



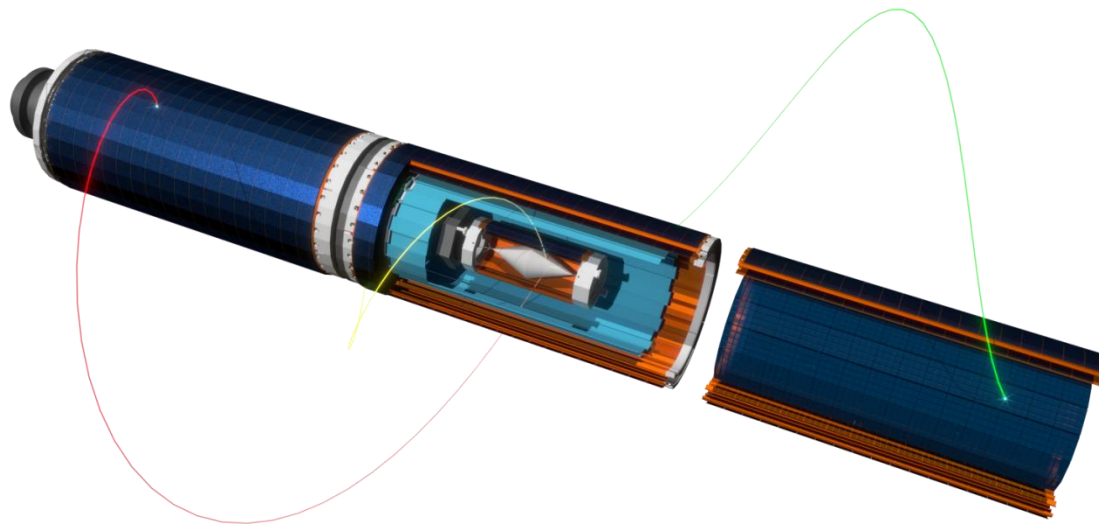
JOHANNES GUTENBERG
UNIVERSITÄT MAINZ



Track reconstruction for the Mu3e experiment

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for the Mu3e collaboration

DPG 2018 @ Würzburg (.03.22, T85.1)



Mu3e Experiment

Mu3e Experiment:

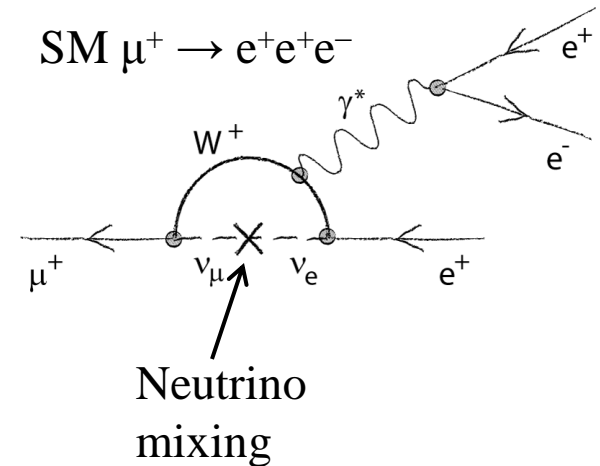
- Search for **Lepton Flavor Violation (LFV)**
 - Decay: $\mu^+ \rightarrow e^+e^+e^-$
 - **Standard Model**: $\text{Br} < 10^{-54}$ (unobservable)
 - Enhanced in **New Physics** models:
 - SUSY, leptoquarks, etc.
 - *Any observed decay will point to NP*
- Location: Paul Scherrer Institute (PSI)
 - Commission in 2019, start in 2020

Current experimental status:

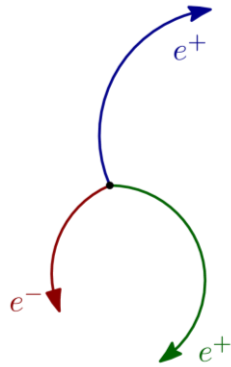
- SINDRUM (1988) [Nucl.Phys.B299\(1988\)1](#)
- $\text{Br}(\mu^+ \rightarrow e^+e^+e^-) < 10^{-12}$ at 90% c.l.

Mu3e aims for sensitivity of one in 10^{15} μ -decays

- Existing beam line: 10^8 μ/s
- With new beam line: one in 10^{16}



Signal

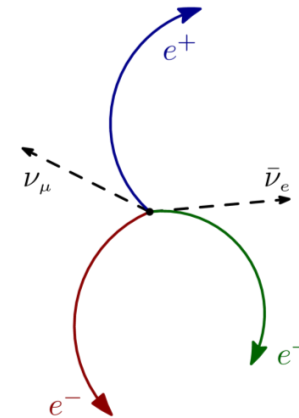
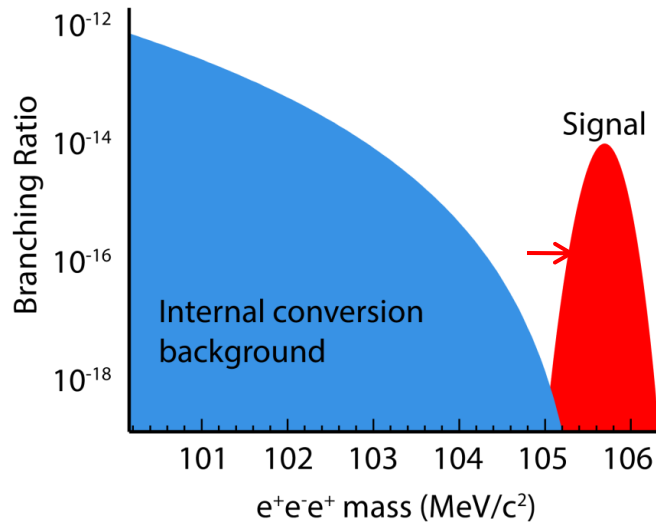


Signal: $\mu^+ \rightarrow e^+e^+e^-$

- Three tracks
- Decay at rest
 - Common vertex
 - Same time
 - $\sum \mathbf{p}_e = 0$
 - $\sum E_e = m_\mu$
 - e^\pm energy < 53 MeV/c

Background:

- Random combinations:
 - $\mu^+ \rightarrow e^+ + 2\nu$
 - e^+/e^- scattering
 - Fake tracks
 - Not same vertex, time, etc.
 - Good vertex/time resolution
- Internal conversion:
 - $\mu^+ \rightarrow e^+e^+e^- + 2\nu$
 - Missing momentum & energy:
 - Need good momentum resolution




Mu3e Detector

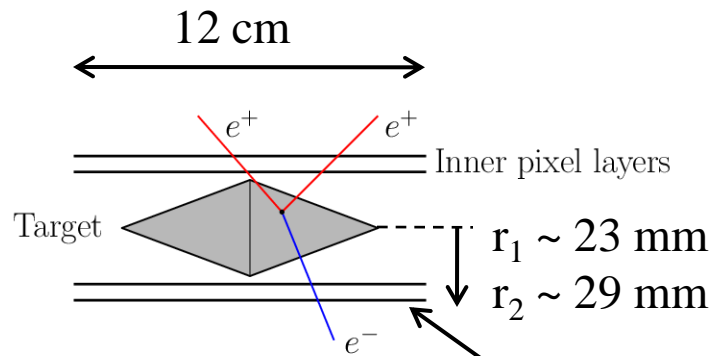
He

$B = 1 \text{ T}$

$O(10^8) \mu^+/s$

μ beam 

$p_\mu \sim 28 \text{ MeV}/c$



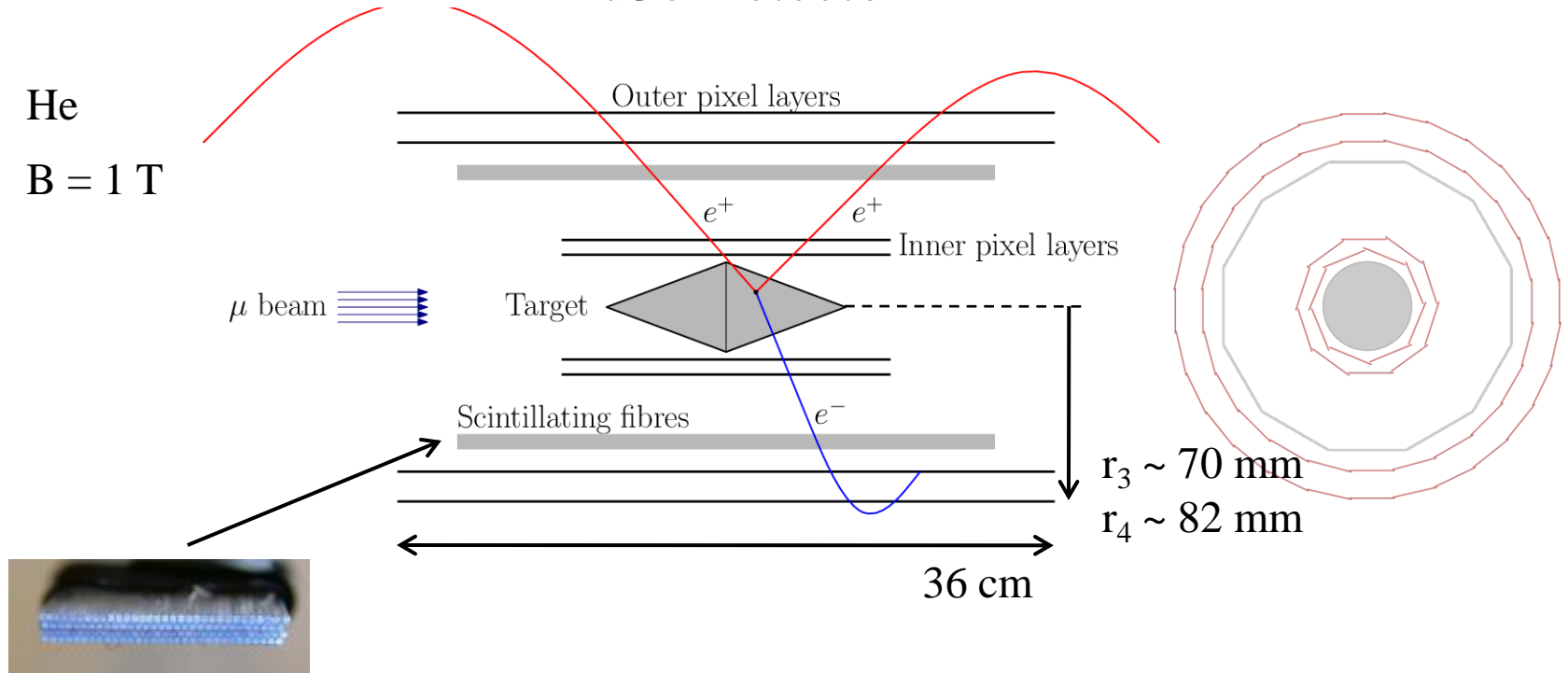
Muons stop and decay on target:

- Double cone hollow target
 - $O(100) \mu\text{m}$ thickness
 - Vertex separation
- Existing beam line at PSI:
 - Continuous muon beam
 - $O(10^8) \mu^+/s$

Inner pixel layers:

- High granularity
- Thin (to reduce MS) & efficient
 - Silicon pixel sensors (HV-MAPS)
- As close as possible to target
 - Pointing to vertex
 - Reduce effect of MS

Mu3e Detector



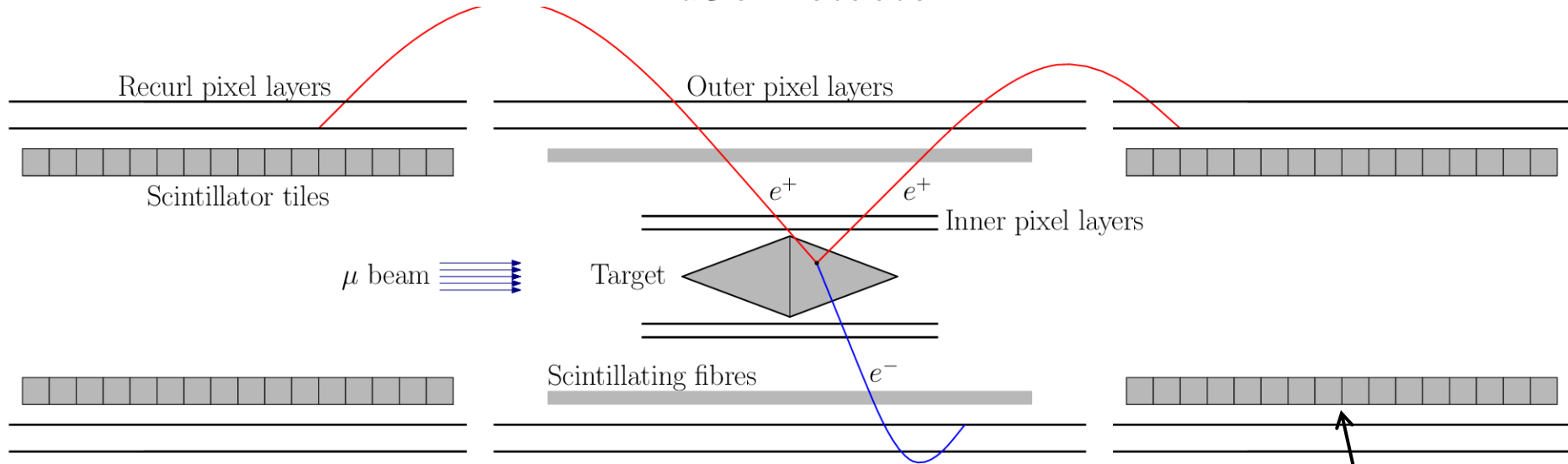
Fibre detector:

- $\sigma_t < 1\text{ ns}$
- Suppress accidental BG
- Charge ID

Two outer pixel layers:

- $B = 1\text{ Tesla}$
- Minimum $p_T \sim 12\text{ MeV}/c$
 - *Limited by outer layer radius*

Mu3e Detector

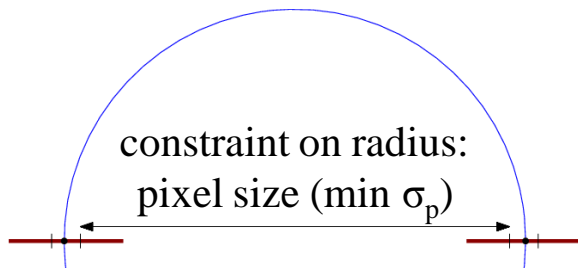
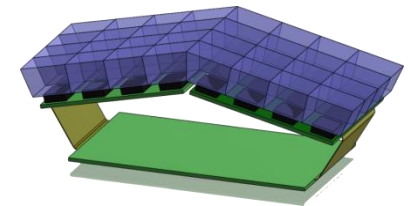


Particles (electrons) bend back in magnetic field:

- Use recurl stations to detect them
- Improve momentum resolution
 - Factor 5-10 improvement

Recurl stations:

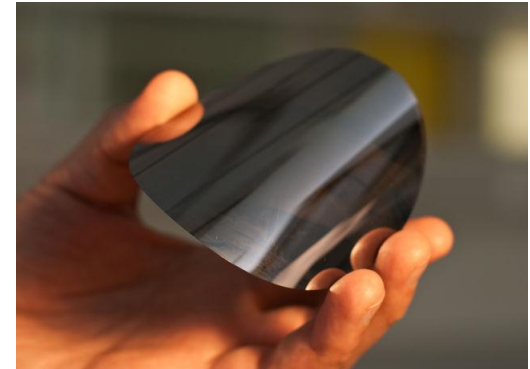
- Two pixel layers (same as central station)
- Tile detector
 - $\sigma_t < 100$ ps
 - Suppress accidentals



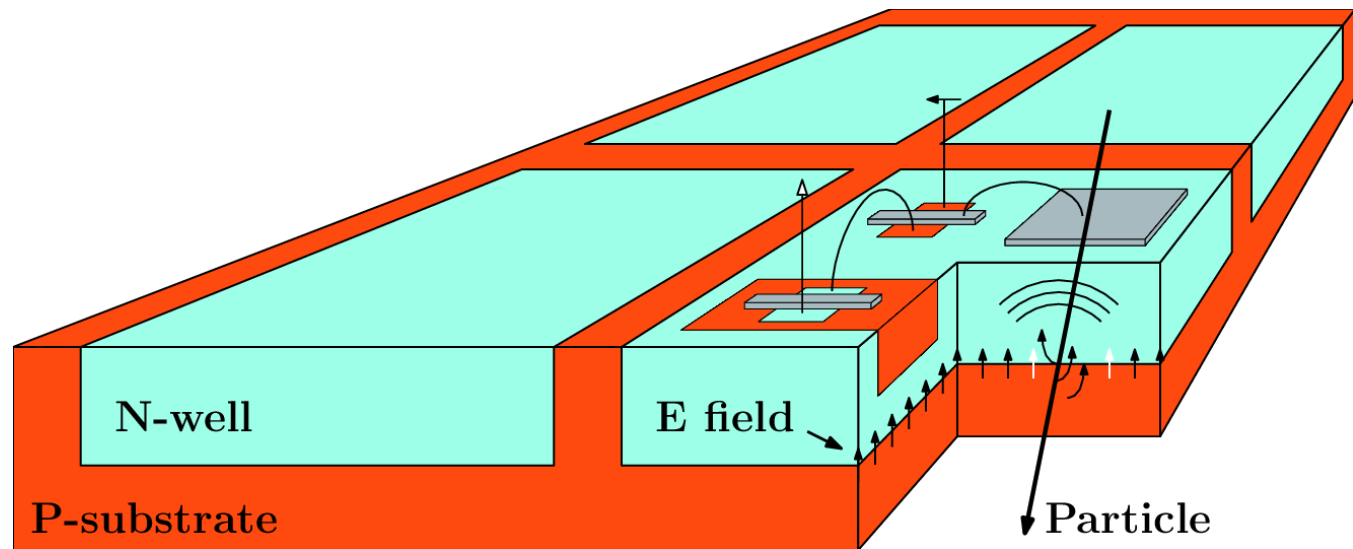
HV-MAPS

High Voltage – Monolithic Active Pixel Sensor:

- Commercially available technology
- Large area ($2 \times 2 \text{ cm}^2$)
- High granularity (pixel $\sim 80 \times 80 \mu\text{m}^2$)
- Thin ($\sim 50 \mu\text{m}$)
- Fast – charge collection via drift (HV, $\sigma_t \sim 15\text{ns}$)
- High efficiency ($> 99\%$)

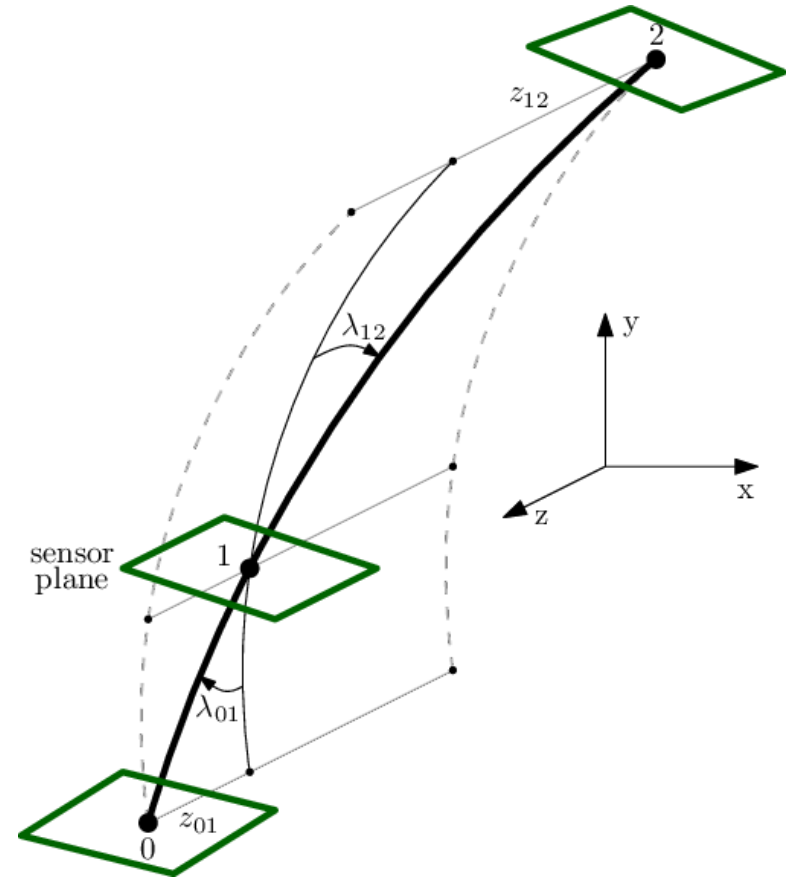


I. Peric, Nucl.Instrum.Meth. A582 (2007) 876



Triplet fit

- Track reconstruction in mag.field
 - Helical trajectory
 - Require minimum 3 hits
- Assuming no pixel uncertainty and no energy loss (Mu3e conditions):
 - One parameter to describe trajectory – curvature r (or momentum)
 - Triplet – trajectory with multiplet scattering (MS) in middle hit
 - MS angles are function of curvature – $\varphi_{ms}(r), \lambda_{ms}(r)$
- Fit – minimize χ^2 (scattering angle):
 - $\chi^2 = \varphi_{ms}^2 / \sigma_{ms,\varphi}^2 + \lambda_{ms}^2 / \sigma_{ms,\lambda}^2$
 - No analytical solution
 - Small MS angles \rightarrow linearization around known solution (circle in xy-plane)



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Track fit

Track/Segment:

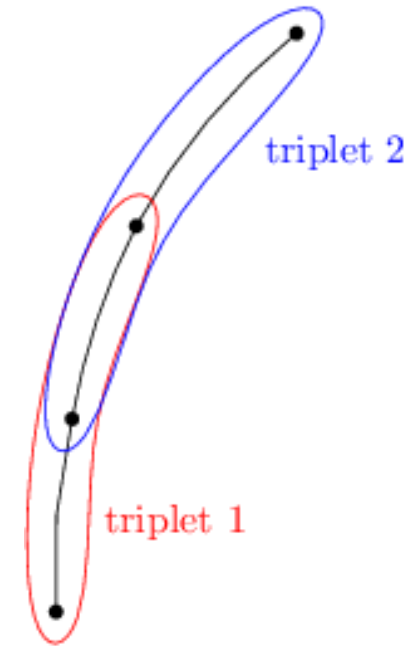
- Sequence of triplets
- 3D radius:
 - Minimize combined χ^2

- Simple solution:
$$r = \frac{\sum r_i / \sigma_i^2}{1 / \sigma_i^2}$$

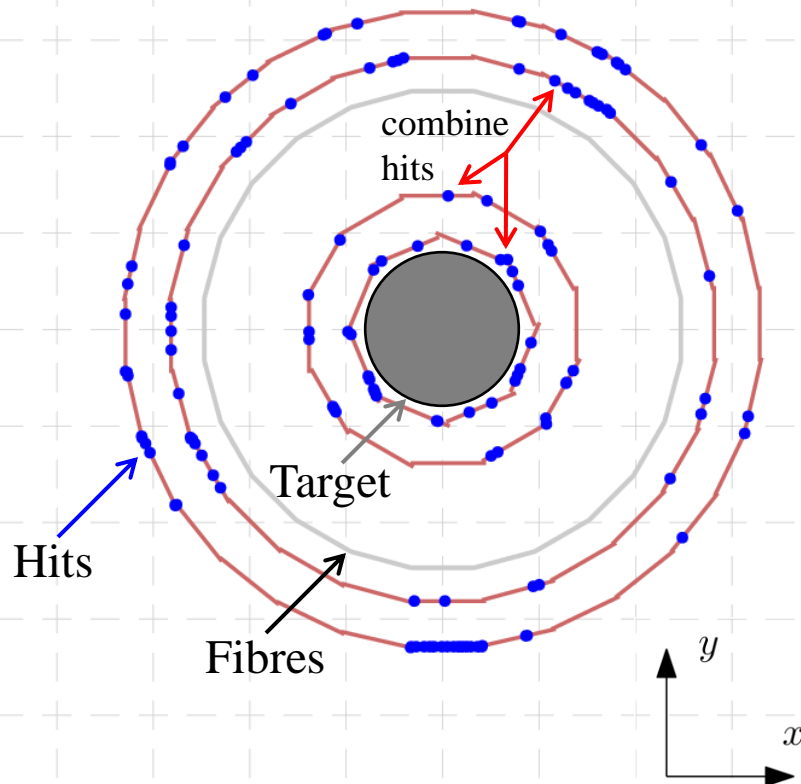
where r_i – individual triplet solutions
(*weighted average*)

Note:

- Theoretically individual triplets can be fitted in parallel and then combined.
- In practice start from seed triplet and then add more hits.



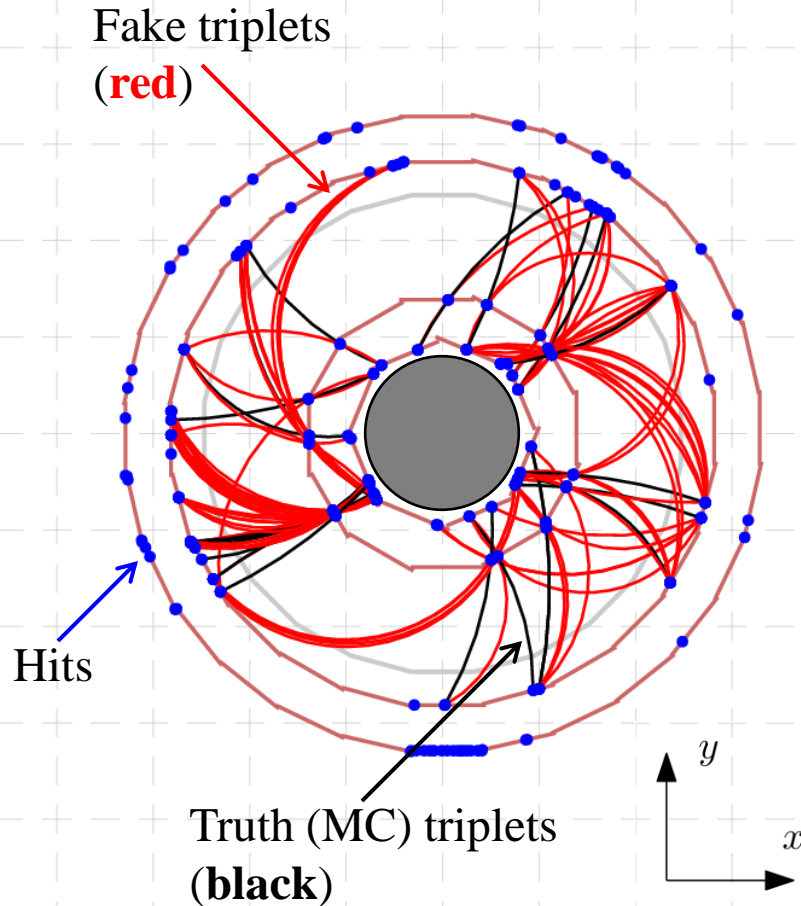
Triples



Make triplets:

- Combine hits of first 3 layers
- n – number of hits per layer
 - Difficulty: $O(n^3)$ combinations
- 10 hits per layer in 50 ns
 - $O(1K)$ combinations per frame
 - 10^{11} per second – large
- Reduce number of fits
 - Geometrical selections (opening angles, etc)

Triples

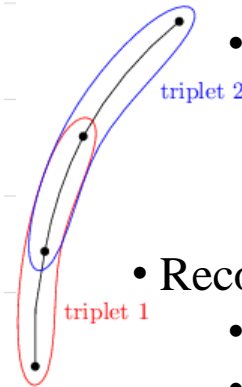
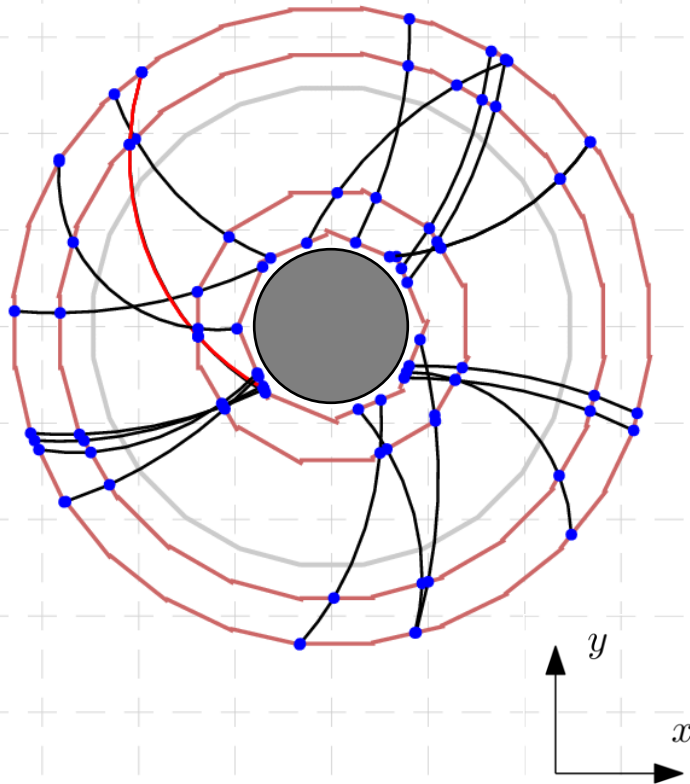


Selections:

- Geometrical
 - Distance between hits, opening angles, etc.
 - Factor 50 reduction in number of fitted combinations
- 10^9 fits per second
 - GPU filter farm
- Reduce background: triplet χ^2
 - Cut on MS angles
- Fake rate (fake combinations per one truth track) ~ 4
 - 10 truth triplets & 40 fakes

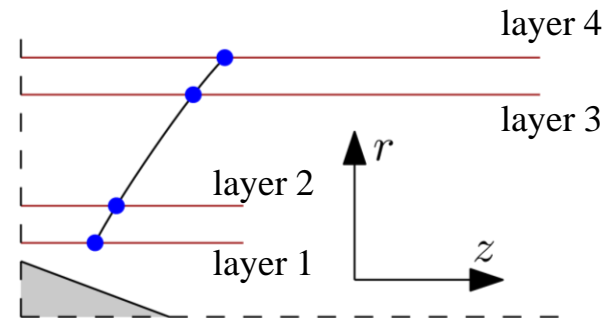
Short tracks

short track:
pair of triplets (4 hits)

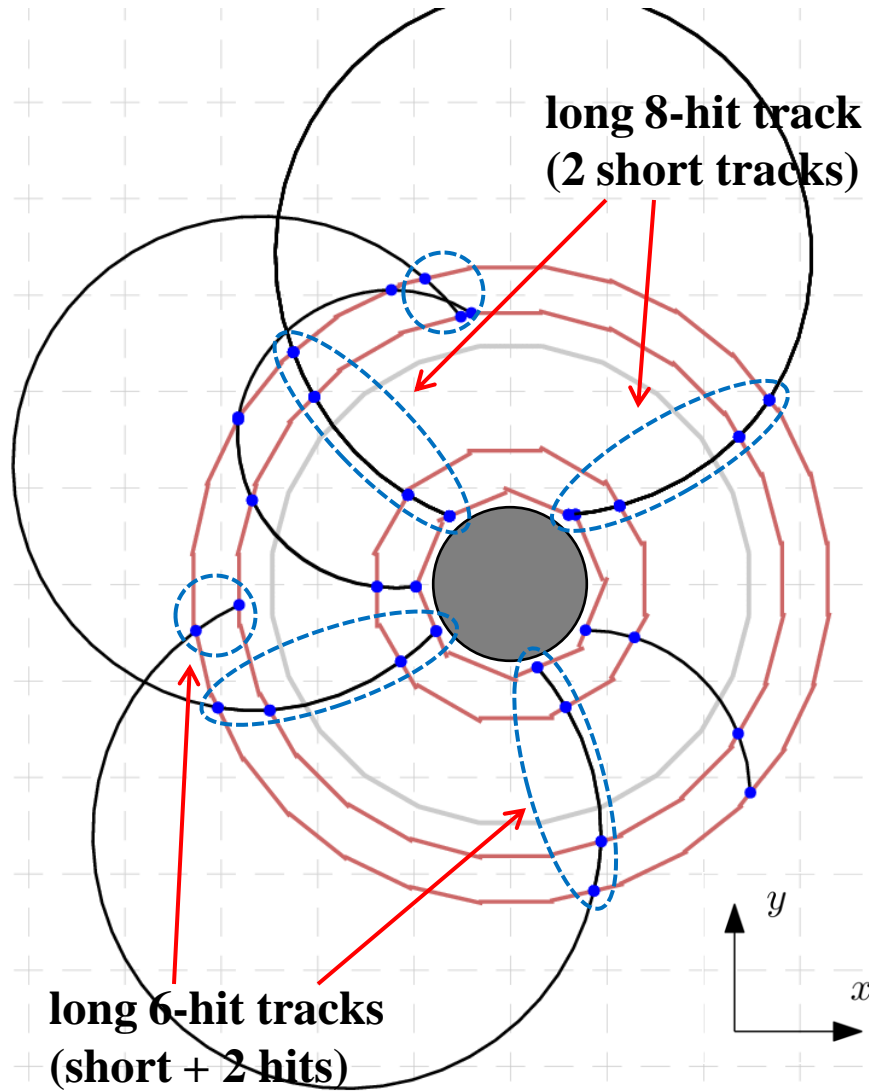


Make short tracks:

- Use triplets as seeds
 - Estimate hit at last layer
 - Lookup in ϕ/z window
- Combine 4 hits (triplet + hit)
 - 2 triplets (2 shared hits)
 - Fit (weighted average)
- Reconstruction frame (50 ns):
 - $O(10)$ short tracks
 - Fake rate $\sim 1.9\%$

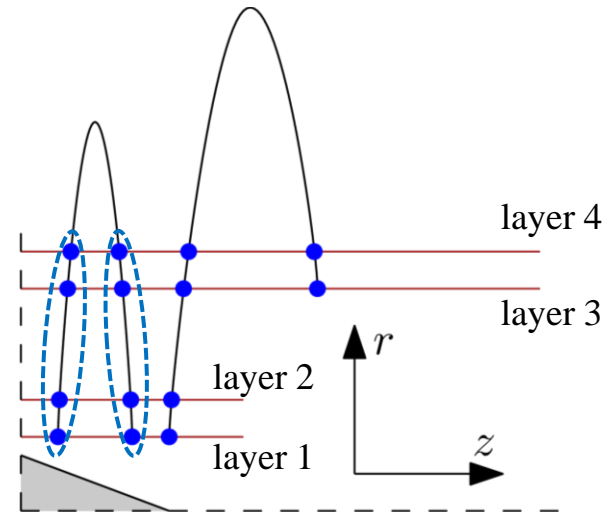


Long 8-hit tracks



Long (6- and 8- hit) tracks:

- Combine short track and pair of hits or two short tracks:
- **Fake rate ~ 10-30%**
 - 1% **true** random combinations
 - Rest – hits from same tracks, different **turns**
- Fibre hits (one per short segment)
 - Reject wrong combinations
 - Charge ID



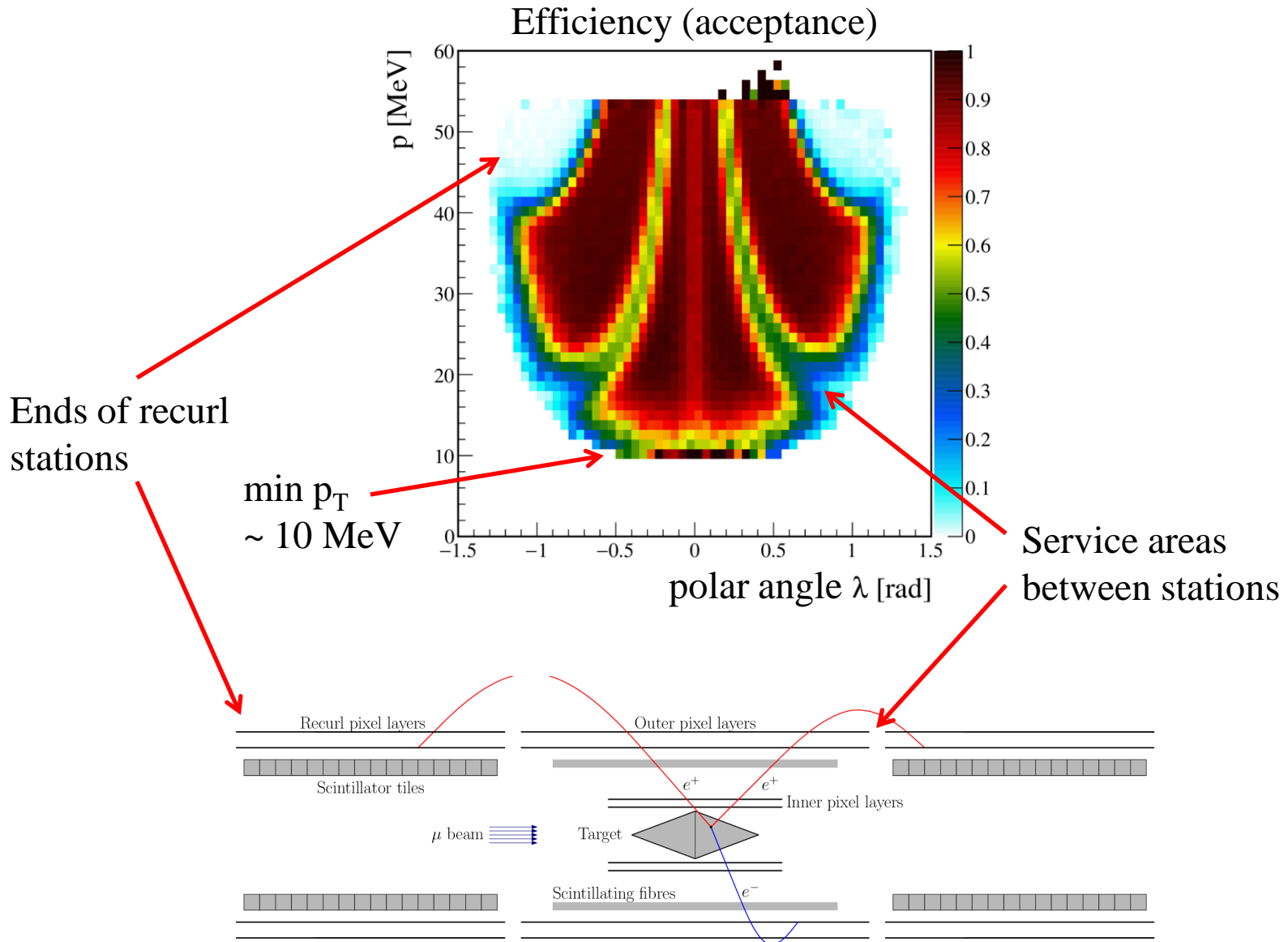
Acceptance & Efficiency

- Full Geant4 simulation of Mu3e detector
 - 50 ns event (frame)
- Decay: $\mu^+ \rightarrow e^+\nu\bar{\nu}$, 10^8 stopped muons per second
 - ~ 5 decays within frame
- Acceptance (geometry):
 - Require minimum 4 hits (1 per layer)
 - p_T – inner/outer layer radius
 - polar angle (λ) – length of detector
 - acceptance $\sim 80\%$

Reconstruction efficiency:

- Short tracks $\sim 95\%$
 - Geometrical cuts and χ^2 cuts
- Long tracks
 - Used for signal reconstruction – vertex fit, etc.
 - 80% of short tracks are reconstructed as long

Long tracks reconstruction efficiency



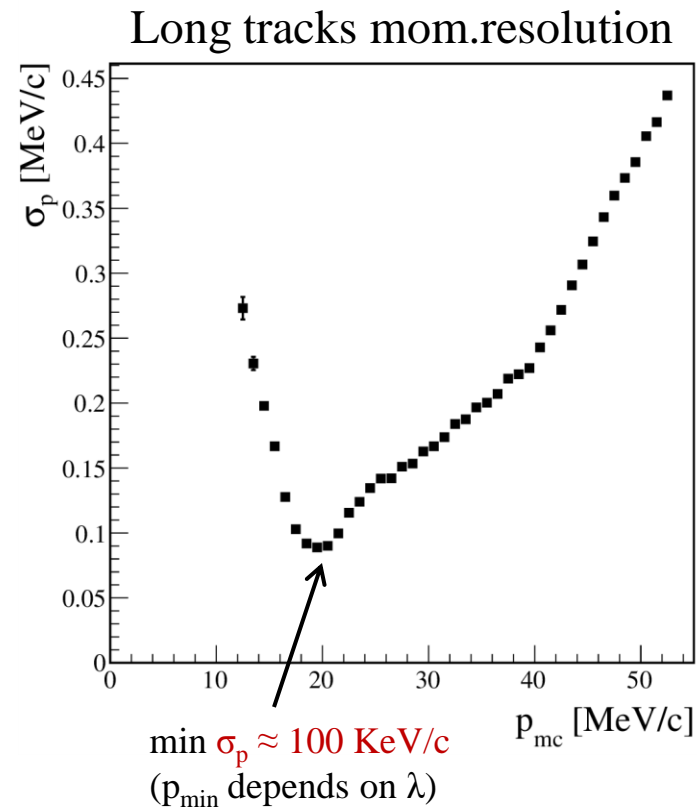
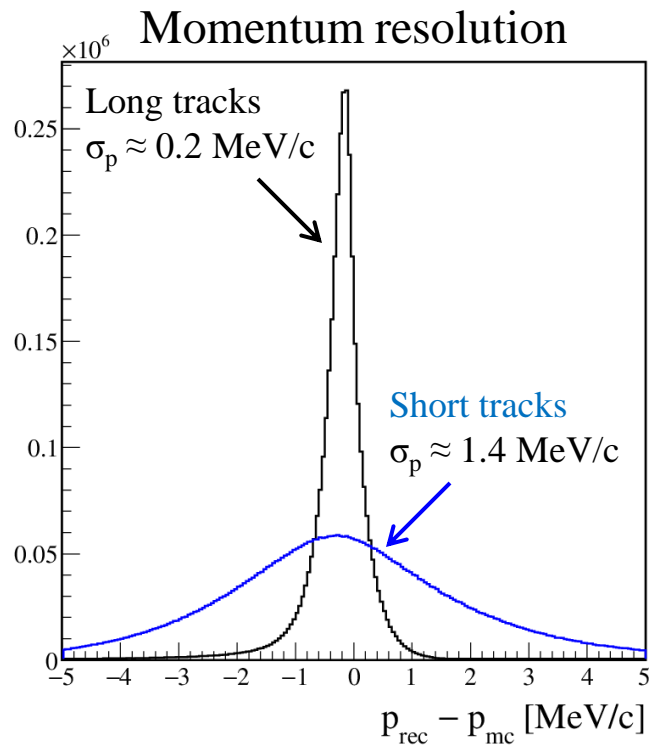
Momentum resolution

Short tracks (4 hits)

- $\langle \sigma_p \rangle \approx 1.4 \text{ MeV/c}$
- Depends linearly on momentum

Long tracks (6 and 8 hits)

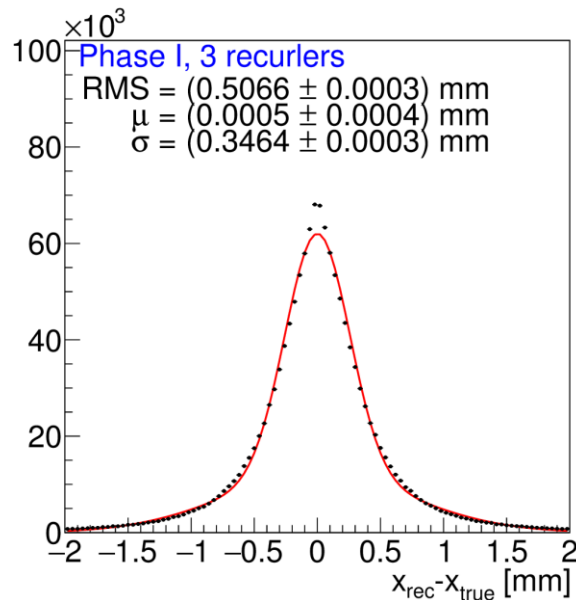
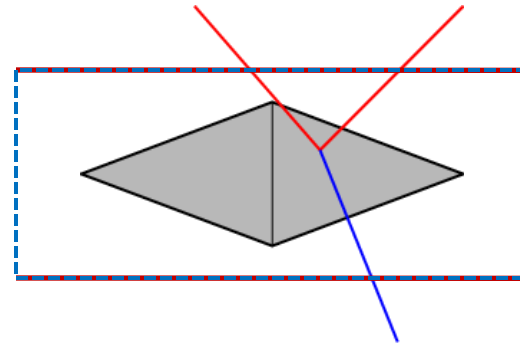
- $\langle \sigma_p \rangle \approx \mathbf{0.2 \text{ MeV/c}}$
- **min $\sigma_p \approx 100 \text{ KeV/c}$**



Vertex fit

Signal – 3 tracks ($e^+e^-e^-$):

- Long (recurl) tracks and/or short tracks
- MS in first layer
- Pixel size & energy loss
- Energy loss in target



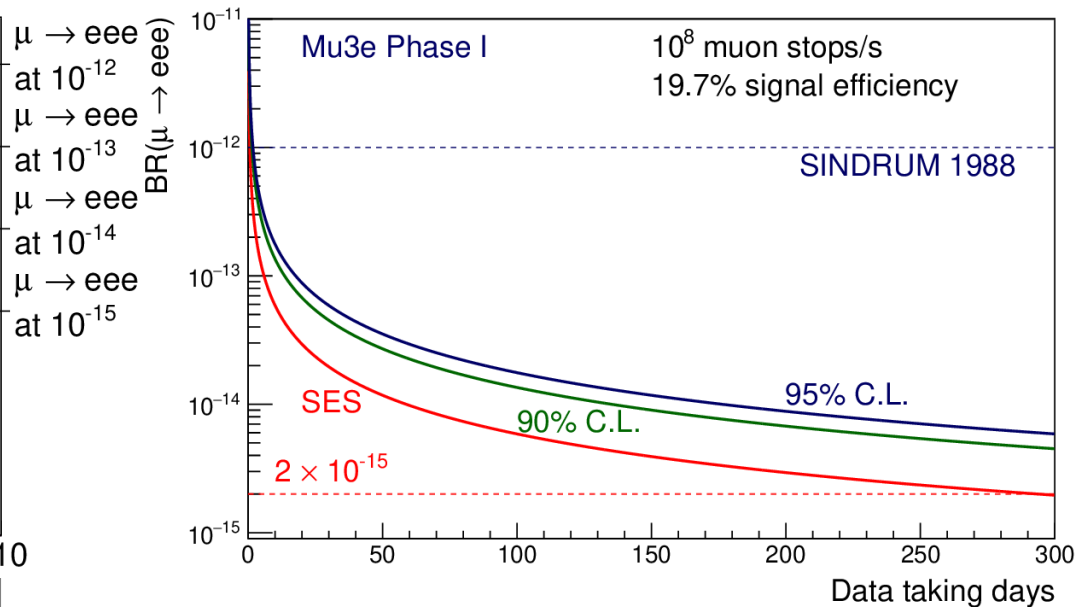
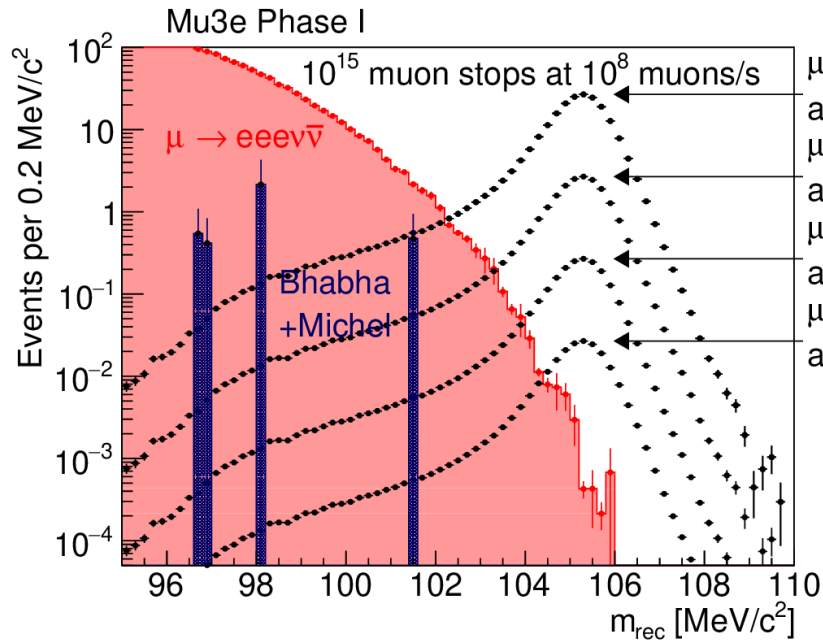
Vertex:

- Constrained to **target area**
 - Or target surface
- Material (first layer & **target**):
 - Scattering, pixel size, energy loss
- Same time at vertex (fibres and/or tiles)
- Vertex resolution:
 - $\sigma_z = 230 \mu\text{m}$ (*limited by MS*)
 - $\sigma_{x,y} = 350 \mu\text{m}$ ($\text{MS} + \sigma_p$)

Signal sensitivity

Phase I detector:

- Main background:
 - Radiative decay (momentum resolution)
 - Bhabha + Michel (vertex resolution)
- Sensitivity:
 - 10^{15} muon stops, one year of data taking
 - $\text{Br} \sim 5 \cdot 10^{-15}$ at 95 c.l.



Summary

Mu3e experiment:

- Search for LFV $\mu^+ \rightarrow e^+e^+e^-$, $\text{Br} < 10^{-15(16)}$

Reconstruction:

- Use triplet fit for track reconstruction
 - Fast, will be used offline and online (GPU filter farm)
 - Good performance
- Require good momentum, space and time resolution & efficiency
 - Short tracks: $\langle \sigma_p \rangle \approx 1.4 \text{ MeV}/c$
 - Long tracks: $\langle \sigma_p \rangle \approx 0.2 \text{ MeV}/c$
 - Fibre and tile time information
- Already meet/exceed Phase I requirements.

Backup

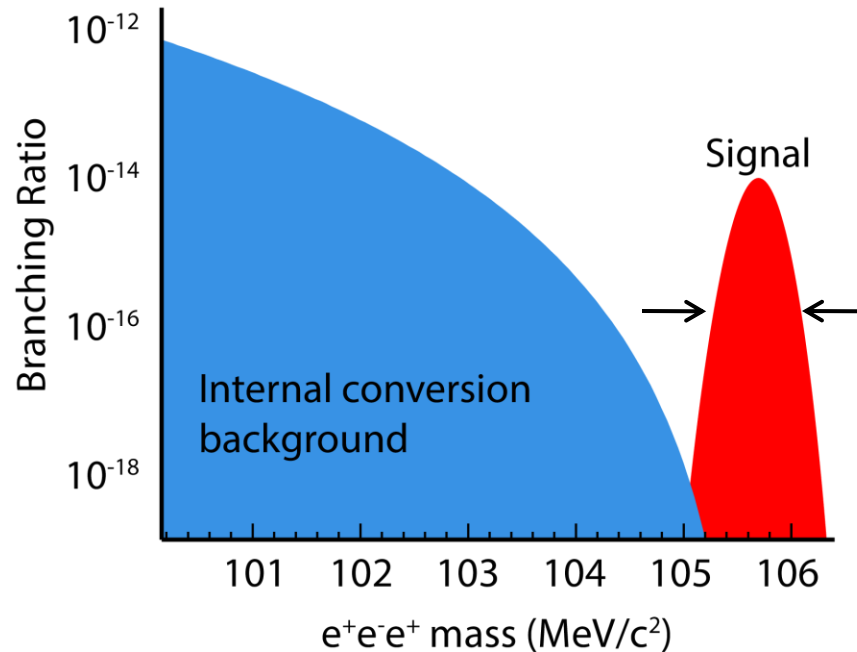
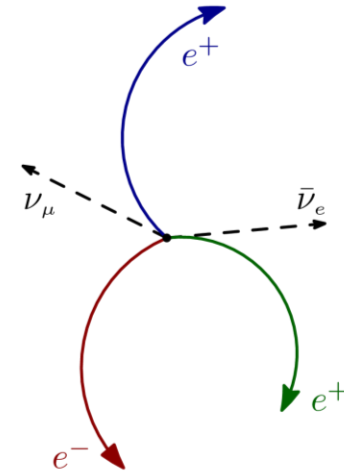
Backgrounds

Internal conversion:

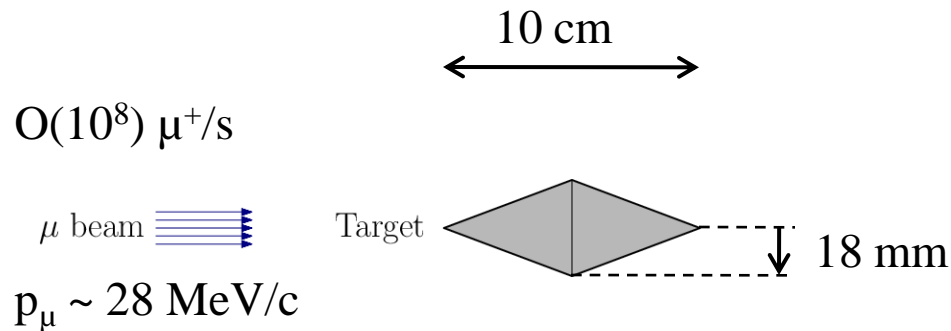


- Missing momentum & energy:

- $\sum \mathbf{p}_e \neq 0$
- $\sum E_e \neq m_\mu$
- Need good momentum resolution



Mu3e Detector

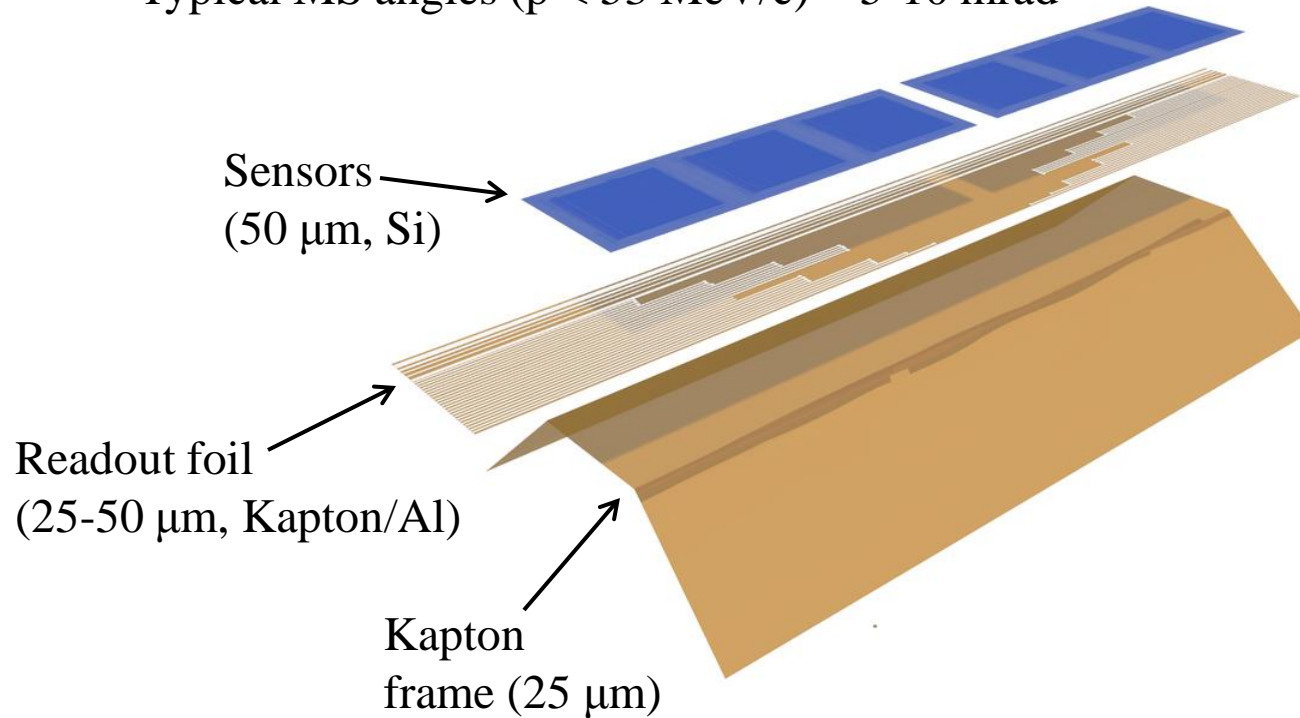


Muons stop and decay at rest on target:

- Existing beam line at PSI:
 - Continuous muon beam
 - $O(10^8) \mu^+/s$
- Double cone hollow target
 - Vertex separation in space
 - 36 mm diameter (to cover beam x-section)
 - $O(100) \mu\text{m}$ thickness

Pixel layers

- Mu3e pixel layers:
 - 2844 sensors (area $\sim 1 \text{ m}^2$)
 - sensor size $2 \times 2 \text{ cm}^2$
 - pixel size $80 \times 80 \mu\text{m}^2$
- **$50 \mu\text{m}$** thick $\sim 0.5 \cdot 10^{-3} X_0$
 - Total thickness (with support) $\sim 1.1 \cdot 10^{-3} X_0$
 - Typical MS angles ($p < 53 \text{ MeV}/c$) $\sim 5\text{-}10 \text{ mrad}$

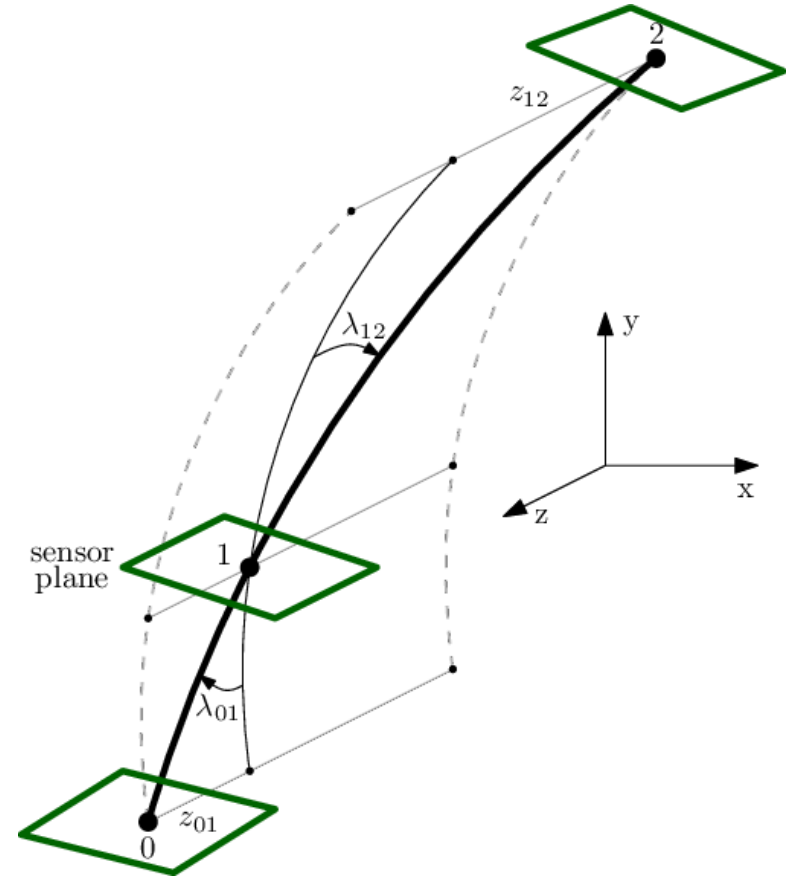


Triplet fit

- "Minimum" track in mag.field
 - **Three** measurements or hits (i.e. in 3 sensor layers)
 - Or **two** helices
- Helix trajectory defined by:
 - Pair of hits (at the end of this helix)
 - And curvature r (or momentum)

Triplet:

- **No hit uncertainty & MS at middle hit**
- **No energy loss ($r = r_1 = r_2$)**
 - MS angles: $\varphi_{\text{ms}}(\mathbf{r})$, $\lambda_{\text{ms}}(\mathbf{r})$
- Fit – minimize χ^2 (scattering angle):
 - $\chi^2 = \varphi_{\text{ms}}^2 / \sigma_{\text{ms},\varphi}^2 + \lambda_{\text{ms}}^2 / \sigma_{\text{ms},\lambda}^2$
 - There is no analytical solution
 - Assume small MS angles
 - Start from "circular" solution in xy-plane and linearize



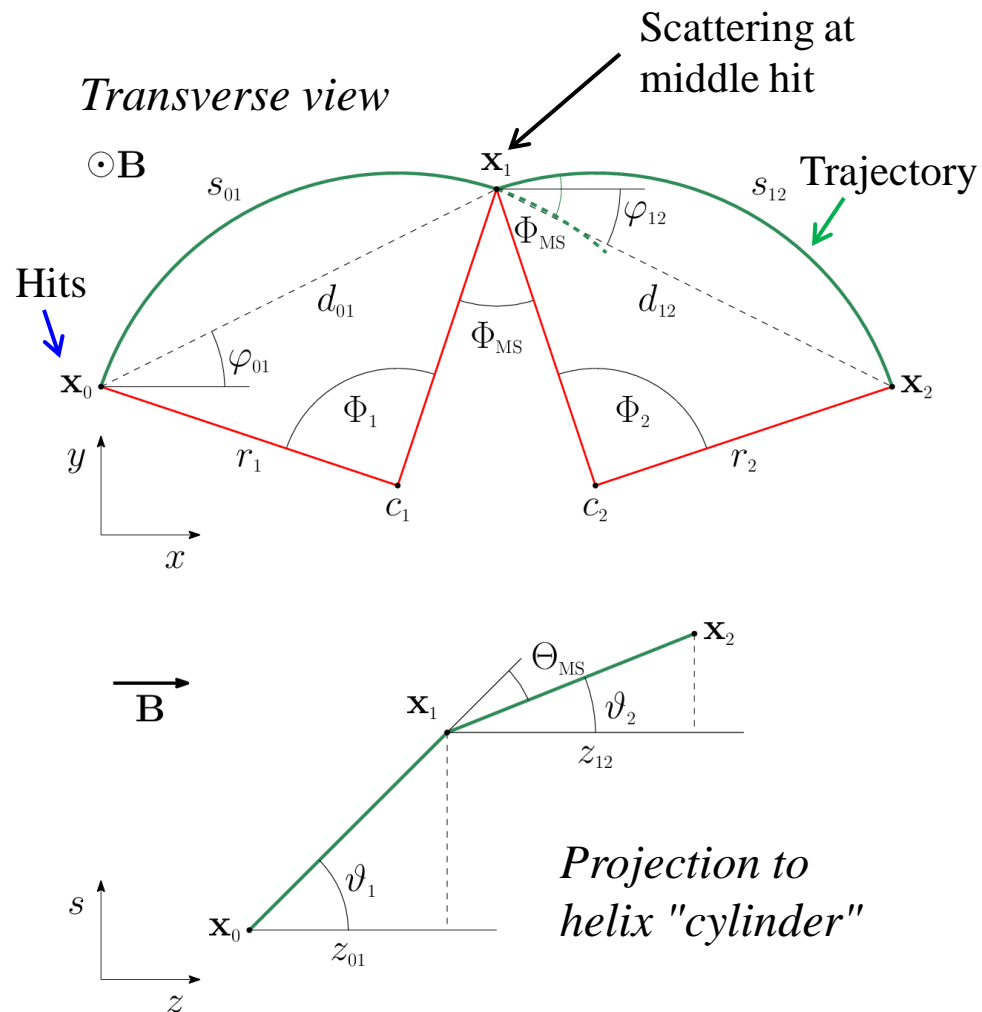
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Triplet fit

- No energy loss ($r = r_1 = r_2$)
- No hit position uncertainty
- Consider only MS at **middle hit**

Triplet:

- 3 hits (3D points) form triplet
 - Combination of 2 helices
- Fully defined by radius r
 - MS angles: $\varphi_{\text{ms}}(r)$, $\lambda_{\text{ms}}(r)$
- Minimize χ^2 (scattering angle):
 - $\chi^2 = \varphi_{\text{ms}}^2 / \sigma_{\text{ms},\varphi}^2 + \lambda_{\text{ms}}^2 / \sigma_{\text{ms},\lambda}^2$
 - There is no analytical solution
 - Assume small MS angles
 - Start from known "circular" trajectory in xy-plane
 - Linearization

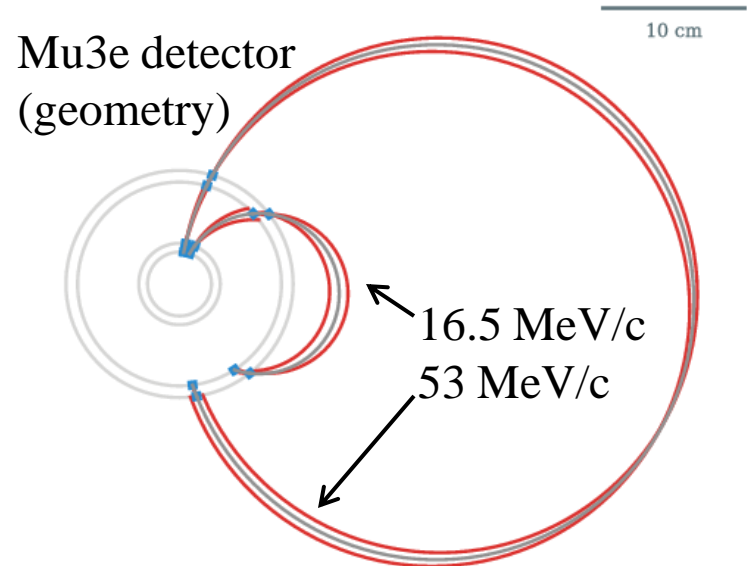


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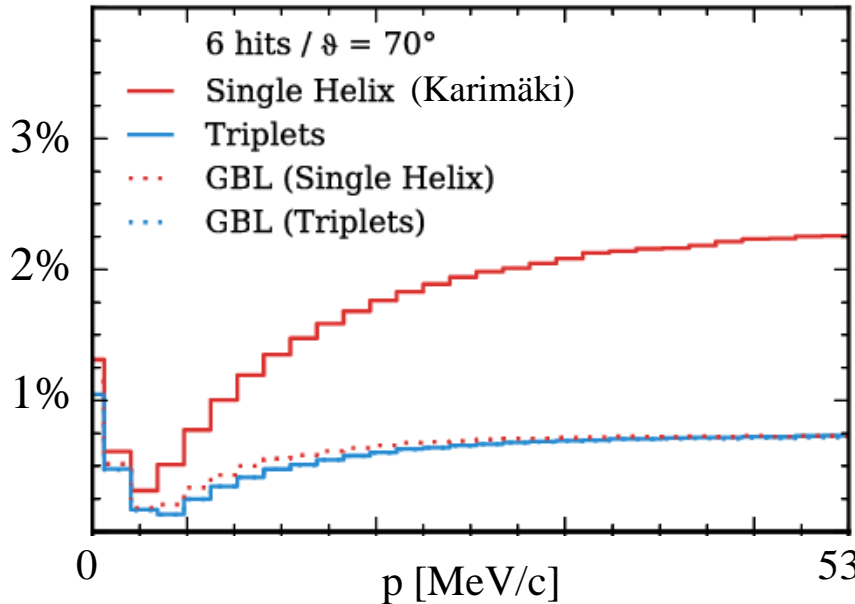
Triplet vs Karimäki vs GBL

- Fit performance depends on tracker geometry.
- Mu3e case:
 - Triplet fit has similar performance as **General Broken Lines** fit
 - Fast, suitable for filter farm (trigger)

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Relative momentum resolution



Single Helix (Karimäki)

- Neglect MS

Triplets

- MS fit

GBL (**General Broken Lines**)

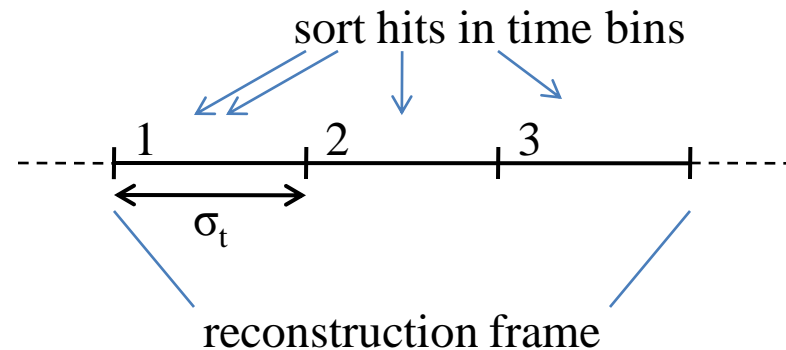
- Fast global track **refit** with full covariance matrix
- Equivalent to Kalman filter

Mu3e track reconstruction

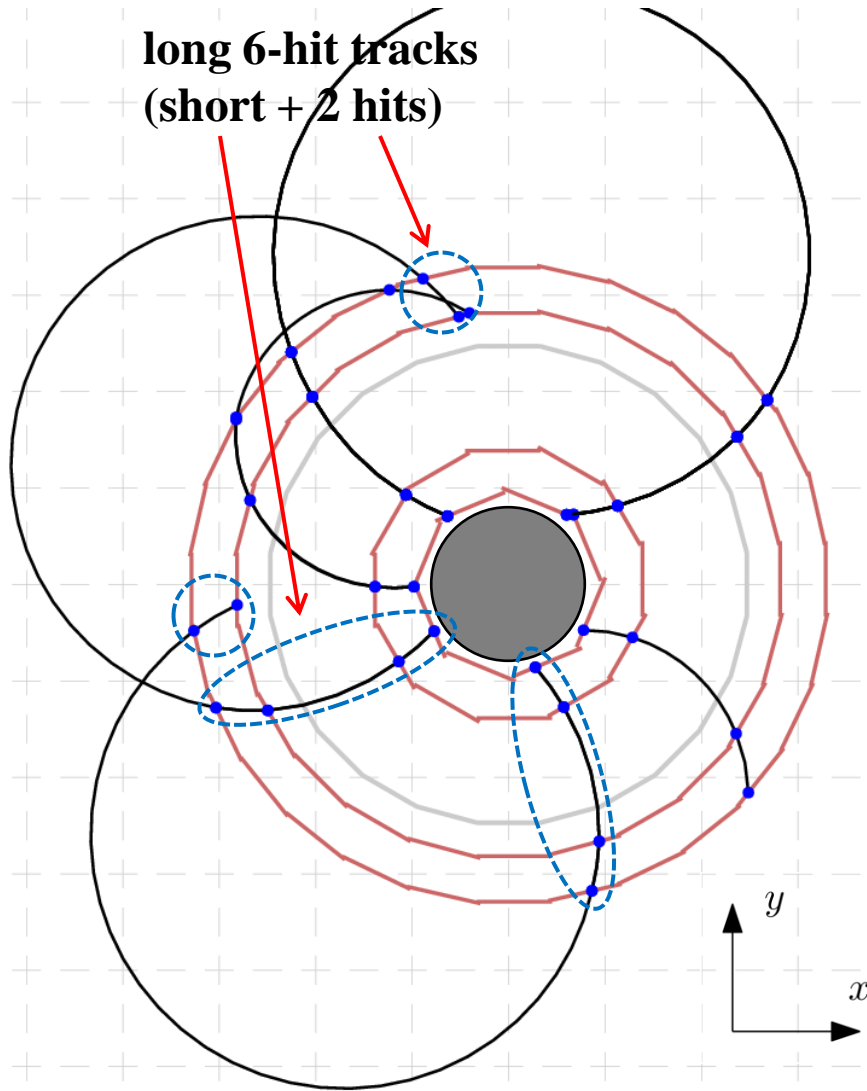
- Track reconstruction is based on triplets
 - Fast and similar performance to GBL (Mu3e environment)
 - The same algorithm will be used offline and online
 - Offline implementation also takes into account pixel size and energy loss (minor fitter change)
- Mu3e readout is essentially continuous:
 - Frame data by combining hits with same (close) timestamps from different system (pixels, fibres, tiles)
 - Reconstruction "frame" of $3 \cdot \sigma_t$

Tracking performance:

- Geant4 simulation of Mu3e detector
- 50 ns reconstruction frame
- 10^8 muon stops, Michel decay

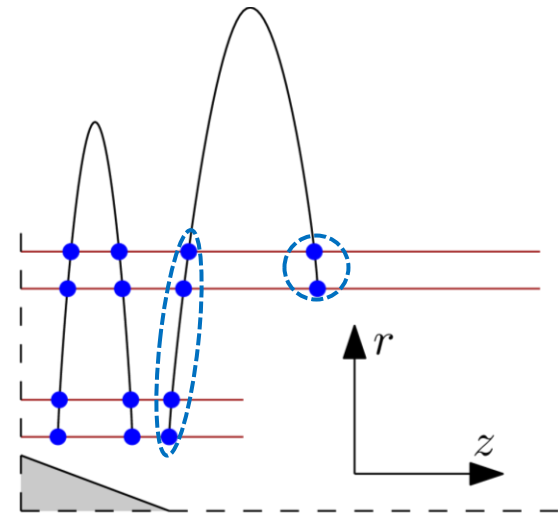


Long 6-hit tracks



Long 6-hit tracks:

- Short track + 2 hits
- Fake rate $\sim 9\%$
 - 1% – random combinations
- Fibre and/or tile hits
 - Reject wrong combinations



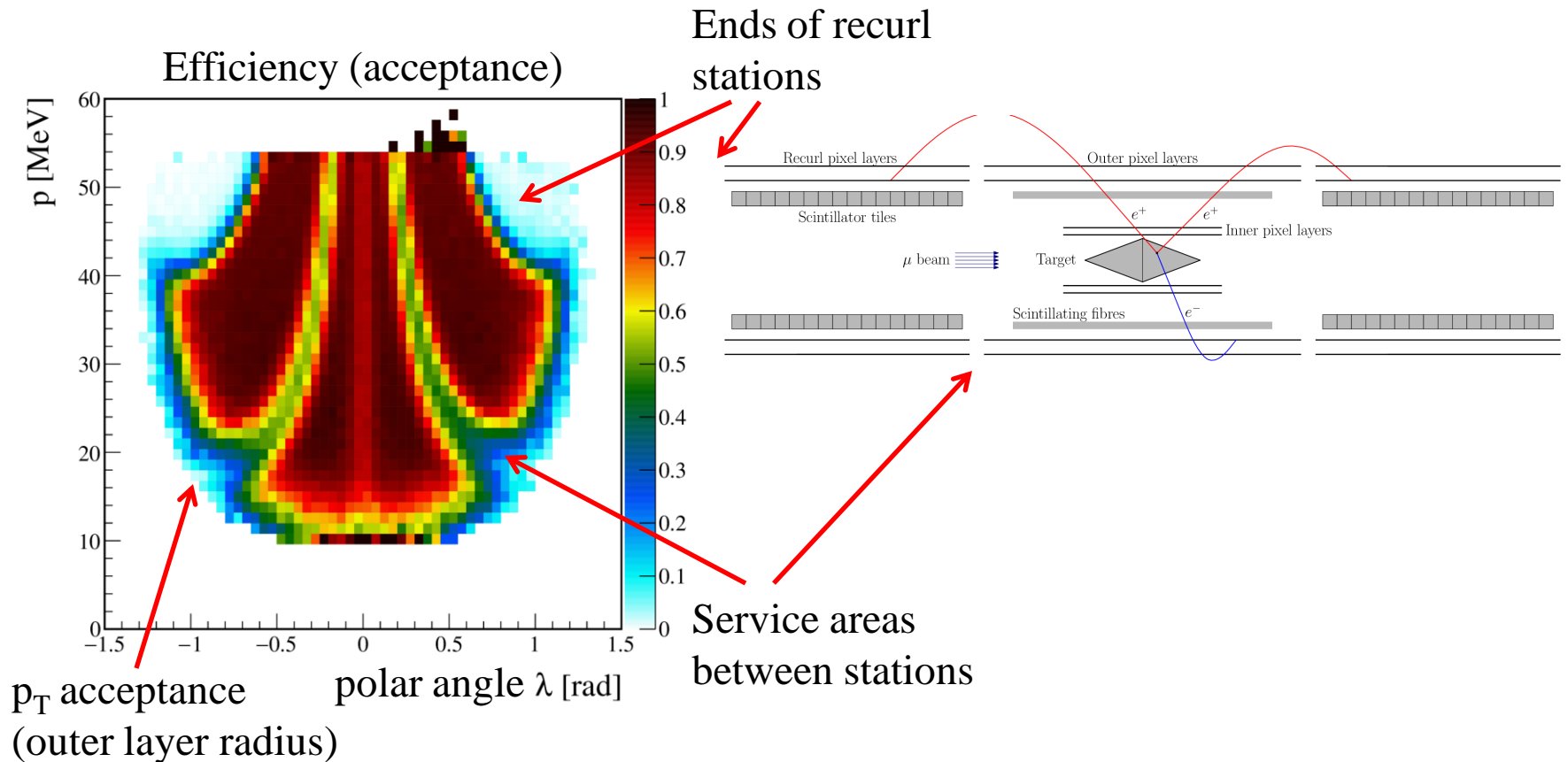
Acceptance & Efficiency

Short tracks (4 hits)

- Geometrical acceptance: 80%
- Reconstruction efficiency: 95%
 - Geometrical cuts and χ^2 cuts

Long tracks (6 and 8 hits)

- 80% of short reconstructed as long
 - Geometry (service areas, etc.)
 - χ^2 cuts

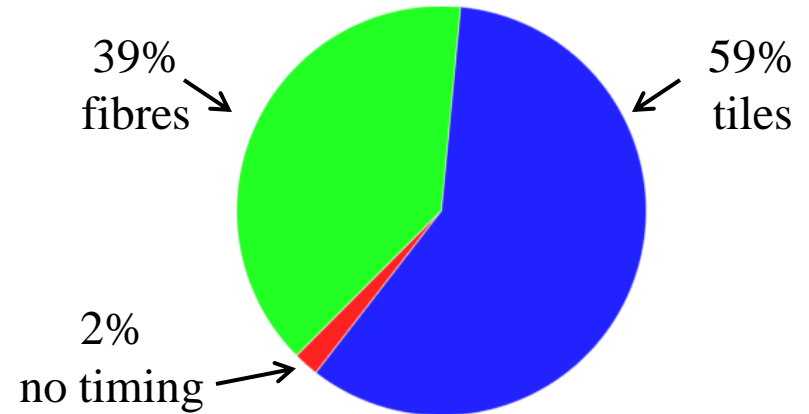


Timing

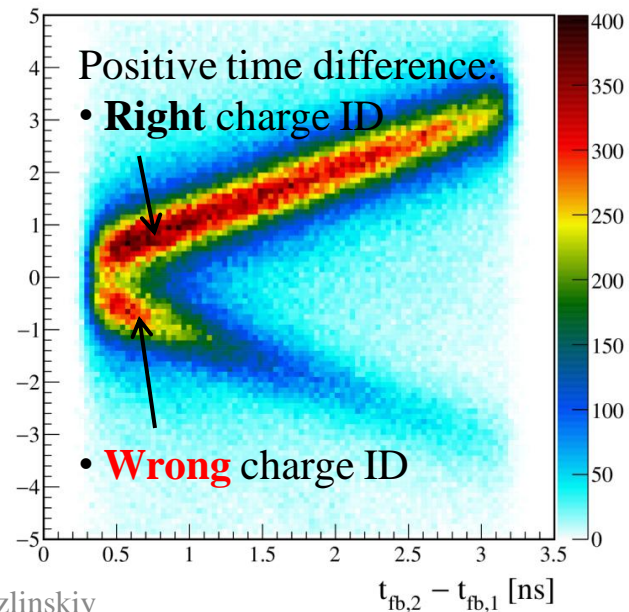
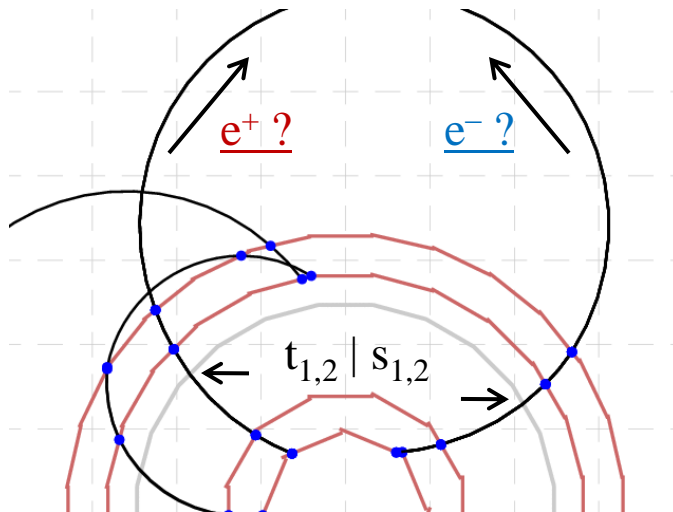
Time information from fibres/tiles:

- Suppress fakes
- Additional vertex constraint
 - Same time at vertex for all tracks
- Charge ID: e^+ or e^-
 - Mainly for long 8-hit tracks
 - Fibre time difference vs path length

Timing information



Ambiguity for central 8-hit tracks



Charge ID

Charge ID for long 8-hit track:

- Unknown direction
- Use fibre time information ($t_2 - t_1$) and path length between fibre hits ($s_2 - s_1$)
 - Should match particle traveling with speed of light

