New Perspectives on Lepton Flavor Violation

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The Standard Model

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i FD\psi + h.c. + \lambda_i y_{ij} h_j \phi^{\dagger} \phi + h.c. + m_\phi^2 \phi^2 - V(\phi) \]
Symmetries of the Standard Model

- Rephasing lepton and quark fields:
  \[ U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} . \]

- **B+L broken** non-perturbatively,
  \[ \Delta B = 3 \quad \wedge \quad \Delta L_e = \Delta L_\mu = \Delta L_\tau = 1 , \]
  but unobservably suppressed at low temperatures.  \cite{tHooft '76}

- Real global symmetry of SM:
  \[ U(1)_{B-L} \times U(1)_{L_\mu-L_\tau} \times U(1)_{L_\mu+L_\tau-2L_e} . \]

- Can even promote to gauge symmetry by adding 3 \( N_R \).
  \cite{Araki, Heeck, Kubo, 1203.4951}
Neutrino oscillations

- Observations of $\nu_\alpha \to \nu_\beta$ prove that $M_\nu \neq 0$ and
  
  \[ U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e} \]
  
  is broken!

- $B - L$ could still be conserved if neutrinos are Dirac.

[Heeck, 1408.6845]

Lepton flavor definitely violated, so where is it?
Neutrino mass $\Rightarrow$ charged LFV?

- **SM + Dirac neutrinos:** *all* LFV is GIM suppressed:

\[
\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu_\alpha \bar{\nu}_\beta)} \simeq \frac{3\alpha_{EM}}{32\pi} \left| \sum_{j=2,3} U_{\alpha j} \frac{\Delta m^2_{j1}}{M^2_W} U^\dagger_{j \beta} \right|^2 < 5 \times 10^{-53}.
\]

[Petcov '77; Cheng & Li '77]

- **SM + heavy seesaw neutrinos:**

\[
\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu_\alpha \bar{\nu}_\beta)} \simeq \frac{3\alpha_{EM}}{8\pi} \left| (m_D M_R^{-2} m_D^\dagger)_{\alpha \beta} \right|^2.
\]

[Cheng & Li '80]

Not true with fine-tuning or structure in $m_D$. 

\[M^2_\nu / M^2_R\]
Neutrino mass ⇔ charged LFV!

- Neutrino-mass induced charged LFV is unobservable.

\[
\text{Observation of CLFV} \rightarrow \text{beyond SM \and beyond } M_\nu! 
\]

- (Only exception: $0\nu\beta\beta$ can probe LFV ($\Delta L_e = 2$) via $M_\nu$.)
- arXiv: many $\nu$-mass models can actually give large LFV:
  - Low-scale/inverse/linear seesaw;
  - SUSY seesaw;
  - Radiative seesaw (Zee-Babu, Ma,…);
- $M_\nu \leftrightarrow$ LFV connection possible but not necessary.
Approximate symmetries

- Flavor still *approximate* symmetry in *charged* lepton sector.
- Unavoidably broken by $M_\nu$, but this is unobservable.

Search for CLFV to learn more about flavor!

- Assuming *heavy new physics*, the best channels are
  
  \[ \ell \to \ell' \gamma, \ \ell \to \ell'' \ell''', \ \mu \to e \text{ conv.}, \ h \to \ell \ell', \ \text{had} \to \ell \ell', \ldots \]

- Organize operators/processes by quantum numbers under
  
  \[ U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e} \cdot \]
\[ \Delta(L_\mu - L_\tau) \]

- \( \bar{e}e \rightarrow \bar{\tau}\tau\tau\mu \)
- \( \bar{\mu}\mu \rightarrow \bar{\tau}\tau \)
- \( \tau \rightarrow \mu\mu\bar{\mu}\bar{\mu} \)
- \( \bar{\mu}\bar{e} \rightarrow \bar{\tau}\tau \)
- \( \bar{\tau} \rightarrow \bar{\tau}\tau \)
- \( \tau \rightarrow \mu\gamma \)
- \( \mu \rightarrow e\gamma \)
- \( \bar{\tau} \rightarrow \bar{e}\gamma \)
- \( \bar{\mu} \rightarrow \bar{e}\gamma \)
- \( e\bar{e} \rightarrow \tau\tau \)
- \( \bar{\tau} \rightarrow \bar{e}e\mu \)
- \( e\bar{e} \rightarrow \tau\tau \)
- \( \tau \rightarrow e\bar{e}\bar{\mu} \)
- \( \tau \rightarrow e\gamma \)
- \( \mu \rightarrow e\gamma \)
- \( \bar{\tau} \rightarrow \bar{\mu}\gamma \)
- \( \bar{\mu} \rightarrow \bar{\mu}\gamma \)
- \( \bar{\mu}\mu \rightarrow \tau\tau \)
- \( \mu\mu \rightarrow \tau\tau \)
- \( \mu\mu \rightarrow \mu\mu\bar{\mu}\bar{\mu} \)
- \( \tau \rightarrow \mu\mu\bar{\mu}\bar{\mu} \)
- \( \bar{\tau} \rightarrow \bar{\mu}\mu\bar{\mu}\bar{\mu} \)

\[ \Delta(L_\mu + L_\tau - 2L_e) \]
Stands for all $\Delta(L_e - L_\mu) = 2$ processes:

<table>
<thead>
<tr>
<th>group</th>
<th>process</th>
<th>current</th>
<th>future</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\mu \rightarrow e\gamma$</td>
<td>$4.2 \times 10^{-13}$</td>
<td>$4 \times 10^{-14}$</td>
</tr>
<tr>
<td>2</td>
<td>$\mu \rightarrow eee$</td>
<td>$1.0 \times 10^{-12}$</td>
<td>$10^{-16}$</td>
</tr>
<tr>
<td>2</td>
<td>$\mu \rightarrow e$ conv.</td>
<td>$\mathcal{O}(10^{-12})$</td>
<td>$10^{-17}$</td>
</tr>
<tr>
<td>2</td>
<td>$h \rightarrow e\mu$</td>
<td>$3.5 \times 10^{-4}$</td>
<td>$2 \times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>$Z \rightarrow e\mu$</td>
<td>$7.5 \times 10^{-7}$</td>
<td>$-$</td>
</tr>
<tr>
<td>2</td>
<td>had $\rightarrow e\mu$ (had)</td>
<td>$4.7 \times 10^{-12}$</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>
Currently being probed.
If you see $\tau \rightarrow \mu \gamma$: still $U(1)(L_\mu + L_\tau - 2L_e)$ symmetry.

$\tau \rightarrow \mu \gamma$, $\tau \rightarrow \mu^- e^- e^+$, $h \rightarrow \mu \tau$, ... with model-dependent rates!
If you see $\tau \to \mu \gamma$ and $\tau \to ee\bar{\mu}$: still $Z_2(e \to -e)$. 

[Heeck, 1610.07623]
# Interpretation of LFV

<table>
<thead>
<tr>
<th>Observation of charged lepton flavor violation</th>
<th>⇒</th>
<th>Remaining symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(L_\alpha - L_\beta) = 2$</td>
<td></td>
<td>$U(1)<em>{L</em>\alpha + L_\beta - 2L_\gamma}$</td>
</tr>
<tr>
<td>$\Delta(L_\alpha + L_\beta - 2L_\gamma) = 6$</td>
<td></td>
<td>$U(1)<em>{L</em>\alpha - L_\beta}$</td>
</tr>
<tr>
<td>$\Delta(L_\alpha + L_\beta - 2L_\gamma) = 6$ and $\Delta(L_\alpha - L_\beta) = 2$</td>
<td></td>
<td>$\mathbb{Z}<em>2$: $\ell</em>\gamma \rightarrow -\ell_\gamma$</td>
</tr>
<tr>
<td>$\Delta(L_\alpha + L_\beta - 2L_\gamma) = 6$ and $\Delta(L_\alpha + L_\gamma - 2L_\beta) = 6$</td>
<td></td>
<td>$\mathbb{Z}<em>3$: $(\ell</em>\alpha, \ell_\beta, \ell_\gamma) \sim (0, 1, 2)$</td>
</tr>
<tr>
<td>$\Delta(L_\alpha - L_\beta) = 2$ and $\Delta(L_\alpha - L_\gamma) = 2$</td>
<td></td>
<td>$-$</td>
</tr>
<tr>
<td>$\Delta(L_\alpha - L_\beta) = 2$ and $\Delta(L_\alpha + L_\gamma - 2L_\beta) = 6$</td>
<td></td>
<td>$-$</td>
</tr>
</tbody>
</table>

- **At least** two orthogonal channels required for **full** LFV.
- Flavor violation by higher units more challenging.
- Easy to build models that single out certain channels, e.g. $\tau^- \rightarrow \mu^-\gamma$ or $\tau^- \rightarrow e^-e^-\mu^+$. 
Example: $\tau^- \rightarrow e^-e^-\mu^+$

- Conserves $L_\mu - L_\tau$, so impose this symmetry.
- Simplest UV model: add SU(2)$_L$ singlet $k^{++}$:

$$\mathcal{L} \supset (g_{\mu\tau} \mu_R^c \tau_R + g_{ee} \bar{e}_R^c e_R) k^{++} + \text{h.c.}$$

- $\tau^- \rightarrow e^-e^-\mu^+$ allowed, everything else forbidden.
- Add $N_R$ and singlet scalars to break $L_\mu - L_\tau$ in $M_R$.
- Could even use symmetry for texture zeroes in $M_\nu$.

[Araki, Heeck, Kubo, 1203.4951]

ν oscillations but approximate symmetry in $\ell^-$ sector.
\[ \tau^- \rightarrow \mu^- \mu^- \mu^- e^+ e^+ ? \]

- Conserves \( L_\mu + 4 L_e - 5 L_\tau \), impose to kill other modes.
- Not difficult to build, but rate is
  \[ \Gamma \propto \langle H \rangle^2 \frac{m_{\tau}^{11}}{m_S^{12}} . \]
- Secretly dimension 10 operator.
- Would need new particles at 10 GeV for observable rate!
  Only possible for neutral fields, otherwise \( Z \rightarrow SS \).

Not pretty...
Baryon number violation
Baryon number violation

- So far assumed $\Delta B = 0$, but can also do LFV with $\Delta B \neq 0$.
- Example: proton decay ($\Delta B = 1$).
- Super-K limits on $p \rightarrow e^+\pi^0, \mu^+\pi^0$ are $10^{34}$ yrs!
Baryon number violation

- So far assumed $\Delta B = 0$, but can also do LFV with $\Delta B \neq 0$.
- Example: proton decay ($\Delta B = 1$).
- Super-K limits on $p \rightarrow e^+\pi^0, \mu^+\pi^0$ are $10^{34}$ yrs!
- More interesting for flavor: $p \rightarrow \ell\ell\ell$:

<table>
<thead>
<tr>
<th>channel</th>
<th>$(\Delta L_e, \Delta L_\mu)$</th>
<th>limit/years</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow e^+e^+e^-$</td>
<td>(1, 0)</td>
<td>$793 \times 10^{30}$</td>
</tr>
<tr>
<td>$p \rightarrow e^+\mu^+\mu^-$</td>
<td>(1, 0)</td>
<td>$359 \times 10^{30}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+e^+e^-$</td>
<td>(0, 1)</td>
<td>$529 \times 10^{30}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+\mu^+\mu^-$</td>
<td>(0, 1)</td>
<td>$675 \times 10^{30}$</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+\mu^+e^-$</td>
<td>$(-1, 2)$</td>
<td>$359 \times 10^{30}$</td>
</tr>
<tr>
<td>$p \rightarrow e^+e^+\mu^-$</td>
<td>$(2, -1)$</td>
<td>$529 \times 10^{30}$</td>
</tr>
</tbody>
</table>

IMB ‘99; SK can improve by $\sim 30$!

Different flavor from $p \rightarrow \ell^+\pi^0$!
Effective operators

[Weinberg, '79 & '80]

Different symmetry properties
Effective operators

- $\Delta B = 1$ proton decay operators:
  - $\text{QQQL}: \quad d=6, \quad \Delta L = 1, \quad \text{e.g.} \ p \to e^+ \pi^0$.
  - $\text{QQLHd}: \quad d=7, \quad \Delta L = -1, \quad \text{e.g.} \ p \to e^- \pi^+ K^+$.
  - $\text{QQLLH}\ell: \quad d=10, \quad \Delta L = 1, \quad \text{e.g.} \ p \to e^+ e^- e^+.$
  - $\text{dddLLLH}: \quad d=10, \quad \Delta L = -3, \quad \text{e.g.} \ p \to e^- \nu \nu \pi^+ \pi^+.$
  - $\text{QudLLLHH}: \quad d=11, \quad \Delta L = 3, \quad \text{e.g.} \ p \to e^+ \nu \nu.$
Effective operators

Impose B+L
Effective operators

- $\Delta B = 1$ proton decay operators:

  - $\text{QQQL}: \quad d=6, \quad \Delta L = 1, \quad \text{e.g.} \ p \to e^+ \pi^0$.

  - $\text{QQLHd}: \quad d=7, \quad \Delta L = -1, \quad \text{e.g.} \ p \to e^- \pi^+ K^+$.

  - $\text{QQQLLH}: \quad d=10, \quad \Delta L = 1, \quad \text{e.g.} \ p \to e^+ e^- e^+ e^-$.

  - $\text{dddLLLH}: \quad d=10, \quad \Delta L = -3, \quad \text{e.g.} \ p \to e^+ e^- \pi^+ \pi^-$.

  - $\text{QudLLLLHH}: \quad d=11, \quad \Delta L = 3, \quad \text{e.g.} \ p \to e^+ e^- \nu \bar{\nu}$.

  

  Imposing $B+L$
Effective operators

Impose B+3L
**Effective operators**

- **$\Delta B = 1$ proton decay operators:**
  - **QQQL:** $d=6$, $\Delta L = 1$, e.g. $p \rightarrow e^+ \pi^0$
  - **QQLHd:** $d=7$, $\Delta L = -1$, e.g. $p \rightarrow e^- \pi^+ K^+$
  - **QQLLLH:** $d=10$, $\Delta L = 1$, e.g. $p \rightarrow e^+ e^- e^+$
  - **dddLLLH:** $d=10$, $\Delta L = -3$, e.g. $p \rightarrow e^- \nu \nu \pi^+ \pi^+$
  - **QuddLLLHH:** $d=11$, $\Delta L = 3$, e.g. $p \rightarrow e^+ \nu \nu$

**Impose $B+3L$**
Effective operators

Impose B-L
Effective operators

• $\Delta B = 1$ proton decay operators:
  - QQQL: $d=6$, $\Delta L = 1$, e.g. $p \to e^+ \pi^0$.
  - QQHd: $d=7$, $\Delta L = -1$, e.g. $p \to e^- \pi^+ K^+$.
  - QQQLHl: $d=10$, $\Delta L = 1$, e.g. $p \to e^+e^-e^+e^-$.
  - ddddLLLH: $d=10$, $\Delta L = -3$, e.g. $p \to e^+e^-\pi^+\pi^-$.
  - QudLLLHH: $d=11$, $\Delta L = 3$, e.g. $p \to e^+\nu\nu$.

Impose $B-L$
Effective operators

Impose $L_e + 2L_\mu - 3L_\tau$
Effective operators

• $\Delta B = 1$ proton decay operators:
  
  - $QQQQL$: $d=6$, $\Delta L = 1$, e.g. $p \to e^+ \pi^0$.
  - $QQLLHd$: $d=7$, $\Delta L = -1$, e.g. $p \to e^- \pi^+ K^+$.
  - $QQQLLHl$: $d=10$, $\Delta L = 1$, e.g. $p \to e^+ e^+ \mu^-$.
  - $dddLLLH$: $d=10$, $\Delta L = -3$, e.g. $p \to e^- \nu_e \bar{\nu}_\mu \pi^+ \pi^+.$
  - $QudLLLHH$: $d=11$, $\Delta L = 3$, e.g. $p \to \mu^+ \bar{\nu}_e \bar{\nu}_e$.

Impose $L_e + 2L_\mu - 3L_\tau$
Lepton-flavored proton decay

\[ \Gamma \propto \langle H \rangle^2 \frac{m_p^{11}}{\Lambda^{12}} \sim (10^{33} \text{ yr})^{-1} (100 \text{ TeV/}\Lambda)^{12}. \]

[Hambye, Heeck, in progress]
Lepton-flavored proton decay

- The decay \( p \rightarrow e^+e^+\mu^- \) or \( p \rightarrow \mu^+\mu^+e^- \) could be dominant!
- Conserves B-L, \( L_\tau \), and \( L_e + 2L_\mu - 3L_\tau \) (or \( L_\mu + 2L_e - 3L_\tau \)).
- 35 \( d=10 \) operators of the form \( \text{QQQL}^cH\ell \).
- Rate suppressed:
  \[
  \Gamma \propto \langle H \rangle^2 \frac{m_p^{11}}{\Lambda^{12}} \sim (10^{33} \text{ yr})^{-1} (100 \text{ TeV}/\Lambda)^{12} .
  \]
- Easy channels, Super-K can probe \( 10^{34} \text{ yrs} \)!
- UV completion @ 100 TeV could show up in flavor physics.
- Other channels, e.g. \( p \rightarrow e^+\pi^0 \), suppressed by \( \nu \) mass.

[Hambye, Heeck, in progress]
\[ p \rightarrow e^+ e^+ \mu^- \]

- Conserves \( L_e + 2L_\mu - 3L_\tau \); Impose!

- Only \( d = 10 \) survives:

\[
\frac{QQQL}{\Lambda^2} + \frac{QQQLH}{\Lambda^3} + \cdots + \frac{QQQL\bar{L}He}{\Lambda^6}.
\]

- Rate very suppressed:

\[
\Gamma \propto \langle H \rangle^2 \frac{m_p^{11}}{\Lambda^{12}} \sim (10^{33} \text{ yr})^{-1} (100 \text{ TeV}/\Lambda)^{12}.
\]

- UV complete with leptoquarks. Analogous for \( p \rightarrow \mu^+ \mu^+ e^- \).

- More extreme: \( p \rightarrow e^+ e^+ e^+ \mu^- \mu^- \) etc., probes \( \Lambda \sim \) few TeV!

Low-hanging fruit for Super-K.
\[ p \rightarrow \mu^+ \mu^+ e^- \]

- Minimal leptoquark example:
  \[ \phi_1 \sim (3, 3, -2/3), \phi_2 \sim (3, 2, 7/3). \]

- \( L_\mu + 2L_e - 3L_\tau \) ensures simple structure
  \[ y_j L_\mu \phi_1 Q_j^c + f_j \bar{u}_j \phi_2 L_e + \lambda \phi_1^2 \phi_2 H. \]

- Also conserves B-L and lepton flavor, but gives lepton non-universality.

- Triplet LQ perfect for \( b \rightarrow s \mu \mu \) anomalies:

  \[ m_{\phi_1} \approx 30 \text{ TeV} \sqrt{y_2 y_3}. \]


Interesting pheno from proton decay @ 100 TeV!
\[ p \rightarrow e^+ e^+ \mu^- \]

- **R-parity violating MSSM:**
  \[
  \lambda_{ijk} L_i L_j \bar{L}_k + \lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k \\
  + \lambda'_{ijk} L_i Q_j \bar{d}_k + \mu'_i L_i H_u .
  \]

- Impose \( L_\mu + 2 L_e + L_\tau \) for \( \lambda' = 0 = \mu' \) and \( \lambda = \lambda_{132} \).

- Add soft terms for sfermion mixing: \( A_{d_{ij}} H_d \tilde{d}_i \tilde{Q}_j - m_{ij}^2 \tilde{L}_i \tilde{L}_j \).

- \( \tau^- \rightarrow e^- \gamma \) and n-n oscillations also allowed.

- Known for neutrino modes \( p \rightarrow e^+ \nu \nu, \mu^+ \nu \nu, K^+ \mu^+ e^- \nu \).  
  [Carlson, Roy, Sher, '95; Bhattacharyya, Pal, '99; Faroughy, Prabhu, Zheng, '15]

- Rich phenomenology.
Summary so far

- SM symmetry: $G = U(1)_{B-L} \times U(1)_{L_\mu-L_\tau} \times U(1)_{L_\mu+L_\tau-2L_e}$.  
- Effective field theory with Majorana $\nu$: 

$$L = L_{SM} + \frac{LLHH}{\Lambda} + \sum_j \frac{O_j}{\Lambda^2} + \sum_j \frac{O'_j}{\Lambda^3} + \sum_j \frac{O''_j}{\Lambda^4} + \ldots$$

Conserves $G$  
Violates $G$  

Could conserve $G$ or subgroup  
⇒ ‘weird’ channels dominate!?
But what if new physics is light?
Simple example: majoron

- 3 singlets $N_R$ + new scalar $\sigma = (f + \sigma^0 + iJ)/\sqrt{2}$.

Break $U(1)_{B-L}$ spontaneously: $\mathcal{L} = -\bar{y}yHN_R - \frac{1}{2}N_R^c \lambda \sigma N_R + \text{h.c.}$

For $M_R \gg m_D$: $M_\nu \simeq -m_D M_R^{-1} m_D^T$

$\simeq 1\text{eV} \left( \frac{m_D}{100\text{GeV}} \right)^2 \left( \frac{10^{13}\text{GeV}}{M_R} \right)$.

[Chikashige, Mohapatra, Peccei, '81; Schechter, Valle, '82]
Majoron couplings

- Tree level coupling only to neutrinos:

\[ \frac{iJ}{2f} \bar{\nu}_\alpha \gamma_5 (m_D^* M_R^{-1} m_D^\dagger)_{\alpha\beta} \nu_\beta = - \frac{iJ}{2f} \sum_k \bar{\nu}_k \gamma_5 m_k \nu_k \]

- One loop:

\[
\frac{iJ}{f} \bar{f} \gamma_5 f \frac{m_f T_3^f}{8\pi^2 v^2} \text{tr} \left( m_D m_D^\dagger \right) \]

\[
\frac{iJ}{f} \ell \alpha \left( \frac{m_\ell}{8\pi^2 v^2} P_R - \frac{m_\alpha}{8\pi^2 v^2} P_L \right) \ell_\beta \left( m_D m_D^\dagger \right)_{\alpha\beta} \]

- Two loop:

\[
\Gamma(J \rightarrow \gamma\gamma) \sim \frac{\alpha^2 \text{tr}(m_D m_D^\dagger)^2}{4096\pi^7} \frac{m_j^2}{v^4 f^2} \left| \sum_f N_c^f T_3^f Q_f^2 g \left( \frac{m_j^2}{4m_f^2} \right) \right|^2
\]

[Heeck, Camilo Garcia-Cely, 1701.07209; see also Pilaftsis '94]
Properties

- Crucial observation: the two matrices are independent!
  \[ \{m_D, M_R\} \leftrightarrow \{M_\nu, m_D m_D^\dagger\}. \]
  [Davidson, Ibarra, hep-ph/0104076]

- $J_{\bar{\ell}\ell'}$ coupling can be large and of arbitrary structure.

- Similar couplings arise for familons or flavor $Z'$.
  [Wilczek, ‘82; Reiss, ‘82; Grinstein, Preskill, Wise, 85; ...]

- Boson not necessarily massless, e.g. pseudo-Goldstone.

- Experimental signature depends on decay channel:
  \[ \ell \rightarrow \ell' J, \quad J \rightarrow \text{inv, } \ell'' \ell''', \gamma\gamma. \]
\[
\ell \rightarrow \ell' \quad \text{J with} \quad \text{J} \rightarrow \text{invisible}
\]

- Standard LFV in seesaw:
  \[
  \frac{\Gamma(\ell \rightarrow \ell' \gamma)}{\Gamma(\ell \rightarrow \ell' \nu \bar{\nu} \ell')} \simeq \frac{3\alpha}{8\pi} \left| (m_D M_R^{-2} m_D^\dagger)_{\ell \ell'} \right|^2.
  \]

- Great signature, but requires light $N_R$.

- With majoron: look for **mono-energetic** lepton:
  
  \[\text{[Pilaftsis, '94; Feng, Moroi, Murayama, Schnapka, '98; Hirsch, Vicente, Meyer, Porod, '09]}\]
  
  \[
  \frac{\Gamma(\ell \rightarrow \ell' J)}{\Gamma(\ell \rightarrow \ell' \nu \bar{\nu} \ell')} \simeq \frac{3}{16\pi^2} \frac{1}{m_\ell^2 f^2} \left| (m_D m_D^\dagger)_{\ell \ell'} \right|^2.
  \]

- If $M_R = \text{diag}(M)$:
  \[
  \frac{\Gamma(\ell \rightarrow \ell' \gamma)}{\Gamma(\ell \rightarrow \ell' J)} \simeq 2\pi \alpha \frac{m_\ell^2}{M^2} \frac{f^2}{M^2} \left\{ \begin{array}{ll}
  \gg 1 \quad \text{for} \quad M \ll f , \\
  \ll 1 \quad \text{for} \quad M \sim f \gg m_\ell .
  \end{array} \right.\]
\[ \mu \rightarrow e \ J \text{ with } J \rightarrow \text{ invisible} \]

- **TWIST, '15**: limits on different anisotropies.
- **Chiral coupling** \( \bar{\mu} P_L e J \) suppresses sensitivity!
  [Heeck, Garcia-Cely, 1701.07209]
- **Bremsstrahlung** is competitive: \( \mu \rightarrow e \ J \gamma \).
  [Goldman et al, '87]
- **Approximate limit**
  \[ \left| \frac{(m_D m_D^\dagger)_{\mu e}}{v_f} \right| \lesssim 10^{-5}. \]
Searches for \( \mu \rightarrow e X \) with Mu3e

- Full reconstruction of all Michel decays is a big challenge for data acquisition.
- \( B(\mu \rightarrow e X) \sim 10^{-8} \) at 90 \% CL.

Mu3e simulation:

- Recurring track in Mu3e
- Required full reconstruction of “recurlers”
\( \mu \to e \ X \) with \( X \to \) visible

- Take \( X \bar{e} \gamma_5 e \ \frac{m_e}{\Lambda_{ee}} \).
- Decay length determines signature.
- Displaced vertex gives new observable. [Heeck, Rodejohann, 1710.02062]
- Muon at rest:

\[
\gamma c\tau \sim \frac{\pi m_\mu \Lambda_{ee}^2}{m_e^2 m_X^2} \sim 2.5 \text{ cm} \left( \frac{\Lambda_{ee}}{100 \text{ GeV}} \right)^2 \left( \frac{10 \text{ MeV}}{m_X} \right)^2.
\]

Sub-GeV \( X \) with ee coupling allowed?
\( \mu \rightarrow e \ X \) with \( X \rightarrow \bar{e}e \)

- Decay length typically below cm. => looks prompt.

- Below beam dump: \( \Lambda_{ee} > 30 \) TeV; mostly invisible, but some DV!

\[
\text{BR}(\mu \rightarrow eX) \text{BR}(X \rightarrow ee)(1 - P(l_{\text{dec}})) \\
\sim \text{BR}(\mu \rightarrow eX) \frac{l_{\text{dec}}}{\gamma c \tau}.
\]

Possible in Mu3e!
\( \mu \to e \ X \ with \ X \to \ \gamma\gamma \)

- Decay length always below cm. \( \Rightarrow \) looks prompt.
- Below beam dump: supernova constraints!
- Prompt channel still interesting, maybe MEG(II) or Mu3e extension?

Muons difficult, taus easier.
\( \tau \rightarrow \ell \ J \) with \( J \rightarrow \) invisible

- ARGUS, ’95; 5e5 taus.
- Belle, ‘16 prelim.; 1e9 taus.
- Also interesting for LFV Z’.
  
  [Heeck, 1602.03810; Altmannshofer et al, 1607.06832]
- Improvement with Belle-II.

\[ \left| \frac{m_D m_D^\dagger}{v f} \right|_{\tau e} \lesssim 6 \times 10^{-3}, \]
\[ \left| \frac{m_D m_D^\dagger}{v f} \right|_{\tau \mu} \lesssim 10^{-3}. \]
\( \tau \rightarrow \ell J \) with \( J \rightarrow \text{invisible} \)

- ARGUS, ‘95; 5e5 taus
- Belle, ‘16 prelim.; 1e9 taus
- Also interesting for LFV \( Z' \).
  
  [Heeck, 1602.03810; Altmannshofer et al, 1607.06832]

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\[ \frac{|(m_D m_D^\dagger)_{\tau e}|}{v f} \lesssim 6 \times 10^{-3}, \]

\[ \frac{|(m_D m_D^\dagger)_{\tau \mu}|}{v f} \lesssim 10^{-3}. \]
**τ → e X with X → visible**

- Tau at rest, higher X boost.
- Arbitrary decay lengths possible.
- Similar for X → ee, μμ, μe.
- Worthwhile in LHCb and Belle (II).

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**[Recent limits: Dolan et al, 1709.00009]**

**[Heeck, Rodejohann, 1710.02062]**

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**Muons difficult, taus easier...**
Summary

- Charged LFV gives info complementary to ν oscillations.
- Not simple yes/no question, need to find out if/how
  \[ U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e} \]
is broken in \( \ell^- \) sector.
  \[ \Rightarrow \text{Need to search all possible channels!} \]
- Non-trivial breaking: \( \tau \to ee\bar{\mu}, \tau \to \mu\mu\bar{e}, p \to e\mu\mu, p \to \mu ee, \ldots \)
- Keep light new physics in mind: \( \ell \to \ell' X, X \to \text{inv, } \ell\ell, \gamma\gamma. \)
- Hope for sign in Mu3e, MEG-II, Belle-II, Mu2e, LHC,…

Still some streetlights to search under!