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Federal Institute of Metrology METAS

# A milestone in the evolution of the International System of Units

Beat Jeckelmann

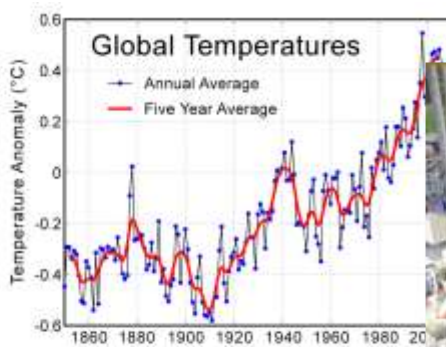
# Metrology: the science and application of measurement

**secure measurements and reference data**

do you get
what you
pay for?

safely below
radiation limits?

data quality for
policy making
sufficient?




**underpin basic science**

do constants
of nature drift with time?

**drive innovation**

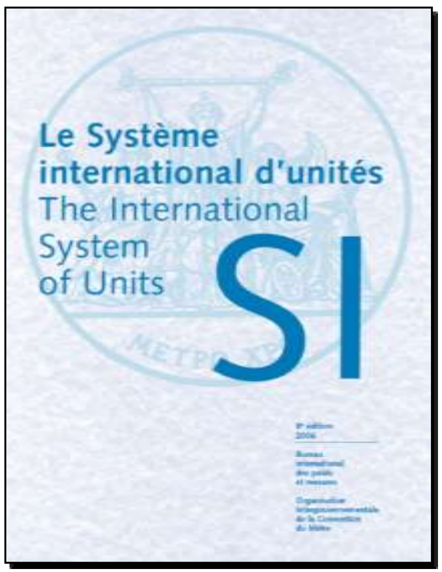
efficiency?
quality control
in production?

precise enough
to be
competitive?


# Worldwide comparability

**Metre Convention**

- Basis for the International System of Units (SI)
- And a global measurement infrastructure




**Mutual recognition arrangement:**

→ 97.6 % of the world economy

→ The cornerstone of international Quality Infrastructure (QI)

→ Basis and condition sine qua non for world wide trade

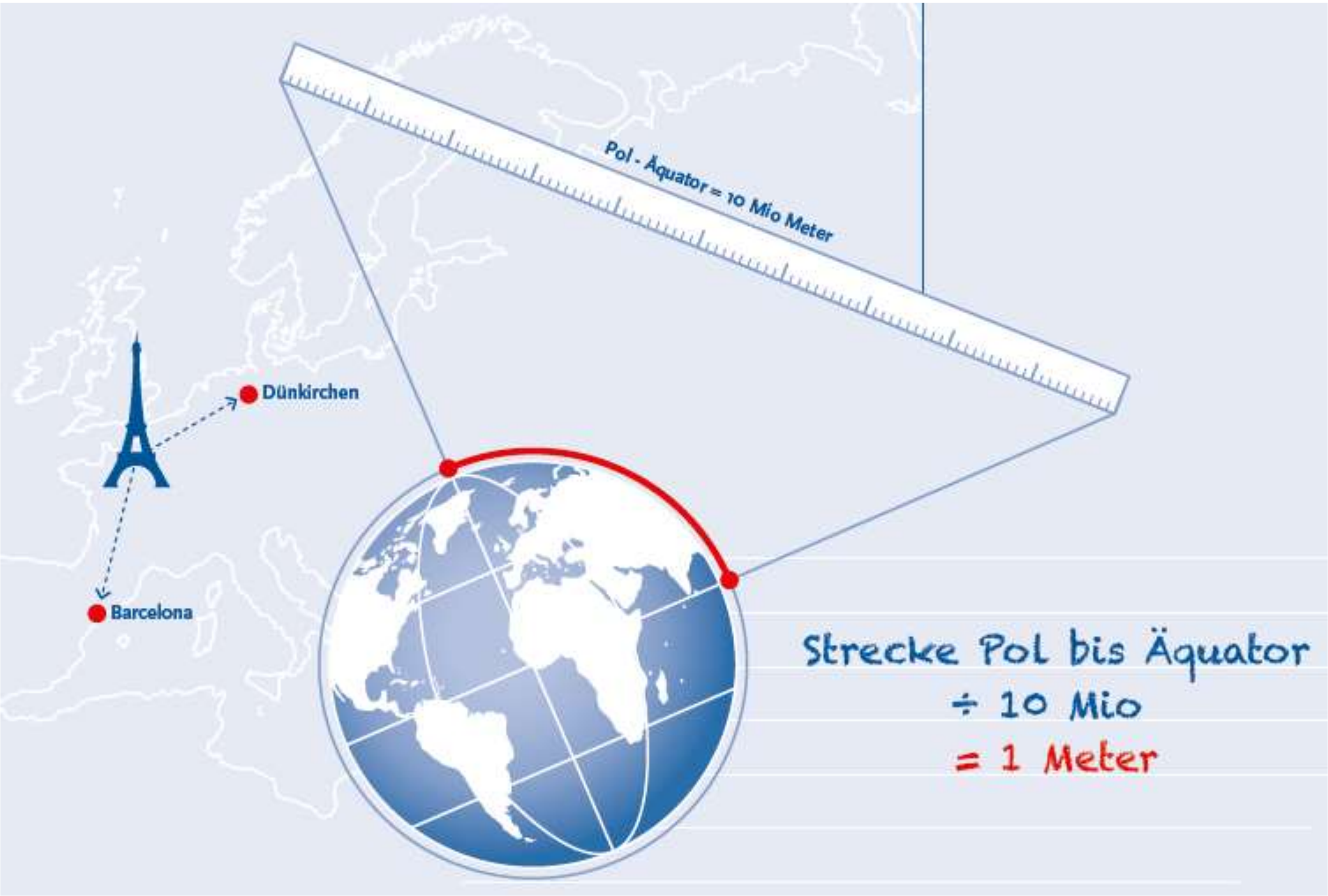
METAS

# Versailles, 16. November 2018……

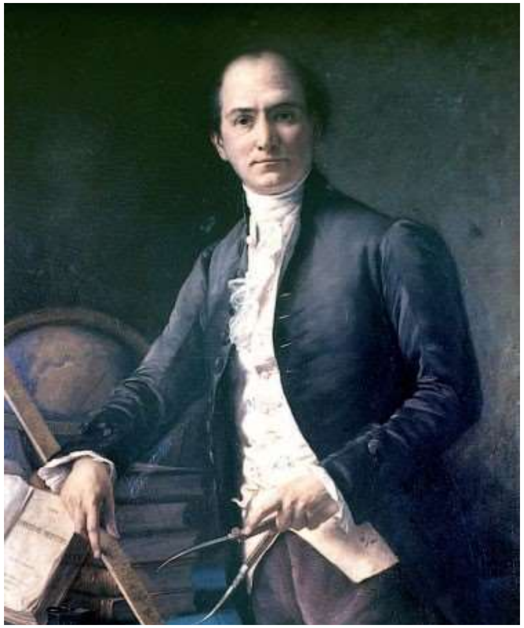
# Outline

- The International System of Units SI: short history; structure
- Shortcomings of the present SI
- The concept of the revised SI
- Challenges:
  - Replacing the kg: Determination of the Planck constant
  - Boltzmann constant
- Conclusions

# 1791: Metric system



# Meridian expedition

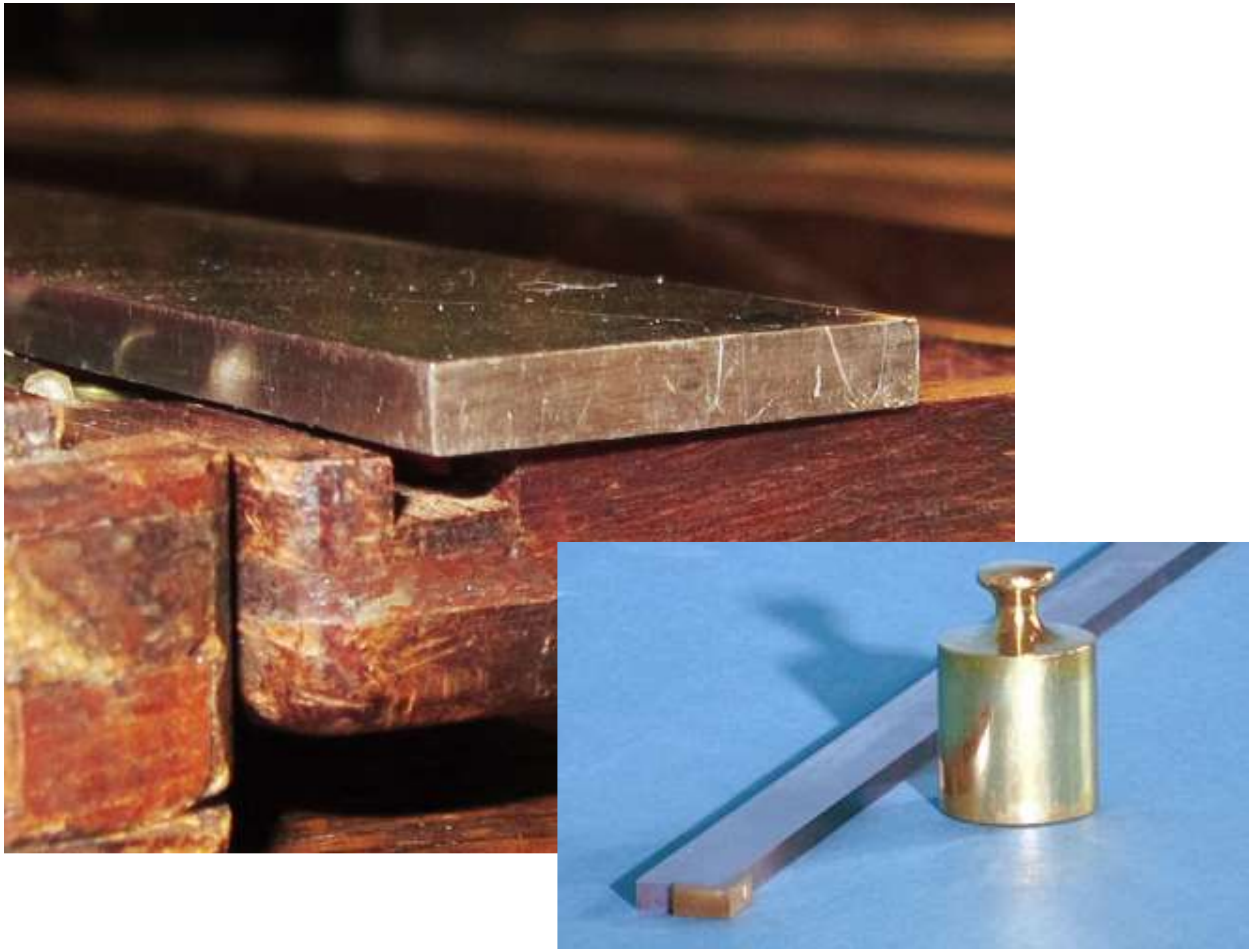


Jean-Baptiste J. Delambre
(northern part)

Pierre Méchains
(southern part)

1792 - 1798
Measurement of the length of the meridian arc through triangulation

# Mètre des archives 1799



1801: Helvetic metre (iron)

# A tous les temps, à tous les peuples….



France, loi du 4 juillet 1837

"...à partir du ***1 janvier 1840***, tous poids et mesures autres que les poids établis par les lois des 18 germinal an III et 19 frimaire an VIII, constitutives du système métrique décimal, seront interdits sous les peines portées par l'article 479 du code pénal ".

# The metre convention 1875

# 1889: Prototype metre and kilogram

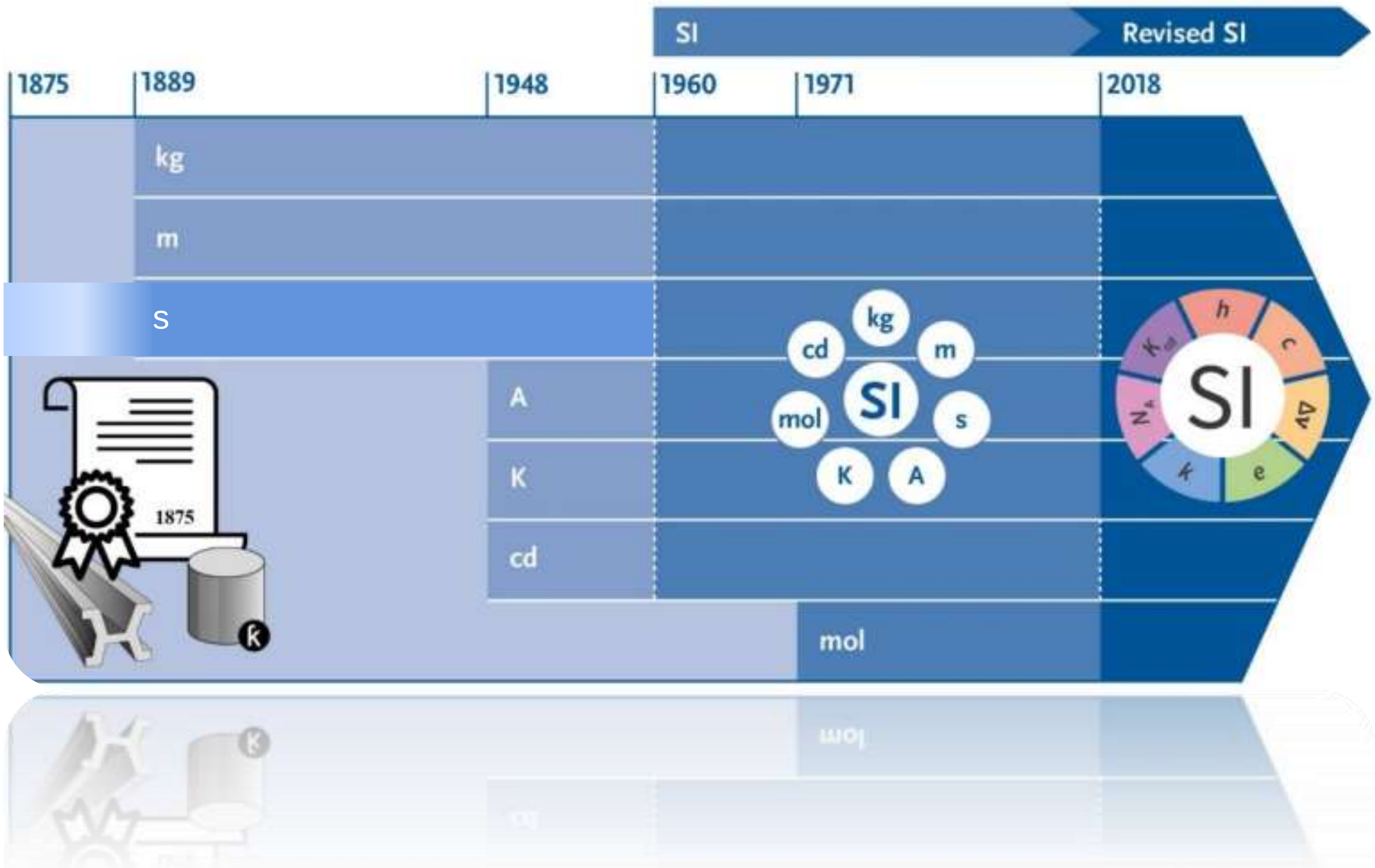
World Geodetic System 1984
meridian arc between pole and equator:
**10001.966 km**

Deviation: 200 ppm

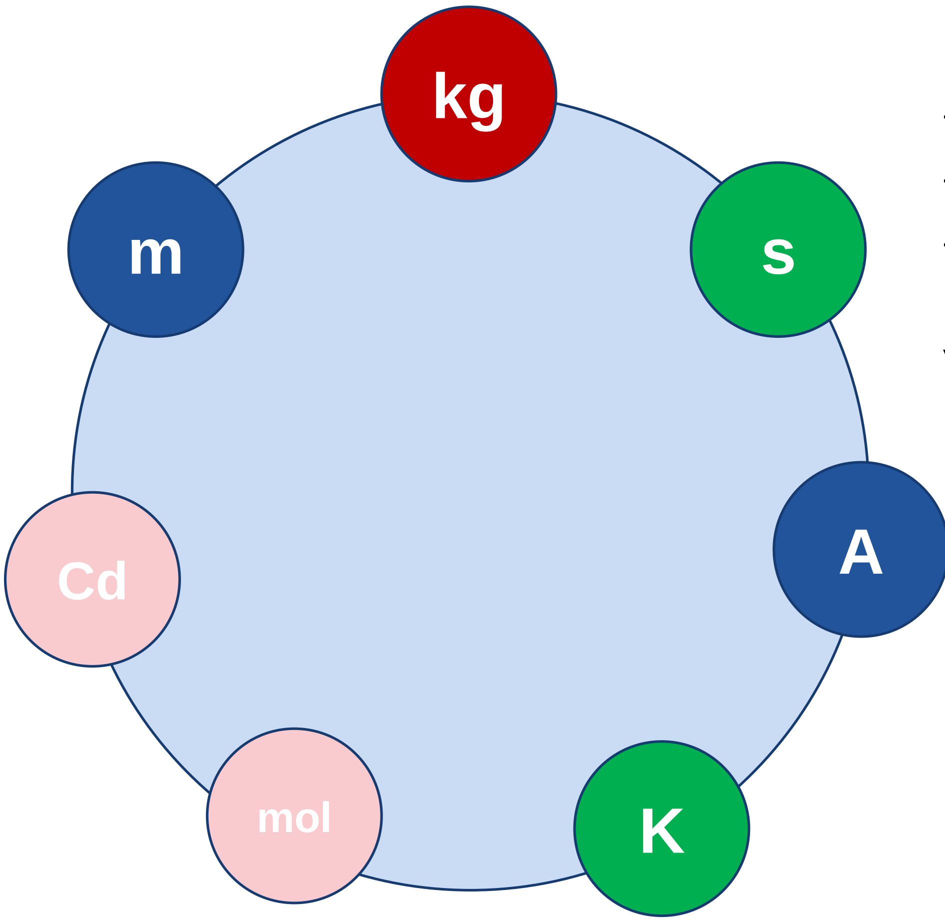

90% Platin, 10% Iridium

1 kg = 1 $dm^3$ water at 4°C

# Short history of the metric system



# The SI today



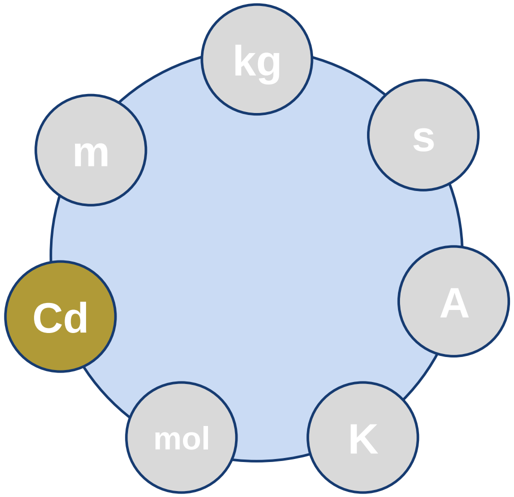

- **7 Base units**
- Values chosen by convention
- Derived units

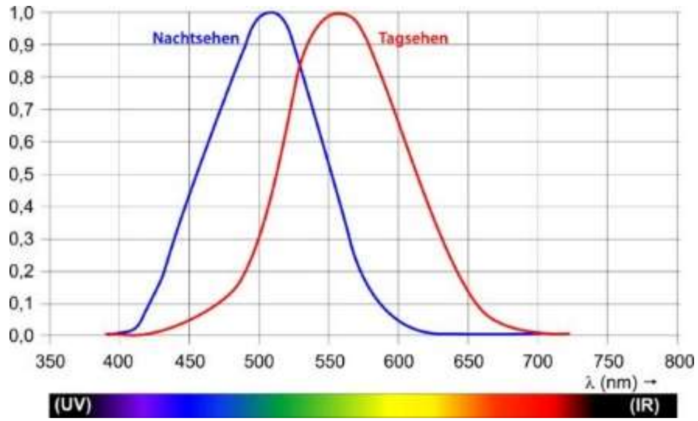
Various types of unit definitions, based on:

- **Artefact**
- **Physical state**
- **Fundamental constant**
- “Technical” constant

# Practical units

Photometric unit
- 1 Candela = Luminous intensity of a source in a given direction,
- monochromatic radiation 540 THz,
- Radiant intensity = 1/683 W/sr

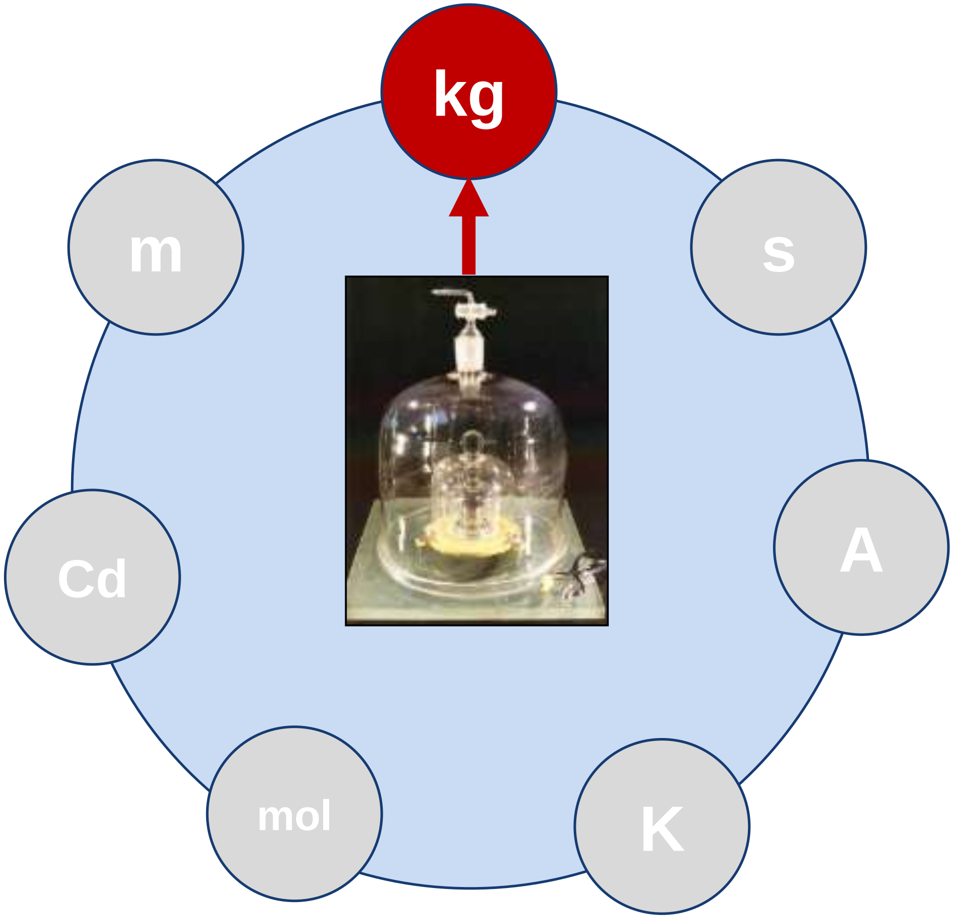



Measurement of light adapted to the human eye

# Unit definitions and shortcomings

The kilogram



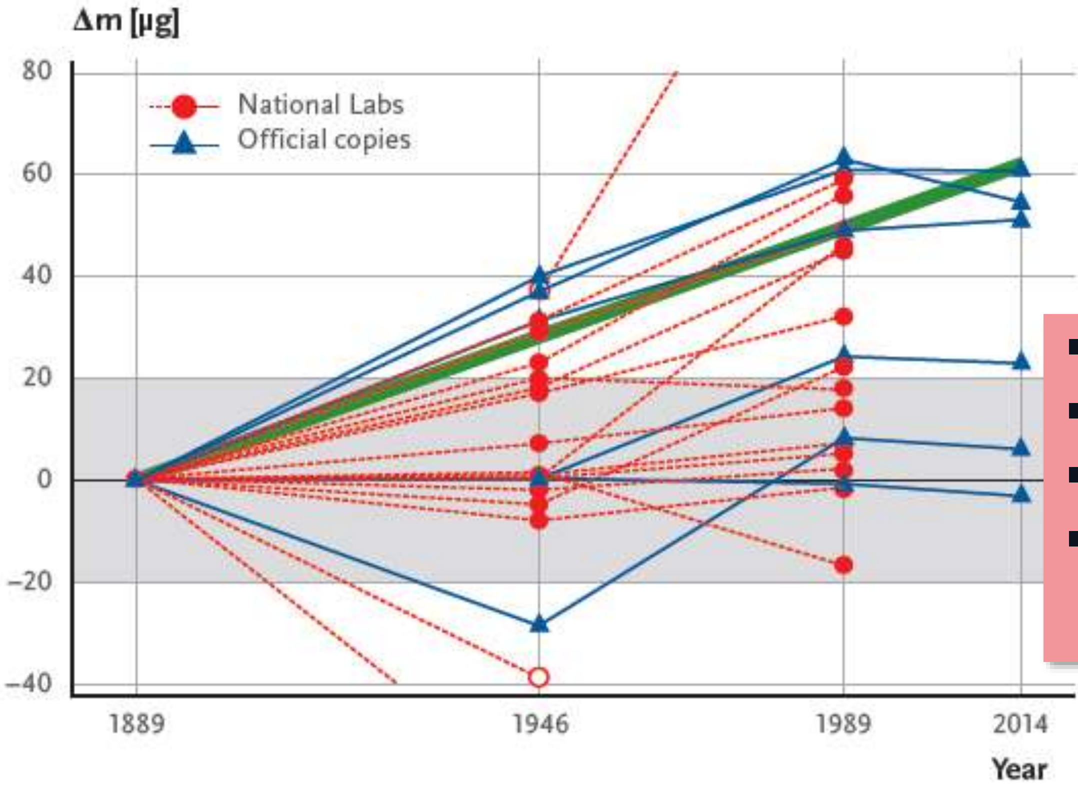
# The kilogram: present definition

"The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram." (CGPM, 1901)

... immediately after cleaning and washing by a specified method (*mise en pratique*, CIPM 1989).

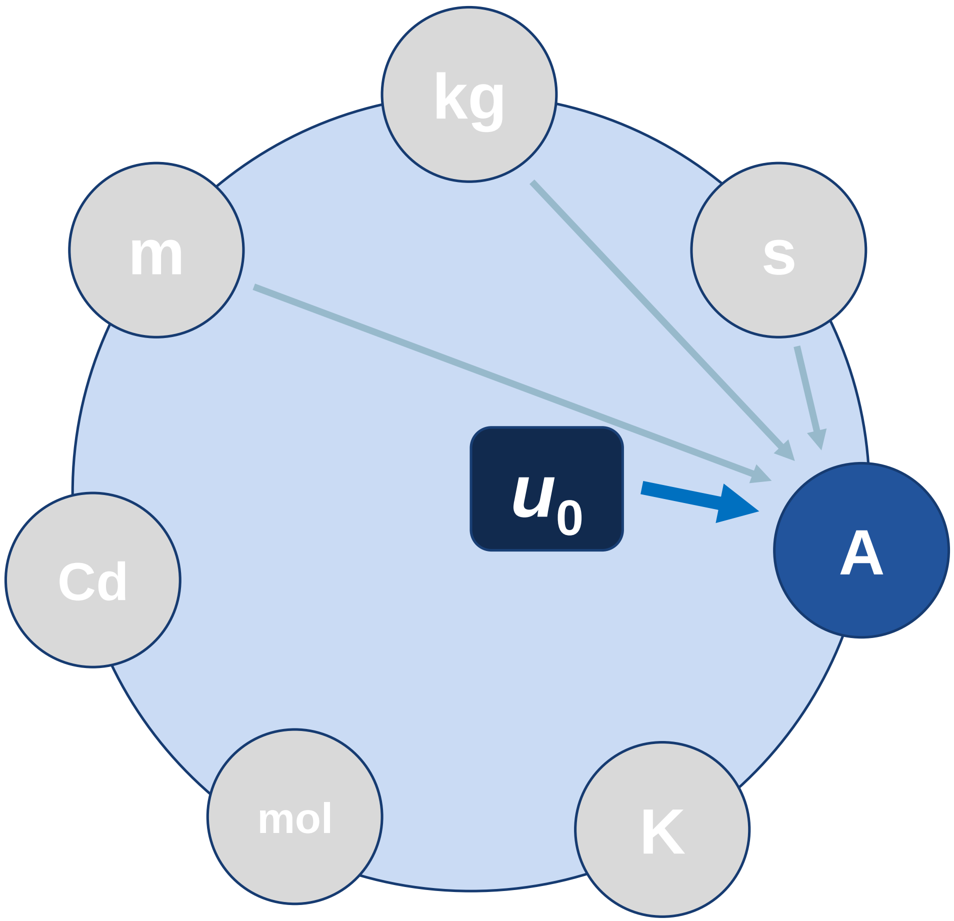
# Periodical verification of the kg

**~ 50 µg / 100 years**

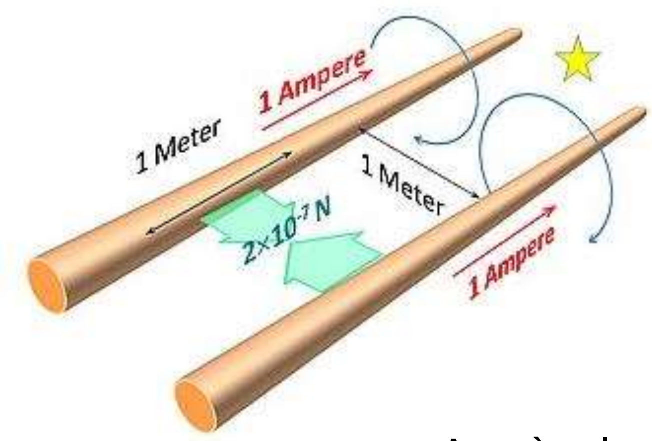



- unique
- local
- instable
- could be contaminated, damaged, stolen,…..

# The ampere



# Ampere definition




$$\frac{F}{l} = 2 \cdot 10^{-7} \ \frac{\mathrm{N}}{\mathrm{m}}$$

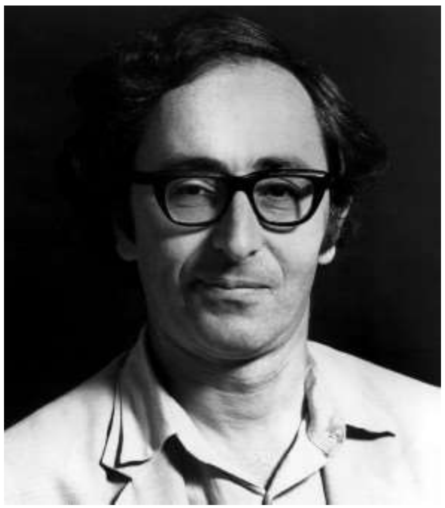
Ampère law for the chosen ideal case: $$\frac{F}{l} = \mu_0 \frac{I^2}{2\pi r}$$

Using the ampere definition and equating mechanical and electrical power (1 Nm/s = 1 VA) leads to:

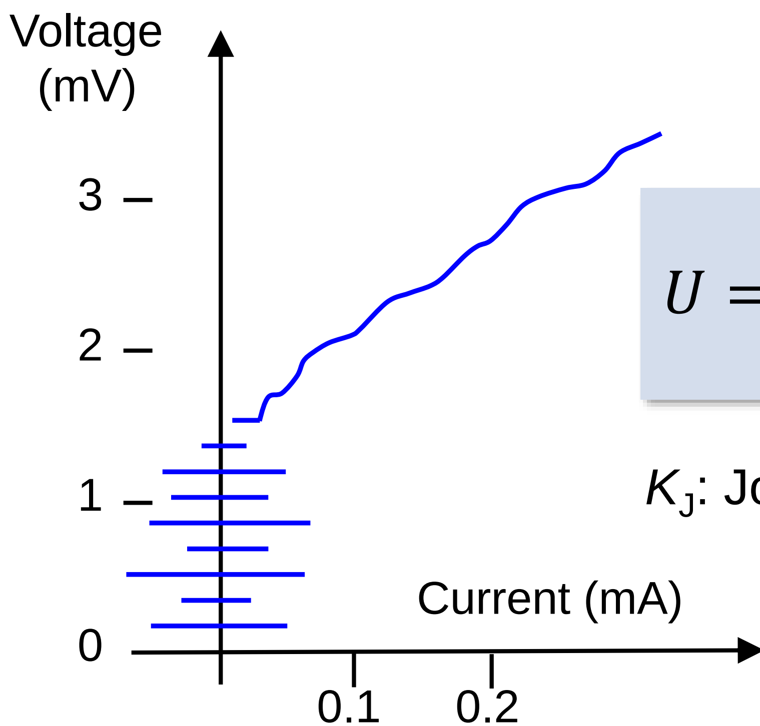
$$\mu_0 = 2\pi \frac{F}{I^2} \frac{r}{l} = 4\pi \cdot 10^{-7} \ \frac{\mathrm{N}}{\mathrm{A}^2}$$

# Electrical quantum standards

- Quantum mechanical effects allow the realization of highly reproducible electrical standards:



**B. Josephson predicts quantized voltage steps in superconductors (1962)** → voltage standard




$$U = \frac{h}{2e} i \cdot f_J = \frac{i}{K_J} f_J$$

$K_J$: Josephson constant

# Quantum voltmeter

## Commercially available

70’000 junctions in series!

PROGRAMMABLE JOSEPHSON VOLTAG

The centre piece of the **AC Quantum Voltmeter** is a 10 Volt programmable Josephson voltage standard circuit

- **Number of Josephson junctions:** 69632
- **Maximum output voltage:** ±10.1 V
- **Operating frequency:** 70 GHz
- **Zero & first order Shapiro step:** 1 mA
- **Bias current:** ±6 mA
- **Voltage increment:** 145 µV

| $V = n \times f / K_{J90}$ | | | |
|---|---|---|---|
| V | Josephson voltage | $K_{J90}$ | Josephson constant |
| n | programmable integer | f | microwave frequency |

TÜV CERT

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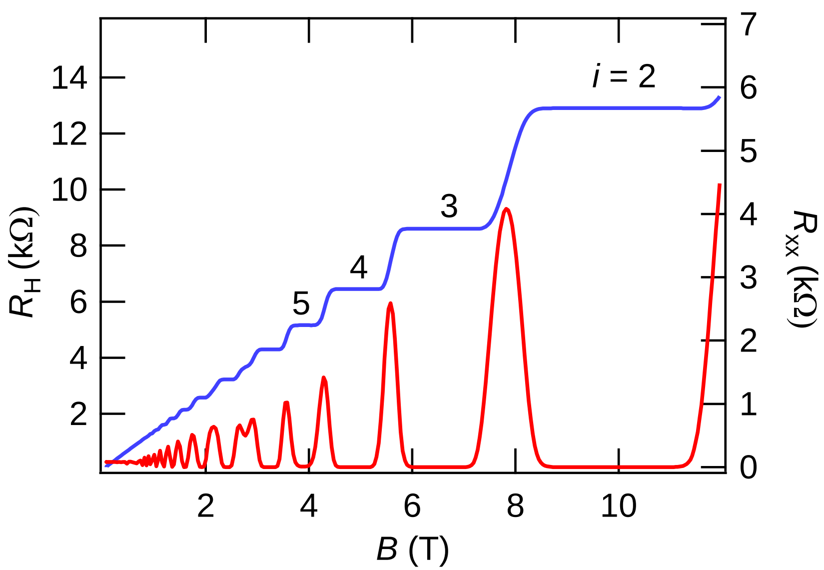
Telefon: +49 3641 2328100 | Fax: +49 3641 2328109
info@supracon.com | www.supracon.com

supracon

# Electrical quantum standards (2)

**K. Von Klitzing discovers the quantum Hall effect in 1980** → Resistance standard




2D electron gas in high magnetic field

$$R_H = \frac{h}{i \cdot e^2} = \frac{R_K}{i}$$

$R_K$: von Klitzing constant

# Practical electrical units

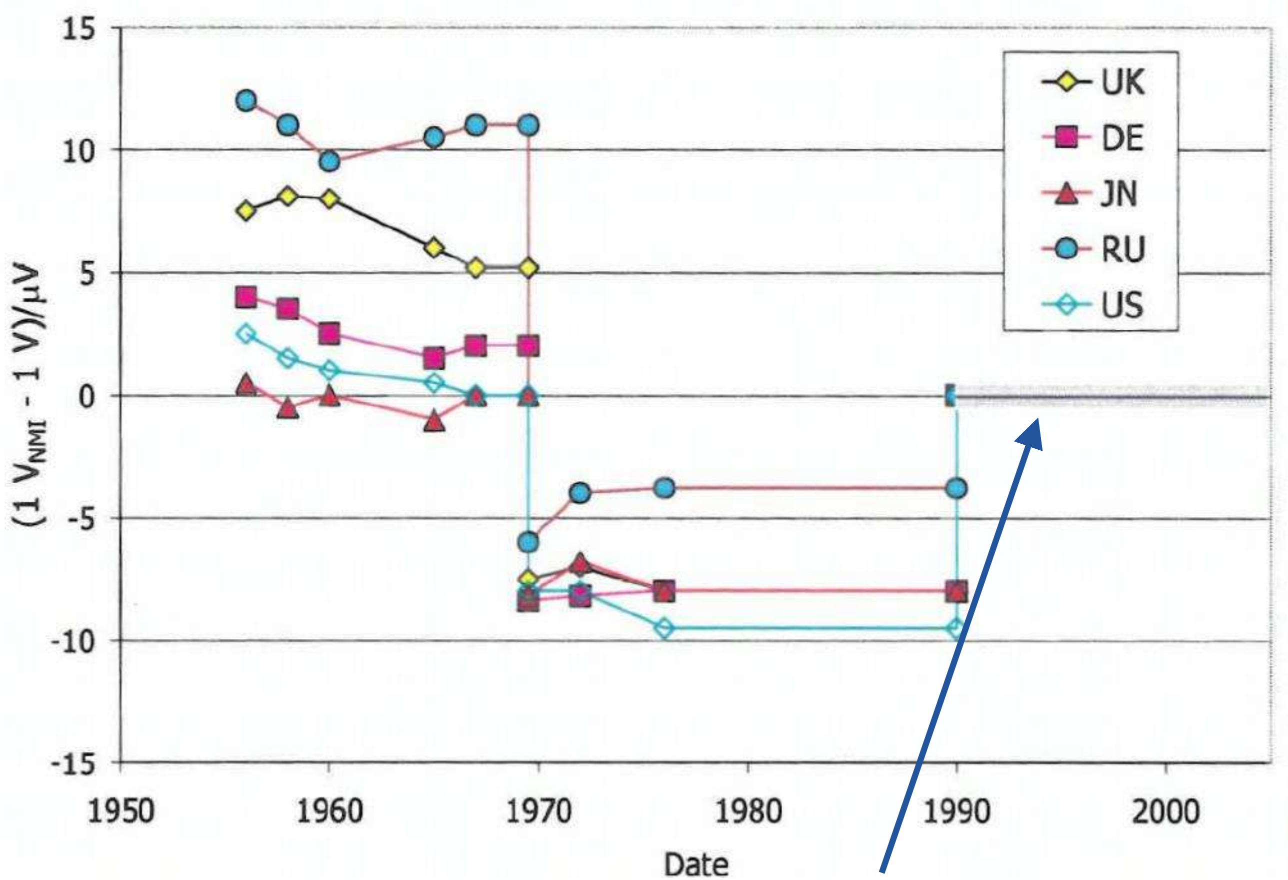
- To make best use of the good reproducibility and worldwide availability of the quantum standards, the conventional values for the Josephson- and von Klitzing constants were introduced as of January 1, 1990.

$K_{J\text{-}90} = 483\,597.9$ GHz/ V $\rightarrow$ rel. uncertainty in the SI : 0.4 ppm
$R_{K\text{-}90} = 25\,812.807\ \Omega$ 0.1 ppm

- Worldwide uniformity and improvement of electrical calibrations as a consequence of the conventional units.
- The uncertainty of the constants does only apply if electrical units are linked with mechanical units.

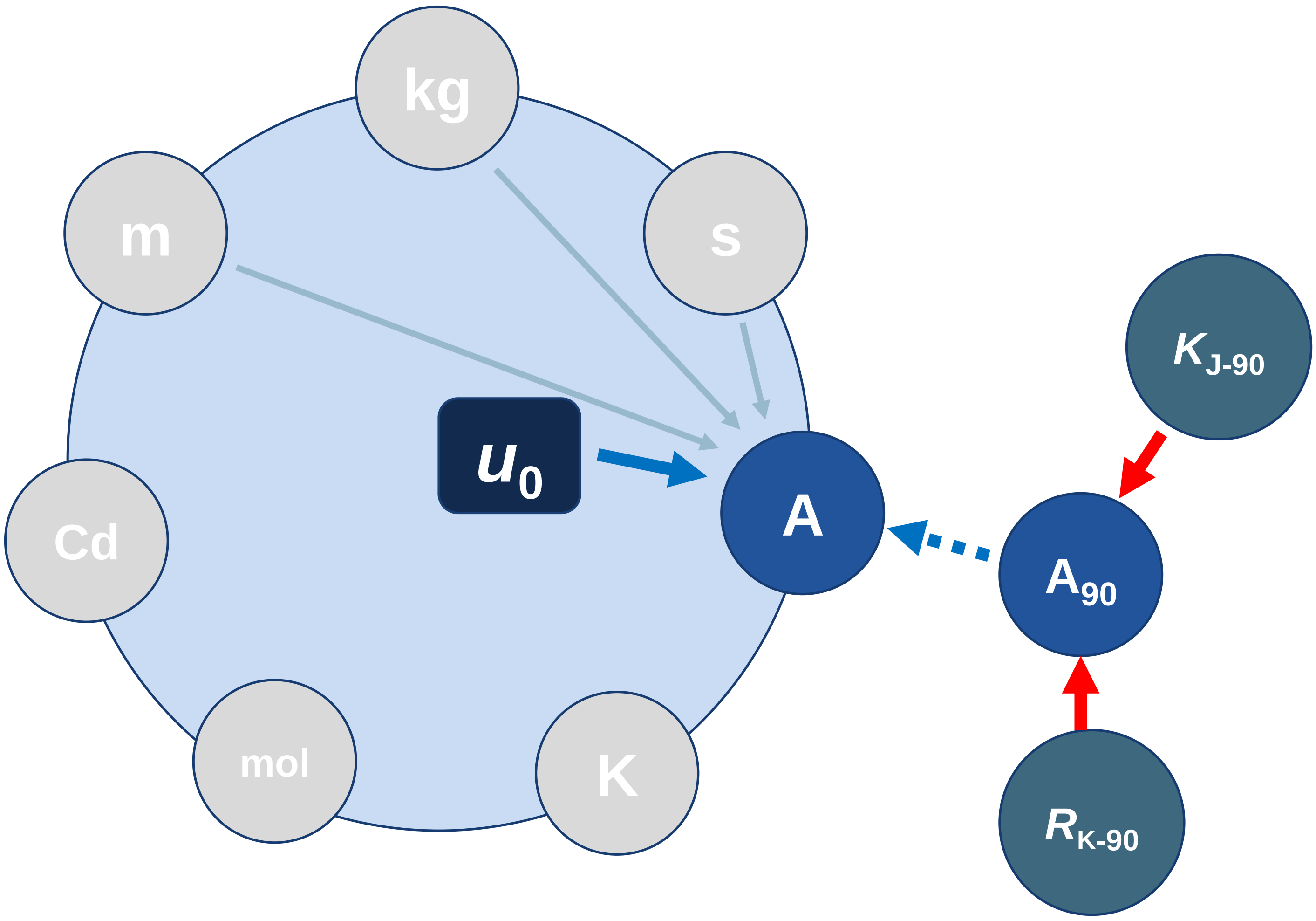
# The effect of the 1990 conventional values

Differences in unit realization in different countries

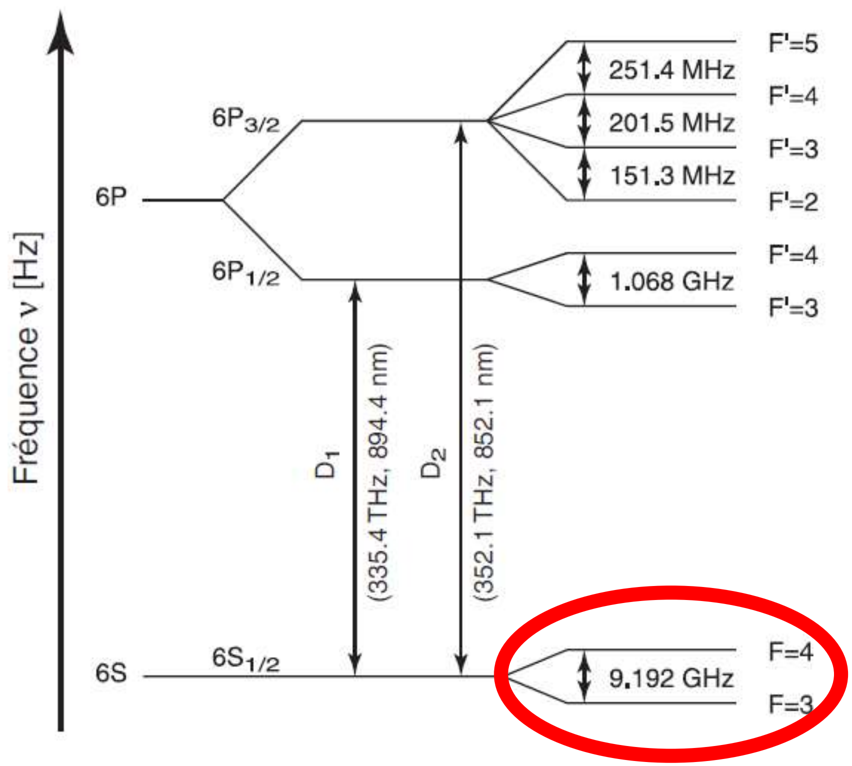


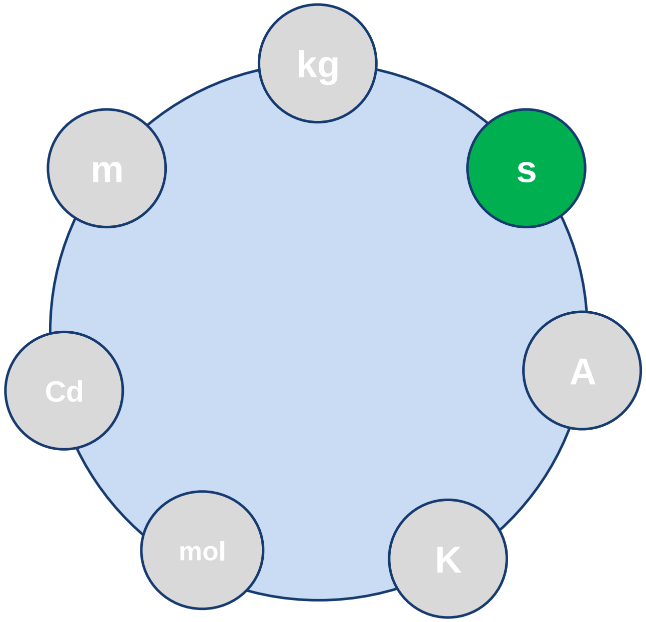

Discovery of a new physical effect allows an **improvement by a factor of 100**

# The practical electrical subsytem



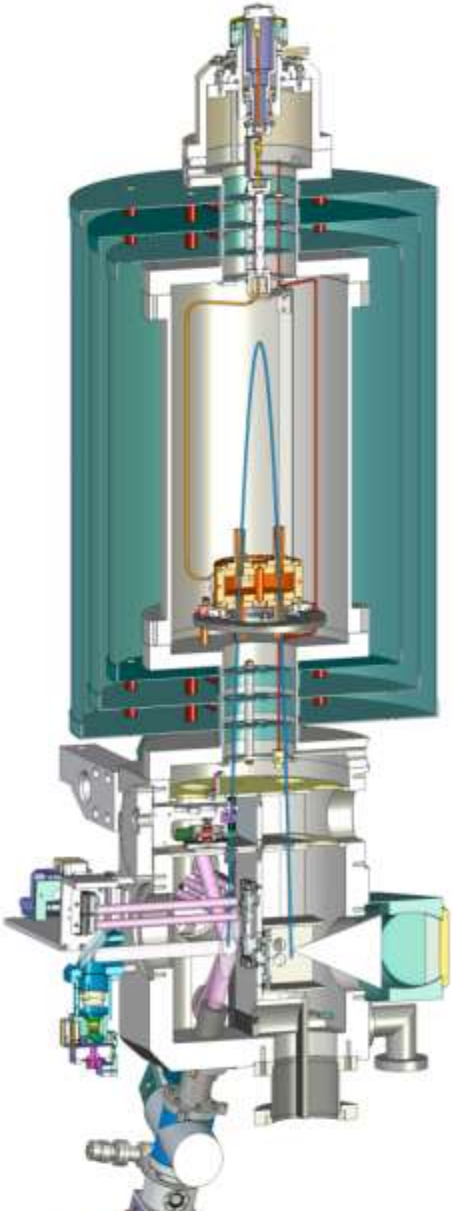
# Unit definition: the second




$$1\ \mathrm{s} = \frac{9\,192\,631\,770}{\Delta\nu\,(^{133}\mathrm{Cs})_{\mathrm{hfs}}}$$

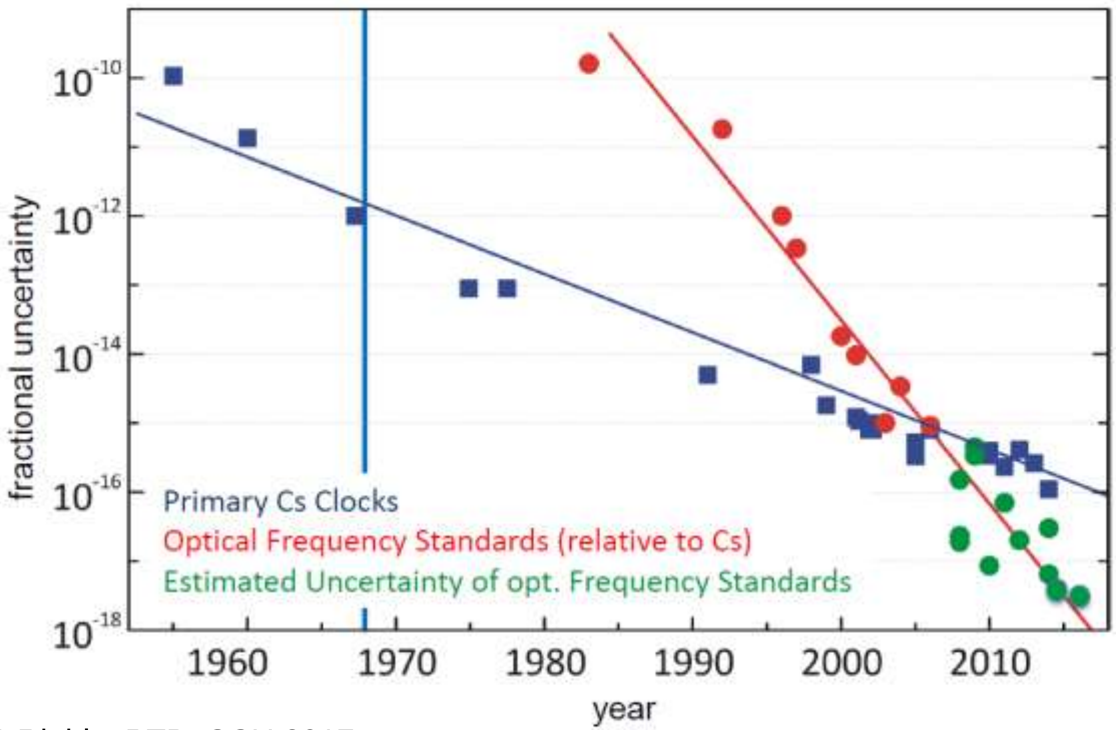
# Realisation of the s: Cs fountain



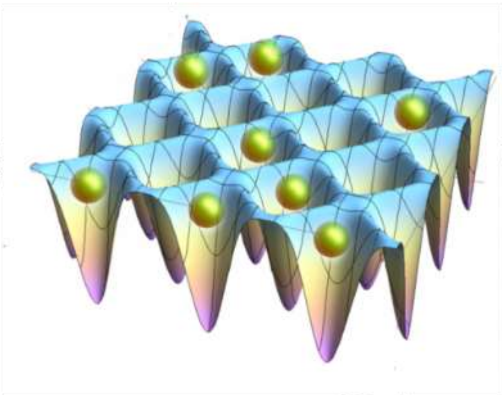
~2×10$^{-16}$

# New definition for the second?

- Properties of the selected Cs transition limit the accuracy of the realization
- Next definition: Frequency of an optical transition (GHz → THz)

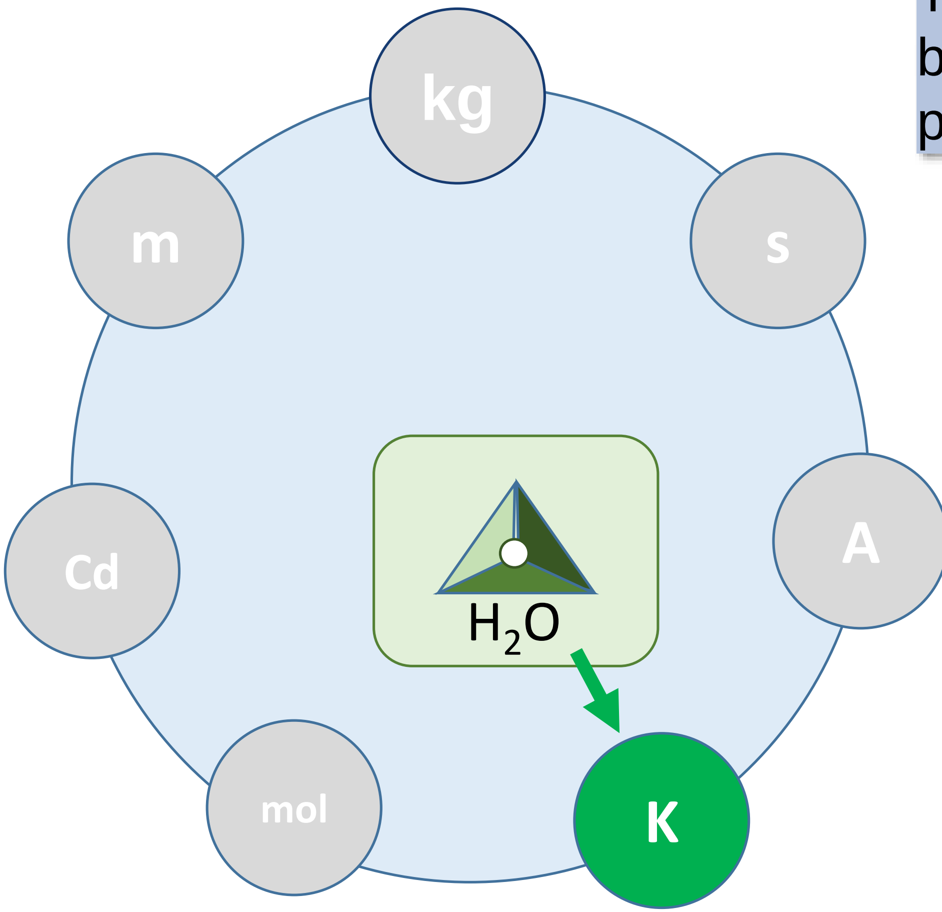



F. Riehle, PTB, CCU 2017



Optical lattice clock, RIKEN

# The kelvin




Temperature scale defined by absolute zero and triple point of water

**Limitations:**

- Impurities in cell
- Isotopic composition of water

# Concept of the revised SI

- Determination of the Planck constant
- Boltzmann constant

## James Clerk Maxwell (1870)

*« If we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or motion or the mass of our planet, but in the wavelength, the period of vibration, and absolute mass of these imperishable and unalterable and perfectly similar molecules. »*

# Units and fundamental constants

## Concept for the revised SI

- Choose seven (fundamental) constants and fix their values.
- presentation of the definition of the entire system using a single statement of the values of the seven chosen constants.
  ⇨ the constants set the scale for the system.
- With the fixed constants and with the help of the laws of physics, all units in the SI may be realized.
- A distinction between base and derived units is no longer necessary. (for reason of continuity with the past however, the concept of base units will be kept).

# Challenge 1: Replacing the kilogram prototype

- Link the unit of mass to fundamental constants
- Experiments needed, that are able to establish this link with $\Delta m/m \leq 10^{-8}$.

- Approach **A**

  "From microscopic to macroscopic world…"

  → count atoms

- Approach **B**

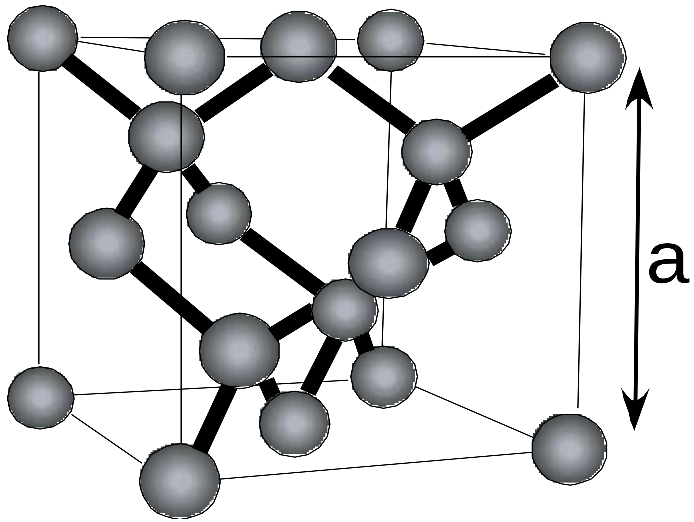
  "establish link between electrical and mechanical quantities via the force laws and benefit from the electrical quantum standards..."

  → quantum Hall effect , Josephson effect

# “Counting” atoms

*Step 1:* Link between microscopic and macroscopic mass

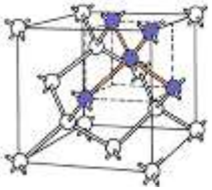
→ Density measurement in a Si single crystal

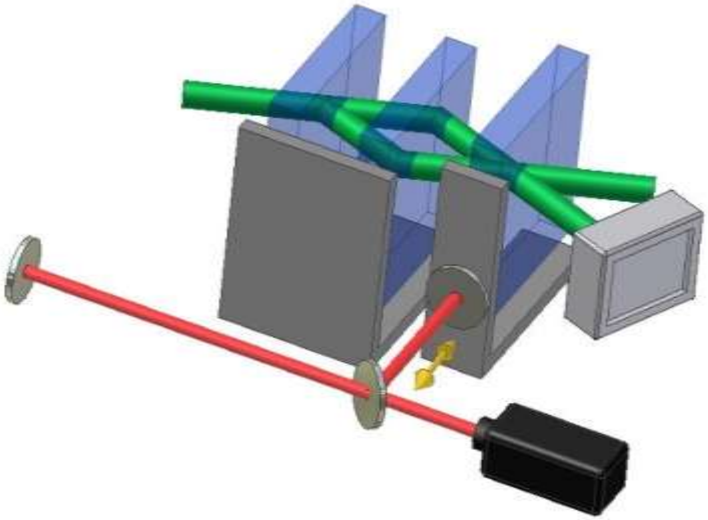



$$n_x = 8\frac{V}{a^3},$$

$$m = n_x \cdot m_{Si}$$

# X-ray crystal density experiment (XRDC)



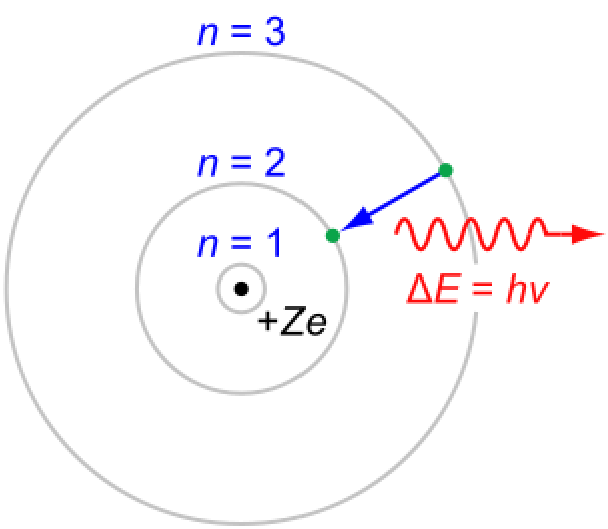


- ***M*(Si),** molar mass: isotopic content ($^{28}Si$, $^{29}Si$, $^{30}Si$) measured using mass spectrometry
- lattice constant ***a*** : X-ray interferometer
- **Volume**: interferometric measurement of the sphere
- ***n*=8** ?: lattice defects, surface effects

Uncertainty using $^{28}Si$ : $2 \times 10^{-8}$

# Link to the Planck constant

Bohr Model of the hydrogen atom

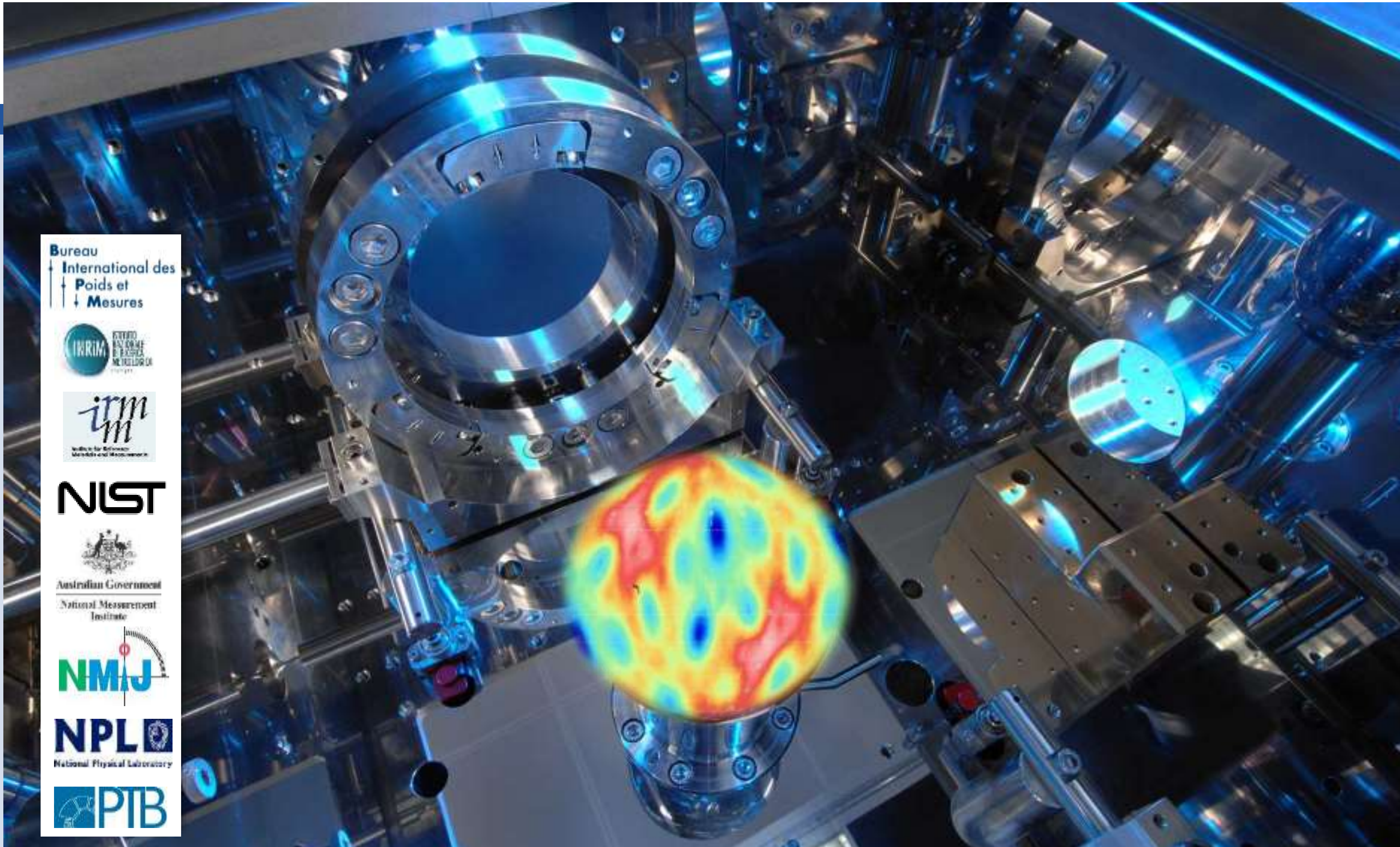



$$\frac{1}{\lambda} = R_\infty \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Rydberg constant

$$R_\infty = \frac{m_e \alpha^2 c}{2h}$$

Si sphere → $m_e = \left( \frac{m_e}{m_{Si}} \right)_r m_{Si}$ → $h = \frac{c\alpha^2}{R_\infty} \left( \frac{m_e}{m_{Si}} \right)_r m_{Si}$

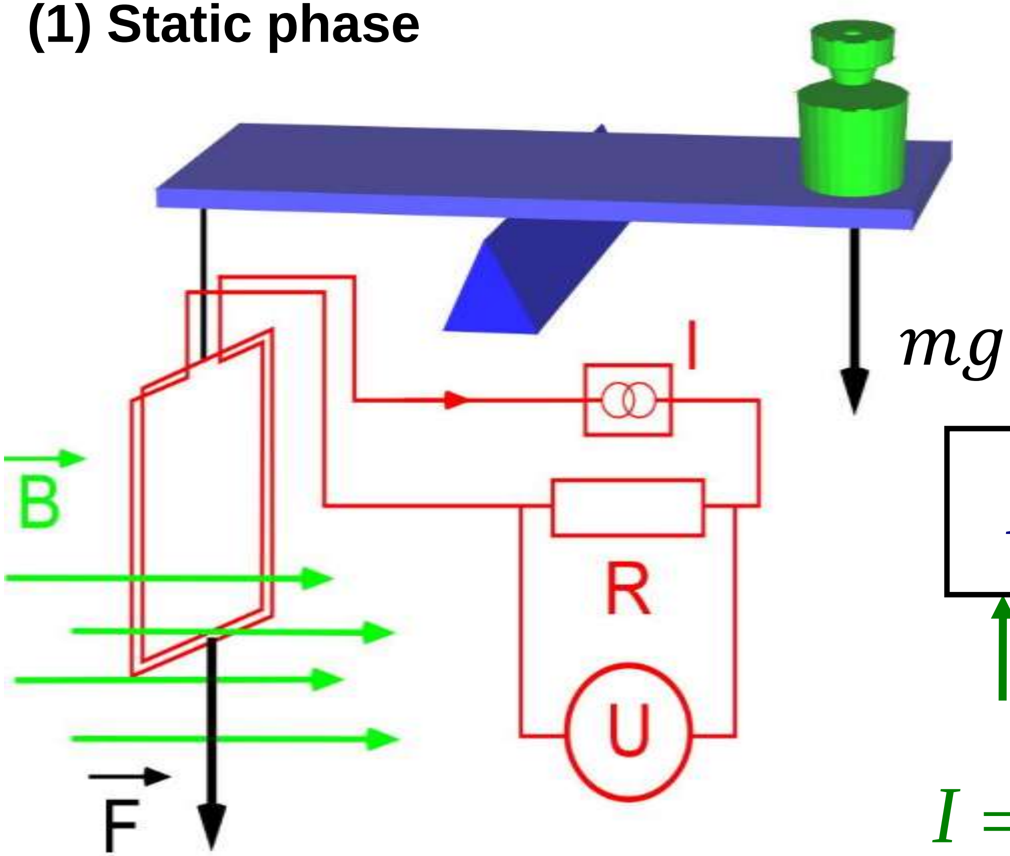



# Deviation in sphere radius: 16 nm (peak to valley)

# Electric approach: Watt balance

Two phase experiment (B. Kibble, 1976)

**(1) Static phase**



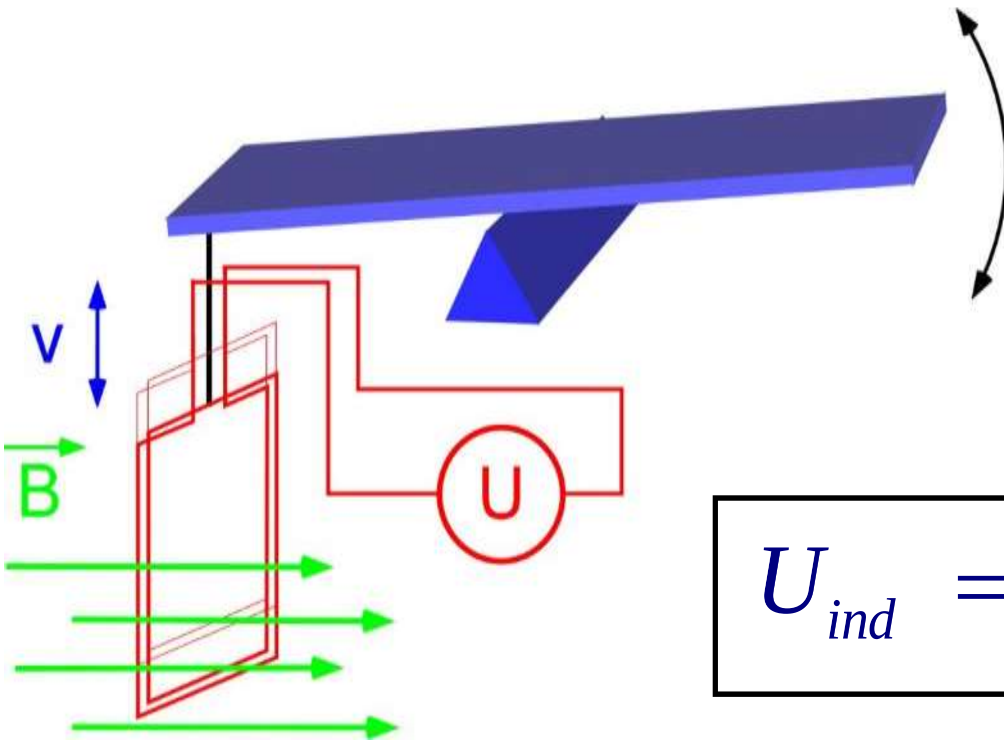

B. Kibble

$$I \cdot \int d\ell \times B = mg$$

$$I = \frac{U_j}{R}$$

(QHE & JVS)

# Watt balance (2): Dynamic phase




$$U_{ind} = v \cdot \int d\ell \times B$$

# Combination of the phases

$$UI = mgv$$

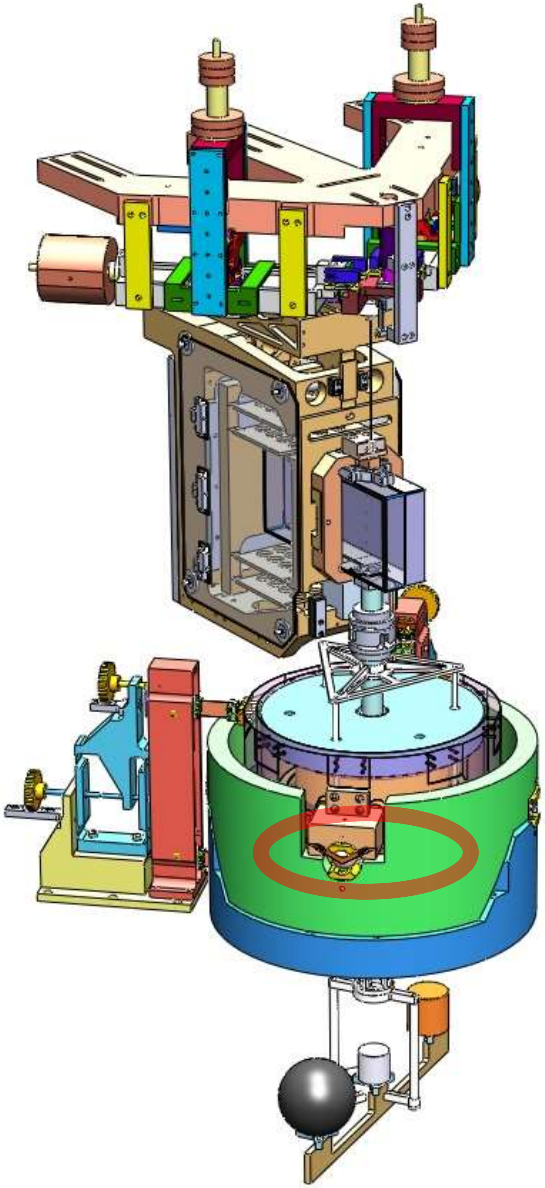
Electrical power

Mechanical power

Electrical quantum effects(QHE & JVS)

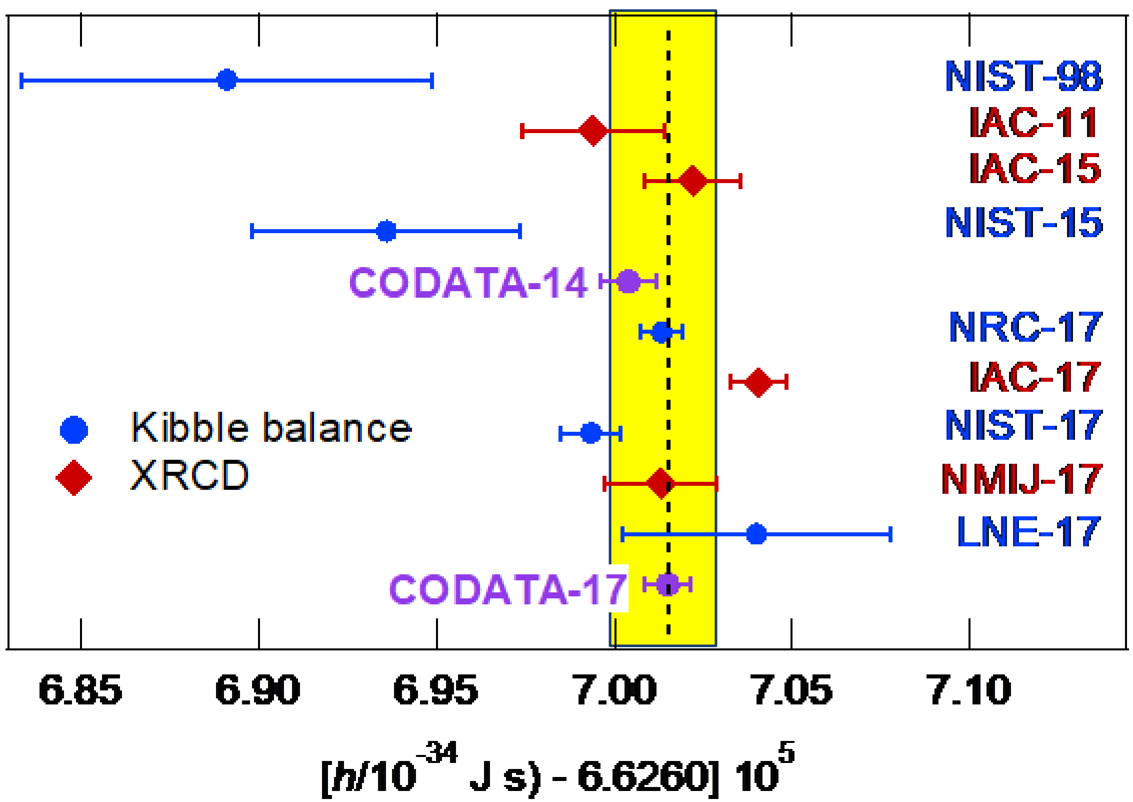
$$m = C\frac{f_{J1}f_{J2}}{gv}h$$

# The METAS watt balance

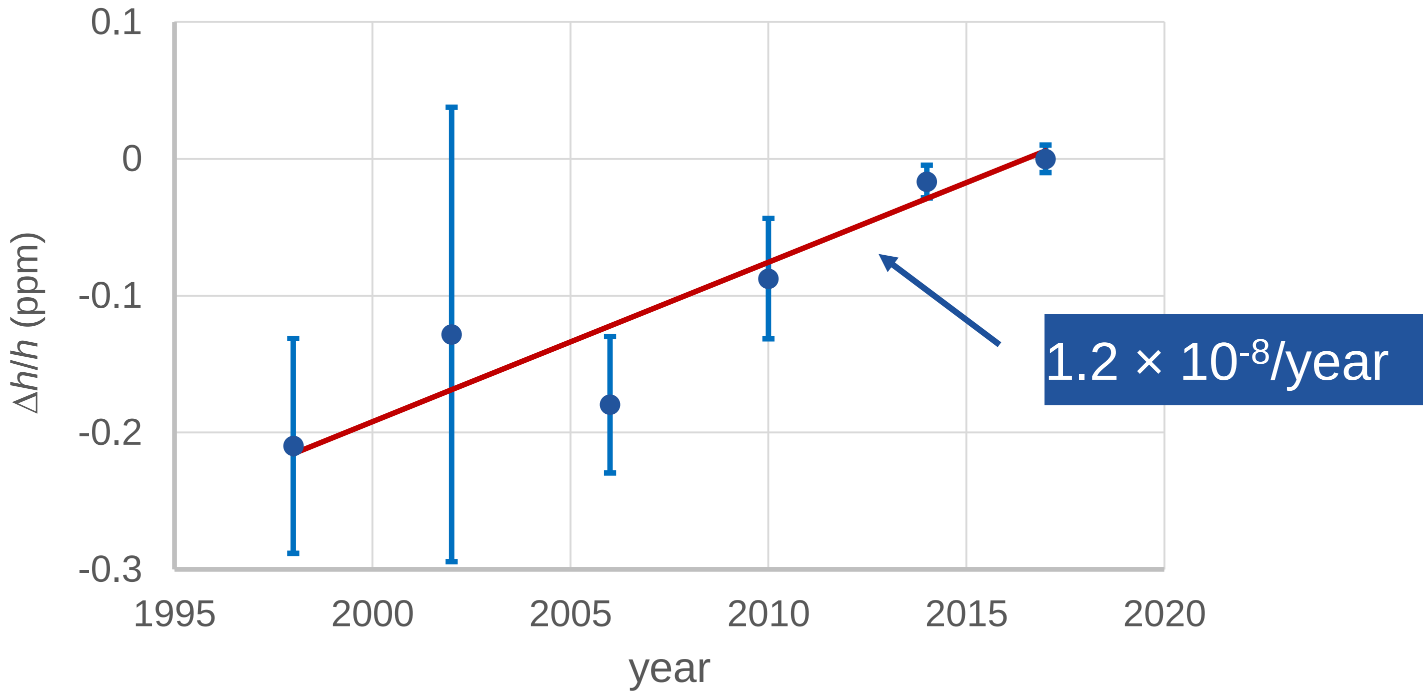


# Status *h*-values

2 ×10$^{-8}$ uncertainty requirement

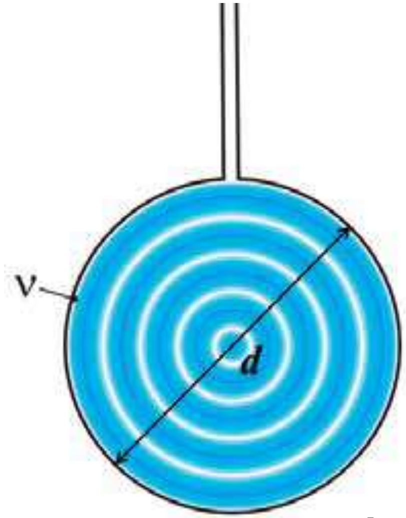


# CODATA- values for *h*



# Challenge 2: Boltzmann constant

NPL




$$E = \frac{3}{2} kT$$

Acoustic gas thermometer

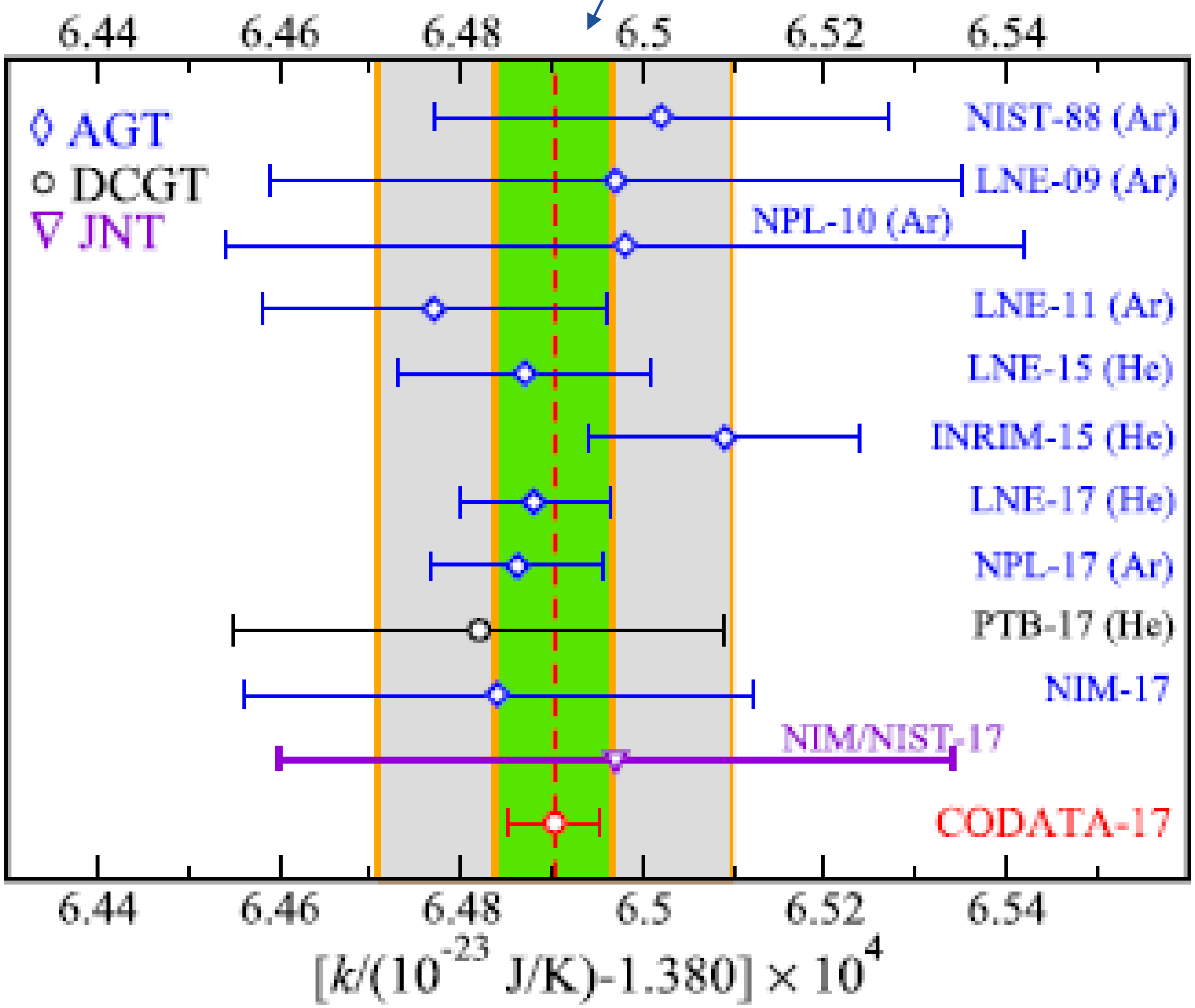
$$\mu_0 = \sqrt{\frac{\gamma_0 N_A k T}{M}}$$

Sound velocity

Specific heat ratio

# Experiments Boltzmann constant

0.5 ×10$^{-6}$ uncertainty requirement




**AGT**: acoustic gas thermometry;

**DCGT**: dielectric constant gas thermometry;

**JNT**: Johnson noise thermometry.

# The seven constants

| *Unit* | | *SI old* |
|---|---|---|
| second | s | $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$ |
| meter | m | $c$ |
| kilogram, | kg | $m(\mathcal{K})$ |
| ampere, | A | $\mu_0$ |
| kelvin, | K | $T_{\text{TPW}}$ |
| mole, | mol | $M(^{12}\text{C})$ |
| candela, | cd | $K_{\text{cd}}$ |

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# CODATA adjustment 2017

Metrologia **55** (2018) L13–L16

https://doi.org/10.1088/1681-7575/aa950a

**Short Communication**

## The CODATA 2017 values of $h$, $e$, $k$, and $N_A$ for the revision of the SI

**D B Newell[1], F Cabiati, J Fischer, K Fujii, S G Karshenboim, H S Margolis, E de Mirandés, P J Mohr, F Nez, K Pachucki, T J Quinn, B N Taylor, M Wang, B M Wood and Z Zhang**

Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants

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Received 2 August 2017, revised 19 October 2017
Accepted for publication 20 October 2017
Published 29 January 2018

CrossMark

**Abstract**

Sufficient progress towards redefining the International System of Units (SI) in terms of exact values of fundamental constants has been achieved. Exact values of the Planck constant $h$, elementary charge $e$, Boltzmann constant $k$, and Avogadro constant $N_A$ from the CODATA 2017 Special Adjustment of the Fundamental Constants are presented here. These values are recommended to the 26th General Conference on Weights and Measures to form the foundation of the revised SI.

Keywords: international system of units, fundamental constants, SI redefinition

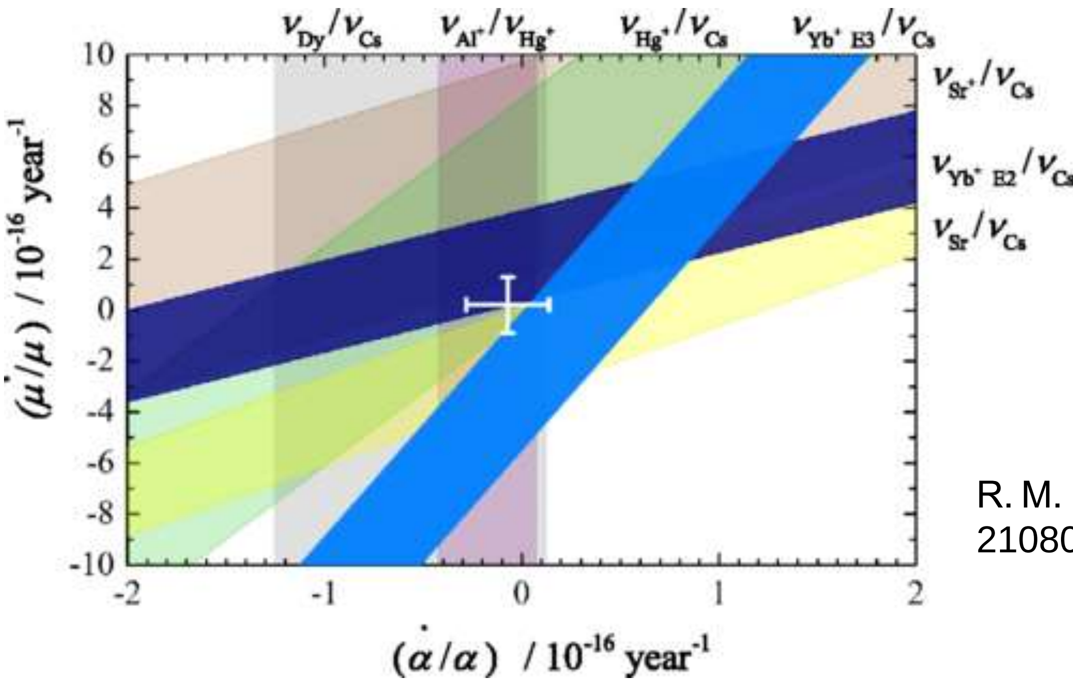
# Definition of the revised SI

**The International System of Units, the SI, is the system of units in which**

- the ground state hyperfine splitting frequency of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ is exactly **9 192 631 770 hertz**,
- the speed of light in vacuum $c$ is exactly **299 792 458 metre per second**,
- the Planck constant $h$ is exactly **6.626 070 15 ×10$^{-34}$ joule second**,
- the elementary charge $e$ is exactly **1.602 176 634 ×10$^{-19}$ coulomb**,
- the Boltzmann constant $k$ is exactly **1.380 649 ×10$^{-23}$ joule per kelvin**,
- the Avogadro constant $N_A$ is exactly **6.022 140 76 ×10$^{23}$ reciprocal mole**,
- the luminous efficacy $K_{cd}$ of monochromatic radiation of frequency **540 ×10$^{12}$ Hz is exactly 683 lumen per watt**.

# Are constants constant?

Frequencies of different clocks depend differently on fundamental constants



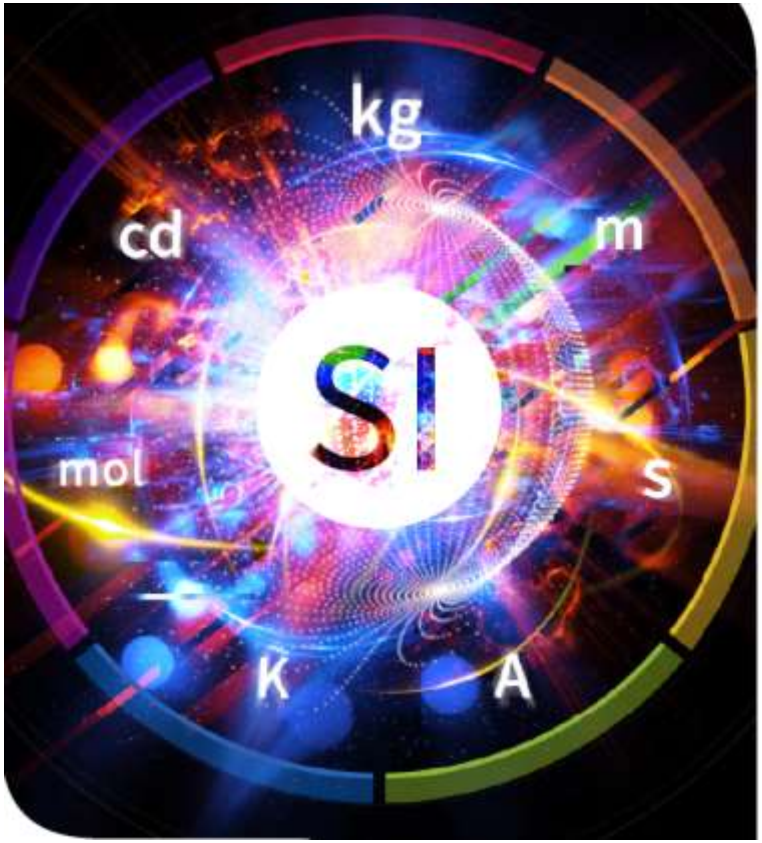

R. M. Godun et al., Phys. Rev. Lett. 113, 210801 – 2014

Constraints placed on time variation of the **fine structure constant α** and the **proton-to-electron mass ratio μ**, from frequency measurements in $^{171}$Yb+, $^{88}$Sr+, $^{87}$Sr, $^{199Hg}$+, Dy and $^{27}$Al+.

# Timeline for the revised SI

- 16. Nov. 2018: Decision at the 26. General Conference for Weights and Measures

- **20. May 2019**:
  World metrology day;
  Practical implementation of the revised SI



# Implementation of the revised SI

- No changes for the units **second, metre and candela**
- The **kg** has to be realized through experiments like the watt balance and the XRCD; The kilogram prototype will have an uncertainty.
- The practical electrical subsystem becomes obsolete; the quantum effects realize the units directly in the SI. As a consequence small step-changes in results will occur.

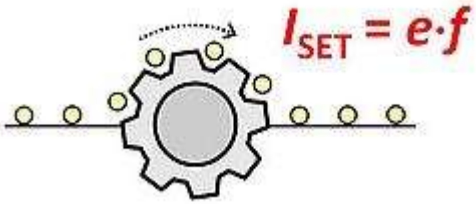
**Voltage: $1.07 \times 10^{-7}$ (rel. change)**
**Resistance: $1.78 \times 10^{-8}$**

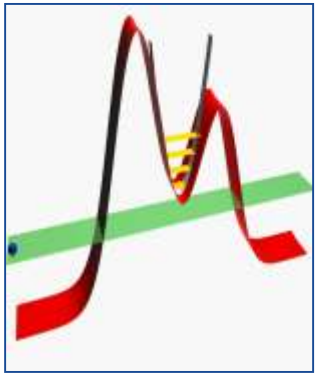
# Electrical units in the revised SI

***h* and *e* fixed:**

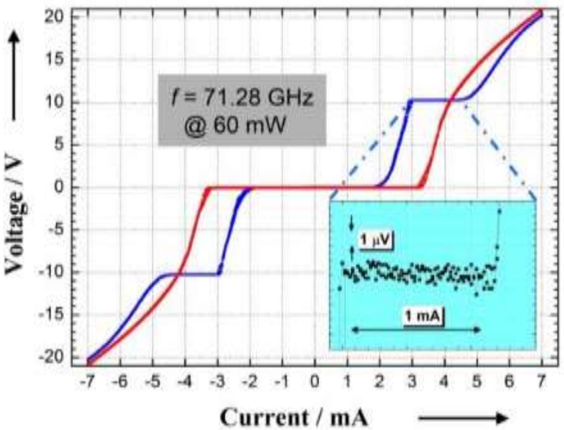
Electrical quantum effects realise the electrical units directly.

ampere



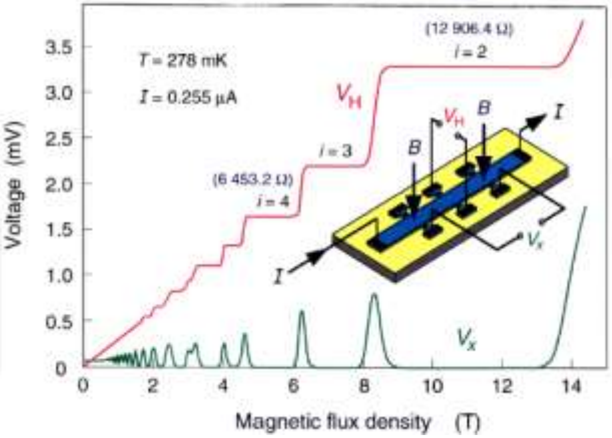
volt




$$U = \frac{h}{2e} i \cdot f_J$$

ohm

$$R_H = \frac{h}{i \cdot e^2}$$



# Implementation (2)

- **Kelvin**: The experiments used for the Boltzmann constant measure now temperature directly. The scale will improve. The temp. of the triple point will have an uncertainty.

- **Mole**: No link to the kilogram anymore; the molar mass of $^{12}C$ has an uncertainty.

# Conclusion


1. The planned revision changes and improves the SI fundamentally.
2. A set of "defining constants" sets-up the system.
3. **The system is independent of specific experiments** - improved insights and technologies make improved unit realizations possible.
4. The realization is possible everywhere.
5. According to current knowledge, the system is stable over time.

METAS


SI International System of Units
kg
m
s
A
K
mol
cd
SI
...for all times and civilisations...
...throughout the Universe...
26e RÉUNION
13-16 NOVEMBRE 2018
26th MEETING
13-16 NOVEMBER 2018

Schweizerische Eidgenossenschaft
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# Thank you very much for your attention