Demystifying Quantum Mechanics
The "ETH Approrach"

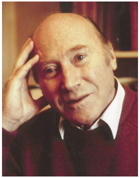
"I leave to several futures (not to all) my garden of forking paths"
J. L. Borges

Great minds of Quantum Mechanics

A. Einstein

W. Heisenberg

P.A.M. Dirac



R. Haag

PSI, March 21, 2019[1]

[1] Jürg Fröhlich, ETH Zurich

Some quotes – for fun

- *"If someone tells you they understand quantum mechanics then all you've learned is that you've met a liar."*
  R. P. Feynman

- *"Anyone who is not shocked by quantum theory has not understood it."*
  N. Bohr

- *"We have to ask what it means!"*
  K. G. Wilson

In my opinion, only Wilson's challenge remains significant.

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In this lecture, I propose to find out which features of quantum theory might be shocking at first sight. I will then attempt to explain why they should not shock you, and *what it all means*. Incidentally, *I don't think I am a liar*

Contents and credits

Contents:

1. What this lecture will be about
2. Why might we be shocked by Quantum Mechanics?
3. Recap of the *"Copenhagen Interpretation"* of QM
4. The "*ETH* Approach" to non-relativistic QM
5. Observations of events in Quantum Mechanics
6. Summary and conclusions
7. [Appendix about relativistic quantum theory]

Credits:

I am indebted to my last PhD student *Baptiste Schubnel* for an enjoyable collaboration, to M. Ballesteros, *Ph. Blanchard* and M. Fraas for cooperation, and to these colleagues as well as many further ones, including some of the *"Bohmians"*, various colleagues at ETH Zurich, and D. Buchholz for useful discussions on QM.

1. What this lecture will be about

New Foundations of Quantum Mechanics are proposed: the

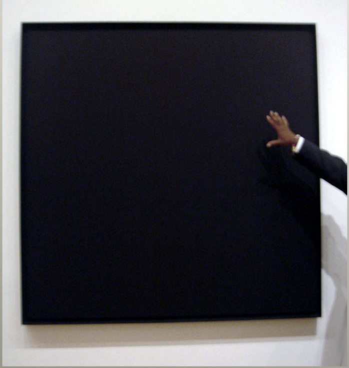
"E T H - Approach to Quantum Mechanics"

where "E" stands for Events, "T" for Trees, and "H" for Histories. This approach enables us to introduce a precise notion of "events" into Quantum Mechanics ($\nearrow$ Haag), explain what it means to observe an event by recording the value of an appropriate physical quantity, and to exhibit the stochastic dynamics of states of isolated open systems featuring events. It adds to the standard formulation of the theory two simple, but fundamental hypotheses concerning the emergence of "events" and their effect on the evolution of states of such systems.

The "ETH - Approach to QM" results in a "Quantum Theory without observers". It does away with "extensions of Quantum Mechanics", all of which have remained unacceptably vague.

A metaphor of Quantum Mechanics

*QM is QM-as-QM and everything else is everything else**



* "The one thing to say about art is that it is one thing. Art is art-as-art and everything else is everything else."

(Ad Reinhardt)

Some say: *"Shut up and calculate!"* But let's abandon this maxim and try to open the *"quantum black box"*!

2. Why might we be shocked by Quantum Mechanics?

Many if not most people appear to be confused about the deeper meaning of Quantum Theory. Asking twenty-five professors of theoretical physics to explain their understanding of the *Foundations of* QM you are likely to get ten different answers most of which contradict some or several of the other ones. – *Worse, generations of students are indoctrinated with erroneous claims about* QM; *e.g., that the Schrödinger-picture dynamics of states and the Heisenberg-picture dynamics of "observables" are equivalent, ...*

Soon one hundred years after the discovery of matrix mechanics by *Heisenberg*, *Born*, *Jordan*, and *Dirac*, this is quite shocking and represents an intellectual scandal, which we had better remove, as soon as possible!

Unfortunately, in 60 minutes I will not reach this goal! (To accomplish something valuable and lasting, one would have to organise a one- or two-semester course on these matters!)

My discussion will be fairly non-technical, yet somewhat daring. I would say, it represents an exercise in *"natural philosophy"*.

A lightening review of unusual or strange features of Quantum Mechanics

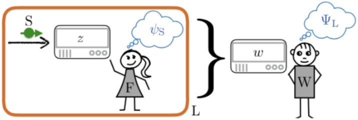
- Bell's Inequalities: Quantum-mechanical correlations between values of physical quantities (e.g., polarisations of a "Bell pair" of photons) measured in two independent labs violate *Bell's inequalities*, in the sense that the numerical range of the qm correlation matrix exceeds the one of the corresponding classical correlation matrix; ($\nearrow$ Tsirelson). – Has been checked experimentally!
- Kochen-Specker Theorem: Shows that the following two assumptions are incompatible with Quantum Mechanics:
  - All physical quantities have definite values at any given time.
  - The values of physical quantities are independent of the measurement context.

  Example: Squares of spin-components of a spin-1 particle. – $\exists$ experiments!

Conclusions: 1. $\nexists$ *"local" theories of commutative hidden variables reproducing the predictions of Quantum Mechanics* (if $\dim(\mathcal{H}) > 2$).
2. Bell-type *"non-locality"* of Quantum Mechanics.

The Schrödinger Equation does not describe the evolution of states of isolated systems featuring events

- Wigner's Friend:



Courtesy of Frauchiger & Renner

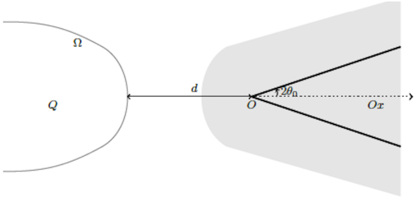
Agent F measures z-component of spin of a silver atom, pepared in state $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$, with outcome $s \in \{\uparrow, \downarrow\}$; state of spin of silver atom *after* measurement given by $\psi_S = |s\rangle$. Instead, W regards the lab containing F as *one big quantum system*, L, in a state evolving according to a Schrödinger Equation $\Rightarrow$ Assigns *pure* state

$$\psi_L = \frac{1}{\sqrt{2}}(|s = \uparrow; D_+, F_+\rangle_L + |s = \downarrow; D_-, F_-\rangle_L)$$

to L, which could be tested by a suitable measurement applied to L. This leads to a contradiction with the description given by F and shows that W *should not use the Schrödinger Eq. to describe the evolution of the state of L*! (More sophisticated versions proposed by Lucien Hardy, Popescu et al., Frauchiger-Renner ...)

And Quantum theory cannot be fully predictive, because ...

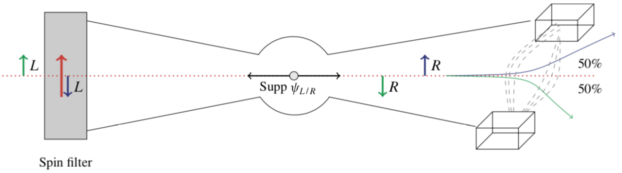
Setup of a Gedanken-Experiment ($\nearrow$ Faupin-F-Schubnel):



$\uparrow$ $\uparrow$

Q = sub-system "confined" to Ω Particle P propagates to the right $\rightarrow$

Time evolution of P ess. *indep.* of Q (cluster props.) $\rightarrow$ Application:



Q:={spin filter $\vee$ particle P'} $\subset \Omega$, shaded area := ess. supp of orbital wave function of P

... Quantum theory is fundamentally probabilistic –

in spite of the *deterministic nature of the Schrödinger Eq.* –
Temporary assumptions (leading to a contradiction):

- P and P': Spin-$\frac{1}{2}$ particles prepared in a *spin-singlet initial state*; spin filter prepared in a poorly known initial state not entangled with initial state of P' and P.
- *Dynamics of state of total system fully determined by Schrödinger equation.* In particular, initial state of spin filter determines whether P' will pass through it or not, (given that the initial state of $P' \vee P$ is a spin-singlet state, with P' and P moving into opposite cones).
- *Correlations between outcomes of spin measurements of P' and of P are as predicted by standard quantum mechanics*, (relying on the "Copenhagen interpretation").

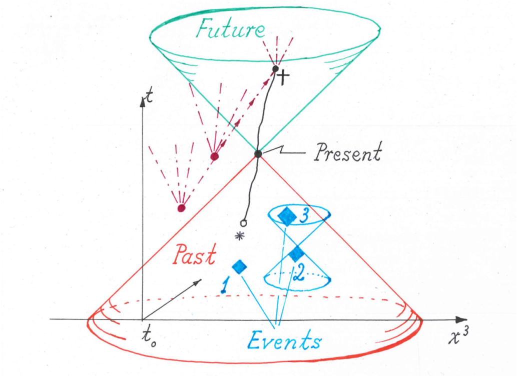
Fact: Heisenberg-picture dynamics of observables (spin, etc.) referring to P is ess. *independent* of dynamics of $Q := \{P' \vee$ spin filter$\}$. This follows from our choice of initial conditions & cluster properties of time evolution! Hence *spin of P ess. conserved before measurement* $\Rightarrow$

Expectation value of spin of $P \approx 0, \forall$ times!

But *this* ***contradicts*** *the third (last) assumption stated above!*

Relativistic theories are not fully predictive, because ...

Space-time with an *event horizon*. (Observer sits at "Present"; is unaware of dangers lurking from outside his past light-cone; he might get killed at †. Events 1 & 2 are space-like separated; event 3 is in the future of 2)



t_0: time right after inflation $\to$ event horizon $\Rightarrow$ initial conditions not fully accessible!

$\boxed{\text{Past} = \text{History of Events} \;/\; \text{Future} = \text{Ensemble of Potentialities}}$

This fundamental structure must be retained in Quantum Mechanics!

3. Recap of the *"Copenhagen Interpretation"* of QM

In the *Copenhagen Interpretation of QM* the state/wave function, Ψ, of a physical system S does not have an "ontological status". According to *Born* and *Heisenberg*: Ψ is merely a mathematical object enabling us to predict probabilities of different possible values a physical quantity $\hat{X}$ can take *in case* this quantity is measured at a certain *sharp time* t. If the measured value of $\hat{X}$ at time t is given by ξ then the state of the system *immediately after* measuring $\hat{X}$ must be taken to be an *eigenstate* corresponding to the eigenvalue ξ of a linear operator $X(t)$ representing $\hat{X}$ at time t, ("projection postulate").The *probability of measuring* $\xi \in spec(\hat{X}) \equiv spec(X(t))$ is given by *Born's Rule*.

In the *absence of measurements/observations*, the q.m. state Ψ of a physical system and its evolution in time mean: *Nothing !* There is essentially *no invariant information* encoded in Ψ and its *Schrödinger-pict.* time evolution, $\Psi(t) = U(t,t')\,\Psi(t')$, $t, t' \in \mathbb{R}$, beyond information on *energy spectra* (of generators of U).

Evolution of states when measurements are made

In the *Heisenberg picture*, states of a system S do *not* evolve in time, *except* at times when a physical quantity, $\hat{X}$, is measured, and then they must be changed in accordance with the

"Projection Postulate":

Assume, for simplicity, that $\hat{X}$ has pure-point spectrum, (possibly only finitely many eigenvalues). Suppose that the operator $X(t)$ represents $\hat{X}$ at time t, and consider its spectral decomposition,

$$X(t) = \sum_{\xi \in spec(\hat{X})} \xi \underbrace{\Pi_\xi(t)}_{spect.proj.}$$

Let Ψ be the state of the system S right *before* $\hat{X}$ is measured at time t. If $\xi \in spec(\hat{X})$ is the measured value of $\hat{X}$ then the state to be used to make predictions of measurement outcomes at *later times* $(> t)$ is determined by the following

Critique of the Copenhagen Interpretation

Projection Rule:

$$\Psi \quad \mapsto \quad \Pi_\xi(t)\Psi / \|\Pi_\xi(t)\Psi\|, \tag{5}$$

where $\|\Pi_\xi(t)\Psi\|^2$ is the *Born probability* of measuring the value ξ. $\exists$ natural generalisation of *Born's Rule*, due to *Lüders, Schwinger* and *Wigner*, that promises to determine the probabilities of entire *histories* of outcomes in arbitrarily many measurements of different physical quantities made at subsequent times, ("*LSW* rule") ...

Critique:

- Mysterious/ominous role of *observers* who perform measurements. Role of "free will" of observers?
- Act of measuring a physical quantity *not* described by theory.
- Measurements take place *instantaneously* $\rightarrow$ unphysical!
- If measurement-acts of observers are not specified theory is totally *un-predictive*.

4. The "*ETH* Approach" to Quantum Theory

Next, we address the question of what is meant by *"events"* featured by isolated systems, and of how they can be recorded (in *direct/projective measurements/observations*). I sketch what I call the "*ETH* Approach" to QM. For simplicity I consider *non-relativistic theories*:
Let S be an isolated physical system. Pure states of S are given by unit rays in a separable Hilbert space $\mathcal{H}_S$; general states by density operators, ω, acting on $\mathcal{H}_S$, with $\omega(A) := Tr(\omega \cdot A)$, for any bd. operator A on $\mathcal{H}_S$.

Time is a fundamental quantity in n.r. physics. The time axis is given by $\mathbb{R}$. Let's suppose the *present time* is t_0, and let I be an arbitrary interval of *future times*, i.e., $I \subset [t_0, \infty)$.

Definition: *"Potential future events"* in an isolated system S – *"potentialities"* – are described by certain *orthogonal projections* acting on $\mathcal{H}_S$ & associated with future time intervals. The *algebra generated by all "potential future events" associated with an interval, I, of future times is denoted by $\mathcal{E}_I$, and we define

$$\mathcal{E}_{\geq t} := \overline{\bigvee_{I \subset [t,\infty)} \mathcal{E}_I}, \quad \text{and} \quad \mathcal{E} := \overline{\bigvee_{t \in \mathbb{R}} \mathcal{E}_{\geq t}}^{\|\cdot\|}, \tag{2}$$

The *"Principle of Diminishing Potentialities"*

(where the algebras $\mathcal{E}_{\geq t}, t \in \mathbb{R}$, are assumed to be *weakly* closed![2])

By definition,

$$\mathcal{E}_I \supseteq \mathcal{E}_{I'} \quad \text{if} \quad I \supseteq I' \,, \qquad \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq t'} \quad \text{if} \quad t' > t \,.$$

The *"Principle of Diminishing Potentialities" (PDP)* is the statement that

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'} \,, \quad \text{whenever} \quad t' > t \geq t_0} \tag{3}$$

In Quantum Mechanics, an *isolated open system* S, including its Heisenberg time-evolution, can be *defined* in terms of a "filtration", $\{\mathcal{E}_{\geq t}\}_{t \in \mathbb{R}}$, of *algebras of future potentialities* satisfying *PDP*. – (Examples!)

Given a state, ω, of S, we set

$$\omega_t := \omega|_{\mathcal{E}_{\geq t}} \,, \quad \text{i.e.,} \quad \omega_t(A) = \omega(A) \,, \forall A \in \mathcal{E}_{\geq t} \,. \tag{4}$$

[2] Considering weakly closed (von Neumann) algebras is convenient, because the spectral projections of any element of the algebra will then also belong to the algebra!

Events

Note that ω might be a *pure* state on $\mathcal{E}$. But, since $\mathcal{E}_{\geq t} \underset{\neq}{\subset} \mathcal{E}$, $\forall t < \infty$, ω_t will generally be a *mixed* state on $\mathcal{E}_{\geq t}$; (entanglement!). This observation opens our eyes/minds towards a clear notion of what might be meant by *"events"* and to a theory of direct/projective observations and recordings of "events".

To render the above *definition* more precise, we say that a *"potential future event"* is given by a family, $\{\pi_\xi | \xi \in \mathcal{X}\}$, of disjoint orthogonal projections contained in an algebra $\mathcal{E}_{\geq t}$, for some $t \geq t_0$, (t_0 = time of "present"), with $\sum_{\xi \in \mathcal{X}} \pi_\xi = \mathbf{1}$.

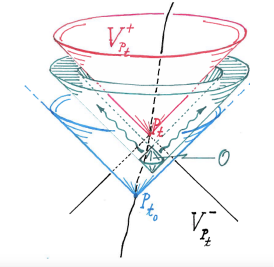
In accordance with the "Copenhagen interpretation" of QM, it appears natural to say that a potential future event $\{\pi_\xi | \xi \in \mathcal{X}\} \subset \mathcal{E}_{\geq t}$ *actually happens* in the interval $[t, \infty)$ of times iff

$$\boxed{\omega_t(A) = \sum_{\xi \in \mathcal{X}} \omega(\pi_\xi \, A \, \pi_\xi), \quad \forall A \in \mathcal{E}_{\geq t} \, ,} \tag{5}$$

i.e., no off-diagonal elements appear on the R.S. of (5) $\rightarrow \omega_t$ is an incoherent superposition of states in the images of the projections π_ξ!

Implications of *PDP* and of entanglement

In RQFT, *PDP* is a consequence of *Huygens' Principle* (Buchholz) and can be understood from the following drawing (see blackboard):



PDP $\rightarrow$ Theory of q.m. Measurements:
Suppose that a physical quantity $\hat{X}$ is measured at time t or later. An operator $X(t) = \sum_\xi \xi \, \Pi_\xi(t) \in \mathcal{E}_{\geq t}$ then represents $\hat{X}$ at times $\geq t$.
Following *"Copenhagen"*, one would argue that, $\forall A \in \mathcal{E}_{\geq t}$:

$$\omega_t(A) = \sum_{\xi \in spec(\hat{X})} \omega_t(\Pi_\xi(t) \, A \, \Pi_\xi(t)) \tag{6}$$

$$\Rightarrow \quad \omega_t([\Pi_\xi(t), A]) = 0, \forall \xi, \quad \text{hence} \quad \omega_t([X(t), A]) = 0\,. \tag{7}$$

The centralizer of a state and its center

Thus, ω_t is an *incoherent superposition* of eigenstates of $X(t)$, and $X(t)$ belongs to what is called the *"centralizer" of the state* ω_t.

Next, we render the meaning of Eq. (5) more precise.

Let $\mathcal{M}$ be a von Neumann algebra, and let ω be a state on $\mathcal{M}$. Given an operator $X \in \mathcal{M}$, we set

$$ad_X(\omega)(A) := \omega([A, X]), \ \forall A \in \mathcal{M}.$$

We define the *centralizer* of a state ω on $\mathcal{M}$ by

$$\mathcal{C}_\omega(\mathcal{M}) := \{X \in \mathcal{M} | ad_X(\omega) = 0\}$$

Note that ω is a normalized trace on $\mathcal{C}_\omega(\mathcal{M})$ ... ! The *center*, $\mathcal{Z}_\omega(\mathcal{M})$, of $\mathcal{C}_\omega(\mathcal{M})$ is defined by

$$\mathcal{Z}_\omega(\mathcal{M}) := \{X \in \mathcal{C}_\omega(\mathcal{M}) | \, [X, A] = 0, \ \forall A \in \mathcal{C}_\omega(\mathcal{M})\}. \tag{8}$$

We are now prepared to introduce a notion of *(actual)* *"events"*.

Events happening around time t

Let S be an isolated open physical system.

Definition: *If ω_t is the state of S on the algebra $\mathcal{E}_{\geq t}$, an "event" is happening at time t iff $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ contains at least two non-zero orth. projections, $\pi^{(1)}, \pi^{(2)}$, which are disjoint, i.e., $\pi^{(1)} \cdot \pi^{(2)} = 0$, and*

$$0 < \omega_t(\pi^{(i)}) < 1\,, \qquad \text{for } \; i = 1, 2\,. \qquad \square$$

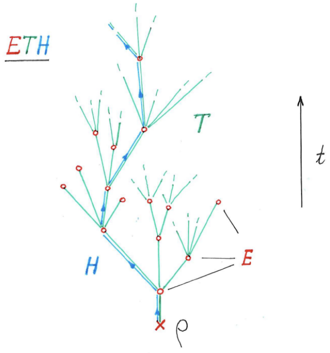
For simplicity suppose that $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ is generated by a family of disjoint orthogonal projections $\{\pi_\xi | \xi \in \mathcal{X}_{\omega_t}\}$, with $\mathcal{X}_{\omega_t} = \text{spec}[\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})]$ a countable set.

"Axiom": *(2^{nd} Law of QMTh) If card$(\mathcal{X}_{\omega_t}) \geq 2$ and $\omega_t(\pi_\xi) \neq 0$, for at least two different points $\xi \in \mathcal{X}_{\omega_t}$, then the state ω_t must be replaced by one of the states $\omega_{t,\xi} := \omega_t(\pi_\xi)^{-1} \cdot \omega_t(\pi_\xi(\cdot)\pi_\xi)\,,$ for some $\xi \in \mathcal{X}_{\omega_t}$ with $\omega_t(\pi_\xi) \neq 0$. The probability, prob$_t(\xi)$, for the state $\omega_{t,\xi}$ to be selected as the state of S right after time t when the event has happened is given by*

$$prob_t(\xi) = \omega_t(\pi_\xi) \quad - \quad Born's Rule \qquad \square \qquad (9)$$

Metaphoric picture of time evolution of states in QM

In the "*ETH* approach", the time-evolution of *states* of a phys. system S is apparently described by a *stochastic branching process*, with branching rules as determined by the above *"Axiom"*, Eq. (9); ($\neq$ "decoherence mumbo-jumbo"!) The following figure illustrates this claim:



t: time, ρ: initial state of S

E: "*E*vents", T: "*T*rees (of possible states of S), H: "*H*istories" (of events/states) – probs. of "*H*istories" deteremined by *Born's Rule*

"I leave to several futures my garden of forking paths"

Note that, in an *autonomous isolated open system* S, all the algebras $(\mathcal{E}_{\geq t})_{t\in\mathbb{R}}$ are *isomorphic* to one another, with $\mathcal{E}_{\geq t} \simeq \mathcal{N}$, where $\mathcal{N}$ is a certain *"universal algebra"*. We define

$$\mathcal{X}_S := \bigcup_{\omega \in \text{phys states of } S} \mathcal{Z}_\omega(\mathcal{N}),$$

the *"non-commutative spectrum"* of the system S.

The above picture "shows" that a *continuous-time stochastic branching process*, with state space $\mathcal{X}_S$ and transition probabilities given by *Born's Rule*, describes the *time evolution of states* of S. (The Schrödinger equation can *only* be used to describe the dynamics of states of (very boring) closed systems *not* featuring any *events*!)

A trajectory of states described by such a process is called a *"history"*. The space of histories of a system S is equipped with probability measures, μ_ω, where ω is an "initial state" on the algebra $\mathcal{E}$ which S has been prepared in; (see blackboard). A measure μ_ω assigns a probability to every history of events that may evolve from the initial state ω. (Naive implementation: Lindblad evolution of density matrices)

5. Detection of events

We have characterised an *isolated open system* S in terms of a filtration of algebras

$$\{\mathcal{E}_{\geq t}\}_{t \in \mathbb{R}} \,,$$

with (see *PDP*)

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'} \,, \qquad \text{whenever} \;\; t' > t} \tag{10}$$

The flow of time in S, (i.e., the time evolution of S in the Heisenberg picture) is encoded in the *proper* embeddings (10), which, in an *autonomous* system S, are completely determined by its *Hamiltonian*.

However, the characterisation of S given in (10) is incomplete! To retrieve physical information from (10) and from our definition of *events*, we must specify operators representing *"phys. quantities"* characteristic of S and – when observed/measured – may *signal the occurrence of events*. Let

$$\mathcal{O}_S := \{\hat{X}_\iota | \iota \in \mathcal{I}_S\} \tag{11}$$

be a list/set of abstract linear operators representing physical quantities characteristic of S; (usually, $\mathcal{O}_S$ isn't a linear space, let alone an algebra).

Measurements of physical quantities

For any operator $\hat{Y} \in \mathcal{O}_S$ and any time t, we specify a concrete self-adjoint operator $Y(t) \in \mathcal{E}_{\geq t}$ representing $\hat{Y}$ at time t; (i.e., $\exists$ a repr. of $\mathcal{O}_S$ by operators on $\mathcal{H}_S$, $\forall t \in \mathbb{R}$). For an *autonomous* system S, the operators $Y(t)$ and $Y(t')$ are conjugated to one another by the *propagator* of S .

Suppose that, at some time t, an *event* happens; i.e., $\exists$ a partition of unity, $\{\pi_\xi | \xi \in \mathcal{X}_{\omega_t}\} \subseteq \mathcal{Z}_{\omega_t} \subset \mathcal{E}_{\geq t}$, by disjoint (commuting) orthogonal projections, as above, containing ≥ 2 elements with positive probability of occurrence representing *possible events* (one of which *actually* <u>*happens*</u>). Let $\hat{Y} \in \mathcal{O}_S$, and let $Y(t) = \sum_{\eta \in spec(\hat{X})} \eta \, \Pi_\eta(t)$ (spectral dec. of Y(t)) be the operator epresenting $\hat{Y}$ at time t. If the "distance"[3]

$$\mathrm{dist}\big(\Pi_\eta(t), \langle \pi_\xi | \xi \in \mathcal{X}_{\omega_t} \rangle\big) \text{ is "very small"} \;\; , \forall \eta \in spec(\hat{Y})\,, \qquad (12)$$

then we say that the *physical quantity* $\hat{Y} \in \mathcal{O}_S$ is *observed/measured* at time t or later, because the state of S just after time t is then an approximate eigenstate of $Y(t)$. The measurement of $\hat{Y}$ is a *signal of an event happening* at time t. ...

[3]defined in terms of "conditional expectations"

6. Summary and conclusions

1. So far, the "*ETH* Approach to QM" is merely a conceptual framework. It is not effective (yet?) when one wants to do concrete q.m. computations. However, it clarifies the *Foundations* of QM and it hopefully helps to dispel those persistent confusions that surround them. It leads to plenty of *very concrete* and interesting-looking problems in functional analysis and *probability theory* (!).

2. I have spent quite a lot of time trying to develop a *relativistic version* of the "*ETH* Approach".
   In fact, if we think that, fundamentally, isolated systems should be *autonomous* we cannot avoid turning towards a *relativistic formulation* of QM. So far, things look good! But I cannot claim to have worked out all the details, yet. Tentative conclusions:

   - Massless modes (*electromagnetic-* and *gravitational field*) play a fundamental role in a quantum theory of *"events"* and of direct (projective) observations/measurements – just like in *Relativity Theory*!)
   - *Huygens' Principle* (as formulated by Buchholz), and hence the *even-dimensionality* of space-time are crucial ingredients.

Theory of Indirect Measurements

3. The *statistics of long sequences of events* observed in an isolated open system S tends to reveal important information about *physical quantities not accessible to direct observation*. This is the basis of a *"Theory of Indirect (Weak) Measurements"* (pioneered by Kraus, Elizur and Vaidman, Maassen and Kümmerer, and others). This theory rests on a clever exploitation of "Entanglement", combined with statistics. It leads to an understanding of phenomena such as the *emission of photons from atoms, radioactive decay of nuclei,* or the *emergence of particle tracks in a cloud chamber, ...* Much recent work!
   Deeper problems are, however, encountered when one studies what I have talked about in this lecture: The *Quantum Theory of Events* and of direct (projective) measurements of physical quantities; and the embedding of Quantum Theory into Relativity Theory.

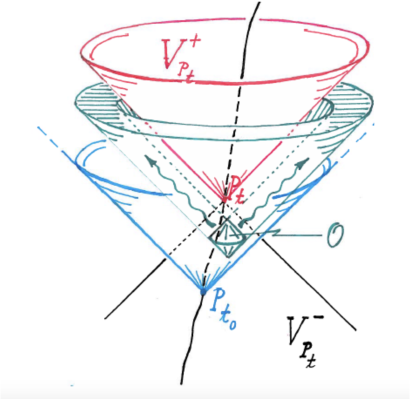
I hope I have convinced you that there are reasons to be optimistic about being able to make progress in this direction.

Thank you for your attention!

7. Appendix about relativistic quantum theory

What will a relativistic formulation of quantum theory tell us about space-time?

First I assume that space-time is Minkowski space, $\mathbb{M}^4$, and, immodestly, that my own proper time is the time of the Universe.



Worldline of JF $\uparrow$

A Theorem of D. Buchholz

Theorem

In an RQFT with massless particles, such as photons, the algebra, $\mathcal{E}_{\geq P_t}$, of all physical quantities ("observables") potentially measurable in the future of the space-time point P_t is of type III_1, and $\mathcal{E}'_{\geq P_t} \cap \mathcal{E}_{\geq P_{t_0}}$ is of type III_1, too, for arbitrary times $t_0 < t$. □

This result is a consequence of "Huygens' Principle" (in the jargon of Buchholz): Photons from the region $\mathcal{O}$ will asymptotically escape along lightcones in the future, $V^+_{P_{t_0}}$, of P_{t_0} but below $V^+_{P_t}$. We cannot catch up with them, anymore, if we have missed them just after they have been emitted. Thus, the *"Principle of Shrinking Potentialities"* (PSP) holds in the form proposed in Eq. (3) of the last Section:

$$\mathcal{E}_{\geq P_{t_0}} \underset{\neq}{\supset} \mathcal{E}_{\geq P_t}, \qquad \text{for } \; t > t_0\,, \tag{8}$$

and we could now follow the arguments outlined in Sect. 5. However, I don't like to be in the center of the Universe; so, let's take JF out of the picture! Before knowing better I propose a formulation of *relativistic local Quantum Theory* with roughly the following features:

A tentative formulation of relativistic local quantum theory

Let $\mathcal{M}$ be some (Hausdorff) topological space. We consider a *fibre bundle*, ${}^{qm}\mathcal{F}$, with base space given by $\mathcal{M}$ and fibre above a point $P \in \mathcal{M}$ given by an ∞-dimensional *-algebra $\mathcal{E}_{\geq P}$ (whose weak closure, also denoted by $\mathcal{E}_{\geq P}$, in any representation determined by a "physical state" can be expected to be of type III_1). All the algebras $\{\mathcal{E}_{\geq P}\}_{P \in \mathcal{M}}$ are assumed to be *isomorphic* to one another.[4]

Definition:

We say that a point $P_0 \in \mathcal{M}$ is *in the past* of a point $P \in \mathcal{M}$, written as $P_0 \prec P$, iff $\exists$ an *injection map* $\iota : \mathcal{E}_{\geq P} \hookrightarrow \mathcal{E}_{\geq P_0}$, (enabling one to identify $\mathcal{E}_{\geq P}$ with a subalgebra of $\mathcal{E}_{\geq P_0}$), and

$$\big(\iota(\mathcal{E}_{\geq P})\big)' \cap \mathcal{E}_{\geq P_0}$$

is inifinite-dimensional. The relation $\prec$ introduces a *partial order* on $\mathcal{M}$. If $P_0 \not\prec P$ and $P \not\prec P_0$ then we say that P_0 and P are *space-like separated*, written as $P_0 \times P$. The relations "$\prec$" and "X" determine a *"causal structure"* on $\mathcal{M}$.

[4] This could be generalized by introducing sheaves of algebras

What are *"events"*?

Let ω be a state on the algebra

$$\mathcal{E}_{\geq \Sigma} := \bigvee_{P' \in \Sigma} \mathcal{E}_{\geq P'} \,,$$

where Σ is a space-like hypersurface contained in $\mathcal{M}$ containing a point $P \in \mathcal{M}$.

Definition:
We say that an *"event"* happens in P iff the center $\mathcal{Z}_{\omega_\Sigma}(\mathcal{E}_{\geq P}) \equiv \mathcal{Z}^P_{\omega_\Sigma}$ of the centralizer, $\mathcal{C}_{\omega_\Sigma}(\mathcal{E}_{\geq P})$, is *non-trivial* and contains at least *two* projections, Π^P_1 and Π^P_2 with the property that

$$0 < \omega_\Sigma(\Pi^P_i) < 1, \qquad \text{for } \; 1 = 1, 2\,.$$

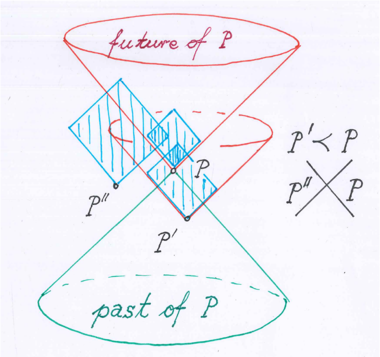
Let $\mathcal{X}^P_{\omega_\Sigma}$ denote the spectrum of $\mathcal{Z}^P_{\omega_\Sigma}$.

"Axiom" (compatibility – locality): If two points, P and P'', of $\mathcal{M}$ are *space-like separated*, and "events", Π^P_ξ and $\Pi^{P''}_\eta$, actually happen in P and P'' then

$$[\Pi^P_\xi, \Pi^{P''}_\eta] = 0, \quad \forall\, \xi \in \mathcal{X}^P_{\omega_\Sigma} \text{ and all } \eta \in \mathcal{X}^{P''}_{\omega_\Sigma}\,. \tag{9}$$

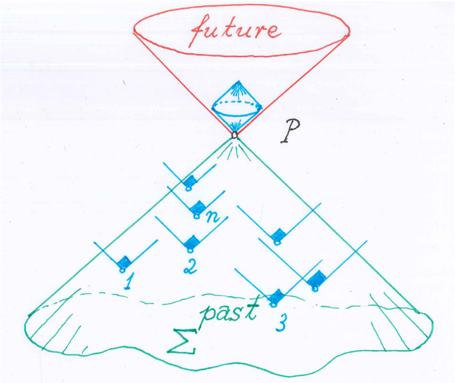
The compatibility axiom

The above axiom is illustrated in the following figure.



However, projections describing events happening in P' and P do *not* commute in general, since P' is in the past of P.
Next, we propose to describe histories of events. We choose a space-like surface Σ in $\mathcal{M}$ with the property that some bounded subset of Σ lies in the past of a point $P \in \mathcal{M}$, as shown in the following figure.

Histories of events



We suppose that a state ω_Σ is prescribed in the past of a space-like surface Σ (choice of initial conditions). Our task is to find out whether all events in the *past* of P but in the future of Σ (so-called "histories"), together with the state ω_Σ, uniquely determine a state, ω_P, on the algebra $\mathcal{E}_{\geq P}$ and, given ω_P, to find out whether an event happens at P.

Probabilities of histories of events

For this purpose, we *assume inductively* that all such events are known: Let $P_1, P_2, \ldots$, be all points in the past of P but *not* in the past of *any* point on Σ with the property that, given initial conditions corresp. to ω_Σ, an event has happened at $P_i, i = 1, 2, \ldots$. With any of these points we can then associate an orthogonal projection $\Pi_{\xi_i}^{P_i}$, $\xi_i \in \mathcal{X}_{\omega_{P_i}}^{P_i} = \mathrm{spec}\big(\mathcal{Z}_{\omega_{P_i}}^{P_i}\big)$. We define *"history operators"*

$$H(P|\omega_\Sigma) := \overrightarrow{\prod_{i=1,2,\ldots}} \Pi_{\xi_i}^{P_i}, \tag{10}$$

where P_i is either in the past of P_{i+1}, or P_i and P_{i+1} are space-like, $\forall i = 1, 2, \ldots$ Thanks to the *compatibility - locality axiom* the operator $H(P|\omega_\Sigma)$ is well-defined! We then set

$$\omega_P(A) := prob\big(H(P|\omega_\Sigma)\big)^{-1} \omega_\Sigma(H(P|\omega_\Sigma)\, A\, H(P|\omega_\Sigma)^*), \tag{11}$$

$\forall A \in \mathcal{E}_{\geq P}$, where

Events and "geometrical structure"

$$\boxed{prob\big(H(P|\omega_\Sigma)\big) := \omega_\Sigma\big(H(P|\omega_\Sigma)\cdot H(P|\omega_\Sigma)^*\big)}$$

Generalized Born Rule

We are now able to answer the question whether an event happens in the space-time point P:
An event happens in P iff the center $\mathcal{Z}_{\omega_P}(\mathcal{E}_{\geq p})$ of the centraliser of ω_P is non-trivial and contains ≥ 2 disjoint orthogonal projections with strictly positive probabilities in ω_P, as predicted by Born's Rule.
This completes the induction step.

The *"compatibility – locality axiom"* can be expected to yield non-trivial constraints on the geometry in the vicinity of two space-like separated points, P and P'', if it is known that $\exists$ events in P and P'' localised in explicitly known regions in the future of P and of P'', respectively. But this will have to be investigated more thoroughly in the future.

Summary and conclusions

- As in the genesis of Special Relativity, the e.m. field, as well as Huygens' Principle play key roles in the genesis of a Quantum Theory solving the "measurements problem" – *which may not have been properly appreciated, so far.*
- As in the genesis of General Relativity, the causal structure of space-time and its curvature play key roles in Quantum Theory:
- The non-commutative nature of Quantum Theory and the "compatibility-locality axiom" determine a *"causal structure"* (but *not* a Lorentzian metric) on $\mathcal{M}$.
- Thanks to the *"Principle of Shrinking Potentialities" (PSP)* and the natural presence of an *"arrow of time"* in the *"ETH approach"* to Quantum Theory, the *"Information Paradox"* and the *"Unitarity Paradox"* appear to dissolve.

I thanks you for your attention !