_\mu = -\frac{e}{2m}\,\overline{\psi}(i\partial^\mu - i\overleftarrow{\partial}^\mu)\psi\, A_\mu - i\frac{e}{4m}\,\overline{\psi}\sigma^{\mu\nu}\psi\, F_{\mu\nu}. \tag{26}$$

We should note that

$$-i\frac{e}{4m}\overline{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}=\frac{e}{2m}\overline{\psi}\left[\mathbf{\Sigma}\cdot\mathbf{B}-i\gamma_5\mathbf{\Sigma}\cdot\mathbf{E}\right]\psi,$$

where

$$\mathbf{\Sigma}\equiv\begin{pmatrix}\boldsymbol{\sigma} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\sigma}\end{pmatrix} \tag{27}$$

Here off diagonal elements are suppressed by $O(v/c)$ relative to diagonal ones. The magnetic moment is defined as the coefficient of $\mathbf{B}$ in the above equation,

$$\boldsymbol{\mu}_Q=Q\left(\frac{e}{2m}\right)\boldsymbol{\sigma},\ \text{and}\ \mu_u=-2\mu_d, \tag{28}$$

where Q is the quark charge and $m_u=m_d=360$ MeV has been assumed. It is clear from eq.(27) that the fermion has a magnetic dipole moment with $g=2$ at tree level in the SM (28). The second term is off-diagonal and there appears the additional P-odd term $\sigma^k p^k$ term in the product of the off diagonal element. The MDM and EDM of particles are defined in the rest frame and we hereafter neglect the off-diagonal element unless it is specified [2]

On the other hand, a true axial vector current, the general form is

$$J^{5\mu}=G_1(k^2)(\overline{u}_2\gamma_5u_1)P^\mu+G_2(k^2)\overline{u}_2\gamma^\mu\gamma_5u_1+G_3(k^2)(\overline{u}_2\gamma_5u_1)k^\mu. \tag{29}$$

In this case, G_3 survives due to chiral symmetry breaking. In weak interactions or higher loops in the SM or in new physics, the current includes both J^μ and $J^{5\mu}$ in general. Thus the following CP odd effective action appears,

$$-i\frac{e}{4m}\overline{\psi}\gamma^5\sigma^{\mu\nu}\psi F_{\mu\nu}=\frac{e}{2m}\overline{\psi}\left[i\mathbf{\Sigma}\cdot\mathbf{E}-\gamma_5\mathbf{\Sigma}\cdot\mathbf{B}\right]\psi.$$

This will be discussed in more detail in connection with the EDM and MDM shortly.

Also we can consider another conserved current like the vector case

$$a(k^2)(\gamma k k^\mu-k^2\gamma^\mu)\gamma^5, \tag{30}$$

which reduces to

$$a(k^2)\mathbf{k}^2\boldsymbol{\sigma}_\perp \tag{31}$$

[2] This is true, especially for measuring EDM by spin precession as for most cases of neutral particles and atoms. It is not so serious for the measurements of EDMs of charged particles and neutral molecules though even in those cases, cooling down indicates a long coherent time is favourable.

in the nonrelativistic limit.

This term is called aqn anapole term, which comes from the second term of

$$A_i(\mathbf{r}) = \int d^3r' \frac{J_i(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}, \tag{32}$$

in the expansion around r, that is,

$$A_i^{(2)}(\mathbf{r}) = \left(\nabla_k \nabla_l \frac{1}{r}\right) T_{ikl} \tag{33}$$

with

$$T_{ikl} = \frac{1}{2}\int d^3r' r'_k r'_l J_i(^\prime). \tag{34}$$

At the loop level in the SM and/or models beyond the SM, the following effective interaction of gauge invariant form can be obtained:

$$\begin{aligned} -i\overline{\psi}_i & \left(A_L^{ij}P_L + A_R^{ij}P_R\right)\sigma^{\mu\nu}\psi_j F_{\mu\nu} \\ &= \frac{-i}{2}(A_L^{ij}+A_R^{ij})\overline{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} + \frac{1}{2}(A_R^{ij}-A_L^{ij})\overline{\psi}\sigma^{\mu\nu}\gamma^5\psi F_{\mu\nu} \\ &\approx (A_L^{ij}+A_R^{ij})\overline{\psi}\mathbf{\Sigma}\cdot\mathbf{B}\psi + i(A_R^{ij}-A_L^{ij})\overline{\psi}\mathbf{\Sigma}\cdot\mathbf{E}\psi.. \end{aligned} \tag{35}$$

Here we have neglected off diagonal parts in the second equalty.

For the electric and magnetic dipole moment, we take zero momentum of the photon. Then the imaginary part of the coefficients of the effective interaction vanish because of the optical theorem (imaginary part of the forward scattering amplitude is given by the sum of possible cuts of intermediate states). We find the anomalous magnetic dipole moment a_ψ and electric dipole moment d_ψ to be

$$a_\psi = \frac{g-2}{2} = -\frac{2m}{\mathrm{e}}\,\Re(A_R^{ii}+A_L^{ii}), \tag{36}$$

$$d_\psi = 2\,\Im(A_R^{ii}-A_L^{ii}). \tag{37}$$

Note that A_L and A_R must include a fermion mass (m_ψ or a fermion mass in the loop) because the effective interaction $\overline{\psi}\sigma^{\mu\nu}\psi$ changes the chirality which can be achieved by adding a mass term in the fundamental Lagrangian. If one of the particles in the loop is much heavier than the others, A_L and A_R are suppressed by the mass. Thus, for large A_L and/or A_R, it is preferred that masses of particles in the loop are similar to each other.

The effective interaction (35) also causes a $\ell_i \to \ell_j\gamma$ decay where the decay rate is given by

$$\Gamma(\ell_i \to \ell_j\gamma) = \frac{m_{\ell_i}^3}{4\pi}\left(|A_L^{ij}|^2 + |A_R^{ij}|^2\right) . \tag{38}$$

Thus EDM and MDM have opposite parities and different on the order of magnitude. However they appears in parallel, and have some similarities also. One of them is their SU(6) property [17] and will be discussed in Appendix **A**.

For an invariant electromagnetic field, EDM, MDM, anapole, and higher n-pole moments appear as the multipole expansions of the Coulomb potential and vector potential. These points are also discussed in Appendix **B**.

Quarks receive additional contributions, which will be discussed for diamagnetic atoms. Here we list the results.

A strong CP violating term connected with the θ vacuum (see Appendix **G**)

$$L_{d=4} = \frac{g_s^2}{64\pi^2}\overline{\theta} G^a_{\mu\nu} G^a_{\rho\lambda} \epsilon^{\mu\nu\rho\lambda} \equiv \frac{g_s^2}{32\pi^2}\overline{\theta} G^a \cdot \tilde{G}^a. \tag{39}$$

In new physics beyond the SM, we have the other P and T violating effective actions: the chromoelectric dipole operator (cEDM)

$$L_C = -\frac{i}{2}\tilde{d}_q \overline{q}\sigma_{\mu\nu}\gamma_5 T^a q G^{\mu\nu a} \equiv -\frac{i}{2}\tilde{d}_q \overline{q}\sigma G \gamma_5 q, \tag{40}$$

and the following dimension 6 operators,

$$L_G = -\frac{1}{6} d_G f_{abc} G^a_{\mu\rho} G^{\rho b}_{\ \ \nu} G^c_{\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma} \equiv -\frac{1}{3} d_G f_{abc} G^a G^b \tilde{G}^c, \tag{41}$$

the so-called Weinberg term [18], and

$$L_{d=6} = \sum C^a_{ij} \overline{\psi_i} O_a \psi_i \overline{\psi_j} O_a \gamma_5 \psi_j. \tag{42}$$

Here ψ_i and ψ_j are leptons and/or nucleons. O_a are scalar, vector, and tensor gamma matrices. We will explain the detailed physical implications in the diamagnetic atom in section 6.

In the SM, weak interactions act with matter and gauge fields in the form,

$$H_{weak} = \overline{\psi} P_L \Gamma_\mu \psi W^\mu \equiv J_\mu W^\mu. \tag{43}$$

However, except for the top quark, fermion masses are small compared to weak bosons masses, They are descrived as the four Fermions coupling

$$H_{weak} = J_\mu J^\mu. \tag{44}$$

This is the case for tree diagrams. If you consider loop diagrams or new physics beyond the SM, we will encounter more general forms. We will discuss this in the Appendix C.

C. Experimental Bounds

We have no experimental signal of the EDM yet but have upper limits. They are as follows [19]. It is very impressive that recently we have a more precise upper limit of the electron EDM from molecule (YBF) than from atom (Tl).

$$\begin{aligned}
d_e \text{ from thallium atom } d(Tl) &= (6.9 \pm 7.4) \times 10^{-28}\,\text{e cm [20]} &(45)\\
d_\mu &= (3.7 \pm 3.4) \times 10^{-19}\,\text{e cm [21]} &(46)\\
d_n &< 2.9 \times 10^{-26}\,\text{e cm} \ \ (90\% C.L.)\text{ [22]} &(47)\\
d({}^{199}Hg) &< 3.1 \times 10^{-29}\,\text{e cm} \ \ (95\% C.L.)\text{ [23]} &(48)\\
d_e \text{ from the molecule } d(\text{YbF}) &= (-2.4 \pm 5.7_{stat} \pm 1.5_{syst}) \times 10^{-28}\,\text{e cm [24]} &(49)
\end{aligned}$$

For reference, we give here the muon anomalous MDM, a non-null signal of new physics beyond the SM.

The deviations of the SM predictions from the experimental result are given by

$$\begin{aligned}
\Delta a_\mu[\tau] &\equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}}[\tau] = 14.8(8.2) \times 10^{-10},\\
\Delta a_\mu[e^+e^-] &\equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}}[e^+e^-] = 30.3(8.1) \times 10^{-10}.
\end{aligned} \tag{50}$$

Here the hadronic contributions to $a_\mu^{\text{SM}}[\tau]$ and $a_\mu^{\text{SM}}[e^+e^-]$ were calculated [25] by using data of hadronic τ decay and e^+e^- annihilation to hadrons, respectively. These values of $\Delta a_\mu[\tau]$ and $\Delta a_\mu[e^+e^-]$ correspond to 1.8σ and 3.7σ deviations from SM predictions, respectively. EDMs and MDMs comes from similar diagrams apart from CP transfomation and the differences of magnitudes in the MDM and EDM stems from the cancellation of diagrams and symmetry.

III. STANDARD MODEL

In this section we give the EDMs of quarks, hadrons, and leptons in the SM framework. Structure of matter multiplets in SM+(Dirac) neutrino is

$$
\begin{aligned}
&Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim \left(3, 2, \frac{1}{6}\right), \\
&u^c = (u_1^c \;\; u_2^c \;\; u_3^c) \sim \left(\overline{3}, 1, \frac{-2}{3}\right), \\
&d^c = (d_1^c \;\; d_2^c \;\; d_3^c) \sim \left(\overline{3}, 1, \frac{1}{3}\right), \\
&L = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim \left(1, 2, \frac{-1}{2}\right), \\
&e^c \sim (1, 1, 1), \\
&\nu^c \sim (1, 1, 0).
\end{aligned} \tag{51}
$$

CP violation occurs a la Kobayashi-Maskawa mechanism, that is, CP phase in CKM mixing matrix for quarks or MNS matrix for leptons. Diagrammatically, it resembles with LFV processes but the former is necessary to incorporate the non-zero Jarlskog parameter. Apart from the uncovered new phenomena in neutrino, the SM has the deficiency of baryon assymmetry. Jarlskog introduced A_{CP} [13] defined by

$$[M_u M_u^\dagger, M_d M_d^\dagger] = i A_{CP}. \tag{52}$$

Here M_u, M_d are up-type and down-type quark mass matrices. The observables are not these matrices but those which are invariant under rebasing and rephasing, that is, eigen values and CKM mixing matrix. By construction, A_{CP} is traceless and Hermitian, and characterizes the effect of CP violation. Its explicit form is

$$\det A_{CP} = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) J_{CP}, \tag{53}$$

where J_{CP} is given by (14). As we will see the detail shortly, one-loop diagram (Fig. (1) gives zero contribution to the EDM. As for second loop (Fig. 2), using this J and denoting by f the Green function of f flavoured fermion [26], the f quark EDM has the form

$$i \sum_{jkl} \Im(V_{jk} V_{lf} V_{jf}^* V_{lk}^*) fjklf = \frac{1}{2} \Im(V_{jk} V_{lf} V_{jf}^* V_{lk}^*) f(jkl - lkj). \tag{54}$$

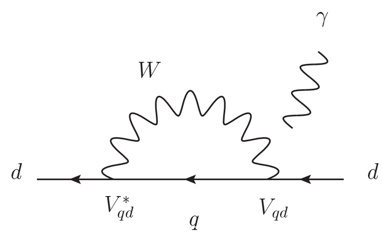


FIG. 1: The diagram for the EDM of d quark at one loop level in the SM.

and $J \approx 3 \times 10^{-5}$is twice the area of unitary triangle $V^*_{ud}V_{ub} + V^*_{cd}V_{cb} + V^*_{td}V_{td}$. One finds that $J/T^{12}_{EW} \leq 10^{-20}$. This falls short of the baryon asymmetry n_B/n_γ by 10^{-10}.

We will show the detail loop by loop in the subsequent sections.

A. Quark EDM

For definiteness, we consider the EDM of d quark. The advantage of d in comparison with u may be that t can be in the loop with the weak interaction to avoid the GIM cancellation [27].

1. One Loop

In the one loop level (Fig. 1), elements of the CKM matrix appear as $|V_{qd}|^2$. Therefore, there is no EDM (imaginary part of coefficient) apparently.

2. Two Loop

At two loop level in the SM, two types of diagrams have an imaginary coefficient potentially. The diagrams are shown in Fig. 2. It is, however, shown that the imaginary part of each diagram vanishes by the summation of contributions from all quarks of internal lines [28].

For the diagrams in Fig. 2, it is clear that q'' must not be d quark because it gives $|V_{q'd}|^2|V_{qd}|^2$ of real value. By the same reason, q' must be different from q''.

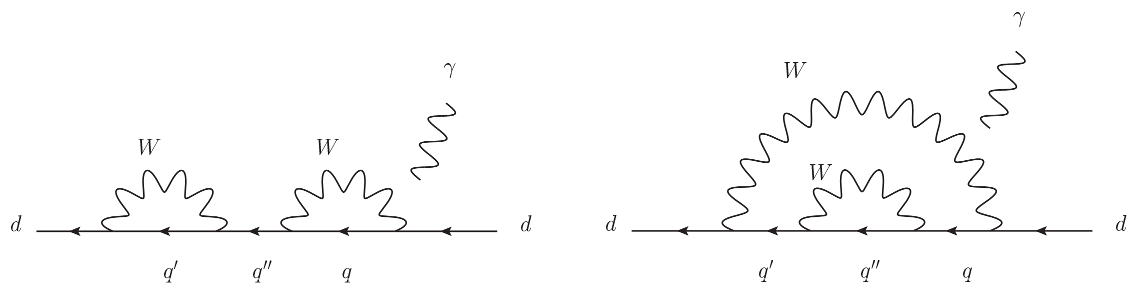


FIG. 2: Diagrams for the EDM of d quark at two loop level in the SM.

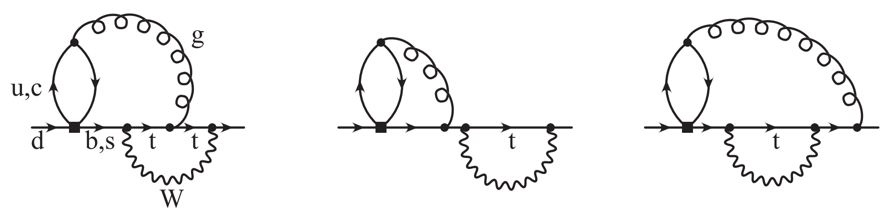


FIG. 3: Diagrams for the EDM of d quark at three loop level in the SM [29].

3. Three Loops

Fig. 3 shows three loops diagrams which contribute to d quark EDM in the SM. The formula for the contribution is given in [29] as

$$\frac{d_d}{\mathrm{e}} \simeq \frac{m_d m_c^2 \alpha_s G_F^2 J_{CP}}{108\pi^5}\left\{ \left(L_{bc}^2 - 2L_{bc} + \frac{\pi^2}{3} \right) L_{Wb} + \frac{5}{8} L_{bc}^2 - \left(\frac{335}{36} + \frac{2}{3}\pi^2 \right) L_{bc} - \frac{1231}{108} + \frac{7}{8}\pi^2 + 8\zeta(3) \right\}, \tag{55}$$

where $L_{ab} \equiv \ln(m_a^2/m_b^2)$. It results in

$$d_d \simeq -10^{-34}\,\mathrm{e\ cm} \tag{56}$$

while the triple log approximation (taking only L^3 term) gives

$$d_d \simeq +10^{-34}\,\mathrm{e\ cm.} \tag{57}$$

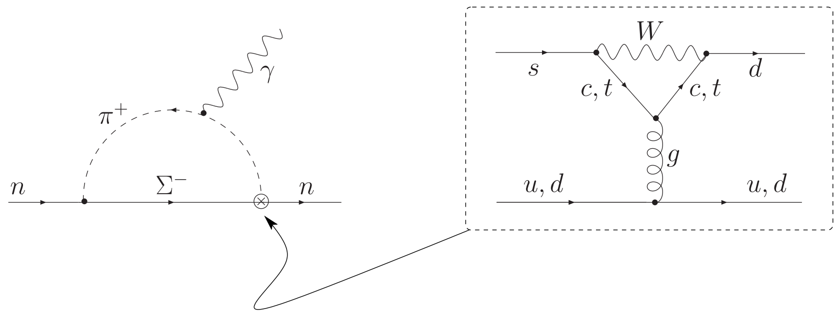


FIG. 4: Diagrams for the EDM of neutron in the SM [30].

B. Neutron EDM

The dominant contribution to the neutron EDM in the SM comes from "two loop" diagram in Fig. 4. The interaction on the left part of the loop is given by the phenomenological interaction hamiltonian

$$H = iG_F m_\pi^2 \overline{u}_n (A + B\gamma^5) u_\Sigma \varphi_\pi \,, \tag{58}$$

$$A = -1.93, \;\; B = -0.65, \tag{59}$$

where u_n, u_Σ, and φ_π stand for wave functions of neutron, Σ^- baryon (dds), and π^+, respectively. The interaction on the right part of the loop is so-called "strong penguin" whose effective operator is given by

$$\mathcal{H}_{\rm pen} = \frac{iG_F \alpha_s(\bar{m})\Delta}{12\sqrt{2}\,\pi} s_{23}s_{13}c_{23}\cos 2\theta_{12}\sin\delta \ln\frac{m_t^2}{m_c^2}\,\bar{s}\gamma_\mu(1-\gamma^5)\lambda^a d \sum_{q=u,d}\bar{q}\gamma^\mu\lambda^a q \tag{60}$$

Note that the $\mathcal{H}_{\rm pen}$ seems to be obtained for $m_c \simeq m_t < m_W$. With these interactions, the neutron EDM was estimated as

$$d_n = d_n^{short} + d_n^{long} \simeq 10^{-32}\,\text{e cm}\,. \tag{61}$$

Here the first is the contribution from Fig. 3 ($O(\alpha_s G_F^2) \;\approx\; 10^{-34}$ ecm) and the second from Fig. 4 [30].

If we incorporate the rephasing invariance of strange wave function, this value is modified to $1.4 \times 10^{-31} \leq |d_n| \leq 9.9 \times 10^{-33}$ e cm [31].

There are new CP violating five and six dimensional operators,

$$\mathcal{L}_{CPV} = \sum_q d_q \overline{q}(\sigma F)\gamma_5 q + \sum_q \tilde{d}_q \overline{q}(\sigma G)\gamma_5 q + wGG\tilde{G} + ... \tag{62}$$

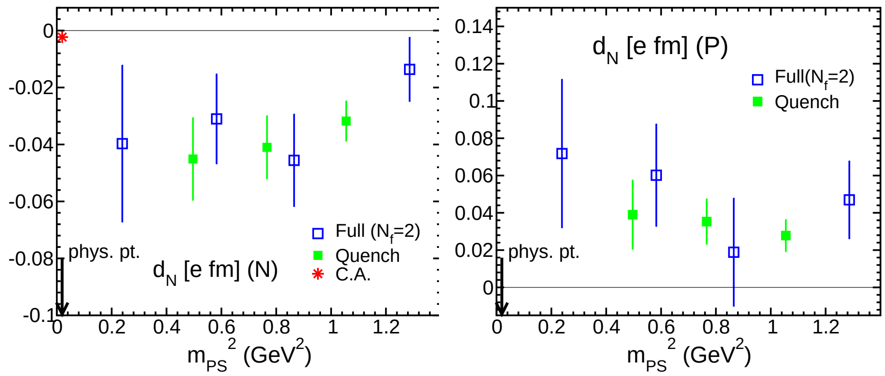


FIG. 5: the EDM as a function of the pseudoscalar meson mass squared m_{PS}^2 for neutrons (left pannel) and protons (right pannel). The arrow shows the physical point of the pion mass squared, $m_\pi^2 = 0.0195\text{GeV}^2$, and the star symbol denotes the the result of the current algebra (C.A.) [32].

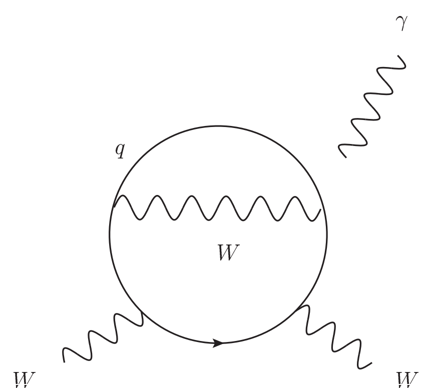


FIG. 6: Diagram for the EDM of W boson at two loop in the SM.

The detail of this contribution will be discussed in the section of diamagnetic atom.

The concrete and model independent calculations are expected in the lattice QCD. We will list only their results. It is also possible to guess the rough estimate of hadron EDM using SU(6). The detailed discussion is given in Appendix **A**. The EDM of neutron is estimated by lattice calculation (see Fig. 5).

C. Lepton EDM

For definitteness, let us concentrate on electron. Similarly to quark EDM, one and two loop diagrams do not contribute to electron EDM. In order to avoid GIM cancelation

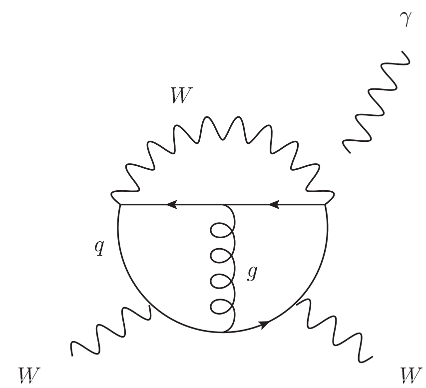


FIG. 7: Three loop diagram which may give a nonzero contribution to the EDM of W boson.

($\propto (m_i^2 - m_j^2)/m_W^2$), CKM matrix is better to be used than Maki-Nakagawa-Sakata matrix (lepton mixing). Two W bosons (at least) should attached to the electron line in order to use a quark loop. Then, the electron EDM is caused by the W boson EDM. It was shown that the W boson EDM vanishes at two loop level [33].

The two loop diagram is shown in Fig. 6.

J_{CP} defined by (14) is antisymmetric under $j \leftrightarrow l$ (54) (corresponding to side line's quarks), whereas it is symmetric in Fig.5. Adding aother loop with gluon (see Fig. 7), the W boson EDM in three loop was estimated as

$$d_W \simeq J_{CP} \left(\frac{1}{16\pi^2}\right)^2 \left(\frac{g^2}{8}\right)^2 \frac{\alpha_s}{4\pi} \frac{\mathrm{e}}{2m_W} \simeq 8 \times 10^{-30}\,\mathrm{e\ cm}\,. \tag{63}$$

The electron EDM in four loop was estimated with d_W as

$$d_e \simeq \frac{g^2}{32\pi^2} \frac{m_e}{m_W} d_W \simeq 8 \times 10^{-41}\,\mathrm{e\ cm}\,. \tag{64}$$

IV. BEYOND THE STANDARD MODEL

In this section we discuss models beyond the SM.

As we have shown in the previous section, the SM predicts rather smaller values of EDMs than the experimental upper limits by roughly ten orders of magnitude. However, we also know that CP violation in the SM is insufficient for baryon asymmetry in the real world. Also we have many direct signals of new physics beyond the SM like neutrino oscillations and muon g-2 etc. Even if we stand in the SM, we have new 4- and 6-dimensional CP violating effective actions like (39) to (42), which have never been discussed so much in the previous section. Then it is very natural to estimate how much such new physics or new models predict the EDMs. In this section we concentrate on new physics beyond the SM. As for the new 4- and 6-dimensional CP violating effective actions, we will discuss in the sections of diamagnetic atoms and molecules.

A. Minimal Supersymmetric Standard Model

In the MSSM, all particles have their SUSY partners; sfermions $\tilde{f}$ (bosons) for fermions fs, Higgsinos (fermions) for Higgs bosons, gauginos (fermions) for gauge bosons, and and another Higgs doublet is added to the SM for recovering the chiral anomaly free condition once broken by this doubling [34]. Yukawa coupling is given by

$$W_{MSSM} = Y_u \overline{u} Q H_u - Y_d \overline{d} Q H_d - Y_e \overline{e} L H_d + \mu H_u H_d \tag{65}$$

SUSY is broken at O(1TeV) by the soft SUSY breaking term which retain hierarchy problem. MSSM is the minimally extended supersymmetric SM and we will consider below cMSSM and νMSSM including light neutrino mass. In general soft breaking terms are

$$\begin{aligned} L_{SB}^{(1)} &= \mu_{Qij}^2 \tilde{Q}_{Li}^\dagger \tilde{Q}_{Lj} + \mu_{uij}^2 \tilde{u}_{Ri}^* \tilde{u}_{Rj} + \mu_{dij}^2 \tilde{d}_{Ri}^* \tilde{d}_{Rj} + \mu_{Lij}^2 \tilde{L}_{Ri}^\dagger \tilde{L}_{Rj} \\ &+ \mu_{eij}^2 \tilde{e}_{Ri}^* \tilde{e}_{Rj} + \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \mu_S^2 S^* S \\ &+ A_{uij} \tilde{u}_{Ri}^* \tilde{Q}_{Li} H_1 + A_{dij} \tilde{d}_{Ri}^* \tilde{Q}_{Li} H_2 + A_{eij} \tilde{e}_{Ri}^* \tilde{Q}_{Li} H_2 \end{aligned} \tag{66}$$

$$L_{SB}^{(2)} = \frac{1}{2}\left(M_3 \tilde{g}^a \tilde{g}^a + M_2 \tilde{W}^a \tilde{W}^a + M_1 \tilde{B}^a \tilde{B}^a + c.c.\right) \tag{67}$$

These include many CP violating phases, in general.

For the loop correction to the Higgs masses, the problem on its quadratic divergence is cancelled by the loop of SUSY partners (different statistics with same coupling).

SUSY particles contribute to fermion EDM at one loop shown in Fig. 8. The neutralinos $\tilde{\chi}^0$ and charginos $\tilde{\chi}^\pm$ are mass eigenstates, and they are linear combinations of Higgsinos and gauginos of $SU(2)_L$ and $U(1)_Y$. The d quark EDM from the diagram is estimated [35] as

$$d_e/e = \frac{1}{2m}(-\frac{1}{3})\frac{4}{3}\frac{\alpha_s}{\pi} v \Im (V_R^{d\dagger} A_D V_L^d)_{11} \frac{\mu m}{(M^2-\mu^2)^2}\left(\frac{1}{2} + 3\frac{\mu^2}{M^2-\mu^2} - \frac{\mu^2(\mu^2+2M^2)}{(M^2-\mu^2)^2} ln \frac{M^2}{\mu^2}\right). \tag{68}$$

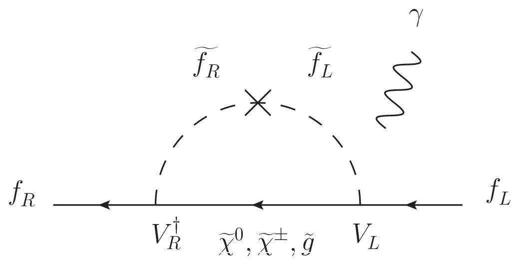


FIG. 8: Diagrams of SUSY contributions to the EDM of fermions. Gluino $\tilde{g}$ contributes only for quark EDM.

Here v is the common vacuum expectation value of H_1 and H_2. m and M are masses of d quark and the universal squark mass (by assumtion), respectively. If we adopt $v\Im(V_R^{d\dagger} A_D V_L^d) \sim M^2$ with rough estimations of $M = 100$ GeV and maximal mixings, we obtain

$$d_d \sim 10^{-22}\,\mathrm{e\ cm}\,. \tag{69}$$

The value is clearly in conflict with the experimental bound on d_n. The naive estimation was done with $M_{\tilde{q}} \simeq 100\,\mathrm{GeV}$ and a sizable CP-violating phase $\sin\phi \sim 1$. Therefore, the contradiction can be resolved by small phase (approximate CP symmetry) and/or heavy masses of SUSY particles. However, we adopt not such fine tuning but the universal soft SUSY breaking (constrained MSSM; cMSSM). $V_{L(R)}$ is the unitary matrix which rotate left (right)-handed weak eigen state, and therefore if $A_{ij} \propto Y_{ij}$, $V_R^{d\dagger} A_D V_L^d$ is real diagonal and the imaginary part of it vanishes. Thus the small EDM leads us to the relations of trilinear terms in (65) and (67)

$$A_u = A_{u0} Y_u, \quad A_d = A_{d0} Y_d, \quad A_e = A_{e0} Y_e. \tag{70}$$

Also we accept the universal soft SUSY breaking which is realized by gravity or gauge mediated SUSY breaking.

$$M_1, M_2, M_3 \sim m_{1/2} \tag{71}$$

$$m_Q, m_L, m_u, m_d, m_e, m_{H_1}, m_{H_2} \sim m_0. \tag{72}$$

CP-violating phases appear only in flavor off-diagonal parts of matrices (hermitian Yukawa

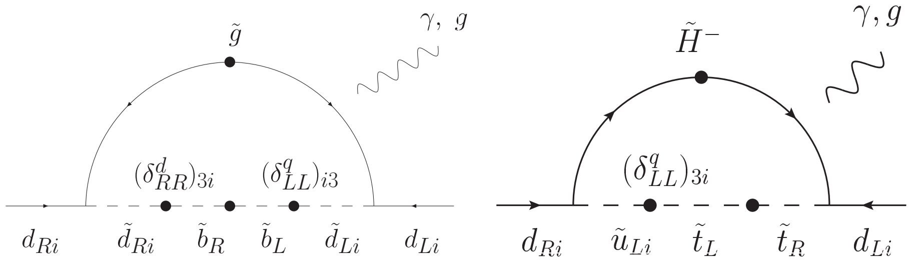


FIG. 9: Diagrams of SUSY contributions to the EDM of fermions in case of flavored CP violation where CP violating phase appears with change of flavor [36].

matrices) and the CP violating effect is suppressed by small mixings only due to RGE.

$$\begin{aligned} \delta^q_{LL} &= \frac{(m^2_{\tilde{q}})_{ij}}{m^2_{\tilde{q}}}, \quad \delta^u_{RR} = \frac{(m^2_{\tilde{u}})_{ij}}{m^2_{\tilde{u}}}, \quad \delta^d_{RR} = \frac{(m^2_{\tilde{d}})_{ij}}{m^2_{\tilde{d}}}, \\ \delta^l_{RR} &= \frac{(m^2_{\tilde{l}})_{ij}}{m^2_{\tilde{l}}}, \quad \delta^e_{RR} = \frac{(m^2_{\tilde{e}})_{ij}}{m^2_{\tilde{e}}}. \end{aligned} \tag{73}$$

The diagrams for the flavored case are shown in Fig. 9. When both left- and right-handed squarks (sleptons) have mixings, they contribute to the EDM in the form:

$$J^{(d_i)}_{LR} = \Im\{\delta^d_{RR} y_d \delta^q_{LL}\}_{ii}, \quad J^{(u_i)}_{LR} = \Im\{\delta^u_{RR} y_u \delta^q_{LL}\}_{ii}. \tag{74}$$

It was shown (see e.g. [36]) that d_d can be $\sim 10^{-25}$-10^{-26} e cm, and then SUSY parameters are constrained by hadronic EDM.

Also there are additional diagrams called Barr-Zee diagrams that contribute to the EDM beyond one loop level (see Fig.10).

Thus the MSSM gives an elegant backgroud but itself does not predict any definite relations between quarks and leptons including all observations in neutrino.

In other word, its predictions are not testified from many constraints from various already known observations. These observations must be complicatedly related in reliable models.

It is a Grand Unified Theory (GUT) which fulfiles these deficiencies [39].

B. Minimal Supersymmetric SU(5) GUT

Here and hereafter "minimal" means the minimum number of Higgs field with renormalizable Yukawa coupling.

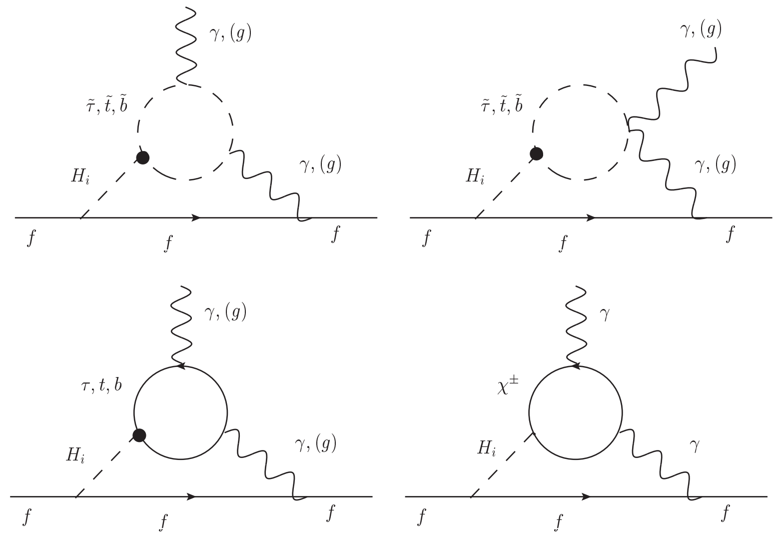


FIG. 10: Barr-Zee diagrams: the H_i lines denote all three neutral Higgs bosons, including CP-violating Higgs-boson mixing, and heavy dots indicate resummation of threshold corrections to the corresponding Yukawa couplings [37].

In SU(5) model [38], matter multiplets in (52) are classified to

$$\mathbf{5}^* = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu \end{pmatrix}_L \qquad \mathbf{10} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}_L . \tag{75}$$

We need two Higgs $\mathbf{5}_H^*$ and $\mathbf{24}_H$, and SU(5) breaks down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ by

$$\mathbf{24}_H = \mathrm{diag}\left(V, V, V, -\frac{3}{2}V, -\frac{3}{2}V\right). \tag{76}$$

Here $\mathbf{5}$ and $\mathbf{10}$ are broken to

$$\mathbf{5} = (1,2)(1/2) + (3,1)(-1/3)), \quad \mathbf{10} = (1,1)(1) + (\overline{3},1)(-2/3) + (3,2)(1/6). \tag{77}$$

Then, $SU(3)_c \times SU(2)_L \times U(1)_Y$ breaks down to $SU(3) \times U(1)_Q$ via

$$\mathbf{5}^* = (0, 0, 0, 0, v/\sqrt{2}). \tag{78}$$

Yukawa coupling has the form,

$$W = \frac{1}{4} f_{ij}^u \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H + \sqrt{2} f_{ij}^d \mathbf{10}_i \mathbf{5}^*{}_j 5_H^* + f_{ij}^\nu \mathbf{5}_i^* \mathbf{1}_j \mathbf{5}_H + M_{ij} \mathbf{1}_i \mathbf{1}_j . \tag{79}$$

Here the products imply

$$\begin{aligned} \mathbf{10}_i\mathbf{10}_j\mathbf{5}_H &= \epsilon_{abcde}\mathbf{10}_i^{ab}\mathbf{10}_j^{cd}\mathbf{5}_H^e \\ \mathbf{10}_i\mathbf{5}^*{}_j\mathbf{5}_H^* &= \mathbf{10}_i^{ab}\mathbf{5}_j^{*a}\mathbf{5}_H^{*b} \ \text{ etc.} \end{aligned} \tag{80}$$

with $a, b = 1, ..., 5$. Then mass matrices have the following forms

$$M^d = M^e = f^d v/\sqrt{2}, \quad M^u = f^u v/\sqrt{2} \tag{81}$$

at GUT scale. This gives nice $b - \tau$ unification. The disparity between their observed masses is supposed to be due to renormalization effect from M_{GUT} to their mass shell. Unfortunately, we can not explain the disparities between the first and second families even if we renormalization effects since it predicts wrong relation

$$m_d/m_s = m_e/m_\mu, \quad m_s/m_b = m_\mu/m_\tau. \tag{82}$$

It also predicts too fast proton decay [40]. Hadronic EDM in SUSY SU(5) was discussed in [41].

Flipped SU(5) changes

$$u^c \leftrightarrow d^c, \quad e^c \leftrightarrow \nu^c \tag{83}$$

and, therefore, we obtain in place of (81)

$$M_u = M_\nu. \tag{84}$$

This does not leads to apparent pathology. Moreover, it is attractive from Doublet-Triplet splitting: Higgs super potential has the form

$$W_H = \mathbf{10} \times \mathbf{10} \times \mathbf{5} + \overline{\mathbf{10}} \times \overline{\mathbf{10}} \times \overline{\mathbf{5}}, \tag{85}$$

which give rise to triplet mass

$$\langle(1,1,0)_{10}\rangle(\overline{3},1;1/3)_{10}(3,1:-1/3)_5 + \langle(1,1;0)_{\overline{10}}\rangle(3,1;-1/3)_{\overline{10}}(\overline{3},1;1/3)_{\overline{5}} \tag{86}$$

but has no doublet mass since $5 + \overline{5}$ has no partner in $10 + \overline{10}$ (Missing partner mechanism). This is a solution to the Doulet-Triplet problem without additional adjoint Higgs. However, flipped SU(5) drives us to unrenormalizable heavy Majorana neutrino mass term,

$$\mathbf{10}_i\mathbf{10}_j\overline{\mathbf{10}}_H\overline{\mathbf{10}}_H/\Lambda \tag{87}$$

for seesaw mechanism.

The other approaches are to introduce unrenormalizable term [42],

$$\begin{aligned} W_Y &= \epsilon_{abcde}\left(f_{1ij}\mathbf{10}_i^{ab}\mathbf{10}_j^{cd}\mathbf{24}_{Hf}^{e}\mathbf{5}_H^{f} + f2ij\mathbf{10}_i^{ab}\mathbf{10}_j^{cf}\mathbf{24}_{Hf}^{d}\mathbf{5}_H^{e}\right)/\Lambda \\ &+ g_{1ij}\mathbf{5}_{Ha}^{*}\mathbf{24}a_{Hb}\mathbf{10}_i^{bc}\mathbf{5}_{jc}^{*}/\Lambda + g_{2ij}\mathbf{5}_{Ha}^{*}\mathbf{10}_i^{ab}\mathbf{24}_{Hc}^{b}\mathbf{5}_{jc}^{*}/\Lambda \\ &+ \Delta Y_5\frac{\langle\mathbf{24}_H\rangle}{\Lambda}\mathbf{5}_i^{*}\mathbf{10}_j\mathbf{5}_H^{*} \end{aligned} \tag{88}$$

or to add another Higgs in Yukawa coupling,

$$Y_{45}\mathbf{5}_i^{*}\mathbf{10}_j\mathbf{45}_H^{*} \tag{89}$$

etc. Unfortunately in SU(5) model, right-handed heavy Majorana neutrino belongs to the singlet and we have no constraint on it. Usually it is assumed to be diagonal but there is no reason to be justified for that. The other undetermined parameter dependence (like $m_0.M_{1/2}, A_0, tan\beta$) crucially depends on this assumption.

These points are remedied in case of renormalizable SO(10) GUT, which is discussed in the next subsection (cEDM and parity odd nuclear interaction).

C. Minimal Supersymmetric SO(10) GUT

In the SO(10) Grand Unified Theory [43], fermions belong to a multiplet of **16** representation as

$$\psi \equiv \left(u_R^r, u_R^g, u_R^b, d_R^r, d_R^g, d_R^b, e_R, \nu_R, u_L^r, u_L^g, u_L^b, d_L^r, d_L^g, d_L^b, e_L, \nu_L,\right)^T . \tag{90}$$

Note that the right-handed neutrino ν_R is included naturally.

So-called minimal renormalizable SO(10) model includes Higgs bosons of **10** and $\overline{\mathbf{126}}$ in Yukawa couplings. This is because

$$\mathbf{16}\times\mathbf{16} = \mathbf{10}+\mathbf{120}+\mathbf{126}. \tag{91}$$

In order to make singlet in Yukawa renormalizable coupling, therefore, Higgs can be $\mathbf{10}, \mathbf{120}, \overline{\mathbf{126}}$. [3] One Yukawa coupling leads to the conclusion that the CKM mass matrix is unity and we needs at least (minimal) two Higgs, $\mathbf{10}+\mathbf{120}$ or $\mathbf{10}+\overline{\mathbf{126}}$. We select

[3] If we relax the renormalizability , different SO(10) models are also possible [44]. However, in this case, we have much less predictivity

the latter set. This is because

$$\overline{\mathbf{126}} = (6,1,1) + (\overline{10},1,3) + (10,3,1) + (15,2,2) \tag{92}$$

under $SU(4)_c \times SU(2)_L \times SU(2)_R$ and the second and third terms play essential role in type I and type II seesaw, respectively. In its SUSY version [45], **126** is necessary to be added. Providing the Higgs VEVs, $H_u = v \sin\beta$ and $H_d = v\cos\beta$ with $v = 174$GeV, the quark and lepton mass matrices can be read off as

$$\begin{aligned} M_u &= c_{10} M_{10} + c_{126} M_{126} \\ M_d &= M_{10} + M_{126} \\ M_D &= c_{10} M_{10} - 3c_{126} M_{126} \\ M_e &= M_{10} - 3M_{126} \\ M_T &= c_T M_{126} \\ M_R &= c_R M_{126} \ , \end{aligned} \tag{93}$$

where M_u, M_d, M_D, M_e, M_T, and M_R denote the up-type quark, down-type quark, Dirac neutrino, charged-lepton, left-handed Majorana, and right-handed Majorana neutrino mass matrices, respectively. Note that all the quark and lepton mass matrices are characterized by only two basic mass matrices, M_{10} and M_{126}, and four complex coefficients c_{10}, c_{126}, c_T and c_R, which are defined as $M_{10} = Y_{10}\alpha^d v\cos\beta$, $M_{126} = Y_{126}\beta^d v \cos\beta$, $c_{10} = (\alpha^u/\alpha^d)\tan\beta$, $c_{126} = (\beta^u/\beta^d)\tan\beta$, $c_T = v_T/(\beta^d v\cos\beta))$ and $c_R = v_R/(\beta^d v \cos\beta))$, respectively. These are the mass matrix relations required by the minimal SO(10) model. The model is very predictive by virtue of the relation between quark Yukawa matrix, lepton Yukawa matrix, and neutrino Majorana matrix. Fig. 11 [45] shows the prediction for the electron EDM $|d_e|$ in the minimal SUSY SO(10) with respect to the universal gaugino mass $M_{1/2}$. The muon EDM $|d_\mu|$ exists above $|d_e|$ by roughly a factor of 10^2. The muon anomalous MDM a_μ and the decay branching ratio of $\mu \to e\gamma$ are also predicted (see Fig. 11).

The effective Lagrangian relevant for the EDM, MDM, and the LFV processes ($\ell_i \to \ell_j\gamma$) is described in (35). $R, L = (1 \pm \gamma_5)/2$ is the chirality projection

$$\mathcal{L}_{\text{eff}} = -i\frac{e}{2} m_{\ell_i} \overline{\ell}_j \sigma_{\mu\nu} F^{\mu\nu} \left(A_L^{ji} P_L + A_R^{ji} P_R \right) \ell_i \ , \tag{94}$$

where $P_{L,R}$ are Left-Right projection operators, and $A_{L,R}$ the photon-penguin couplings of 1-loop diagrams in which chargino-sneutrino and neutralino-charged slepton are propagating.

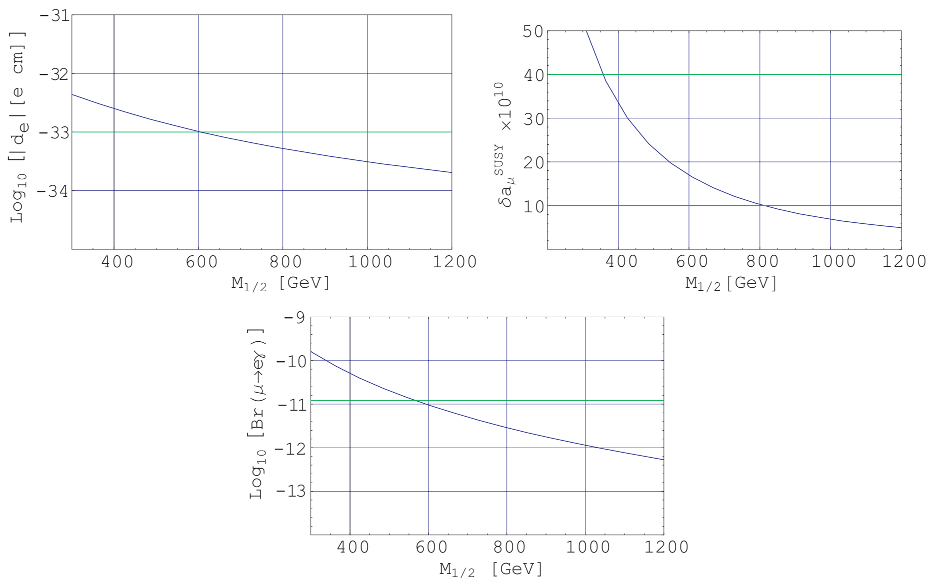


FIG. 11: The predictions for the electron EDM $|d_e|$, the muon anomalous MDM δa_μ (50), and the decay branching ratio of $\mu \to e\gamma$ n the minimal SUSY SO(10) with respect to the universal gaugino mass $M_{1/2}$ [45].

it should be noted that we have changed the normalization of $A_{L,R}$ from (35) by $\frac{em_l}{2}$. The explicit formulas of $A_{L,R}$ etc. used in our analysis are summarized in [45] [46]. If the diagonal components of $A_{L,R}$ have imaginary parts, the EDMs of the charged leptons are given by

$$d_{l_i}/e = -m_{l_i} \Im (A_L^{ii} - A_R^{ii}) \tag{95}$$

in the new normalization. The rate of the LFV decay of charged-leptons is given by

$$\Gamma(\ell_i \to \ell_j \gamma) = \frac{e^2}{16\pi} m_{\ell_i}^5 \left(|A_L^{ji}|^2 + |A_R^{ji}|^2 \right) \ , \tag{96}$$

while the real diagonal components of $A_{L,R}$ contribute to the anomalous magnetic moments of the charged-leptons such as

$$\delta a_{\ell_i}^{SUSY} = \frac{g_{\ell_i} - 2}{2} = -m_{\ell_i}^2 \Re \left[A_L^{ii} + A_R^{ii} \right] \ . \tag{97}$$

In order to clarify the parameter dependence of the decay amplitude, we give here an approximate formula of the LFV decay rate [46],

$$\Gamma(\ell_i \to \ell_j \gamma) \sim \frac{e^2}{16\pi} m_{\ell_i}^5 \times \frac{\alpha_2}{16\pi^2} \frac{|\left(\Delta m_{\tilde{\ell}}^2\right)_{ij}|^2}{M_S^8} \tan^2\beta \ , \tag{98}$$

where M_S is the average slepton mass at the electroweak scale, and $\left(\Delta m_{\tilde{\ell}}^2\right)_{ij}$ is the slepton mass estimated in Eq. (99). We can see that the neutrino Dirac Yukawa coupling matrix plays the crucial role in calculations of the LFV processes. We use the neutrino Dirac Yukawa coupling matrix of Eq. (100) in our numerical calculations. In the leading-logarithmic approximation, the off-diagonal components ($i \neq j$) of the left-handed slepton mass matrix are estimated as

$$\left(\Delta m_{\tilde{\ell}}^2\right)_{ij} \sim -\frac{3m_0^2 + A_0^2}{8\pi^2}\left(Y_\nu^\dagger L Y_\nu\right)_{ij} \;, \tag{99}$$

where the distinct thresholds of the right-handed Majorana neutrinos are taken into account by the matrix $L = \log[M_G/M_{R_i}]\delta_{ij}$.

Unlike the muon MDM, quark and lepton EDM has still null observation. This is of course due to tiny CP violation and due to the cancellation of diagrams where γ (gluon) couple with slepton (squark) and where they do with Higgsino (gluino) in (Fig.9) [47].

If we consider gauge mediation scenario for SUSY breaking, $A_0 \approx 0$. In the basis where both of the charged-lepton and right-handed Majorana neutrino mass matrices are diagonal with real and positive eigenvalues, the neutrino Dirac Yukawa coupling matrix at the GUT scale is found to be [48]

$$Y_\nu = \begin{pmatrix} -0.000135 - 0.00273i & 0.00113 + 0.0136i & 0.0339 + 0.0580i \\ 0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\ -0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i \end{pmatrix} . \tag{100}$$

Semi-Leptonic LFV processes was discussed in [49].

Thus the EDM, MDM, lepton flavor violations etc. are all closely connected, which are expected to be explained universally by GUT. However, we do not adhere to the special model in this review and also discuss more phenomenological models in the following subsections.

These may be the remnants from GUT or may be independent on GUT. For instance, the adjoint representation of SO(10), **45** is decomposed into

$$\mathbf{45} = (1,3,1) + (3,1,1) + (15,1,1) + (6,2,2) \tag{101}$$

under $SU(4)_c \times SU(2)_L \times SU(2)_R$ and leads to Left-Right symmetric model, $g_L = g_R$. Also **10** representation is decomposed into

$$\mathbf{10} = (1,2,2) + (6,1,1), \tag{102}$$

which leads us to two Higgs $SU(2)_L$ doublets under the SM. Also $\overline{\mathbf{126}}$ is

$$\overline{\mathbf{126}} = (6,1,1) + (10,3,1) + (\overline{10},1,3) + (15,2,2). \tag{103}$$

If $(\overline{10},1,3)$ ($(10,3,1)$) has vev, it gives type I (type II, or Higgs triplet Model) seesaw model.

In the following we consider these models independently on GUT principally. [4]

D. Two Higgs Doublet Model

Most of models beyond the SM has some new Higgs bosons. As the simplest extension of the Higgs sector of the SM which has only one Higgs doublet H_1, another Higgs doublet H_2 is introduced in the Two Higgs Doublet Model [51]. There are several types of the model depending on which doublet couples with which fermion:

type I (SM-like)	:	H_1 couples with all fermions
		H_2 decouples with fermions
type II (MSSM-like)	:	H_1 couples with down-type quarks and charged leptons
		H_2 couples with up-type quarks
type III (general)	:	both of Higgs doublets couple with all fermions
etc. etc.		

If CP-violating term exists in the Higgs potential, e.g. $(H_1^\dagger H_1)(H_1^\dagger H_2)$ with an imaginary coefficient, there appears the mixing between CP-even (H^0) and CP-odd (A^0) neutral Higgs bosons. Then these Higgs bosons can contribute to the EDM (see Fig.12). The mixing between H^0 and A^0 provides also CP-violating electron-nucleon effective interactions ($\bar{e}i\gamma^5 e\bar{N}N$, etc.) which will contribute to the atomic EDM.

Barger et al. [52] gave large d_μ close to the propose experiments. It should be remarked that their values are calculated in units of $\Im Z$ where

$$\text{Amplitude}(q^2) = \sum_n \frac{\sqrt{2}G_F Z}{q^2 + m_{H^n}^2}, \tag{104}$$

and it is probable that $|\Im Z| \approx 0$. Indeed, the masses of neutral and charged Higgses and phases are tightly constrained from $R_b \equiv \frac{\Gamma(Z\to b\bar{b})}{\Gamma(Z\to hadrons)}$, $\Gamma(b\to s\gamma)$, $\overline{B}^0 - B$ mixing, ρ parameter etc., and we should take those constraints into account [53].

[4] He et al. discussed neutron EDM in those models [50].

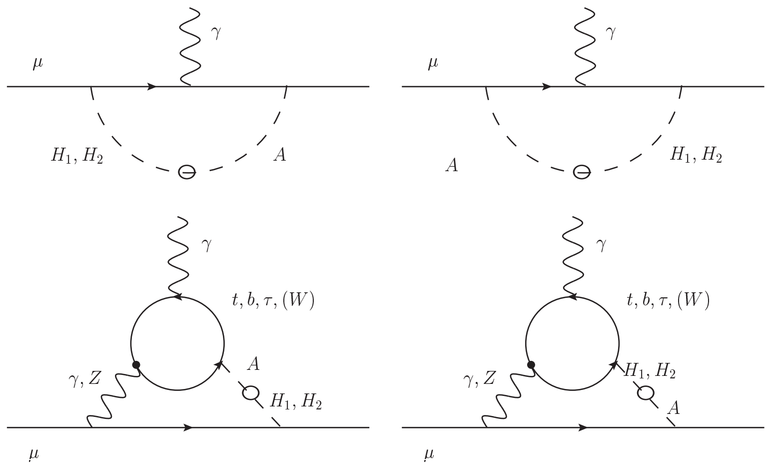


FIG. 12: Diagrams of muon EDM in two Higgs doublet model [52].

E. Higgs Triplet Model

In the Higgs Triplet Model [54], we introduce a SU(2) triplet $Y = 2$ scalar as

$$\Delta \equiv \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \quad \mathcal{L}_{tripletYukawa} = -h_{\alpha\beta}\overline{L_\alpha^C} i\sigma^2 \Delta P_L L_\beta + h.c. \tag{105}$$

This model generates neutrino masses without right-handed neutrinos with the triplet vacuum expectation value v_Δ which is given by the explicit breaking of the lepton number. This model is very predictive because of a clear relation

$$m_{\alpha\beta} = \sqrt{2} v_\Delta h_{\alpha\beta}, \tag{106}$$

where $m_{\alpha\beta}$ denotes the Majorana mass matrix for neutrinos. There is no new interaction with quarks and no effect on quark EDM. Unfortunately, this model can not give a large contribution to lepton EDM also because of the absence of the new interaction with right-handed fermions. For example, one loop diagram for electron has a factor of $|h_{\alpha e}|^2$ (similarly to Fig. 1).

F. Left-Right Symmetric Model

Left-Right model [55] is used in a varieties of ways and needed to be clarified. If we consider it as a remnants from SO(10), $SO(10) \to SU(4)_c \times SU(2)_L \times SU(2)_R$, it satisfies at v_{PS} energy scale

$$g_L = g_R \tag{107}$$

and PS model is unified at M_{GUT} as

$$\frac{M_4}{\alpha_4} = \frac{M_{2L}}{\alpha_{2L}} = \frac{M_{2LR}}{\alpha_{2R}} = \frac{M_{1/2}}{\alpha_{GUT}}. \tag{108}$$

Also mixing matrices of left-handed and right-handed fermions are same. Of course these constraints are realized at v_{PS} but deviate from as energy goes down to the SM scale by renormalization effects.

However, if we consider a model of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, we are free from the above constraints. For instance, in the framework of SO(10) GUT, v_R is order of $O(10^{12})$GeV.However if we go apart from GUT but still consider that the mixing matrix of right-handed quarks V_R has a similar structure as that of left-handed quarks V_L, the lower limit of M_{WR} is relaxed to $M_{WR} > 1.6$ TeV [56] Moreover we may go beyond restricting on V-A and V+A interactions only and may consider general form;

$$\begin{aligned} L_{\mu\to e\nu\overline{\nu}} &= -\frac{4G_F}{\sqrt{2}}\Big[g^S_{RR}(\overline{e}_R\nu_{eL})(\overline{\nu_\mu}_L\mu_R) + g^S_{RL}(\overline{e}_R\nu_{eL})(\overline{\nu_\mu}_R\mu_L) \\ &+ g^S_{LR}(\overline{e}_L\nu_{eR})(\overline{\nu_\mu}_L\mu_R) + g^S_{LL}(\overline{e}_L\nu_{eR})(\overline{\nu_\mu}_R\mu_L) \\ &+ g^V_{RR}(\overline{e}_R\gamma^\mu\nu_{eR})(\overline{\nu_\mu}_R\gamma_\mu\mu_R) + g^V_{RL}(\overline{e}_L\gamma^\mu\nu_{eL})(\overline{\nu_\mu}_L\gamma_\mu\mu_L) \\ &+ g^V_{LR}(\overline{e}_L\gamma^\mu\nu_{eL})(\overline{\nu_\mu}_R\gamma_\mu\mu_R) + g^V_{LL}(\overline{e}_L\gamma^\mu\nu_{eL})(\overline{\nu_\mu}_R\gamma_\mu\mu_R) \\ &+ \frac{g^T_{RL}}{2}(\overline{e}_R\sigma^{\mu\nu}\nu_{eL})(\overline{\nu_\mu}_R\sigma_{\mu\nu}\mu_L) + \frac{g^T_{LR}}{2}(\overline{e}_L\sigma_{\mu\nu}\nu_{eR})(\overline{\nu_\mu}_L\sigma_{\mu\nu}\mu_R) + H.c.\Big] \end{aligned} \tag{109}$$

The Left-Right Symmetric Model (the LR Model) is an extension of the SM to restore the parity symmetry in the original Lagrangian. The LR Model is based on the gauge group of $SU(3)\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$. The charge of $U(1)$ in the LR model has a clear meaning as the difference between the baryon number B and the lepton number L in contrast with the mysterious hypercharge Y in the SM. Similarly to $SU(2)_L$ doublet in the SM, the right-handed fermions compose doublet of $SU(2)_R$ in the LR Model. Therefore, the right-handed neutrinos ν_R are introduced naturally as $SU(2)_R$ partners of right-handed charged leptons. After the spontaneous breaking of $SU(2)_R\otimes U(1)_{B-L}$ to $U(1)_Y$, the hypercharge is given by

$$Y/2 = I_{3R} + (B-L)/2. \tag{110}$$

Since electric charge is connected by

$$Q = I_{3L} + Y/2, \tag{111}$$

(110) implies the charge quantization, which is one of great achievements of [55].

Since we require the parity symmetry to the theory, the gauge coupling of $SU(2)_R$ must be the same as the one of $SU(2)_L$: $g_2 \equiv g_{2L} = g_{2R}$. The Higgs field which gives the Yukawa terms is a complex bidoublet of $SU(2)_L \otimes SU(2)_R$ with $B - L = 0$. The bidoublet field can be expressed as

$$\Phi \equiv \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \tag{112}$$

which transform as $\Phi \to \Phi' = U_L \Phi U_R^\dagger$ under $SU(2)_L$ and $SU(2)_R$.

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \tag{113}$$

with $\kappa \neq \kappa'$ gives rise to the breaking of L-R symmetry. However, Φ is neutral (B-L=0) and $U(1)_{B-L}$ is not broken. So we need another Higgs. Usually, two complex triplet fields (Δ_L for $SU(2)_L$ and Δ_R for $SU(2)_R$) with $B - L = 2$ are also introduced to generate Majorana neutrino masses (see also the Higgs Triplet Model in Sect. IV E).

$$\Delta_L = (3, 1, 2) \quad \Delta_R = (1, 3, 2) \tag{114}$$

Then, the gauge symmetry breaking proceeds as follows: first Δ_R^0 acquires vev v_R, leading to $SU(2)_L \times U(1)_Y$ with (110), which furthermore breaks to $U(1)_Q$ by the vev of Φ.

The triplet Yukawa coupling for Δ_R must be equal to the coupling for Δ_L because of the parity symmetry which is spontaneously broken by v_R. Thus we have

$$v_R \gg \kappa, \ \kappa' \gg v_L \tag{115}$$

Thus the two Higgs doublets model (not of all but its measure part) and Higgs triplet model in the previous subsections are combined together in left-right model.

Their vevs v_R and v_L are different to each other and from κ and κ'.

Figure 13 shows one of diagrams which contribute to the EDM of the electron. In a simple case where there is only one flavor of leptons, the electron EDM is estimated [57] as

$$|d_e| < \begin{cases} 8.2 \times 10^{-27} \dfrac{|\mathrm{Im}(m_D)|}{\mathrm{MeV}} \text{ e cm} & \text{for } \left(\dfrac{m_R}{m_W}\right)^2 \gg 1, \\ 3.3 \times 10^{-26} \dfrac{|\mathrm{Im}(m_D)|}{\mathrm{MeV}} \text{ e cm} & \text{for } \left(\dfrac{m_R}{m_W}\right)^2 \ll 1, \end{cases} \tag{116}$$

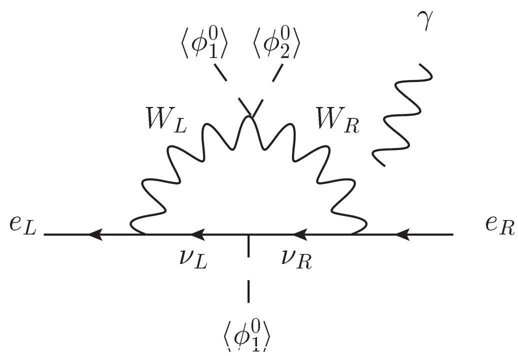


FIG. 13: One of diagrams of gauge boson contributions to the electron EDM in LR symmetric model. Gauge bosons contribute to quark EDM also.

where m_D denotes the Dirac mass of the neutrino ($m_D\overline{\nu_L}\nu_R$) and m_R, $m_W = 80\,\text{GeV}$ are the masses of heavy right-handed neutrino and the light W boson, respectively.

The contributions of Higgs bosons in the LR model are not significant [58].

Another important contribution of W_R in CP violation may be the neutrinoless double beta decay [59]

G. Fourth Family Model

We set the quarks of the fourth family [60] as

$$(t', b')^T. \tag{117}$$

The mixing angles and CP violating phases for N families are given by

$$N^2 - (2N-1) = \frac{N(N-1)}{2} + \frac{(N-1)(N-2)}{2}, \tag{118}$$

where the first term is mixing angles and the second CP phases (see Appendix **D** for Majorana fermion case).

If we consider 4-generation SM (SM4) [26], we can construct new Jarlskog parameter in place of (53)

$$A_{(234)} = (m_{t'}^2 - m_t^2)(m_{t'}^2 - m_c^2)(m_t^2 - m_c^2)(m_{b'}^2 - m_b^2)(m_{b'}^2 - m_s^2)(m_b^2 - m_s^2)J_{(234)}, \tag{119}$$

If we take heavy quark masses t' and b' in the range of 300 to 600 GeV, $A_{(234)}/T_{EW}^{12}$ can be of order n_B/n_γ.

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(10). \tag{120}$$

Thus SM4 enhances CP violation and therefore the EDM also.

We may consider inside the loop only $t,\ t',\ b'$ heavy and identify as

$$c = u = U, \quad d = s = b = D. \tag{121}$$

Then two loop diagram of Fig.2 vanishes for u quark but survives for d,s quarks, giving

$$d_d \approx \lambda^7 \frac{\alpha_s}{4\pi}\frac{\alpha_W}{4\pi}\frac{1}{16\pi^2}G_f m_d \frac{m_t^2}{m_W^2} \approx 3\times 10^{-32} ecm, \tag{122}$$

where λ is the Wolfenstein parameter, $\lambda = |V_{us}| = 0.22$. This is only two orders of magnitude larger than the SM result of section (III A 3). However, if we consider the chromoelectric dipole moment of the s quark,

$$\begin{aligned}\tilde{d}_s &\approx \Im(V_{ts}^* V_{tb} V_{hb}^* V_{hs})\frac{\alpha_s}{4\pi}\frac{\alpha_W}{4\pi}\frac{1}{16\pi^2}G_F m_s \frac{m_t^2}{m_s^2} \\ &\approx \lambda^5 \frac{\alpha_s}{4\pi}\frac{\alpha_W}{4\pi}\frac{1}{16\pi^2}G_F m_s \frac{m_t^2}{m_s^2}.\end{aligned} \tag{123}$$

Using the estimation of the relation between d_N and $\tilde{d}_s$ by [61], we have [26]

$$d_N \approx -\frac{1}{2}\tilde{d}_s \approx 5\times 10^{-30} \text{ e cm}. \tag{124}$$

H. Extra Dimensions

The motivations for extra dimensions are diverse for both SUSY and non-SUSY. There are many SUSY breaking scenarios. The extra dimension makes the geometrical SUSY breaking possible. The gauge supermultiplets propagate in the bulk, we get gaugino mass

$$M_a \approx \frac{\langle F\rangle}{M_5^2 R_5}, \tag{125}$$

which is called gaugino mediation.

Even if the theory itself is CP invariant, it may be violated by extending the theory to extra dimensions. This is because the compactification of the extra dimensions does not respect the symmetry in general. The CP phases come either from the boundary condition of extra dimensions (One of Scherk-Scwarz mechanisms [62]) or vev of fifth gauge field (Hosotani mechanism [63]) [64] [65].

$$L = i\overline{\psi}\gamma^N(\partial_N - ieA_N)\psi - M\overline{\psi}\psi. \tag{126}$$

Inclusion of a torsion term results in a nonminimal term

$$\kappa\overline{\psi}\sigma F\psi \tag{127}$$

or the Fermion mass term via Hosotani mechanism

$$\overline{\psi}(M + i\gamma_5 X_4)\psi \tag{128}$$

with

$$X_4 \equiv \int dy A_4. \tag{129}$$

Here y is the coordinate of the extra dimension. The rotation of mass term gives rise to

$$\kappa'\overline{\psi}\sigma F\gamma_5\psi. \tag{130}$$

The concrete constraints from the observation are given, for instance [66],

$$d_n = \frac{4}{3}d_d - \frac{1}{3}d_u = \frac{4}{3}d_d, \tag{131}$$

simply because they did not consider up quark and

$$d(KK) \sim -2.3 \times 10^{-23}(Rm_W)^2 \text{ [e cm]}. \tag{132}$$

Since $\frac{4}{3}d(KK)$ must be less than the experimental upper limit, we have

$$\frac{1}{R} > 33m_W \simeq 2.6\text{[TeV]}. \tag{133}$$

V. THE EDMS OF ATOMS

The origin of the difficulties of the measurement of electron EDM is due to the absence of resonance unlike the neutron. A possible way is to perform the resonance experiment on neutral atom and to interpret the result in terms of electron EDM or hadron EDM. These object has very tiny values and let's consider the effect linear in the EDM. In the subsequent atomic and molecular experiments we treat an internal electic field $\mathbf{E}_{int}$ induced by atom or molecule as well as an external elctric field $\mathbf{E}$. This $\mathbf{E}$ induces the EDM $e\mathbf{r}_i$ with the intrinsic $\sum_i \beta\boldsymbol{\sigma}_i \equiv \sum_i \mathbf{d}_e^i$. Thus the total Hamiltonian is a sum of unperturbative P,T-even term,

$$H_0 = \sum_i c\boldsymbol{\alpha}\cdot\boldsymbol{p}_i + \beta_i mc^2 + V_{nucl}(r_i) + \sum_{i<j} V_C(r_{ij}), \tag{134}$$

and T,P-odd term

$$H_{PTV} = -\sum_i \mathbf{d}_e^i \cdot \mathbf{E}_{int}^i - \sum_i \mathbf{d}_e^i \cdot \mathbf{E} - e\sum_i \mathbf{r}_i \cdot \mathbf{E}. \tag{135}$$

The last term of H_0 is a two-body interaction and can not be solved exactly.

The first and third terms are P-odd and the second P-even. So the first and second order energy shift are given by

$$E_m^1 = -\sum_i \langle m_0|\mathbf{d}_e^i|m_0\rangle \cdot \mathbf{E} \tag{136}$$

and

$$E_m^2 = \sum_{n\neq m}\sum_i \left\{ \frac{\langle m_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|n_0\rangle\langle n_0|e\mathbf{r}^i\cdot\mathbf{E}|m_0\rangle}{E_m^0 - E_n^0} + \frac{\langle m_0|e\mathbf{r}^i\cdot\mathbf{E}|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|m_0\rangle}{E_m^0 - E_n^0} \right\}. \tag{137}$$

Here $|m_0\rangle$ is an eigen state of H_0, it should be remarked that, as will be shown in (327), EDM appears as the coefficient of the energy shift linear on the external electric field. So

$$E_m = E_m^1 + E_m^2 \equiv -\mathbf{d}'\cdot\mathbf{E}, \tag{138}$$

where

$$\begin{aligned}\mathbf{d}' = \langle m_0|\mathbf{d}_e^i|m_0\rangle \; - \; &\sum_{n\neq m}\sum_i \left\{ \frac{\langle m_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|n_0\rangle\langle n_0|e\mathbf{r}^i|m_0\rangle}{E_m^0 - E_n^0} \right. \\ &\left. + \frac{\langle m_0|e\mathbf{r}^i|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|m_0\rangle}{E_m^0 - E_n^0} \right\}.\end{aligned} \tag{139}$$

However this $\mathbf{d}'$ vanishes as follows.

$$\begin{aligned}&\langle m_0|e\mathbf{r}^i\cdot\mathbf{E}|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{E}_{int}|m_0\rangle = i\langle m|\mathbf{r}^i\cdot\mathbf{E}|n\rangle\langle n|\mathbf{d}_e^i\cdot[\mathbf{p}^i,\ H_0]|m_0\rangle \\ &= i\langle m_0|\mathbf{r}^i\cdot\mathbf{E}|n_0\rangle\langle n_0|\mathbf{d}_e^i\cdot\mathbf{p}|m_0\rangle(E_m - E_n)\end{aligned} \tag{140}$$

Using the communtation relation $[r_i, p_j] = i\hbar\delta_{ij}$ the second term of (139) cancells with the first term. This is the famous Sciff theorem [67]. Since the expectation value of

$$\boldsymbol{\Sigma}\cdot\mathbf{E}_{int} = [\boldsymbol{\Sigma}\cdot\nabla,\ H_0] \tag{141}$$

does not contribute to a linear Stark effect, the residual EDM is becomes

$$V_{EDM} = -d_e(\beta - 1)\mathbf{\Sigma}\cdot\mathbf{E}_{int} \tag{142}$$

and the residual energy shift is

$$\begin{aligned}\Delta E &= -d_e\langle m_0|(\beta-1)\mathbf{\Sigma}\cdot\mathbf{E}|m_0\rangle - 2d_e\sum_{n\neq m}\frac{\langle m_0|\mathbf{r}\cdot\mathbf{E}|n_0\rangle\langle n_0|(\beta-1)\mathbf{\Sigma}\cdot\mathbf{E}_{int}|m_0\rangle}{E_m - E_n} \\ &= -\mathbf{d}(atom)\cdot\mathbf{E} \end{aligned}\tag{143}$$

The first term has no enhancement factor unlike the the second term and is much smaller than the second term, and

$$\mathbf{d}(atom) = -2d_e\sum_{n\neq m}\sum_i\frac{\langle m_0|\mathbf{r}^i|n_0\rangle\langle n_0|(\beta-1)\mathbf{\Sigma}^i\cdot\mathbf{E}_{int}|m_0\rangle}{E_m - E_n} \tag{144}$$

d_{atom} has the large value when these states are almost degenerate. However, this enhancement is reflected in quite different ways in paramagnetic atoms and diamagnetic atoms. Though (143) itself is rather universal, H_{PTV} is variant. One example is P,T-odd Nucleon-electron interaction like (see Appendix **C** for the detail)

$$+ iG_{S'}\overline{N}N\overline{L}\gamma_5 L + iG_{P'}\overline{N}\gamma_5 N\overline{L}L. \tag{145}$$

There are the other CP violating effective interactions (see the last part of section IIB). Another important interaction is due to Schiff moment. The origin of Schiff moment itself is not unique.

As we mentioned, in the nonrelativistic Hamiltonian for a system of particles of finite size, there is no interaction energy of first order in the EDM if there is no misalignment of charge and moment distribution. Schiff also indicated in [67] how this theorem is violated by relativistic (Breit equation $O\left((v/c)^2\right)$) and the misalignment (the Schiff moment), where v is the velocity of electron or nucleon. Prior to this discovery, Salpeter [68] indicated that radiative corrections of $O\left((v/c)^3\right)$ enhances Hydrogen EDM.

Sandars pointed out that relativistic effect of electron EDM in heavy alkali atom gives large atomic EDM [69].

Let us proceed to discuss in more detail for Hydrogen-like atom. The EDM has, by definition, odd parity and naively vanishes between the states with same parity.

For nonrelativistic case, its energy levels are

$$E = -\frac{mZ^2\alpha^2}{2n^2}. \tag{146}$$

Here n is the principal quantum number, and this energy is degenerate n^2-ply, $\sum_l = 0^{n-1}(2l+1) = n^2$ (see Eq.(159)).

If we consider the relativistic effects (spin effects) the degenerate energy levels are split into n fine-structure components at different j [70]. Let us obtain the relativistic terms w.r.t. $O(v/c)$ (see Appendix **E** for relativistic expansion and Appendix **F** for nonrelativistic approximation for more detail).

At the first order of v/c, we obtain the Pauli equation

$$i\hbar\frac{\partial\varphi}{\partial t} = H\varphi = \left[\frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + e\Phi - \frac{e}{2mc}\boldsymbol{\sigma}\cdot\mathbf{B}\right]\varphi. \tag{147}$$

In further approximation of $O\left((v/c) - 2\right)$, we assume $\mathbf{B} = 0$ (*i.e.* $\mathbf{A} = 0$), we get

$$H = \frac{\mathbf{p}^2}{2m} + e\Phi - \frac{\mathbf{p}^4}{8m^2c^2} - \frac{e\hbar}{4m^2c^2}\boldsymbol{\sigma}\cdot[\mathbf{E}\times\mathbf{p}] - \frac{e\hbar^2}{8m^2c^2}\nabla\cdot\mathbf{E}. \tag{148}$$

If $\mathbf{E}$ is centrally symmetric,

$$\mathbf{E} = -\frac{\mathbf{r}}{r}\frac{d\Phi}{dr}. \tag{149}$$

The spin-orbit interaction (the fourth term) becomes

$$V_{sl} = \frac{e\hbar}{4m^2c^2r}\boldsymbol{\sigma}\cdot[\mathbf{r}\times\mathbf{p}]\frac{d\Phi}{dr} = \frac{\hbar^2}{2m^2c^2r}\frac{dU}{dr}\mathbf{l}\cdot\mathbf{s}. \tag{150}$$

For many electron case of atomic number Z,

$$V_{sl} = \sum \alpha_a \mathbf{l}_a \mathbf{s}_a, \tag{151}$$

where

$$\alpha_a = \frac{\hbar^2}{2m^2c^2r_a}\frac{dU(r_a)}{dr_a}, \tag{152}$$

$$|U(r_a)| \approx \frac{Ze^2}{a_B} \approx \frac{Z^2me^4}{\hbar^2}, \tag{153}$$

and, therefore,

$$\alpha \approx Z^4\left(\frac{e^2}{\hbar c}\right)^2\frac{me^4}{\hbar^2}. \tag{154}$$

For given total $\mathbf{L}$ and $\mathbf{S}$, the averaged V_{SL} is

$$V_{SL} = A\mathbf{S}\cdot\mathbf{L}, \tag{155}$$

$$\mathbf{L}\cdot\mathbf{S} = \frac{1}{2}[J(J+1) - L(L+1) - S(S+1)]. \tag{156}$$

Since the value of $\mathbf{L}$ and $\mathbf{S}$ are same for a multiplet, energy splitting is given by the Lande's interval rule,

$$\Delta E_{J,J+1} = AJ. \tag{157}$$

Then we obtain

$$\begin{aligned} & 1s_{1/2} \\ & \left(2s_{1/2},\ 2p_{1/2}\right), \quad 2p_{3/2} \\ & \left(3s_{1/2}, 3p_{1/2}\right), \quad \left(3p_{3/2}, 3d_{3/2}\right), \quad 3d_{5/2}. \end{aligned} \tag{158}$$

The remaining degeneracy is removed by the hyper fine-structure components caused by the radiative correction (Lamb shift [71]). So using this hyper fine splitting, we can obtain large atomic EDM [68].

To estimate atomic EDM we need two informations. One is that of atomic wave functions and another is that of P (or T) violating interactions.

Atomic EDM are due to those of constituents, electrons and nucleons. For electron EDM it is very important that the atom has an unpaired electron, and electron EDM is proportional to Z^3 [69] (For the review, see [10][11].). If there is no unpaired electron (diamagnetic atom), we can measure quark (or hadron) EDM. For proton EDM, nucleus has an unpaired proton. In this case polarized molecule takes an important role [72].

We are dealing with many electrons system and the electron wave functions are not in general exact. In this case the expectation values of the EDM depends on the representation.

$$< b|\mathbf{r}|a > \equiv \mathbf{r}_{ba} = \frac{1}{E_b - E_a} < b|H_0\mathbf{r} - \mathbf{r}H_0|a >, , \tag{159}$$

where $H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V$. Inserting this into (159), we obtain

$$\begin{aligned} \mathbf{r}_{ba} &= \frac{1}{E_b - E_a} < b|\nabla^2\mathbf{r} - \mathbf{r}\nabla^2|a > = -\frac{i}{m\omega_{ba}}\mathbf{p}_{ba} && (160) \\ &= \frac{1}{m\omega_{ba}^2}(\nabla V)_{ba}. && (161) \end{aligned}$$

These three representations are of course equivalent. However, if we use the approximate wave functions, these values are different in general. So we must be careful what is the origins of discrepancies, due to different approximations or to representations [73].

A. Relativistic Effects

The relativistic equation of atom with CP violating interaction (ξ term) is

$$\left[\gamma_\mu\left(\partial_\mu - \frac{ie}{\hbar c}A_\mu\right) - i\frac{mc}{\hbar}\right]u = \xi\frac{e}{4mc^2}\gamma_5\gamma_\mu\gamma_\nu F_{\mu\nu}u. \tag{162}$$

Electron EDM breaks the CP invariance and CP violating energy equation is

$$(H_0 + H')u = Eu. \tag{163}$$

Here H_0 is the Hamiltonian of the original single electron Dirac equation in the external field,

$$H_0 = m\beta c^2 + \boldsymbol{\alpha}\cdot(c\mathbf{p} - e\mathbf{A}) + e\phi \tag{164}$$

which leads to (134) in the static limit and H' is CP violating interaction Hamiltonian of the right-hand side of (162) [68] [5]

$$\begin{aligned} H' &= \xi\frac{e\hbar}{2mc}\beta\left(\boldsymbol{\Sigma}\cdot\mathbf{E} + i\boldsymbol{\alpha}\cdot\mathbf{B}\right) \\ &\approx -\xi\frac{e\hbar}{2mc}\beta\boldsymbol{\Sigma}\cdot\nabla\phi, \end{aligned} \tag{165}$$

where $\boldsymbol{\Sigma}$ is defined by (27) and

$$\boldsymbol{\alpha} \equiv \beta\boldsymbol{\gamma} = \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & \mathbf{0} \end{pmatrix}. \tag{166}$$

ξ is dimensionless constant which measures the EDM in units of the Bohr magneton. Thus $\xi \ll 1$ implies that the EDM is small compared with e times the Compton wavelength. The last approximation in (165) comes from the relativistic suppression due to the mixing of the upper half with the lower one. It should be remarked that $\boldsymbol{\sigma}\cdot\mathbf{E}$ is T-odd (T is time reversal operator) but $\boldsymbol{\alpha}\cdot\mathbf{B}$ (the suppressed term) T-even.

We consider a hydrogen-like atom with charge Z, where $\mathbf{E} = \frac{Ze}{r^2}\mathbf{e}_r$.

$$H' = -\xi Z\alpha r^{-2}s_r \quad \text{with } s_r = \boldsymbol{\sigma}\cdot\mathbf{e}_r/2 \tag{167}$$

[5] This term only contributes to the intrinsic EDM and we denote H', distinguishing it from H_{PTV} of (135).

in atomic units $e = m = \hbar = 1$. The Hamiltonian of the above single electron system in the mean field approximation is generalized for many electrons system as [69]

$$H'_0 = \sum_i \left[\beta_i mc^2 + \boldsymbol{\alpha}_i \cdot c\mathbf{p}_i + e\phi_i\right] + \sum_{j\neq k} \frac{1}{2}\left[\frac{e^2}{r_{jk}} + B_{jk}\right]. \tag{168}$$

Here suffix indicates the quantity of i'th electron and B_{jk} is the relativistic corrections. Taking the retard potential into consideration, Lagrangian can be described up to order $\mathrm{O}((\frac{v}{c})^2)$ as

$$\frac{e^2}{r_{jk}} + B_{jk} \equiv \frac{e^2}{r_{jk}}\left[1 - \frac{1}{2}\left(\boldsymbol{v}_j \cdot \boldsymbol{v}_k + \frac{(\boldsymbol{v}_j \cdot \mathbf{r}_{jk})(\boldsymbol{v}_k \cdot \mathbf{r}_{jk})}{r_{jk}^2}\right)\right]. \tag{169}$$

Further higher order $\geq O\left((\frac{v}{c})^3\right)$ corrections come from photon emission (Breit interaction) [74] and

$$\begin{aligned} U_{jk} = {} & \frac{e^2}{r_{jk}} - \pi\left(\frac{e\hbar}{mc}\right)^2 \delta(\mathbf{r}_{jk}) - \frac{e^2}{2m^2c^2 r_{jk}}\left(\mathbf{p}_j\mathbf{p}_k + \frac{(\mathbf{r}_{jk}\mathbf{p}_j)(\mathbf{r}_{jk}\mathbf{p}_k)}{r_{jk}^2}\right) \\ & \frac{e^2\hbar}{4m^2c^2r_{jk}^3}\left(-(\boldsymbol{\sigma}_j + 2\boldsymbol{\sigma}_k)[\mathbf{r}_{jk}\mathbf{p}_j] + (\boldsymbol{\sigma}_k + 2\boldsymbol{\sigma}_j)[\mathbf{r}_{jk}\mathbf{p}_k]\right) \\ & + \frac{1}{4}\left(\frac{e\hbar}{mc}\right)^2\left(\frac{\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k}{\mathbf{r}_{jk}^3} - 3\frac{(\boldsymbol{\sigma}_j\mathbf{r}_{jk})(\boldsymbol{\sigma}_k\mathbf{r}_{jk})}{r_{jk}^5} - \frac{8\pi}{3}\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k\delta(\mathbf{r}_{jk})\right) \end{aligned} \tag{170}$$

The second line corresponds to spin-orbit interaction and the third spin-spin interaction.

If we incorporate the spin of nucleus, the degeneracy of $\mathbf{J}$ is split (hyperfine structure),

$$V_{iJ} = a\mathbf{i} \cdot \mathbf{J}, \tag{171}$$

where $\mathbf{i}$ and $\mathbf{J}$ are the spin of nucleus and total angular momentum of electron envelope, respectively. However, in this hyperfine splitting dominant contribution comes from magnetic dipole and electric quadrupole and does not play important role in the EDM.

Thus the linear Stark appears as relativistic effects in duplicate meanings, i.e. $1 - \beta$ and B_{jk} components. Many particle interaction effects are due to this relativistic effect as well as due to the other nonrelativistic excitation effects.

We proceed to the detailed calculation of single electron case (167) [68]. The operator s_r commutes with $\mathbf{M}^2$ and M_z, but has odd parity, and

$$< l = j \pm \frac{1}{2}, j, m|s_r|l = j \mp \frac{1}{2}, j, m >= \frac{1}{2}. \tag{172}$$

Let the radial part of $u = r\chi_{nl}$. Then it satisfies

$$\left\{\left[2\mu\phi - d^2/dr^2\right] + l(l+1)r^{-2} - 2\mu E_{nl}\right\}\chi_{nl} = 0, \tag{173}$$

where μ is the reduced mass of electron and set equal to unity in the subsequent equations. It follows from (172) that

$$< n, l_+, j, m|H'|n', l_-, j, m >= -\xi Z\alpha(2j+1)^{-1}(E_n - E'_n) \int dr \chi_{nl_+}\chi_{n_rl_-} \quad (174)$$

with $l_\pm = j \pm \frac{1}{2}$. Therefore, naively the first-order perturbation vanishes. The exception is discussed in section **5.2**. The second-order perturbation energy is obtained by use of (174)

$$\Delta E_{n,l_\pm,j} = (\xi Z\alpha/(2j+1))^2 \int dr \chi_{n,l_\pm} \left[E_n + \frac{1}{2}\frac{d^2}{dr^2} + \frac{Z}{r} - \frac{1}{2}\frac{l_\mp(l_\mp+1)}{r^2}\right] \chi_{nl_\pm}. \quad (175)$$

Using

$$< r^{-2} >= \frac{Z^2}{n^3(l+1/2)}, \quad (176)$$

we obtain

$$\Delta E_{n,l_\pm,j} = \pm Z^4\xi^2 n^{-3}(2j+1)^{-1}(l_\pm + \frac{1}{2})^{-1}\alpha^2 Ry \quad (177)$$

with $Ry = \frac{me^4}{2\hbar^2} = 13.6\text{eV}$.

B. Peculiar Property of Paramagnetic Atom

The atomic enhancement factor defined by

$$K \equiv d_{atom}/d_e \quad (178)$$

is given by [75]

$$K = \sum_m \frac{4(Z\alpha)^3 r_{m0}\hbar c}{(J+1)a_B^2\gamma(4\gamma^2-1)(N_0N_m)^{3/2}(E_m - E_0)} \quad (179)$$

for alkali atom. Here the sum is taken over the excited state m, and N_0, N_m are effective principal quantum number defined in (190). $\gamma = \sqrt{(j+1/2)^2 - Z^2\alpha^2}$ and r_{0m} is electric dipole radial integral,

$$r_{nl,n'l'} =< n, l|r|n', l-1 >= \sqrt{l} \int_0^\infty R_{n',l-1}R_{n,l}r^3 dr \quad (180)$$

in units of $a_B = \frac{\hbar^2}{me^2}$:Bohr radius. We will derive (179) (see Eq.(193)). We start with the general relativistic arguments. For Diamagnetic atoms the dominant contribution to atomic EDM comes from that of nucleus, which will be discussed later.

In hydrogen-like atom, states of different angular momentum l with fixed principal number n are degenerate in nonrelativistic approximation. The eigen functions with external field are the superposition of the field-free function with different l-values, which gives the linear Stark effect. Let us write the Dirac spinor in the form

$$u_\pm = r^{-1}\left(\chi_{2\pm}(r)\eta_{jl_\pm}, \quad -i\chi_{1\pm}(r)\eta_{jl\mp}\right)^T. \tag{181}$$

χ_i satisfy the following equations,

$$\begin{aligned} \frac{d\chi_1}{dr} - \kappa\frac{\chi_1}{r} &= \left[\frac{mc}{\hbar}\left(1-\frac{E}{mc^2}\right) - \alpha\frac{Z}{r}\right]\chi_2 \\ \frac{d\chi_2}{dr} + \kappa\frac{\chi_2}{r} &= \left[\frac{mc}{\hbar}\left(1+\frac{E}{mc^2}\right) + \alpha\frac{Z}{r}\right]\chi_1, \end{aligned} \tag{182}$$

where

$$\kappa = \mp(j+\frac{1}{2}) \text{ for } j = l \pm \frac{1}{2}. \tag{183}$$

Using (165) and (172), we obtain

$$\langle n, j, l_+, m|H'|n, j, l_-, m\rangle = -\frac{1}{2}\xi Z\alpha\int_0^\infty drr^{-2}(\chi_{2+}\chi_{2-} - \chi_{1+}\chi_{1-}). \tag{184}$$

$\chi_{2\pm}$ and $\chi_{1\pm}$ are related to each other as (182) and we obtain

$$4\langle l_+|H'|l_-\rangle = \xi Z\alpha^3\int_0^\infty dr(D_+\chi_+)r^{-2}(D_-\chi_-), \tag{185}$$

where

$$D_\pm = \frac{d}{dr} \pm \frac{j+1/2}{r}. \tag{186}$$

The exact Dirac wave functions with given n, l, j are [70] [76]

$$\begin{aligned} \frac{\chi_2}{r} &= -\frac{\sqrt{\Gamma(2\gamma+n_r+1)}}{\Gamma(2\gamma+1)\sqrt{n_r!}}\sqrt{\frac{1+\epsilon}{4N(N-\kappa)}}\left(\frac{2Z}{Na_B}\right)^{3/2} e^{-\frac{Zr}{Na_B}}\left(\frac{2Zr}{Na_B}\right)^{\gamma-1} \times \\ &\times\left[-n_r F\left(-n_r+1, 2\gamma+1, \frac{2Zr}{Na_B}\right) + (N-\kappa)F\left(-n_r, 2\gamma+1, \frac{2Zr}{Na_B}\right)\right] \end{aligned} \tag{187}$$

and

$$\begin{aligned} \frac{\chi_1}{r} &= -\frac{\sqrt{\Gamma(2\gamma+n_r+1)}}{\Gamma(2\gamma+1)\sqrt{n_r!}}\sqrt{\frac{1-\epsilon}{4N(N-\kappa)}}\left(\frac{2Z}{Na_B}\right)^{3/2} e^{-\frac{Zr}{Na_B}}\left(\frac{2Zr}{Na_B}\right)^{\gamma-1} \times \\ &\times\left[n_r F\left(-n_r+1, 2\gamma+1, \frac{2Zr}{Na_B}\right) + (N-\kappa)F\left(-n_r, 2\gamma+1, \frac{2Zr}{Na_B}\right)\right]. \end{aligned} \tag{188}$$

Here F is the confluent hypergeometric function and n_r radial quantum number, the number of nodes of radial part of the wave function,

$$n_r = \frac{\alpha Z\epsilon}{\sqrt{1-\epsilon^2}} - \gamma, \; n = n_r + |\kappa| \tag{189}$$

with $\epsilon = \frac{E}{mc^2}$, and

$$N = \sqrt{n^2 - 2n_r(|\kappa| - \sqrt{\kappa^2 - \alpha^2 Z^2})}. \tag{190}$$

Here we have used

$$1 - \epsilon^2 = \frac{\alpha^2 Z^2}{N^2} \tag{191}$$

and normalized $\lambda r = \frac{Z}{N a_B} r$ as in the hypergeometric functions. N is called apparent principal quantum number and

$$E_{nl} = -\frac{m Z^2 \alpha^2}{2N^2}. \tag{192}$$

Using these forms, (184) finally reads, for instance [68],

$$\langle 2s_{1/2}|H'|2p_{1/2}\rangle = \frac{\xi Z^3 \alpha(\gamma - 1)}{2\gamma(2\gamma - 1)(\gamma + 1)(2\gamma + 1)^{1/2}} Ry, \tag{193}$$

where γ takes the value $\sqrt{1 - Z^2\alpha^2}$ in this case. For small $Z\alpha$, it is reduced to

$$\langle 2s_{1/2}|H'|2p_{1/2}\rangle = \frac{\xi Z^5 \alpha^3}{8\sqrt{3}} Ry. \tag{194}$$

For more general case

$$< j, l_+|H'|j, l_- >= -\frac{4(Z\alpha)^3}{\gamma(4\gamma^2 - 1)(N_+ N_-)^{3/2}} Ry. \tag{195}$$

For heavy alkali atom, for instance, cesium, [9]

$$K(Cs) = d(Cs)/d_e = -\frac{16}{3}\frac{Z^3\alpha^2 r(6s, 6p_{1/2})}{a_B^2 \gamma(4\gamma^2 - 1)(N_s N_p)^{3/2}} \frac{Ry}{E(6p_{1/2}) - E(6s)} = 118. \tag{196}$$

The radial integral is experimentally known [77]

$$r(6s, 6p_{1/2}) = \int_0^\infty dr r^3 R_{60}(r) R_{61}(r) = 5.5 a_B. \tag{197}$$

(196) should be checked with the experimental result [24].

For Francium (Z=87, 7s→ $7p_{1/2}$), $K(Fr)$ is estimated as 873. However, as was stated in [10], (179) is not applicable for atoms with complex configurations and requires electrons' correlations. Such calculations are performed in, for instance, [78] and $K(Fr)$ is modified to 895.

There are some discrepancies on the estimate of enhancement factor of K of Thallium [Xe]$4f^{14}5d^{10}6s^2 6p^1$ [79] [80] [81]. The discrepancy seems to come from the starting assumptions. [80] considered that Thallium has three valence electrons, $6s^2 6p^1$, whereas [81] considered it having one valence electron. If we adopt [80],

$$d(^{205}Tl) = -582(20) d_e \tag{198}$$

or if we take [81],

$$d(^{205}Tl) = -466 d_e \to d_e < 1.6 \times 10^{-27}\text{e cm}. \tag{199}$$

In preparing this revised version, an interesting paper has just appeared [82] which asserts that this discrepancy disappears, converging to $K = -573$.

Xe has closed electron shell of $5s^2\ 5p^6$ but we may one elctron of $5p$ state excited to $5p^5\ 6s^1$, which resembles with that of Cs, [Xe]$6s^1$, whose enhancement factor was estimated to $K(^{133}Cs) = 114$ [83] or 120.53 [84].

As for ^{129}Xe, the lowest excited state with a $6s$ electron has a enhancement value $K(^{129}Xe^*) = 120$ [75][85] or 111 [86], and

$$d_e < 3.2 \times 10^{-24} e\ cm. \tag{200}$$

Using these results, Ellis et al. considered the maximal EDMs of nuclei [87].

C. Chiral Condensate

Before discussing diamagnetic atom, we will briefly resume QCD chiral dynamics [88]. This is because hadronic matrix elements are described in terms of quark condensates by using operator product expansion.

Let us begin with the following effective action (see Appendix **G** for the implication of the effective action).

$$L = \overline{\psi}(\gamma^\mu D_\mu + M)\psi - \frac{\alpha_s}{4} G_{\mu\nu}\tilde{G}^{\mu\nu}, \tag{201}$$

where

$$D_\mu = \partial_\mu + i g_s A^a_\mu \lambda^a. \tag{202}$$

This action is invariant under $SU(3)_L \times SU(3)_R$ transformations in the limit of $m_u = m_d = m_s = 0$. That is,

$$Q_L\psi = e^{i\alpha^a\lambda^a}\psi, \quad Q_R\psi = e^{i\beta^a\lambda^a\gamma_5}\psi, \tag{203}$$

where u, d, s quarks constitute SU(3) group,

$$\psi = (u, d, s)^T \tag{204}$$

and we have the following conserved currents

$$j^\mu_{L,R} = \overline{\psi}_{L,R}\lambda^a\gamma^\mu\psi_{L,R}. \tag{205}$$

Here λ^a are the Gell-Mann's 3×3 matrices and ψ_L (ψ_R) are left-handed (right-handed) part of ψ.

$$j^{a\mu} = j_L^{a\mu} + j_R^{a\mu}, \tag{206}$$

$$j_5^{a\mu} = j_L^{a\mu} - j_R^{a\mu}. \tag{207}$$

So we have the conserved currents and conserved charges Q_a and Q_{5a}. They satisfy the algebras

$$[Q_a, Q_b] = if_{abc}Q_c, \quad [Q_{5a}, Q_b] = if_{abc}Q_{5c}, \quad [Q_{5a}, Q_{5b}] = if_{abc}Q_c. \tag{208}$$

However, this group is not exact and they are spontaneously broken to

$$Q_a|0\rangle = 0, \quad Q_{5a}|0\rangle \neq 0. \tag{209}$$

Thus there appear 8 pseudo Nambu-Goldstone bosons. Pseudo implies that the original chiral symmetry (Q_5 transformation) is not exact. It is broken by

$$H_{SB} = m_u\overline{u}u + m_d\overline{d}d + m_s\overline{s}s. \tag{210}$$

This can be rewritten as

$$\begin{aligned} H_{SB} &= (m_u + m_d + m_s)(\overline{u}u + \overline{d}d + \overline{s}s)/3 \\ &+ (m_u - m_d)(\overline{u}u - \overline{d}d)/2 \\ &+ (2m_s - m_u - m_d)(2\overline{s}s - \overline{u}u - \overline{d}d)/6. \end{aligned} \tag{211}$$

Here the first line is an SU(3) invariant, the second breaks isospin SU(2), the third responsible for the SU(3) symmetry.

$$M_\pi^2 = (m_u + m_d)B + O(m_q^2 \ln m_q), \tag{212}$$

$$M_{K^\pm}^2 = (m_u + m_s)B + O(m_q^2 \ln m_q), \tag{213}$$

$$M_{K^0}^2 = (m_d + m_s)B + O(m_q^2 \ln m_q). \tag{214}$$

Here $B = -\frac{2}{f_\pi^2}\langle 0|\overline{q}q|0\rangle$ with pion decay constant $f_\pi = 133$MeV, and we have used the chiral limit

$$f_\pi = f_K, \quad \langle 0|\overline{u}u|0\rangle = \langle 0|\overline{d}d|0\rangle = \langle 0|\overline{s}s|0\rangle. \tag{215}$$

Adler-Bell-Jackiw axial vector current anomaly [89] and its non-Abelian version is

$$\partial_\mu j^{5\mu} = 2i \sum_{q=u,d,s} m_q\overline{q}q + \frac{N_f}{8\pi^2}\left(F\tilde{F} + G^a\tilde{G}^a\right). \tag{216}$$

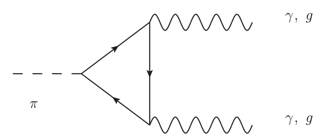


FIG. 14: Chiral anomaly induces $\pi^0 \to \gamma\gamma$ via $if_0\pi F\tilde{F}$ interaction.

with the number of flavour N_F. Since

$$\frac{N_f}{8\pi^2}G^a\tilde{G}^a = 2N_f\partial^\mu K_\mu \tag{217}$$

with

$$K_\mu \equiv \frac{1}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}A^{\nu a}\left(\partial^\rho A^{\sigma a} + \frac{1}{3}gf^{abc}A^{\rho b}A^{\sigma c}\right). \tag{218}$$

In the limit of $m_u = m_d = m_s = 0$, the axial current $j^{5\mu}$ gets conserved again by replacing $j^{5\mu}$ as

$$\tilde{j}_{5\mu} = j_{5\mu} - 2N_f K_\mu. \tag{219}$$

Thus quark condensate is essential for chiral symmetry breaking. Nevertheless, anomalous term is crucial to the presence of $\pi^0 \to 2\gamma$. The neutral axial vector current gives the modified PCAC relation,

$$\partial^\mu j^0_{5\mu} = f_\pi m_\pi^2 \pi^0 + \frac{\alpha}{4\pi}F\tilde{F}. \tag{220}$$

We will come back to this problem in the next subsection. U(1) problem concerning QCD condensate is discussed in Appendix **H**.

D. Peculiar Property of Diamagnetic Atom

Diamagnetic atom has no unpaired electron, and the main contribution of atomic EDM comes from the misalignment between charge and the EDM distribution of nucleus. Thus hadronic part of the atomic EDM manifests itself through the Schiff moment [67][90].

$$H_{atom} = H_{electron} + H_{nucleus} + \sum_{i=1}^{Z}(e\Phi(\mathbf{r}_i) - e\mathbf{r}_i \cdot \mathbf{E}) - \mathbf{d}_N \cdot \mathbf{E}, \tag{221}$$

where $\mathbf{r}_i$ are the i'th electron coordinates and $\mathbf{d}_N$ is the nuclear EDM. $\mathbf{E}$ means the external $\mathbf{E}$. It should be remarked for diamagnetic atoms that the Stark term due to electron' intrinsic spin term (the first term of (135)) is replaced by that of nucleon. $\Phi(\mathbf{r})$ is the nuclear electrostatic potential given by

$$\Phi(\mathbf{r}) = \int \frac{\rho(\mathbf{x})d^3x}{|\mathbf{r}-\mathbf{x}|}, \tag{222}$$

where $\rho(\mathbf{x})$ is the charge density of nucleus.

Here it is important to notice [91]

$$\frac{i}{m}\left[\sum_{i=1}^{Z}\mathbf{p}_i, H_{atom}\right] = -e\sum_{i=1}^{Z}\nabla_i\Phi(\mathbf{r}_i) + Ze\mathbf{E}, \tag{223}$$

where $\mathbf{p}_i$ are the momentum of atomic electrons, and the first term is the average electric field induced by atomic electrons. The expectation value of this commutator in the energy eigenstate vanishes and we may add

$$V = \mathbf{d}_N\cdot\mathbf{E} - \frac{1}{eZ}\sum_{i=1}^{Z}\mathbf{d}_N\cdot\nabla_i\Phi(\mathbf{r}_i) \tag{224}$$

to H_{atom} as far as we consider the expectation value. This implies we may change

$$-\mathbf{d}_N\cdot\mathbf{E} \to -(\mathbf{d}_N - \langle\mathbf{d}_N\rangle)\cdot\mathbf{E}. \tag{225}$$

So the expectation value is zero. This is another statement of the Schiff theorem. From the first term of (223), we should consider the interaction of atomic electrons with the nucleus,

$$\Phi(\mathbf{r}) - \frac{1}{Ze}\langle\mathbf{d}_N\rangle\cdot\nabla\Phi(\mathbf{r}) \tag{226}$$

as the screened electrostatic potential. Therefore, the atomic EDM reads

$$\mathbf{d}_{atom} = \sum_n \frac{\left\langle 0|e\sum_i^Z \mathbf{r}_i|n\right\rangle\left\langle n|e\sum_i^Z\left(\Phi(\mathbf{r}_i) - \frac{1}{Ze}\langle\mathbf{d}_N\rangle\cdot\nabla\Phi(\mathbf{r}_i)\right)|0\right\rangle}{E_0 - E_n} + h.c. \tag{227}$$

Using the charge distributions

$$\begin{aligned}\int\rho(\mathbf{x})d^3x &= Z|e|, \quad \int\mathbf{x}\rho(\mathbf{x})d^3x = \langle\mathbf{d}_N\rangle, \\ \int x^2\rho(\mathbf{x})d^3x &= Z|e|\left\langle x^2\right\rangle_{ch}, \quad \int(x_kx_{k'} - \frac{1}{3}\delta_{kk'}x^2)\rho(\mathbf{x})d^3x = Z|e|\langle Q_{kk'}\rangle \quad \text{etc}\end{aligned} \tag{228}$$

$$\left\langle 0_N\left|e\Phi(\mathbf{r}) - \frac{1}{Z}\langle\mathbf{d}_N\rangle\cdot\nabla\Phi(\mathbf{r})\right|0_N\right\rangle = -\frac{Ze^2}{|\mathbf{r}|} + 4\pi e\mathbf{S}\cdot\nabla\delta(\mathbf{r}) + \ldots \tag{229}$$

Here ... indicates electric octupole and higher pole contributions, and $\mathbf{S}$ is the famous Schiff moment [92], (The detailed derivation is given in Appendix **B**)

$$\mathbf{S}^{ch} = \frac{e}{10}\sum_{p=1}^{Z}\left(r_p^2 - \frac{5}{3}\langle r^2\rangle_{ch}\right)\mathbf{r}_p. \tag{230}$$

The $\langle Q_{kk'}\rangle$ vanishes for ^{199}Hg, ^{129}Xe, ^{225}Ra.

There is another Schiff moment $\mathbf{S}^{nucl}$ due to the misalignment between the charge distribution and the EDM distribution of nucleus, whose derivation is given in Appendix **I**.

Corresponding to these situations, we should consider (230) more generally

$$\mathbf{S} = \frac{1}{10}\sum_{N}^{A}\sum_{i} e_i\left((\mathbf{r}_N + \boldsymbol{\rho}_i)^2 - \frac{5}{3}\langle r^2\rangle_{ch}\right)(\mathbf{r}_N + \boldsymbol{\rho}_i). \tag{231}$$

Here $\mathbf{r}_N$ is a N'th nucleon position and $\boldsymbol{\rho}_i$ is the position of the ith charge inside the N'th nucleon, and

$$\sum_i e_i = e_N, \quad \sum_i e_i\boldsymbol{\rho}_i = \mathbf{d}_N. \tag{232}$$

Retaining the term up to linear term of $\boldsymbol{\rho}$, we have

$$\mathbf{S} = \mathbf{S}^{ch} + \mathbf{S}^{nucl} \tag{233}$$

with

$$\mathbf{S}^{ch} = \frac{1}{10}\sum_{N}^{A} e_N\left(r_N^2\mathbf{r}_N - \frac{5}{3}\langle r^2\rangle_{ch}\mathbf{r}_N\right), \tag{234}$$

$$\mathbf{S}^{nucl} = \frac{1}{6}\sum_{N}^{A}\mathbf{d}_N(r_N^2 - \langle r^2\rangle_{ch}) + \frac{1}{5}\sum_{N}^{A}\left(\mathbf{r}_N(\mathbf{r}_N\cdot\mathbf{d}_N) - \frac{1}{3}\mathbf{d}_N r_N^2\right). \tag{235}$$

Usually $\mathbf{S}^{nucl}$ is considered to be small compared with S^{ch}. The mean value of $\mathbf{S}^{ch}$ is nonzero only in the presence of P- and T=odd nucleon-nucleon interactions.

For the arguments of hadronic EDM, we must represent hadronic CP violating interactions from those of (39) and (40). This will be discussed in the last part of this subsection (see (277)). They are described as

$$\mathcal{L}_{\pi NN} \equiv g^{(0)}_{\pi NN}\overline{N}\tau^a N\pi^a + g^{(1)}_{\pi NN}\overline{N}N\pi^0 + g^{(2)}_{\pi NN}(\overline{N}\tau^a N\pi^a - 3N\tau^3 N\pi^0). \tag{236}$$

Here $g^{(i)}_{\pi NN}$ $(i = 1, 2, 3)$ are CP odd coupling constants, whereas we denote the CP even strong πNN coupling constant as $G_{\pi NN}(= 13.5)$. The Schiff moment due to this coupling

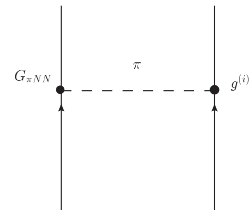


FIG. 15: One $g^{(i)}$ coupling induces effective CP-odd NN interaction, which give rise to S^{ch}.

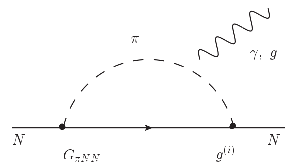


FIG. 16: One $g^{(i)}$ coupling induces $\mathbf{d}_N$, which gives rise to S^{nucl}.

is calculated as follows. (236) gives rise to both S^{ch} and S^{nucl}. P- and T-odd NN potential has the form via Fig.15 and its effective potential is given by

$$W(\mathbf{r}_a - \mathbf{r}_b) = \frac{G_{\pi NN} m_\pi^2}{8\pi m_N} \big\{ \big[g^{(0)}(\tau_a \cdot \tau_b) - \frac{g^{(1)}}{2}(\tau_a^z + \tau_b^z) + g^{(2)}(3\tau_a^z \tau_b^z - \tau_a \cdot \tau_b) \big] (\boldsymbol{\sigma}_a - \boldsymbol{\sigma}_b)$$
$$- \frac{g^{(1)}}{2}(\tau_a^z - \tau_b^z)(\boldsymbol{\sigma}_a + \boldsymbol{\sigma}_b) \big\} \cdot (\mathbf{r}_a - \mathbf{r}_b) \frac{exp(-m_\pi |\mathbf{r}_a - \mathbf{r}_b|)}{m_\pi |\mathbf{r}_a - \mathbf{r}_b|^2} \big[1 + \frac{1}{m_\pi |\mathbf{r}_a - \mathbf{r}_b|} \big]. \tag{237}$$

Here we have suppressed the subscript πNN in $g^{(i)}$. $\mathbf{d}_j$ is via Fig.16 and

$$\mathbf{d}_j = \frac{e G_{\pi NN}}{4\pi^2 m_N} \ln \frac{m_N}{m_\pi} (g^{(0)} - g^{(2)}) \boldsymbol{\sigma}_j \tau_j^z. \tag{238}$$

Given T and P-odd perturbation, let us calculate Schiff moment [93] in

$$H = H_0 + H_{res}. \tag{239}$$

Here

$$H_0 = T + V_{00} + V_{11} \tag{240}$$

is unperturbative one-particle Hamiltonian and exactly solvable and

$$H_{res} = W + V_{22} + V_{13} + V_{31} + V_{04} + V_{40}. \tag{241}$$

W is the pseudoscalar interaction (237) and V the Skyrme interaction [94]. Subscripts (ij) refer to the final and initial numbers of quasiparticles.

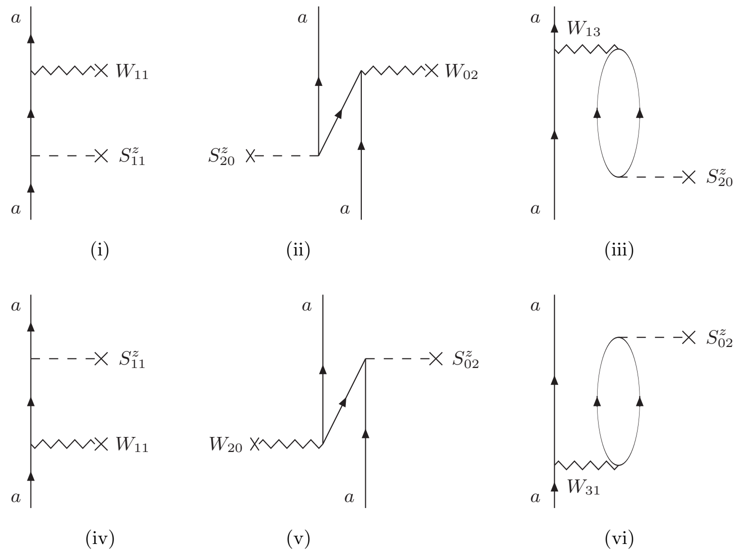


FIG. 17: First-order quasiparticle diagrams contributing to the Schiff moment [95] . The broken line represents the action of the Schiff operator, the zig-zag line represents the P- and T-odd interaction. The looped line in higher order diagram represents a generic Skyrme interaction.

Let us assume that in the 0'th order approximation, the state is $\Phi_a = |\alpha\rangle$, and define Q by

$$Q \equiv \sum_{\beta \neq \alpha} |\beta\rangle\langle\beta|. \tag{242}$$

Then perturbed wave function is given by

$$\Psi_a = \left(1 + \frac{Q}{\epsilon_a - H_0} H_{res} + \frac{Q}{\epsilon_a - H_0} H_{res} \frac{Q}{\epsilon_a - H_0} H_{res} + ...\right) \Phi_a. \tag{243}$$

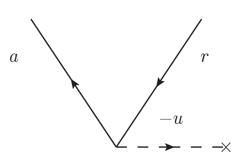
This is the Brillouin-Wigner expansion and ϵ_a is the single quasiparticle energy of the valence nucleon.

So in the first order perturbation of S^z, we obtain

$$\langle\Psi_a|S^z|\Psi_a\rangle = N^{-1}\langle\Phi_a|\left[1 + H_{res}\left(\frac{Q}{\epsilon_a - H_0}\right) + ...\right] S^z \left[1 + \left(\frac{Q}{\epsilon_a - H_0}\right) H_{res} + ...\right]|\Phi_a\rangle. \tag{244}$$

The first-order quasiparticle (Goldstone) diagram is given in Fig.17 [95].

Here the Goldstone diagram implies that



$$= \frac{|\alpha_a^r\rangle\langle r| - u|a\rangle}{\epsilon_a - \epsilon_r}. \tag{245}$$

TABLE I: Calculated coefficients a_i and b for ^{199}Hg. The units are e fm^3. The last two references include the Skyrme interaction SkO$'$. Five results of Ban et.al. are due to Hartee-Fock and Hartree-Fock-Bogoliubov approximations. SLy4, SIII et al. indicate several Skyrme interactions.

	a_0	a_1	a_2	b
Dmitriev-Sen'kov 2003 [96]	-0.0004	-0.055	0.009	–
de Jesus-Engels (averaged) [95]	0.007	0.071	0.018	–
Ban et al [97]				
SLy4(HF)	-0.013	0.006	0.022	-0.003
SIII(HF)	-0.012	-0.005	0.016	-0.004
SV(HF)	-0.009	0.0001	0.016	-0.002
SLy(HFB)	-0.013	0.006	0.024	-0.007
SkM*(HFB)	-0.041	0.027	0.069	-0.013

Higer order quasiparticle calculations need some elaborate code and should be referred to [97], and we simply list the final results

$$\langle \Psi_a | S^z | \Psi_a \rangle \equiv S = (a_0 + b) G_{\pi NN} g^{(0)} + a_1 G_{\pi NN} g^{(1)} + (a_2 - b) G_{\pi NN} g^{(2)}, \tag{246}$$

where the coefficients a_i specify S^{ch} and b does S^{nucl} defined by (233). The numerical results of a_i and b for ^{199}Hg are given in Table I.

In this Table, the first two groups considered that EDM of nucleons $\mathbf{d}_N$ is independent on $\mathcal{L}_{\pi NN}$, whereas Ban considered $\mathbf{d}_N$ is related as (238).

In the former approach, nucleon polarization effects must be added to a_i contributions [98]. Thus the results are not affirmative yet. The results are divergent not only among different groups but also in the same group due to different approximation methods. Thus it is difficult task to estimate the nucleon EDM from that of diamagnetic atom. Let us consider some cases. For ^{199}Hg, Numerical calculation is [99]

$$d(^{199}\text{Hg}) = -2.8 \times 10^{-17} \left(\frac{S}{e\ fm^3} \right) \text{e cm}. \tag{247}$$

The value of of the Schiff moment of $d(^{199}Hg)$ can be presented as a sum of proton and neutron EDMs [100]

$$S = s_p d_p + s_n d_n \tag{248}$$

with $s_p = 0.20 \pm 0.02\ fm^2$ and $s_n = 1.895 \pm 0.035\ fm^2$.

Combining the experimental value [101] (see more up-to-date data in [23])

$$d(^{199}\mathrm{Hg}) < 2.1 \times 10^{-28}\mathrm{e\ cm} \tag{249}$$

with (248), we obtain

$$|d_p| < 3.8 \times 10^{-24}\ \mathrm{e\ cm}\ \ |d_n| < 4.0 \times 10^{-25}\ \mathrm{e\ cm.} \tag{250}$$

For ^{129}Xe case, numerical calculation is [102]

$$d(^{129}Xe) = 0.38 \times 10^{-17} \left(\frac{S}{e\ fm^3} \right) e\ cm \tag{251}$$

The measurement is [103]

$$d(^{129}Xe) = (-0.3 \pm 1.1) \times 10^{-26} e\ cm. \tag{252}$$

From (252) value, [104] obtained

$$|d_p| \leq 4 \times 10^{-21}\mathrm{e\ cm}\ \ |d_n| \leq 1 \times 10^{-21}\mathrm{e\ cm.} \tag{253}$$

General arguments of CP violating four Fermi coupling are given at Appendices **C** and **D**.

1. *cEDM and parity odd nuclear interaction*

In this subsection we give a very short review of chiral symmetry and its breaking in strong interactions since it has many problems. Let us start with the conserved axial-vector current (CAC) hypothesis [105],

$$\partial_\mu j^\alpha_{5\mu}(x) = 0. \tag{254}$$

Of course CAC requires $m_\pi = f_\pi = 0$ and is not realistic. However, it makes clear to understand how to break chiral symmetry. (254) leads us to

$$\langle N'|j^\alpha_{5\mu}|N\rangle = \sqrt{\frac{m_N^2}{EE'}} F_A(t) \left[i\overline{u}'\gamma_\mu\gamma_5 \frac{\tau^\alpha}{2} u + 2m_N \frac{q_\mu}{q^2} \overline{u}'\gamma_5 \frac{\tau^\alpha}{2} u \right]. \tag{255}$$

Nambu aserted [106] that $1/q^2$ in the second term of (255) should be interpreted as

$$\frac{1}{q^2} = \lim_{m_\pi \to 0} \frac{1}{q^2 - m_\pi^2}. \tag{256}$$

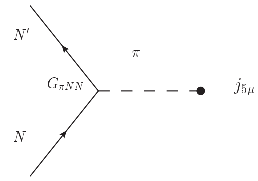


FIG. 18: Nambu's interpretation of pion dominance.

This corresponds to the diagram (Fig. (18)), which can be written as

$$G_{\pi NN}\overline{u}'\gamma_5\tau^\alpha u\frac{f_\pi q^\mu}{q^2-m_\pi^2}. \tag{257}$$

Comparing (257) with (255) where (256) is inserted, we get

$$f_\pi G_{\pi NN}=\frac{g_A}{g_V}m_N. \tag{258}$$

This is the Goldberger-Treiman's relation [107]. Here use has been made of

$$\langle 0|j_{5\mu}^\alpha(0)|\pi^\alpha\rangle = i\frac{f_\pi}{\sqrt{2\omega}}p^\mu. \tag{259}$$

$$\begin{aligned}\langle N'|\partial_\mu j_5^{\mu\alpha}|N\rangle &= -i\sqrt{\frac{m_N^2}{EE'}}F_A(t)\left[-2m_N\overline{u}'\gamma_5\frac{\tau^\alpha}{2}u-2m_N\frac{q^2}{q^2-m_\pi^2}\overline{u}'\gamma_5\frac{\tau^\alpha}{2}u\right]\\ &= \sqrt{\frac{m_N^2}{EE'}}F_A(t)\left(\frac{m_Nm_\pi^2}{q^2-m_\pi^2}\right)i\overline{u}'\gamma_5\tau^\alpha u.\end{aligned} \tag{260}$$

Substituting the equation of motion of π,

$$(\Box+m_\pi^2)\pi^\alpha=j^\alpha, \tag{261}$$

into (260), we obtain

$$\langle N'|\pi^\alpha|N\rangle = -\frac{\langle N'|j^\alpha|N\rangle}{q^2-m_\pi^2}\approx -i\sqrt{\frac{m_N^2}{EE'}}\frac{G_{\pi NN}\overline{u}'\gamma_5\tau^\alpha u}{q^2-m_\pi^2}. \tag{262}$$

Assuming the matrix elements vary little between $t=0$ and $t=m_\pi^2$, we obtain

$$\begin{aligned}\langle N'|\partial_\mu j_5^{\mu\alpha}(0)|N\rangle &\approx \frac{g_A}{g_V}\frac{m_N}{G_{\pi NN}}m_\pi^2\langle N'|\pi^\alpha|N\rangle\\ &= f_\pi m_\pi^2\langle N'|\pi^\alpha|N\rangle.\end{aligned} \tag{263}$$

Thus we obtain the PCAC condition

$$\partial_\mu j_5^{\mu\alpha}=f_\pi m_\pi^2\pi^\alpha \tag{264}$$

and (220) with chiral anomaly.

Next, we proceed to discuss the path from the presence of the strong EDM of dimension 5 (cEDM and θ term) to the effective CP-odd $g^{(i)}$.

Let us write the operator defined by (40) as O and consider the following amplitude

$$q_\mu M_\mu \equiv q_\mu \int d^4x e^{-iqx} \langle N| T\left(j^\alpha_{5\mu}(x) O(0)\right)|N'\rangle. \tag{265}$$

From the definition of time ordered product, the right-handed side is rewritten

$$\begin{aligned} q_\mu M_\mu \; &= \; -i \int d^4x e^{-iqx} \{ \langle N| T\left(\partial_\mu j^\alpha_{5\mu}(x) O(0)\right)|N'\rangle \\ & -i\delta(x_0)\langle N| \left[j^\alpha_{50},\ O(0)\right]|N'\rangle \}. \end{aligned} \tag{266}$$

Using (264), we obtain [108]

$$q_\mu M_\mu = -f_\pi \langle \pi^\alpha N|O(0)|N'\rangle - i\langle N|[Q_5(0), O(0)]|N'\rangle \tag{267}$$

or equivalently

$$\begin{aligned} \lim_{q\to 0}\sqrt{2\omega}\langle \pi^\alpha N|O(0)|N'\rangle = -\frac{i}{f_\pi}\langle N|[Q_5(0), O(0)]|N'\rangle \\ -\lim_{q\to 0}\frac{q_\mu}{f_\pi}\int d^4x e^{iqx}\langle N|T(J_{5\mu}(x)O(0))|N'\rangle. \end{aligned} \tag{268}$$

Substituting the concrete form of $O(0)$ as (40) into the above equation, we obtain

$$\begin{aligned} \text{RHS of (268)} = \frac{1}{f_\pi}\langle N|\tilde{d}_u(\overline{u}G\sigma u - m_0^2\overline{u}u) - \tilde{d}_d(\overline{d}G\sigma d - m_0^2\overline{d}d)|N\rangle \\ +\frac{m_*}{f_\pi}\left[2\overline{\theta} + m_0^2\left(\frac{\tilde{d}_u}{m_u} + \frac{\tilde{d}_d}{m_d} + \frac{\tilde{d}_s}{m_s}\right)\right]\langle N|\overline{u}u - \overline{d}d|N\rangle \end{aligned} \tag{269}$$

with

$$m_* = \frac{m_u m_d}{m_u + m_d}, \quad m_0^2 = \frac{\langle 0|g_s\overline{q}G\sigma q|0\rangle}{\langle \overline{q}q\rangle}. \tag{270}$$

Here if we use the Peccei-Quinn mechanism [109], the second term of (269) vanishes in the following way [110].

$$L = \frac{\alpha_s}{8\pi} a G\tilde{G}, \tag{271}$$

where a is axion field and $G\tilde{G} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G^b_{\mu\nu}G^b_{\alpha\beta}$. When there exists cEDM, axion potential becomes

$$V_{eff}(a) = K_1 a + \frac{1}{2}K a^2. \tag{272}$$

Here

$$K \equiv -\lim_{k\to 0}\int d^4x e^{ikx}\langle 0|T\left(\frac{\alpha_s}{8\pi}G\tilde{G}(x)\frac{\alpha_s}{8\pi}G\tilde{G}(0)\right)|0\rangle, \quad (273)$$

$$K_1 \equiv -\lim_{k\to 0}\int d^4x e^{ikx}\langle 0|T\left(\frac{\alpha_s}{8\pi}G\tilde{G}(x)\sum i\frac{\tilde{d}_q}{2}g_s\overline{q}G\sigma\gamma_5 q(0)\right)|0\rangle. \quad (274)$$

Due to [110]

$$\begin{aligned} K &= m_*\langle 0|\overline{q}q|0\rangle, \\ K_1 &= -\frac{1}{2}m_*\sum_{q=u,d,s}\frac{\tilde{d}_q}{m_q}\langle 0|ig_s\overline{q}G\sigma\gamma_5 q|0\rangle. \end{aligned} \quad (275)$$

So

$$\frac{\partial V_{eff}}{\partial a} = K_1 + Ka = 0 \quad (276)$$

leads to vanishing of the second term of (269).

Finally we obtain [108]

$$\begin{aligned} g^{(0)}_{\pi NN} &= \frac{\tilde{d}_u + \tilde{d}_d}{f_\pi}\langle N|H_u - H_d|N\rangle \\ g^{(1)}_{\pi NN} &= \frac{\tilde{d}_u - \tilde{d}_d}{f_\pi}\langle N|H_u + H_d|N\rangle. \end{aligned} \quad (277)$$

Here

$$H_q = g_s\overline{q}G\sigma q - m_0^2\overline{q}q. \quad (278)$$

Thus $g^{(2)}_{\pi NN}$ vanishes if we impose Pecei-Quinn symmetry. In the absence of Pecei-Quinn symmetry, there appears $g^{(2)}_{\pi NN}$ [111]. The contribution of mixing of η with π was also considered in [111].

VI. THE EDMS OF MOLECULES

In this section we consider heteronuclear diatomic molecule which has permanent dipole moment. Polar paramagnetic molecules have stronger enhancement factors than paramagnetic atoms. Diamagnetic molecules are more sensitive to nuclear P,T violation than diamagnetic atoms.

There are many advantageous points in molecule [112]. Firstly, the polar molecule is polarized by a modest laboratory electric field E_{lab} but has a vast internal electric field E_{int}.

This implies the hugely enhanced stark effect and small fake magnetic field of $\frac{\mathbf{v}\times\mathbf{E}_{lab}}{c}$ in comparison with atomic case. Secondly, there appears very small energy interval between nuclear rotation levels of opposite parity, which is roughly 10^{-3}Ry as will be discussed. Also g-factor can be very small etc.

In general the electric dipole moment $\mathbf{D}$ is defined by

$$\mathbf{D} = e\left(\sum_i Z_i\mathbf{R}_i - \sum_j \mathbf{r}_j\right), \tag{279}$$

where $\mathbf{R}_i$ and $\mathbf{r}_j$ are coordinates of nucleons and electrons composing molecule. For heteronuclear molecule

$$\mathbf{D}_a =< a|\mathbf{D}|a >\neq 0 \tag{280}$$

and $\mathbf{D}$ has the permanent electric dipole moment.

So the behaviours of heteronuclear molecule and homonuclear molecule are different.

First we begin with diatomic molecule with total spin $\mathbf{S} = 0$ case.

We first give the general rules of diatomic molecule.

In diatomic molecule, the field has axial symmetry along the two nuclei. Hence the projection of $\mathbf{L}$ (total orbital angular momentum of electrons) on this axis which is denoted by Λ is conserved.

The motion of molecule is composed of the orbital motion of electrons, vibrations and rotation of nucleus, They interact complicatedly but their interactions are approximated as independent motions as the 0'th approximation (Born-Oppenheimer approximation)

$$\psi = \psi_e\psi_v\psi_r \tag{281}$$

and total energy is, therefore,

$$E = E_e + E_v + E_r. \tag{282}$$

They are electronic energy ($\approx Ry$), and vibration and rotation energies of nucleus, respectively. Let us consider the nuclear motions of diatomic molecule. First we begin with the case of total spin (mainly of electrons) $\mathbf{S} = 0$. E_v is considered as a harmonic oscillator and its energy are estimated from

$$M\omega_N^2 a^2 \approx \frac{\hbar^2}{ma^2} \equiv E_e. \tag{283}$$

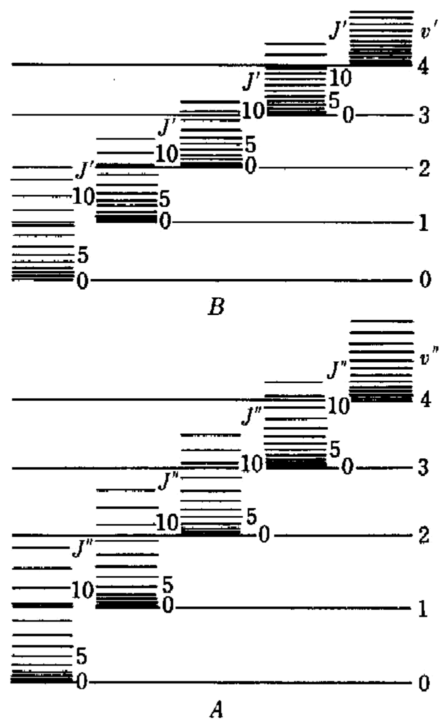


FIG. 19: The vibration (v', v'') and rotation (J', J'') terms of electron states A and B [112].

Here a is the distance between nucleus. M and m are the reduced mass of nuclei and electron mass, respectively. Therefore

$$E_v = \hbar\omega_N \approx \left(\frac{m}{M}\right)^{1/2} E_e. \tag{284}$$

Whereas the rotation energy is

$$E_r = B(\mathbf{K} - \mathbf{L})^2, \tag{285}$$

where $\mathbf{K}$ and $\mathbf{L}$ are total angular momentum of molecule and electron angular momentum, respectively, and

$$B(r) = \frac{\hbar^2}{2Mr^2} = \frac{\hbar^2}{2I}. \tag{286}$$

$\mathbf{K}$ and the axial component of $\mathbf{L}$ are conserved.

$$E_r = \frac{\hbar^2}{2I} l(l+1) \approx \frac{m}{M} J Ry. \tag{287}$$

This is much less than the atomic energy interval in general. We show in Fig. 19 the typical spectroscopies.

TABLE II: Angular momenta of diatomic molecule and their projections to molecular axis. $\mathbf{n}$ is the unit vector of molecular axis.

angular momentum	notation	z component
electron spin	$\mathbf{S}$	Σ
electron orbital angular momentum	$\mathbf{L}$	Λ
nucleus orbital angular momentum	$\mathbf{N}$	0
total angular momentum (without spin)	$\mathbf{K} = \Lambda\mathbf{n} + \mathbf{N}$	Λ
total angular momentum (with spin)	$\mathbf{J} = \mathbf{\Omega} + \mathbf{N}$	Ω

Λ-doubling:

In (285) $\mathbf{K}^2$ and $\mathbf{L}^2$ terms depend on $|\Lambda|$, and $\mathbf{K}\cdot\mathbf{L} \propto B^{2\Lambda} \approx (m/M)^{2\Lambda}$ [113].

Hence off diagonal parts are neglected, and $+\Lambda$ and $-\Lambda$ states are degenerate.

When we take the relativistic effect into consideration we have another coupling of Spin of electrons $\mathbf{S}$ (usually nucleon spin can be neglected) with orbital angular momentum of electrons $\mathbf{L}_e$ and of nucleons $\mathbf{L}_N$. The most important energy shift is $A(r)\mathbf{L}_e \cdot \mathbf{S}$.

Selection rule in the electric dipole transition:

$$|J' - J| \leq 1 \leq J + J' \tag{288}$$

$$+ \to -, \quad - \to + \tag{289}$$

To obtain molecular spectra, we must consider the interactions among the above three terms; electron term E_e, nuclear vibration E_v, and rotation E_r.

The interaction between E_e and E_r is especially important.

First we consider E_e for static nucleus. Unlike atomic case, conserved are not total orbital angular moment $\mathbf{L}$ and spin $\mathbf{S}$ of electrons but their projection to molecular axis

$$\mathbf{J}_z \equiv \Lambda + \Sigma = \Omega \tag{290}$$

which takes the values over $\Lambda + S, \Lambda + S - 1, ..., \Lambda - S$. These states are described as $^{2S+1}\Lambda_\Omega$. For example, $^2\Pi_{1/2}$, $^2\Pi_{3/2}$ for the states with $\Lambda = 1, S = 1/2$.

For atomic fine structure is given by (157), whereas the fine structure for diatomic molecule

$$\Delta E = \frac{dA\Lambda\Sigma}{d\Sigma} = A\Lambda = const. \tag{291}$$

We call this relativistic interactions spin-axis interaction, which is composed of spin-orbit, spin-spin interactions, as well as the spin and orbital interactions with the rotation of molecule. Corresponding to the relative magnitudes of these interactions, we can classify molecule energy levels as follows [113] [114]. We define the magnitudes of interactions as follows.

LA: the coupling of orbital angular momentum with the axis (the electric interaction between the two atoms in the molecule).

SA: the coupling of spin angular momentum with the axis.

ΔE_r: the intervals between rotational levels.

If the distances between terms with different Λ are larger than both the intervals of fine structures $(2S+1)$ and rotational structures, they are further classified into

- Hund's case a $\quad LA \gg SA \gg \Delta E_r$

  In this case, $\Lambda,\ \Sigma,\ \Omega$ are well defined and electron state is expressed as ${}^{2\Sigma+1}\Lambda_\Omega$. For the $a \to a$ transition,

$$\Sigma' - \Sigma = 0, \quad \Omega' - \Omega = 0, \quad \pm 1 \tag{292}$$

$$\nu_L = \Delta T \gg \nu_s = \frac{\partial T}{\partial \Sigma} = A\Sigma \gg \nu_J = B_v(2J+1) \tag{293}$$

$$U_J(r) = U(r) + A(r)\Omega + B(r)\overline{(\mathbf{J}-\mathbf{L}-\mathbf{S})^2}. \tag{294}$$

  Here the third term is a perturbation.

  $\mathbf{L_e}$ and $\mathbf{S}$ precess around the internuclear axis z implying that Λ and Σ are conserved quantum numbers. The total energy is described as

$$E = E_e + A_e\Omega + \hbar\omega(v+1/2) + B_e\{J(J+1) - 2\Omega^2\}. \tag{295}$$

  In this review we are interested in the transition between parity odd rotation levels of the same electron term.

- Hund's case b $\quad LA \gg \Delta E_r \gg SA$

  Σ is not defined. Here the effect of the rotation of the molecule predominates over the multiple splitting and total angular momentum $\mathbf{J}$ and the sum $\mathbf{K} = \mathbf{L} + \mathbf{N}$ are conserved. In this case, $\mathbf{S}$ is almost free from molecule (the vector $\mathbf{K} + \mathbf{S}$) precessing around $\mathbf{J}$, and Σ is not conserved)

$$\nu_K \gg \nu_s, \tag{296}$$

$$|K' - K| \le 1 \le K + K', \tag{297}$$

$$H_0 = H_e + B\mathbf{K}^2 \tag{298}$$

with $\mathbf{K} = \Lambda\hat{z} + \mathbf{N}$ and $\mathbf{J} = \mathbf{K} + \mathbf{S}$.

$$U_K(r) = U(r) + B(r)K(K+1) + A(r)\Lambda\frac{(J-S)(J+S+1)}{2K(K+1)} \tag{299}$$

with

$$K = \Lambda, \Lambda+1, ... \tag{300}$$

Here the third term is perturbation. The total energy is

$$E = U_e + \hbar\omega_e\left(\frac{1}{2}\right) + B_e K(K+1) + A_e\Lambda\frac{(J-S)(J+S+1)}{2K(K+1)}. \tag{301}$$

- Hund's case c $SA \gg LA \gg \Delta E_r$

  Only Ω is well defined. This is the case where the coupling of $\mathbf{L}$ with the axis is small compared with the spin-orbit coupling.

$$H_0 = H_e + H_{ls} + B\mathbf{J}^2. \tag{302}$$

- Hund's case d $\Delta E_r \gg LA \gg SA$

  This is the case where the coupling of $\mathbf{L}$ with the axis is small in comparison with the intervals in E_r.

$$H_0 = H_e + B\mathbf{N}^2 - B(J^+l^- + J^-l^+). \tag{303}$$

- Hund's case e $SA \gg \Delta E_r \gg LA$.

A. Paramagnetic Molecule

As we will show, there are a variety of paramagnetic atoms, for instnce, HgF, YbF, TlO whose electrons configurations are ^{70}Yb=[Xe]$4f^{14}6s^2$, ^{80}Hg=[Xe]$4f^{14}5d^{10}6s^2$,

^{81}Tl=[Xe]$4f^{14}5d^{10}6s^26p^1$. The selection rules of transitions are

$$S' - S = 0, \tag{304}$$

$$\Lambda' - \Lambda = 0, \ \ \pm 1 \tag{305}$$

$$\Sigma^+ \to \Sigma^+, \ \ \Sigma^+ \to \Sigma^+ \ \ \text{for } \Lambda = 0. \tag{306}$$

For BiS molecule [115], electron configuration of Bi is [Xe]$4f^{14}5d^{10}6s^26p^3$ and Bi^{++} has one unpaired electron. The electric field of S leads to a mixing of parity odd states:

$$|\Omega\rangle = |1/2\rangle = a|s_{1/2}, \Omega\rangle + b|p_{1/2}, \Omega\rangle + c|p_{3/2}, \Omega\rangle. \tag{307}$$

Here $\Omega = J_z = 1/2$. So

$$\langle\frac{1}{2}|\frac{1}{2}\rangle = -2ab\frac{4(Z\alpha)^2 Z|e|d_e}{\gamma(4\gamma^2-1)a_B^2(N_sN_{p1/2})^{3/2}}. \tag{308}$$

For total J, angular momentum of nuclei takes two values, $N_1 = J+1/2$ and $N_2 = J+1/2$, so the characteristic energy splitting between P-odd states is

$$\Delta E_r = BN_2(N_2+1) - BN_1(N_1+1) = 2B(J+1/2), \tag{309}$$

which is, for BiS, four to six orders of magnitude smaller than the case of heavy atom.

$$\mathbf{d} = \frac{2\omega d_M \langle\omega|H_d|\omega\rangle}{\Delta E_{J,\eta}}\frac{\mathbf{J}}{J(J+1)} \tag{310}$$

and

$$K = \frac{d}{d_e} = 3\times 10^7 \frac{(-1)^{J+1/2}\eta}{(J+1/2)(J+1)}. \tag{311}$$

The effective electric field on the valence electron is proportional to KE_{ind} for polar paramagnetic molecule. So it is very advantageous to measure molecular EDM.

Recently the most stringent upper limit of d_e was reported by using YbF [24]. Yb belongs to the rare-earth elements and its electron configuration is [Xe]+$4f^{14}6s^2$ and Yb$^+$ ion constitutes paramagnetic molecule. f electrons' interaction with the axis of molecule is weakened by the deep position of the f electrons and classified as Hund's c class. Their result is

$$d_e = (-2.4 \pm 5.7_{stat} \pm 1.5_{sys}) \times 10^{-28}\text{e cm} \tag{312}$$

which sets the upper limit

$$|d_e| < 10.5 \times 10^{-28} \text{ e cm}. \tag{313}$$

The other experiment using ThO [116] is also very interesting since a modest laboratory electric field $E_{lab} \leq 100$ V/cm fully polarizes a ThO whose internal electric field E_{mol} is 100GV/cm. (The electron configuration of Thorium is Th=[Rn]$6d^2 7s^2$.) This gives another advantage for polar molecules. Furthermore, the triplet state $^3\Delta_1$ of ThO gives the merit of g-factor cancellation (see Eq. (330)). Also for the other molecules we can expect g-factor cancellation, where g-factors are defined by the ratio of spin rotation energy H_E and $\mu_B B_z$. Here H_E is given by

$$H_E = \beta \mathbf{J}^2 + \Delta \mathbf{S}' \cdot \mathbf{J} - D\mathbf{n} \cdot \mathbf{E}, \tag{314}$$

where $\mathbf{S}'$ is the effective spin and $\mathbf{S}' = \mathbf{S}$ for Hund's case b.The detail of meanings of right-hand side is given in [117]. The expectation value of H_E crosses zero at a specific value of electric field and the molecule becomes insensitive to magnetic field at that point.

One of the problems for molecular EDM is the difficulty of laser cooling compared with atomic case. This may be solved by first cooling composite atoms and next combining them by the Feshbach resonance [118] and optical trap methods [119] [120]. The theoretical problem is to calculate matrix elements in Dirac-Coulomb + higher order approximation (see Appendix (J)).

B. Diamagnetic Molecule

We will consider TlF as an example of diamagnetic molecule. In searching for molecular EDM, we have two tasks. One is to derive d_{mol} from CP-odd elementary N-N and/or N-e interactions. Another is to deduce d_p and d_n from the observed d_{mol}.

The electron configuration of Tl atom is [Xe]$4f^{14}5d^{10}6s^2 6p^1$ and Tl^+ has a closed electron shell. Tl^+ forms also incomplete shell $6s6p$ instead of $6s^2$,

$$|\Omega\rangle = |6s, \Omega\rangle + \beta \left(-\frac{2\Omega}{\sqrt{3}} |6p_{1/2}, \Omega\rangle + \sqrt{\frac{2}{3}} |6p_{3/2}, \Omega\rangle \right) \tag{315}$$

with

$$\beta = \frac{2}{\sqrt{3}} \frac{Ry}{E_{6s} - E_{6p}} \frac{a^2 r(6s, 6p)}{r_1^2} = 0.27. \tag{316}$$

Here $\Omega = \pm 1/2$, and $r(6s, 6p)$ is the radial integral defined by (180) whose value is 2.3.

Using (F4) [9]

$$\langle s_{1/2}|H|p_{1/2}\rangle = \frac{Gm^2\alpha^2}{\sqrt{2}\pi}\frac{Z^2R}{(N_sN_p)^{3/2}}Ry\{\gamma(Zk_{1p}+Nk_{1n}) - 4j\frac{2+\gamma}{3}\langle k_{2p}\sum_p \boldsymbol{\sigma}_p + k_{2n}\sum_n \boldsymbol{\sigma}_n\rangle\}, \tag{317}$$

where R is the relativistic factor

$$R = \frac{4}{\Gamma^2(2\gamma+1)}\left(\frac{a_B}{2Zr_0A^{1/3}}\right)^{2-2\gamma} \tag{318}$$

with $r_0 = 1.2$fm. As for the nuclear matrix element,

$$\langle k_{2p}\sum_p \boldsymbol{\sigma}_p + k_{2n}\sum_n \boldsymbol{\sigma}_n\rangle \approx k_{2p}\frac{\mathbf{I}}{I} \tag{319}$$

since a valence proton in Tl atom is $s_{1/2}$. Reference [12] goes further to get

$$S(Tl) = -\frac{2\pi}{3}(r_q^2 - r_d^2)d_p, \tag{320}$$

where $r_q,\ r_d$ are defined by (I17). From the experimental limit [121],

$$S_{exp}(Tl) \leq 0.8\times 10^{-8} efm^3 \tag{321}$$

we obtain

$$d_p \leq 10^{-22}\text{e cm}. \tag{322}$$

See (250) and (253) for diamagnetic atom. The numerical calculations were estimated along the following line of thoughts [122]: assuming Born-Oppenheimer approximation, total wave function of TlF is described as

$$\Psi = \psi_n(\mathbf{r_n})\psi_e(\mathbf{r}_i)\psi_R(\mathbf{r}_N,\mathbf{I}). \tag{323}$$

Here $\psi_n(\mathbf{r_n})$ describes the motion of Tl nucleus,$\psi_e(\mathbf{r}_i)$ does F nucleus and electrons with respect to the center of mass of Tl nucleus, and $\psi_R(\mathbf{r}_N,\mathbf{I})$ the spin and motion of Tl nucleus.

Let us integrate over ψ_e and take (B5) into account. We obtain

$$\langle H_{edm}\rangle = D\langle\psi_R\psi_n|\mathbf{a}\cdot\sum_n\left(\frac{q_n}{Z}\boldsymbol{\sigma} - \frac{\mathbf{d}_n}{D}\right)\left(\int_{r_i=0}^{r_n}\psi_e^*\sum_i\frac{Y_{10}^i(\Theta,\Phi)}{r_i^2}\psi_e d^3r_i\right)|\psi_R\psi_n\rangle, \tag{324}$$

where

$$D\boldsymbol{\sigma} = \langle\psi_n|\sum_n \mathbf{d}_n|\psi_n\rangle. \tag{325}$$

For the present approximation (B2), ψ_e is given by

$$\psi_e = \Pi_i \psi_i(r_i) = \Pi_i \sum_l a_l^i r_i^l Y_{lm}^i(\theta_i, \phi_i) \tag{326}$$

and so on.

Anyhow, analytical studies are restricted and we may need more elaborate numerical calculations as was done in the case of atomic structures or much more than that case. However, it is certain that unknown but very fruitful frontiers are expanding in front of this field. Many experiments are preparing or ongoing. In these situations, theoretical developments are strongly awaited.

VII. SUMMARIES AND DISCUSSION

We have explored the EDMs of quarks, leptons, hadrons, atoms, and molecules. First we studied the SM predictions on the EDM and showed that those are far from the present experimental upper limits. We have direct signals of new physics beyond the SM from neutrino oscillations and muon g-2, and many indirect ones like baryon asymmetry, DM etc.

Among them, the CP deficiency in baryon asymmetry $\eta \equiv \frac{n_B}{n_\gamma} \approx 10^{-10}$ is especially important for searching for new physics. Namely, we can not reproduce η via CKM CP violating phase only even if we incorporate CP violation due to a θ term and other radiative corrections in the SM framework like $G\tilde{G}$ etc.

In order to estimate the deviation of phenomena from the SM, we have tried to estimate them first in the SM precisely, including the effects of the above mentioned extra terms.

Next we have explored many theories beyond the SM by focusing on the EDM of elementary particles.

The MSSM and two Higgs doublet model, for instance, give rather large values of EDMs. However, those values are mainly due to the ambiguities of the theories themselves. It is important to see whether such values are checked to be consistent with the other phenomena or not. We think those points are still very insufficient. More predictive models like the renormalizable minimal SO(10) GUT discussed in Section **4.3** often give more stringent values which are still several orders smaller than the present upper limits.

However, the situation is not so pessimistic. Some hope comes from unprecedented collaborations with atomic and molecular physics and elementary particles mainly via brilliant developments of laser physics. Most impressive is the new upper limit of the electron EDM from polarized molecule YbF. As for paramagnetic atoms, theoretical calculations have been developed and seems to be convergent. Whereas, for diamagnetic atoms there are still large discrepancies (Table I). Lattice QCD is very promising but it is not convergent in the limit of $m_\pi = 0$ (Fig.5). However, it is certain that these situations have been improved rapidly. The large parts of such progress have been and will be done by the collaboration of a wide field of physics and chemistry. The mutual close relationships among particle, atomic, and molecular physics require the wide range of studies over these regions.

We hope that this review gives some help for these difficult tasks.

This review is restricted in theoretical part and we have not discussed many excellent ideas on the experimental side. The latter is very attractive but is beyond the scope of this review simply due to the author's ability. We only briefly explain the mechanism and a list of experiments though it is not exhaustive.

The procedure for the EDM measurement is as follows. First a static external electric field $\mathbf{E}$ is applied parallel to magnetic field $\mathbf{B}$. The energy splitting is measured as a spin precession frequency $\nu_{\uparrow\uparrow}$. Next we change $\mathbf{E}$ unti-parallel to $\mathbf{B}$ whose precession frequency is denoted by $\nu_{\uparrow\downarrow}$. Namely,

$$\begin{aligned} h\nu_{\uparrow\uparrow} &= 2\boldsymbol{\mu}\cdot\mathbf{B} + 2\mathbf{d}\cdot\mathbf{E} \\ h\nu_{\uparrow\downarrow} &= 2\boldsymbol{\mu}\cdot\mathbf{B} - 2\mathbf{d}\cdot\mathbf{E} \end{aligned} \tag{327}$$

and

$$h\Delta\nu = 4\mathbf{d}\cdot\mathbf{E}. \tag{328}$$

Its sensitivity is given by

$$\delta d = \frac{h}{2e}\frac{1}{K}\frac{1}{E}\frac{1}{\sqrt{N\tau T}} \tag{329}$$

Here N: number of sample, τ: coherence time, and T: measuring time. K is an enhance factor for paramagnetic atoms and molecules given in (179) and $K \propto Z^3\alpha^2$. E is a magnitude of an internal electric field. So experiments try to get larger values of $K,\ E,\ N,\ \tau,\ T$. We only list up ongoing and planned experiments (see Table 3). We have still more species, solids like $GGG,\ Gd_2Ga_5O_{12},\ Gd_3Fe_5O_{12},\ PbTiO_3,\ Gd_3Ga_5O_{12}$, solid He, liquid Xe (see Table nEDM Collaboration). Please refer to the corresponding sections for the terminologies in the experimental features. A few comments are in order. + signature at PbO molecule implies

TABLE III: A list of ongoing and planned experiments searching for EDM. Superscript * indicates estimated sensitivity.

Species	Group name	Features
muon		
d_μ	FNAL	10^{-21} e cm* (2015)
	J-PARC	10^{-24} e cm* (2015) with spin frozen technique
	PSI	3-4 orders below current limit* (spin frozen technique)
neutron (all 10^{-28} e cm*)		
d_n	ILL (Grenoble)	$\|d_n\| < 2.9 \times 10^{-26}$ ecm (90% C.L.) [22]
	PSI (Zurich)	comagnetometer outside of test material
	SNS (Oak Ridge)	^{3}He as comagnetometer
	KEK-RCNP (Japan)	^{129}Xe as comagnetometer [123]
deuteron		
d_D	KVI/BNL/COSY	10^{-29} e cm*
paramagnetic Atom		
Cs	Amherst College	d(Cs)=$(-1.8 \pm 6.7 \pm 1.8) \times 10^{-24}$ ecm [124]
	LBNL	highly improved magnetic shielding [125]
Tl	Berkeley	$d_e < 1.6 \times 10^{-27} (90\% C.L.)$ e cm [20]
Fr	CYRIC(Tohoku Univ.)	K(Fr)=895 EDM measurement starts on 2014 [126].
Ra	KVI (Groningen)	magneto-optical trap [127]
diamagnetic atom		
^{199}Hg	Seattle	$d(^{199}Hg) < 3.1 \times 10^{-29}$ (95% C.L.)[23]
Ra	Argonne/KVI	large enhancement $d(Ra)/d(Hg) \approx 10^{2-3}$
Xe	@nEDM Collaboration	polarized liquid Xe droplets
	Tokyo Institute of Technology	artificial feedback mechanism [128]
	Princeton	liquid cell
	Univ. Mainz	$d(^{129}Xe) \approx 10^{-30}$ e cm*
Rn/Xe	Michgan	$d(^{129}Xe) = (+0.7 \pm 3.3) \times 10^{-27}$ ecm [129]
Rn	Rn EDM Collaboration	octupole enhancement of 400-600
paramagnetic molecule		
YbF	Hinds (Imperial College) et al.	the most sringent limit of d_e[24]
ThO	ACME Collaboration	g-factor cancellation at $^3\Delta_1$ [116]
PbO	DeMille (Yale) et al.	g-factor cancellation at metastable $^3\Sigma_1^+$ [130]
PbF	Shafer-Ray (Oklahoma) et al.	g-factor cancellation at $^2\Pi_{1/2}$ [131]
HfF^+	Cornell group	trapped molecular ions in rotating electric field [132]
HgF/BaF		same electron configuration as YbF
RaF	KVI	high W_a parameter [133]
FrSr	Aoki (Tokyo) et al.	ultra cold molecule/3D optical lattice [134]
diamagnetic molecule		
TlF	Hinds (Yale) et al.	the measured $\Delta\nu = (1.4 \pm 2.4) \times 10^{-4}$ Hz[135]
YbHg	Takahashi (Kyoto) et al.	ultra cold molecule/3D optical lattice

the parity under the mirror reflction (reflection under arbitrary plane including molecule axis) (see Eq.(306)). As for g-factor cancellation in molecular EDMs, ThO and the others' cancellation mechanisms are different: the former is due to

$$\mu = (\Lambda + g\Sigma)\mu_B \approx 0 \text{ for } \Lambda = 2 \tag{330}$$

and the latters are due to Eq.(314).

This table is far from being exhaustive but reflects some prospect from a theoretical physicist.

For more detail see, for instance, ECT* Workshop: Violations of Discrete Symmetries in Atoms and Nuclei. Nov 15- 19, 2010 [136].

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Appendix A: SU(6) and Dipole moments

Both magnetic dipole moment and electric dipole moment are proportional $eQ\boldsymbol{\sigma}$.

So we can obtain the information of the ratio of $d_p/d_n = \mu_p/\mu_n$ from SU(6) in the light quark (u,d,s) base [17] if CP violation in the EDM does not affect SU(3) symmetry. They are both represented as

$$\langle 56|35|56\rangle \tag{A1}$$

Here baryons belong to 56-representation since irreducible representation of $qqq = 56$ and we use that dipole moments are the generator of SU(6).

$$\begin{aligned}|p,1/2\rangle \;=\; &\frac{\sqrt{2}}{6}\left\{|uud\rangle\left(2|++-\rangle-|+-+\rangle-|-++\rangle\right)\right.\\ &+|udu\rangle\left(2|+-+\rangle-|-++\rangle-|++-\rangle\right)\\ &\left.+|duu\rangle\left(2|-++\rangle-|++-\rangle-|+-+\rangle\right)\right\},\end{aligned} \tag{A2}$$

$$\begin{aligned}Q\sigma_3|p,1/2\rangle \;=\; &\frac{\sqrt{2}}{6}\left\{\frac{2}{3}|uud\rangle\left(2|++-\rangle-|+-+\rangle+|-++\rangle\right)\right.\\ &+\frac{2}{3}|uud\rangle\left(2|++-\rangle+|+-+\rangle-|-++\rangle\right)\\ &-\frac{1}{3}|uud\rangle\left(-2|++-\rangle-|+-+\rangle-|-++\rangle\right)\\ &\left.+\text{cyclic permutations}\right\},\end{aligned} \tag{A3}$$

$$\begin{aligned}&\langle p,1/2|Q\sigma_3|p,1/2\rangle\\ &=3\frac{2}{36}\left(\frac{2}{3}(4+1-1)+\frac{2}{3}(4-1+1)-\frac{1}{3}(-4+1+1)\right)=1.\end{aligned} \tag{A4}$$

The corresponding neutron dipole moments are given by the exchange of $u\leftrightarrow d$, resulting to $\frac{2}{3}\leftrightarrow-\frac{1}{3}$. Therefore,

$$\begin{aligned}&\langle n,1/2|Q\sigma_3|n,1/2\rangle\\ &=3\frac{2}{36}\left(-\frac{1}{3}(4+1-1)-\frac{1}{3}(4-1+1)+\frac{2}{3}(-4+1+1)\right)=-\frac{2}{3},\end{aligned} \tag{A5}$$

and

$$\frac{d_p}{d_n}=\frac{\mu_p}{\mu_n}=-\frac{3}{2}. \tag{A6}$$

The experimental values of MDM of proton and neutron are [19]

$$\mu_p=2.792847356\pm0.000000023,\quad \mu_n=-1.91304273\pm0.00000045 \tag{A7}$$

and the coincidence with SU(6) prediction is good up to quantum corrections. For the EDM, compare with the result of lattice calculations Fig.5.

Appendix B: Multipole expansions

We will study the multipole expansions of electromagnetic potential $A_\mu=(\phi,\mathbf{A})$ due to the charged system of finite size. The electric and magnetic fields are defined by

$$\mathbf{E}=-\frac{\mathbf{1}}{\mathbf{c}}\frac{\partial\mathbf{A}}{\partial\mathbf{t}}-\mathbf{grad}\phi,\quad \mathbf{H}=\mathbf{rot}\mathbf{A}. \tag{B1}$$

Let us assume (as as in the experimental environment) that the electromagnetic field is static, that is, the field is time independent. In such case, $\mathbf{E}$ ($\mathbf{H}$) is determined only by $\mathbf{A}$ (ϕ). Let us consider a stational motion of chaged particles where e_a charged particles are located at $\mathbf{r}_a$ and study how the obserber at $\mathbf{R}$ feels vector potential A_μ.

$$\phi(\mathbf{R}) = \sum_a \frac{e_a(\mathbf{r}_a)}{|\mathbf{R}-\mathbf{r}_a|}, \quad \mathbf{A}(\mathbf{R}) = \sum_a \frac{e_a \mathbf{v}_a(\mathbf{r}_a)}{|\mathbf{R}-\mathbf{r}_a|}. \tag{B2}$$

Here we have neglected the retarded effect of fast particles. If we included it, charged distribution has the retarded time dependence and we should replace the arguments as,

$$t \to t - \frac{|\mathbf{R}-\mathbf{r}_a|}{c}, \quad |\mathbf{R}-\mathbf{r}_a| \to |\mathbf{R}-\mathbf{r}_a| - \frac{\mathbf{v}\cdot(\mathbf{R}-\mathbf{r}_a)}{c}. \tag{B3}$$

If the scale $\mathbf{R} \gg \mathbf{r}_a$, (B2) is expanded around $\mathbf{R}$,

$$\begin{aligned} \phi &= \frac{\sum_a e_a}{R} - \sum e_a(\mathbf{r}_a\cdot\nabla)\frac{1}{R} + \sum e_a e_b r_a^i r_b^j \partial_i\partial_j \frac{1}{R} + ... \\ &= \phi^{(0)} + \phi^{(1)} + \phi^{(2)} + ... \end{aligned} \tag{B4}$$

Then $\phi^{(l)}$ is given by

$$\phi^{(l)} = \frac{1}{R^{l+1}} \sum_{m=-l}^{m=l} \sqrt{\frac{4\pi}{2l+1}} Q_{lm}^{(e)} Y_{lm}^*(\Theta, \Phi), \tag{B5}$$

where

$$Q_{lm}^{(e)} = \sum_a e_a r_a^l \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\theta_a, \varphi_a). \tag{B6}$$

gives electric 2^l-pole moment. The superscript (e) indicates electric moment distinguishing magnetic counterpart (see (B25)). Its continuous representation is

$$Q_{lm}^{(e)} = \sqrt{\frac{4\pi}{2l+1}} \int d^3r \rho(\mathbf{r}) r^l Y_{lm}(\frac{\mathbf{r}}{r}). \tag{B7}$$

This comes from

$$\frac{1}{|\mathbf{R}-\mathbf{r}|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{r^l}{R^{l+1}} \frac{4\pi}{2l+1} Y_{lm}^*(\Theta, \Phi) Y_{lm}(\theta, \phi). \tag{B8}$$

First few normalized spherical harmonics Y_{lm} are

$$\begin{aligned} Y_{00} &= 1/\sqrt{4\pi}, \\ Y_{10} &= i\sqrt{3/(4\pi)}\cos\theta, \quad Y_{1\pm1} = \mp i\sqrt{3/(8\pi)}\sin\theta e^{\pm i\phi}, \\ Y_{20} &= \sqrt{5/(16\pi)}(1-3\cos^2\theta), \\ Y_{2,\pm1} &= \pm\sqrt{15/(8\pi)}\cos\theta\sin\theta e^{\pm i\phi}, \quad Y_{2,\pm2} = -\sqrt{15/(32\pi)}\sin^2\theta e^{\pm 2i\phi} \quad \text{etc.} \end{aligned} \tag{B9}$$

For instance $Q^{(e)}_{1m}$ constitute electric dipole moment

$$Q^{(e)}_{10} = id_z, \quad Q^{(e)}_{1\pm1} = \mp\frac{i}{\sqrt{2}}(d_x \pm id_y). \tag{B10}$$

Analogously, vector potential $\mathbf{A}$ is expanded as

$$A_i(\mathbf{R}) = \int \frac{J_i(\mathbf{r})}{|\mathbf{R}-\mathbf{r}|}d^3r = A^{(0)}_i + A^{(1)}_i + A^{(2)}_i + ... \tag{B11}$$

For instance $A^{(2)}_i$ is

$$A^{(2)}_i = \left(\nabla_k\nabla_l\frac{1}{R}\right)T_{ikl}, \tag{B12}$$

where

$$T_{ikl} = \frac{1}{2}\int d^3r r_k r_l J_i(r). \tag{B13}$$

The identity

$$\int d^3r\nabla_m(r_i r_k r_l J_m) = 0 \tag{B14}$$

leads us to

$$\int d^3r(r_k r_l J_i + r_i r_l J_k + r_i r_k J_l) = 0, \tag{B15}$$

where use has been made

$$\partial_m J_m = 0. \tag{B16}$$

This identity gives 10 constrains. Since T_{ikl} has 6×3 freedoms and $18-10=8$ physical freedoms are remained. We will show that five of eight freedoms constitute $M2$ moment and the remaining three do anapole moment. It goes from subtracting (B15) from (B13) that

$$T_{ikl} = -\frac{1}{3}\epsilon_{ikm}\epsilon_{mnr}\int d^3r r_l r_n J_r = -\frac{1}{3}\epsilon_{ikm}\int d^3r r_l M_m, \tag{B17}$$

where

$$M_m = \epsilon_{mnr}r_n J_r. \tag{B18}$$

Dividing the T_{ikl} of (B17) into symmetric and antisymmetric parts w.r.t. l, m, we obtain

$$\text{Symmetric part of } (B17) = -\frac{1}{6}\epsilon_{ikm}M_{lm} \tag{B19}$$

with

$$M_{lm} = \int d^3r(r_l\epsilon_{mnr} + r_m\epsilon_{lnr})r_n J_r. \tag{B20}$$

This gives magnetic quadrupole moment. Whereas,

$$\text{Anti-symmetric part of } (B17) = \frac{1}{6}\int d^3r \left[\delta_{il}\left(r_k(r_mJ^m) - r^2J_k\right) + \delta_{kl}\left(J_ir^2 - r_i(r_mJ^m)\right)\right]. \tag{B21}$$

Here we use the identity obtained from contracting (B15) w.r.t. k and l

$$\int d^3r(r^2J_i + 2r_ir_mJ^m) = 0 \tag{B22}$$

Then the anti-symmetric part becomes

$$\text{Anti-symmetric part of } (B17) = \frac{1}{4\pi}(\delta_{il}a_k - \delta_{kl}a_i), \tag{B23}$$

where

$$a_i = -\pi\int d^3r r^2J_i \tag{B24}$$

is called anapole moment.

General expression for magnetic photon corresponding to electric counterpart (B7) is

$$Q^{(m)}_{lm} = \frac{1}{l+1}\sqrt{\frac{4\pi}{2l+1}}\int d^3r[\mathbf{r}\times\mathbf{l}]\cdot\nabla(r^lY_{lm}) \tag{B25}$$

and called 2^l-pole magnetic moment (For relativistic case l is replaced by $j = |\mathbf{l}+\mathbf{s}|$).

Appendix C: C,P,T-transformations of Fermi coupling

We consider the four fermions (current-current) coupling. Here it is concerned with the transformation property of Fermion and not with the detailed dynamics, we consider it as $\overline{N}\hat{O}N\overline{L}\hat{O}'L$, where N, L are spinors and $\hat{O}$ and $\hat{O}'$ are a combinations of gamma matrices. The most general form are

$$\begin{aligned}
&G_S\overline{N}N\overline{L}L + G_P\overline{N}\gamma_5L\overline{N}\gamma_5L\\
&+G_V\overline{N}\gamma_\mu N\overline{L}\gamma^\mu L + G_A\overline{N}\gamma_\mu\gamma_5N\overline{L}\gamma^\mu\gamma_5L + G_T\overline{N}\sigma_{\mu\nu}N\overline{L}\sigma^{\mu\nu}L\\
&+G_{V'}\overline{N}\gamma_\mu N\overline{L}\gamma^\mu\gamma_5L + G_{A'}\overline{N}\gamma_\mu\gamma_5N\overline{L}\gamma^\mu L\\
&+iG_{S'}\overline{N}N\overline{L}\gamma_5L + iG_{P'}\overline{N}\gamma_5N\overline{L}L + iG_{T'}\epsilon^{\mu\nu\rho\sigma}\overline{N}\sigma_{\mu\nu}N\overline{L}\sigma_{\rho\sigma}L.
\end{aligned} \tag{C1}$$

The first two lines costitute Lorentz scalars, the third line P-odd and the fourth line P,T-odd terms. Imaginar i in the last line comes from the Hermiticity of action. The last term of the fourth line are also expressed as

$$\overline{N}\sigma_{\mu\nu}N\overline{L}\sigma^{\mu\nu}\gamma_5L \text{ or } \overline{N}\sigma_{\mu\nu}\gamma_5N\overline{L}\sigma^{\mu\nu}L \tag{C2}$$

since

$$\epsilon^{\mu\nu\rho\sigma}\gamma_\mu = -i\gamma_5\gamma^\nu\gamma^\rho\gamma^\sigma. \tag{C3}$$

C,P,T conjugations are defined by

$$C\psi(t,\mathbf{r}) = \gamma^2\psi^*(t,\mathbf{r}), \tag{C4}$$

$$P\psi(t,\mathbf{r}) = i\gamma^0\psi(t,-\mathbf{r}), \tag{C5}$$

$$T\psi(t,\mathbf{r}) = i\gamma^3\gamma^1\psi^*(-t,\mathbf{r}). \tag{C6}$$

The fourth line of (C1) is T-odd since

$$\overline{N}N \to -\overline{N}N, \quad \overline{L}\gamma_5 L \to \overline{L}\gamma_5 L \quad etc. \tag{C7}$$

under T-transformation.

Appendix D: CP phases in general L-R model and generation number

Type I (canonical) seesaw is composed of N left-handed and N right-handed neutrino. Right-handed neutrino has heavy Majorana mass term.

$$L_{Yukawa} = -\overline{\nu_R}M_D\nu_L - \frac{1}{2}\{\overline{(\nu_L)^c}M_L\nu_L + \overline{(\nu_R)^c}M_R\nu_R\} + h.c. \tag{D1}$$

This is described in terms of mass eigen vectors,

$$L_M = (\overline{N^{(1)}}, \overline{N^{(2)}})\begin{pmatrix} U^{(1)} & U^{(2)} \\ V^{(1)*} & V^{(2)*} \end{pmatrix}^T \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}\begin{pmatrix} U^{(1)} & U^{(2)} \\ V^{(1)*} & V^{(2)*} \end{pmatrix}\begin{pmatrix} N^{(1)} \\ N^{(2)} \end{pmatrix} + h.c. \tag{D2}$$

$$\nu_{lL} = \sum_{j=1}^{2N} U_{lj}N_{jL}, \quad \nu_{lR} = \sum_{j=1}^{2N} V_{jl}N_{jR}, \tag{D3}$$

where $l = 1, ..., N$.

So $N \times 2N$ unitary maqtrices U and V are decomposed into $N \times N$ matrices,

$$U = \left(U^{(1)}, U^{(2)}\right)^T, \quad V = \left(V^{(1)}, V^{(2)}\right)^T. \tag{D4}$$

In the SM $m_{\nu_l} = 0$ and there is no mixing in neutrino sector.

For Dirac neutrino case, $U^{(2)} = V^{(2)} = 0$. As we mentioned above, there exist $(N-1)(N-2)/2$ phases in this case.

For Majorana neutrino case, $U^{(2)} = V^{(1)} = 0$ when there exist both left-handed (L-type $N_1, ..., N_N$) and right- handed neutrino (R-type $N_{N+1}, ..., N_{2N}$). In this case $2N \times 2N$ unitary $V^{(2)} = 0$ is added for only left-handed Majorana neutrino case.

$$H_W = \frac{G}{\sqrt{2}} \left[j_{La}^{\dagger} j_L^a + \lambda j_{Ra}^{\dagger} j_R^a + \kappa \left(j_{La}^{\dagger} j_R^a + j_{Ra}^{\dagger} j_L^a \right) \right], \tag{D5}$$

where

$$j_{La} = \sum_l \bar{l}(x) \gamma_a (1 - \gamma_5) \nu_{lL}(x), \tag{D6}$$

$$j_{Ra} = \sum_l \bar{l}(x) \gamma_a (1 + \gamma_5) \nu_{lR}(x) \tag{D7}$$

with $l = e, \mu, ...$ Mass matrices has rebasing and rephasing symmetries, which does not change physics.

We start with N generation of quarks (Dirac fermions). $N \times N$ unitary matrix has N^2 real numbers. Of these, 2N-1 is absorbed by rephasing of 2N left-handed and right-handed quarks. An orthogonal $N \times N$ orthogonal matrix has $N(N-1)/2$ Euler angles. The remaining $(N-1)(N-2)/2$ is the number of phase parameters. Kobayashi-Maskawa predicted that there must at least three generations to incorporate CP phase in mass matrix [2]. If we relax this arguments to include Majorana neutrino we can use only rephasing of charged lepton in MNS mixing matrix whose freedom is N. Therefore, the number of phases in MNS matrix is $N^2 - N(N-1)/2 - N = N(N-1)/2$.

If we furthermore generalize the above arguments to include heavy right-handed neutrino [59],

$$\nu_{lL} = \sum_{j=1}^{2N} U_{lj} N_{jL}, \quad \nu_{lR} = \sum_{j=1}^{2N} V_{jl} N_{jR}, \tag{D8}$$

where $l = 1, ..., N$.

So $N \times 2N$ unitary maqtrices U and V are decomposed into $N \times N$ matrices,

$$U = \left(U^{(1)}, U^{(2)} \right)^T, \quad V = \left(V^{(1)}, V^{(2)} \right)^T. \tag{D9}$$

In the SM $m_{\nu_l} = 0$ and there is no mixing in neutrino sector.

For Dirac neutrino case, $U^{(2)} = V^{(2)} = 0$. As we mentioned above, there exist $(N-1)(N-2)/2$ phases in this case.

Appendix E: Expanion in power of $1/c$

Relativistic equation of Fermion in an external electromagnetic foeld is

$$(\gamma(p - eA) + m)\,\psi = 0. \tag{E1}$$

Let us study the relativistic effect as the deviation from the Schroedinger prescription which is obtained by expanding (E1) in power of $1/c$. For that purpose we must exclude mc^2 from energy, which implies to replace ψ to ψ'

$$\psi = \psi' e^{-imc^2 t/\hbar} \tag{E2}$$

and

$$\left(i\hbar\frac{\partial}{\partial t} + mc^2\right)\psi' = \left[c\boldsymbol{\alpha}\cdot\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right) + \beta mc^2 + e\Phi\right]\psi'. \tag{E3}$$

Substituting

$$\psi' = \begin{pmatrix} \phi' \\ \chi' \end{pmatrix} \tag{E4}$$

into (E3), Dirac spinor is reduced to two component Weyl spinors]

$$\left(i\hbar\frac{\partial}{\partial t} - e\Phi\right)\phi' = c\boldsymbol{\sigma}\cdot\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)\chi', \tag{E5}$$

$$\left(i\hbar\frac{\partial}{\partial t} - e\phi + 2mc^2\right)\chi' = c\boldsymbol{\sigma}\cdot\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)\chi'. \tag{E6}$$

Retaining only the term $2mc^2\chi'$ in the second equation, we obtain

$$\chi = \frac{1}{2mc}\boldsymbol{\sigma}\cdot\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)\phi. \tag{E7}$$

Substituting this into the first equation, we finally obtain the famous Pauli equation,

$$i\hbar\frac{\partial\phi}{\partial t} = \left[\frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + e\Phi - \frac{e}{2mc}\boldsymbol{\sigma}\cdot\mathbf{H}\right]\phi. \tag{E8}$$

The current density is

$$\mathbf{j} = c\psi^*\boldsymbol{\alpha}\psi = c(\phi^*\boldsymbol{\sigma}\chi + \chi^*\boldsymbol{\sigma}\phi). \tag{E9}$$

Substituting χ of (E7) into it, we obtain

$$\mathbf{j} = \frac{i\hbar}{2m}(\phi\nabla\phi^* - \phi^*\nabla\phi) - \frac{e}{mc}\mathbf{A}\phi^*\phi + \frac{\hbar}{2m}\nabla\times(\phi^*\boldsymbol{\sigma}\phi). \tag{E10}$$

In the presence of the EDM (see (30)), $\mathbf{j}$ includes pseudo-vector part,

$$\mathbf{j}_d = id_N \nabla \times (\psi^* \boldsymbol{\gamma} \psi), \tag{E11}$$

which in two components approximation is reduced to

$$\mathbf{j}_d = \frac{\mathbf{d}}{2m} \nabla \times [\phi' \boldsymbol{\sigma} \times (\mathbf{p}' + \mathbf{p})\phi], \tag{E12}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the momenta of ϕ and ϕ', respectively.

Appendix F: Nonrelativistic approximation

In the heavy nucleon limit, nucleon bilinear forms are approximated as

$$\begin{aligned} \overline{N}(x)\gamma_0 N(x) &= \delta(\mathbf{r}), \quad \overline{N}(x)\boldsymbol{\gamma} N(x) = 0, \\ \overline{N}(x)\gamma_0\gamma_5 N(x) &= 0, \quad \overline{N}(x)\boldsymbol{\gamma}\gamma_5 N(x) = -\boldsymbol{\sigma}_N \delta(\mathbf{r}). \end{aligned} \tag{F1}$$

We are interested in P-odd and T-odd weak interaction in the Fermi coupling between electron and nucleons. In the heavy nucleon limit (F1) these interactions are limited in the following forms,

$$H = \frac{G}{\sqrt{2}} \left(k_1 \overline{N} N \overline{e} i\gamma_5 e + k_2 \frac{1}{2} \epsilon_{\kappa\lambda\mu\nu} \overline{N} \sigma_{\kappa\lambda} N \overline{e} \sigma_{\mu\nu} e \right). \tag{F2}$$

In the nonrelativistic (heavy nucleon mass) limit it reduces to

$$H = i\frac{G}{\sqrt{2}} \delta(\mathbf{r})(k_1 \gamma_0 \gamma_5 + 4k_2 \boldsymbol{\sigma} \cdot \boldsymbol{\gamma}). \tag{F3}$$

Here you should consider H is sandwiched by electron wave functions. For the case of a nucleus of charge Z and mass number A, it gives [9]

$$H = i\frac{G}{\sqrt{2}} \delta(\mathbf{r}) \left[(Zk_{1p} + Nk_{1n})\gamma_0\gamma_5 + 4\left(k_{2p} \sum_p \boldsymbol{\sigma}_p + k_{2n} \sum_n \boldsymbol{\sigma}_n \right) \cdot \boldsymbol{\gamma} \right], \tag{F4}$$

$$\begin{aligned} \langle s_{1/2}| \; H \; |p_{1/2}\rangle = g\frac{Z^2 R}{(N_s N_p)^{3/2}} Ry & \left[\gamma(Zk_{1p} + Nk_{1n}) - 8\mathbf{j}\frac{2+\gamma}{3} \langle k_{2p} \sum_p \boldsymbol{\sigma}_p + k_{2n} \sum_n \boldsymbol{\sigma}_n \rangle \right. \\ & \left. + \; 8\mathbf{j}(1-\gamma)\langle k_{2p} \sum_p \left(\mathbf{n}_p(\boldsymbol{\sigma}_p \mathbf{n}_p) - \frac{1}{3}\boldsymbol{\sigma}_p \right) + k_{2n} \sum_n \left(\mathbf{n}_n(\boldsymbol{\sigma}_n \mathbf{n}) - \frac{1}{3}\boldsymbol{\sigma}_n \right) \rangle \right], \end{aligned} \tag{F5}$$

The above arguments can be applied for both paramagnetic and diamagnetic atoms. Let us apply the above arguments to Cs, Tl, and Xe* atoms [9], corresponding to the arguments in Section **5.2** in the presence of (F2), The wave function for Cs is described as

$$\begin{aligned} |\overline{6s_{1/2}, F}\rangle &= |6s_{1/2}, F\rangle - 3.7 \times 10^{-11} \Big[0.41k_{1p} + 0.59k_{1n} \\ &+ 0.74 \times 10^{-2} \left(F(F+1) - \frac{33}{2}\right) k_{2p}\Big] |6p_{1/2}, F\rangle, \end{aligned} \tag{F6}$$

and, therefore,

$$\begin{aligned} d(Cs) &= e\langle\overline{6s_{1/2}, F}|z|\overline{6s_{1/2}, F}\rangle = -ea_B \times 1.34 \times 10^{-10} \\ &\times \left[0.41k_{1p} + 0.59k_{1n} + 0.74 \times 10^{-2} \left(F(F+1) - \frac{33}{2}\right) k_{2p}\right]. \end{aligned} \tag{F7}$$

Here F is the total angular momentum of the atom. The observed value [137] is

$$d(Cs) = (-1.8 \pm 6.7 \pm 1.8) \times 10^{-24}\ |e|cm. \tag{F8}$$

For Tl

$$d(\text{Tl}) = ea_B \cdot 0.96 \times 10^{-9}(0.4k_{1p} + 0.6k_{1n} - 2 \cdot 10^{-3}k_{2p}). \tag{F9}$$

For Xe*

$$d(\text{Xe}^*) = -1.3 \cdot 10^{-10} ea_B(0.41k_{1p} + 0.59k_{1n}). \tag{F10}$$

Appendix G: Strong CP violation

In the QCD world, the true vacuum is described by the θ vacuum,

$$|\theta\rangle \equiv \sum_n e^{-in\theta}|n\rangle, \quad (n = \text{integer}). \tag{G1}$$

$$\begin{aligned} \langle\theta'|e^{-iHt}|\theta\rangle &= \sum_{n,m} e^{im\theta'} e^{-in\theta}\langle m|e^{-iHt}|n\rangle \\ &= \sum_{m,n} e^{-i(n-m)\theta} e^{im(\theta'-\theta)} \int [dA]_{n-m} e^{i\int Ld^4x} \\ &= \sum_{\nu} e^{-i\nu\theta} \int [dA]_{\nu} e^{i\int Ld^4x} \end{aligned} \tag{G2}$$

Using that A_n gives

$$n = \frac{1}{16\pi^2} \int d^4x Tr\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right) \tag{G3}$$

and substituting (G3) into (G2) we obtain

$$\langle\theta'|e^{-iHt}|\theta\rangle = \sum_\nu \int [dA]_\nu e^{i\int L_{eff}\ d^4x} \tag{G4}$$

with

$$L_{eff} = L + \frac{\theta}{16\pi^2}\int d^4x Tr\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right) \tag{G5}$$

Appendix H: U(1) problem

θ term in (201) comes from the fact that the vacuum in QCD is $\theta\rangle$, whereas $G\tilde{G}$ term in (216) does from quantum anomaly, occurring irrelevant to Abelian and non Abelian. In this appendix we will show that these two terms are closely related and lead us to solve U(1) problem.

In the following discussions we consider mass zero quark limit, and $N_f = 3$, up, down, strange quarks. Chiral invariant action has originally $U_L(3)\otimes U_R(3)$ symmetry. If, as we have considered, QCD vacuum is quark condensate

$$\langle\overline{u}u\rangle = \langle\overline{d}d\rangle = \langle\overline{s}s\rangle, \tag{H1}$$

action symmetry is reduced to the flavor symmetry U(3) and generates 3^2 NG bosons. They are $\pi^\pm,\ \pi^0,\ K^\pm,\ K^0,\ \overline{K}^0,\ \eta_8$, and η_0. Here the first eight particles constitute octet and the last a singlet. The observed mass eigen states, η and η' particles, are the linear combinations of η_8 and η_0, and their masses are $m_\eta = 550$MeV, $m_{\eta'} = 958$ MeV. Weinberg showed [138] that the observed $m_{\eta'}$ is too heavy for predicted NG boson,

$$m_{\eta'} \leq \sqrt{3}m_\pi. \tag{H2}$$

This is one of the U(1) problems. Another is concerned with $\eta\to\pi^+\pi^-\pi^0$ process.

Let us explain these problems [139]: The octet axial vector currents satisfy

$$\partial^\mu J^a_{5\mu} = f_a m_a^2 \phi^a \quad (a = 1, ..., 8) \tag{H3}$$

and

$$\begin{aligned}\delta_{ab}m_a^2 f_a^2 &= i\frac{m_b^2-k^2}{m_b^2}ik_\nu\int d^4x e^{-ikx}\langle 0|T\left(\partial^\mu J^a_{5\mu}(0)\partial^\nu J^b_{5\nu}(x)\right)|0\rangle \\ &= i\frac{m_b^2-k^2}{m_b^2}\left\{ik_\nu\int d^4x e^{-ikx}\langle 0|T\left(\partial^\mu J^a_{5\mu}(0) J^b_{5\nu}(x)\right)|0\rangle\right. \\ &+ \left.\int d^4x e^{-ikx}\langle 0|\delta(x_0)\left[\partial^\mu J^a_{5\mu}(0),\ J^b_{50}(x)\right]|0\rangle\right\}.\end{aligned} \tag{H4}$$

In the low energy limit, if there is no massless pole, this reduces to

$$\delta_{ab} m_a^2 f_a = i \int d^4x \langle 0|\delta(x^0)[J_{50}^b(x), \partial^\mu J_{5\mu}^a(x)]|0\rangle \tag{H5}$$

Whereas, isosinglet axial vector current constitute ABJ anomaly (216). The isosinglet can be described as a sum of SU(3) octet and singlet,

$$J_{5\mu} = \frac{1}{\sqrt{3}} J_{5\mu}^{(8)} + \sqrt{\frac{2}{3}} J_{5\mu}^{(0)} \tag{H6}$$

with

$$J_{5\mu}^{(8)} = \frac{1}{\sqrt{3}}(\overline{u}\gamma_\mu\gamma_5 u + m_d \overline{d}\gamma_\mu\gamma_5 d - 2\overline{s}\gamma_\mu\gamma_5 s). \tag{H7}$$

$$J_{5\mu}^{(0)} = \sqrt{\frac{2}{3}}(\overline{u}\gamma_\mu\gamma_5 u + m_d \overline{d}\gamma_\mu\gamma_5 d + \overline{s}\gamma_\mu\gamma_5 s) \tag{H8}$$

Taking (216), (218), and (219) into considerations, we obtain the same equation for isosinglet case as (H5) by replacing $J_{5\mu}$ with $\tilde{J}_{5\mu}$,

$$\begin{aligned} m_0^2 f_0^2 &= i\frac{m_0^2 - k^2}{m_0^2} \left\{ ik_\nu \int d^4x e^{-ikx} \langle 0|T\left(\partial^\mu \tilde{J}_\mu^5(0)\tilde{J}_\nu^5(x)\right)|0\rangle \right. \\ &+ \left. \int d^4x e^{-ikx} \langle 0|\delta(x_0)\left[\partial^\mu \tilde{J}_\mu^5(0),\ \tilde{J}_0^5(x)\right]|0\rangle \right\}. \end{aligned} \tag{H9}$$

Here f_0 is the isoscalar meson decay constant. If there is no zero mass pole like the octet cases, this relation is same as the octet case (H5) except for $J_{5\mu}$ replaced by $\tilde{J}_{5\mu}$, and we obtain

$$m_0^2 f_0^2 = m_\pi^2 f_\pi^2. \tag{H10}$$

So, if SU(3) is good symmetry, it goes from (H6) and (H10) that

$$f_0 \geq \frac{1}{\sqrt{3}} f_\pi, \tag{H11}$$

which directly leads to (H2). However if any massless particle couples to $\tilde{J}_\mu^5$, then first term of (H5) does not vanish and we can evade (H10) [140]. 't Hooft showed that this is indeed the case if we take θ vacuum into consideration correctly [141]. Also Witten proposed a solution compatible with quark condensate [142]:

$$m_{\eta^0}^2 = \frac{4N_f^2}{f_{\eta^0}} \left(\frac{\partial^2 E_\theta}{\partial\theta^2}\right)_{\theta=0}, \tag{H12}$$

where

$$\left(\frac{\partial^2 E_\theta}{\partial\theta^2}\right)_{\theta=0} = \frac{1}{N_c^2}\left(\frac{1}{16\pi^2}\right)^2 \int d^4x \langle T\left(Tr(G(x)\tilde{G}(x))Tr(G(0)\tilde{G}(0))\right)\rangle \tag{H13}$$

$$\langle \pi^+\pi^-\pi^0|\eta\rangle = \frac{m_u - m_d}{F_\pi m_q} \lim_{k\to o}\langle \pi^+\pi^-|\partial^\mu J_{5\mu}(k)|\eta\rangle. \tag{H14}$$

The right-hand side vanishes due to momentum conservation. However, it is experimentally observed as $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 200$eV. This process is occurred via SU(2) violating operator [143]

$$\mathcal{L} = \frac{1}{2}(m_u - m_d)(\overline{u}u - \overline{d}d) \tag{H15}$$

and

$$\langle 3\pi|\mathcal{L}|\eta\rangle \to \frac{(m_u - m_d)A}{\sqrt{2}f_\pi^2}, \tag{H16}$$

where

$$\begin{aligned} A &\equiv \langle \pi\pi|(m_u\overline{u}\gamma_5 u + m_d\overline{d}\gamma_5 d)|\eta\rangle \\ &= \frac{1}{2i}\langle \pi\pi|\partial^\mu \tilde{J}_\mu^5(0)|\eta\rangle. \end{aligned} \tag{H17}$$

So this process is suppressed by axial vector current conservation even if $mu \neq m_d$. This is another U(1) problem.

$$L_{QCD} \to L_{QCD} + 2N_f\varphi\partial^\mu K_\mu \tag{H18}$$

under the chiral transformation

$$\psi' = e^{i\gamma_5\varphi}\psi. \tag{H19}$$

Appendix I: Schiff moment

There are several origins for the Schiff moment. Here we discuss the Schiff moment induced by the nuclear EDM when the charge and the EDM distributions (ρ_q and ρ_d, respectively) in the nucleus are different [12].

The interaction of the electron with the dipole moment of finite size nucleus is

$$V_s = \int d^3r'[\rho_d(\mathbf{r}') - \rho_q(\mathbf{r}')\mathbf{d}_N \cdot \nabla' \frac{-e}{|\mathbf{r} - \mathbf{r}'|}, \tag{I1}$$

$$V_s = \frac{1}{2}e\int d^3r'[\rho_d(\mathbf{r}') - \rho_q(\mathbf{r}')d_{N,l}r'_m r'_n \nabla_l\nabla_m\nabla_n \frac{1}{r}. \tag{I2}$$

Here we may assume [144]:

ρ_q is spherically symmetric.

$\mathbf{d_N}$ coincides with the EDM of valence nucleon, $\mathbf{d_N} = d_{p,n}\boldsymbol{\sigma}$.

ρ_d is due to the valence nucleon.

Then

$$V_s = \frac{1}{2} e d_{p,n} \int d^3 r' 4\pi r^2 \left[\rho_d(\mathbf{r'}) \langle \sigma_l n_m n_n \rangle - \rho_q(r') \frac{1}{3} \delta_{mn} \langle \sigma_l \rangle \right] \nabla_l \nabla_m \nabla_n \frac{1}{r}, \tag{I3}$$

where $\mathbf{n} = \mathbf{r}'/r'$. Let us divide $\nabla_l \nabla_m \nabla_n$ as

$$\left[\nabla_l \nabla_m \nabla_n - \frac{1}{5}(\delta_{lm}\nabla_n + \delta_{mn}\nabla_l + \delta_{nl}\nabla_m)\Delta \right] + \frac{1}{5}(\delta_{lm}\nabla_n + \delta_{mn}\nabla_l + \delta_{nl}\nabla_m)\Delta. \tag{I4}$$

The first term corresponds to the electron interaction with the 2^3-pole moment of the nucleus.

$$\begin{aligned} \text{The second term} &= \left[\rho_d \langle \sigma_l n_m n_n \rangle - \rho_q \frac{1}{3} \delta_{mn} \langle \sigma_l \rangle \right] \frac{1}{5} (\delta_{lm}\nabla_n + \delta_{mn}\nabla_l + \delta_{nl}\nabla_m) \\ &= - \left[\frac{1}{3} \rho_q \langle \boldsymbol{\sigma} \rangle - \frac{1}{5} \rho_d \langle 2\boldsymbol{\sigma} \cdot \boldsymbol{nn} + \boldsymbol{\sigma} \rangle \right] \cdot \nabla \end{aligned} \tag{I5}$$

Here we use [6] [144]

$$\begin{aligned} \boldsymbol{\sigma} \cdot \boldsymbol{nn} &= \frac{1}{3}\boldsymbol{\sigma} - \frac{\sqrt{8\pi}}{3} [Y_2 \otimes \boldsymbol{\sigma}]_{(1)} \\ \boldsymbol{\sigma} &= \sqrt{4\pi} [Y_0 \otimes \boldsymbol{\sigma}]_{(1)} \\ 2\boldsymbol{\sigma} \cdot \boldsymbol{nn} + \boldsymbol{\sigma} &= \sqrt{4\pi} \left(\frac{5}{3} [Y_0 \otimes \boldsymbol{\sigma}]_{(1)} - \frac{2\sqrt{2}}{3} [Y_2 \otimes \boldsymbol{\sigma}]_{(1)} \right) \end{aligned} \tag{I6}$$

where

$$[Y_l \otimes \chi]_{j,m} = \sum_{m_l, m_s} \langle l, m_l; \frac{1}{2}, m_s | j, m \rangle Y_{l,m_l}(\theta, \phi) \chi_{m_s} \tag{I7}$$

with the Clebsch-Gordan coefficient $\langle l, m_l; \frac{1}{2}, m_s | j, m \rangle$ related with 3j-symbol

$$\langle k_1, q_1, k_2, q_2 | K, Q \rangle = (-1)^{k_1 - k_2 + Q} \sqrt{2K+1} \begin{pmatrix} k_1 & k_2 & K \\ q_1 & q_2 & -Q \end{pmatrix}. \tag{I8}$$

The following equation is the Wigner-Eckart theorem (the definition of reduced matrix element) $\langle\ ||\ ||\ \rangle$,

$$\langle \kappa m | O_{JM} | \kappa m' \rangle = \frac{1}{\sqrt{2j+1}} \langle jmJM | jm' \rangle \langle \kappa || O_J || \kappa \rangle \tag{I9}$$

[6] The following arguments are indebted to discussions with T. Sato

where κ is defined by (183) and

$$|\kappa\rangle \equiv |[Y_l(\boldsymbol{n}) \otimes \chi]_{(j)}\rangle. \tag{I10}$$

It should be noted that the reduced matrix has no dependence on $m,\ m'.\ M$.

For $J = 1$ case

$$\langle\kappa|\vec{O}|\kappa\rangle = \langle\kappa|\vec{J}|\kappa\rangle \frac{\langle\kappa||\vec{O}||\kappa\rangle}{\langle\kappa||\vec{J}||\kappa\rangle}. \tag{I11}$$

$$\langle\kappa||\vec{J}||\kappa\rangle = \sqrt{J(J+1)(2J+1)} \tag{I12}$$

$$\begin{aligned}
\langle\kappa||[Y_l \otimes \sigma]_{(1)}||\kappa\rangle \ &= \ 2|\kappa|(-1)(-1)^{|\kappa|}(j,\frac{1}{2};j,-\frac{1}{2}|1,0) \\
&\times \begin{cases} \dfrac{1-2\kappa}{\sqrt{3}} & \text{for } l=0 \\ -\dfrac{2(1+\kappa)}{\sqrt{6}} & \text{for } l=2 \end{cases} \qquad (\text{I13}) \\
&= \frac{1}{2}\sqrt{\frac{2j+1}{j(j+1)}} \times \begin{cases} 1-2\kappa \ \ \text{for} \ \ l=0 \\ -\sqrt{2}(1+\kappa) \ \ \text{for } l=2 \end{cases} \qquad (\text{I14})
\end{aligned}$$

Thus we obtain

$$\begin{aligned}
&\left\{\rho_d\langle\sigma_l n_m n_n\rangle - \rho_q\frac{1}{3}\delta_{mn}\langle\sigma_l\rangle\right\}\frac{1}{5}\left[\delta_{lm}\nabla_m + \delta_{mn}\nabla_l + \delta_{nl}\nabla_m\right] \\
&= \left[\frac{1}{3}\rho_q(\kappa-\frac{1}{2}) - \frac{1}{5}\rho_d(\kappa-\frac{3}{2})\right]\frac{\langle\mathbf{j}\rangle\cdot\nabla}{j(j+1)} \qquad (\text{I15})
\end{aligned}$$

So

$$V_s = \frac{ed_{p,n}}{2}\left[r_q^2\frac{1}{3}(\kappa-\frac{1}{2}) - r_d^2\frac{1}{5}(\kappa-\frac{3}{2})\right]\frac{\mathbf{j}\cdot\nabla}{j(j+1)}4\pi\delta(\mathbf{r}), \tag{I16}$$

where the mean squared radii was defined by

$$r_{q,d}^2 \equiv \int d^3r'\ r'^2\rho_{q,d}(\mathbf{r}'). \tag{I17}$$

In this derivation we assumed that the nuclear charge is uniformly distributed over a sphere of radius $r_0 = 1.2\times10^{-13}A^{1/3}$ cm, and $r_q^2 = \frac{3}{5}r_0^2$. Also we may assume $r_d^2 = r_q^2$ [9]. Then we get the final expression for the Schiff moment.

$$\mathbf{S} = d_{p,n}r_0^2\frac{4\pi}{25}\frac{(\kappa+1)\mathbf{j}}{j(j+1)}. \tag{I18}$$

Appendix J: Effective Hamiltonian in molecule

We have said that there appears huge internal electric field $\mathbf{E}_{int}$ in polar molecule. Here we consider how to estimate E_{int}. The Dirac-Coulomb Hamiltonian is

$$H_0 = \sum_i \{c\boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i mc^2 + V_{nucl}(\mathbf{r}_i)\} + \sum_{i<j} \frac{1}{\mathbf{r}_{ij}} \tag{J1}$$

and P,T-odd perturbation (the intrinsic part of H_{PTV}) is

$$H' = -d_e \sum_i \beta_i \boldsymbol{\sigma} \cdot \mathbf{E}_i^{int}. \tag{J2}$$

with

$$\mathbf{E}_{i,int} = -\nabla_i \left(V_{nucl}(\mathbf{r}_i) + 2\sum_{i>j} \frac{e^2}{r_{ij}} \right). \tag{J3}$$

Here electric field is given by Eq.(B1) with $\phi = \sum_i \{V_{nucl}(\mathbf{r}_i)\} + \sum_{i<j} \frac{1}{\mathbf{r}_{ij}}\}$. We are considering a static field, $\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = 0$, and H' is represented as

$$H' = d_e \sum_i [\beta_i \boldsymbol{\sigma}_i \cdot \nabla_i,\ H - T] \tag{J4}$$

Here T is kinetic term of electron

$$T = \sum_i \{c\boldsymbol{\alpha}_i \cdot \mathbf{p}_i + \beta_i mc^2\}. \tag{J5}$$

The expectation value w.r.t. the eigen function of H_0 gives

$$\langle\Psi| \sum_i [\beta_i \boldsymbol{\sigma}_i \cdot \nabla_i,\ H] |\Psi\rangle = 0 \tag{J6}$$

Whereas,

$$\sum_i [\beta_i \boldsymbol{\sigma}_i \cdot \nabla_i,\ T] = i \sum_i \sum_j \{[\beta_i \boldsymbol{\sigma}_i \cdot \mathbf{p}_i, \boldsymbol{\alpha}_j \cdot \mathbf{p}_j] + [\beta_i \boldsymbol{\sigma}_i \cdot \mathbf{p}_i,\ (\beta_j - 1)m_j c]\}$$

Here the second term vanishes and the first term gives

$$i \sum_i \sum_j \{[\beta_i \boldsymbol{\sigma}_i \cdot \mathbf{p}_i, \boldsymbol{\alpha}_j \cdot \mathbf{p}_j] = \begin{cases} \sum_i 2\beta_i \gamma_5 \mathbf{p}_i^2 & \text{for } i = j. \\ 0 & \text{for } i \neq j. \end{cases}$$

So

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} \tag{J7}$$

Thus H' of (J4) is rewritten as [145]

$$H'_{eff} = 2icd_e \sum_i \beta_i \gamma_5 \mathbf{p}_i^2 \tag{J8}$$

and we obtain finally

$$2ic\langle\psi_0|\beta\gamma_5 p^2|\psi_0\rangle = -4cp^2 \Im(\varphi^\dagger \chi). \tag{J9}$$

Thee enhancement factor is given by

$$K = \sum_n \frac{\langle\psi|2ic\beta_i\gamma_5 \mathbf{p}_i^2|\phi_n\rangle\langle\phi_n|\sum_i ez_i|\psi\rangle}{E - E_n} + h.c. \tag{J10}$$

So the detailed calculations are reduced to the electron wave functions in atoms and molecules. For molecular case, unfortunately, only H_2^+ can be solved in the Born-Oppenheimer approximation [146]. However, its perturvation expansion around atomic level is also interesting since this method is applicable to the other diatomic molecule [147]. For more detailed explanation for diatomic case, see [148].

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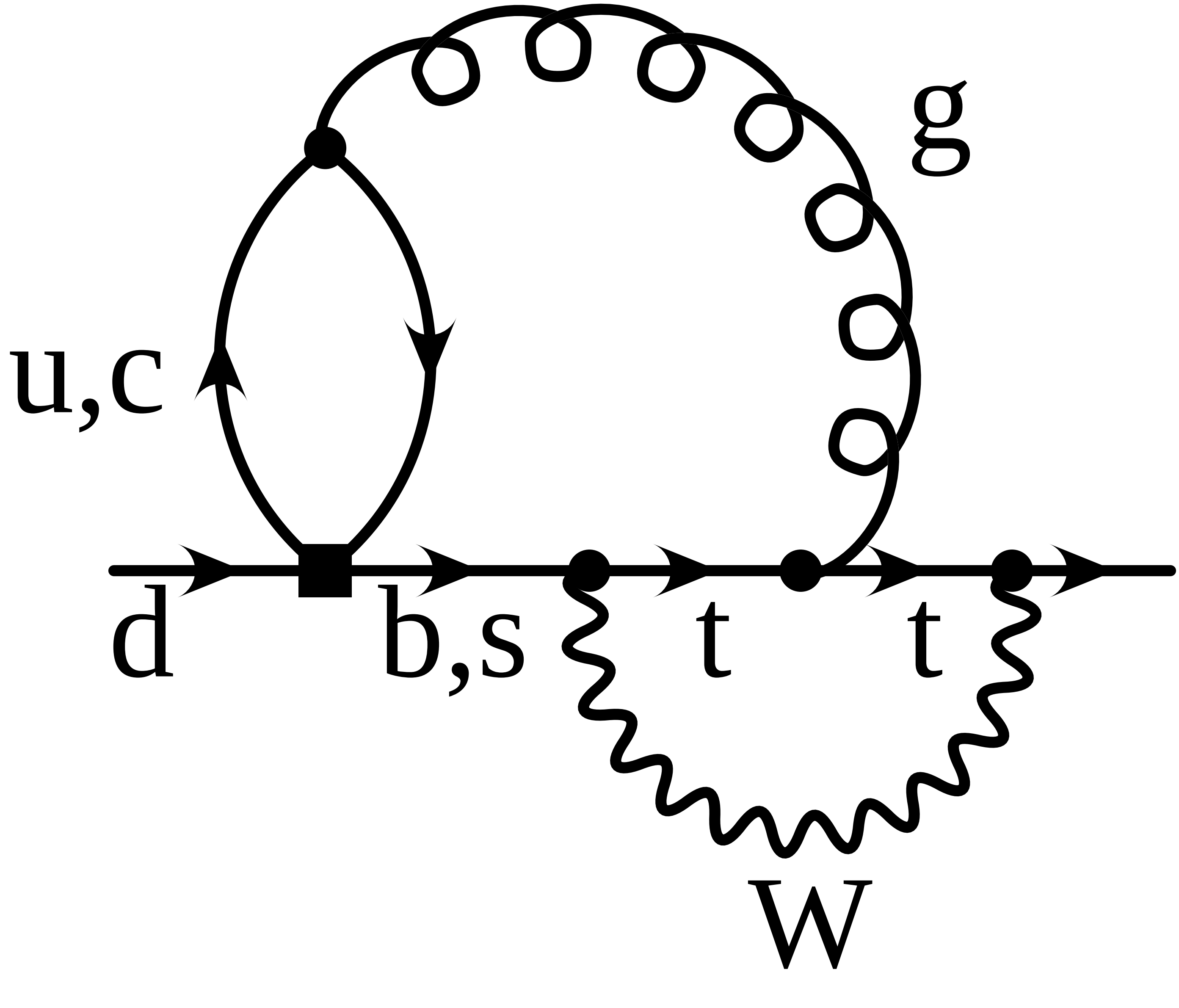

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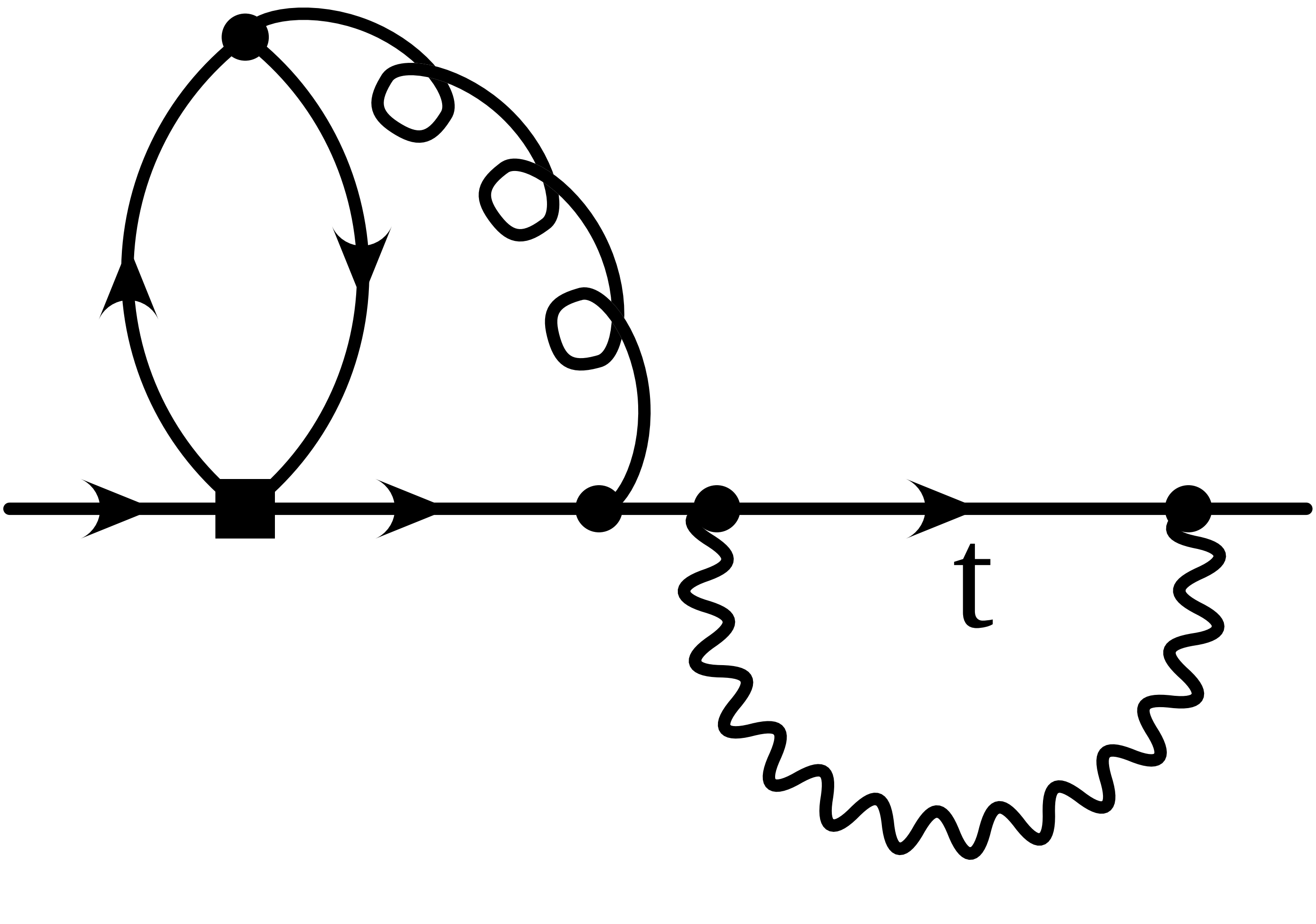
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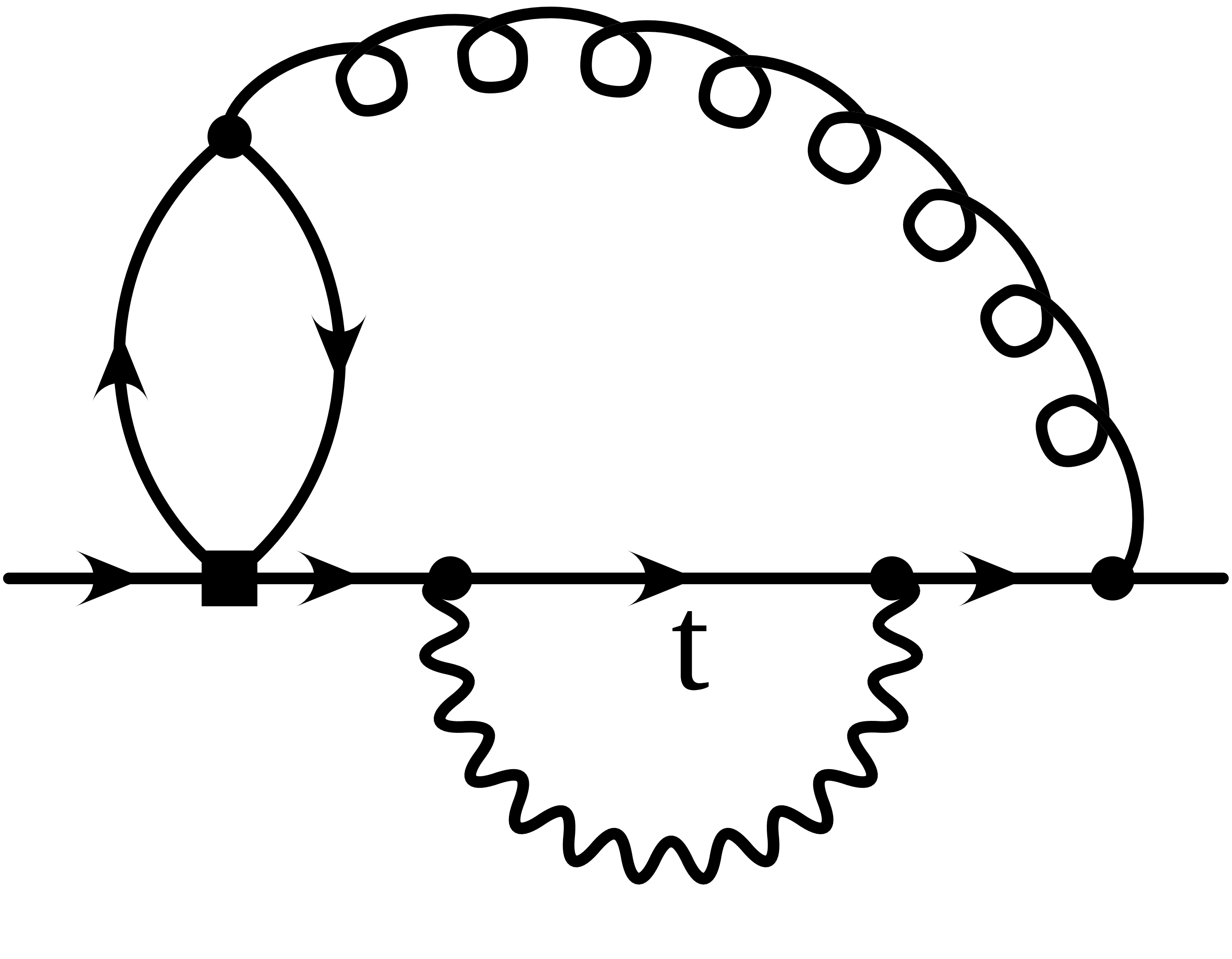
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