Conversion of Bound Muons: Lepton Flavour and Number Violation

Tanja Geib

+ Alexander Merle: *Phys. Rev. D93 (2016) 055039* → technical details on $\mu^- \rightarrow e^-$
+ Alexander Merle: *arXiv:1612.00452* → technical details on $\mu^- \rightarrow e^+$

Max Planck Institute for Physics

PSI Seminar, December 16, 2016
Today’s Agenda:

- What happens in a $\mu^− - e^+$ conversion?
- What are similarities and differences when considering $\mu^− - e^−$ and $\mu^− - e^+$ conversion?
- How to tackle $\mu^− - e^−$ conversion (using the example of a realisation via doubly charged scalars)?
- Employing the complementarity between collider and low energy physics to increase the testability $\rightarrow$ Results based on the example case
- How to tackle $\mu^− - e^+$ conversion (using the example of a realisation via doubly charged scalars)?
- Discovery potential for $\mu^− - e^+$ conversion
- Open issues $\rightarrow$ where do we need to improve in order to get reliable predictions?
- Summary and Outlook
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**μ−e Conversion**

What happens in a $\mu^- e^{\pm}$ conversion? → experimentally a two-step process

**First Step:** $\mu^-$ is captured in an ‘outer’ atomic shell, and subsequently de-excites to the 1s ground state.

**Second Step:** $\mu^-$ is captured by the nucleus and reemits an $e^{\pm}$.

→ we only consider "coherent" conversion: initial and final state nucleus are in ground state.
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**μ−e Conversion**

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![Diagram](mu-e_conversion_diagram.png)

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Energy Scales of the Process

- muon **bound** in **1s state** with binding energy
  \[ \epsilon_B \approx \frac{m_\mu}{m_e} \cdot 13.6 \text{ eV} \cdot Z \ll m_\mu \xrightarrow{Z \leq 100} \text{non-relativistic} \]

- consider "coherent" process \(\rightarrow\) initial and final nucleus in **ground state**
  + in good approximation: both nuclei at rest
  \[
  \Rightarrow E_e = m_\mu - \epsilon_B + E_i - E_f \sim \mathcal{O}(100 \text{ MeV})
  \]
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  \]

- \(\Rightarrow e^{\pm}\) is **relativistic** particle under influence of Coulomb potential:
  \[ E_e \approx E_\mu \approx m_\mu \text{ and } m_e \approx 0 \]

- for 4-momentum transfer \(q' = p_e - p_\mu\)
  In this set-up \(\Rightarrow q'^2 \approx -m_\mu^2\)
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\[ \mu^- - e^- \text{ vs } \mu^- - e^+ \text{ Conversion} \]

\[ 0\nu\beta\beta \]

\[ \mu^- - e^- \text{ conv.} \]

LNV-Alternatives:
- \[ \mu^- - \mu^+ \text{ conversion} \]
- \[ K^+ \rightarrow \pi^+ \mu^+ \mu^- \]

LFV-Alternatives:
- \[ \mu \rightarrow e + \gamma \]
- \[ \mu \rightarrow 3e \]

from

\[ \mu^- - e^+ \]

- needs to occur at two nucleons to achieve \( \Delta Q = 2 \) → similar to \( 0\nu\beta\beta \)
- around 40% of the process’ total are g.s. → g.s.

\[ \mu^- - e^- \]

- occurs at single nucleon \((\Delta Q = 0)\)
- dominated by coherent process

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Further investigations needed:
- confirm/obtain the percentage that takes place "coherently"
- derive a more involved spectrum for the positrons
μʻ− eʻ vs μʻ− eʻ+ Conversion

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- \( 0\nu\beta\beta \): Analogous EFT treatment
- \( \mu^- e^+ \) conversion
- LNV-Alternatives: \( \mu^- \mu^+ \) conversion
  - \( K^+ \rightarrow \pi^+ \mu^- \mu^- \)
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\( 0\nu\beta\beta \text{ from } TG, \text{ Merle, Zuber Phys.Lett. B764 (2017) 157} \)

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Improvements from Upcoming Experiments

Snapshot on current limits and sensitivities of upcoming experiments:

**past:** SINDRUM II for $^{48}\text{Ti}$ (1993), $^{208}\text{Pb}$ (1995), $^{197}\text{Au}$ (2006)

**future:** DeeMee for $^{28}\text{Si}$, COMET and Mu2e (taking data $\sim 2018$) for $^{27}\text{Al}$, PRISM/PRIME for $^{48}\text{Ti}$

$\rightarrow$ improvements can be transferred to $\mu^-\rightarrow e^+$ conversion

$\rightarrow$ sensitivities on both processes will increase by several orders of magnitude in the foreseeable future

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Improvements from Upcoming Experiments

Snapshot on **current limits** and **sensitivities of upcoming experiments**:

### Future sensitivity for $\mu^-\rightarrow e^+$ conversion

<table>
<thead>
<tr>
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<th>Pb-208</th>
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<th>Ti-48</th>
<th>Si-28</th>
<th>Al-27</th>
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How to tackle $\mu^– e^–$ conversion (using the example of a realisation via \textit{doubly charged scalars})?
Effective theory of a doubly charged scalar singlet
based on King, Merle, Panizzi JHEP 1411 (2014) 124

Minimal extension of SM:

- only **one** extra particle: $S^{++}$
  - lightest of possible new particles (UV completion e.g. Cocktail model)
  - reduction of input parameters
- tree-level coupling to SM (to charged right-handed leptons)
  - LNV and LFV!
- effective Dim-7 operator (necessary to generate neutrino mass)

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - V(H, S)$$
$$+ (D_\mu S)^\dagger (D^\mu S) + f_{ab} \overline{\ell_{Ra}}^c \ell_{Rb} S^{++} + \text{h.c.} - \frac{g^2 v^4 \xi}{4 A^3} S^{++} W^- W^- + \text{h.c.}$$
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\( \mu^- - e^- \) Conversion: Universally Valid for Models Involving Doubly Charged Singlet Scalars \( \text{based on TG, Merle Phys.Rev. D93 (2016) 055039} \)

\( \mu^- - e^- \) conversion realised at **one-loop** level

\( \mu^- e^- \) conversion realised at one-loop level

relevant diagrams
Different Contributions to $\mu^- e^-$ Conversion

- estimate nuclear radius: $R = r_0 A^{1/3} \sim \mathcal{O}(10^{-15} \text{ m})$
- reduced Bohr radius: $a_0 \frac{m_e}{m_\mu} \sim \mathcal{O}(10^{-13} \text{ m})$
- estimate interaction range: $r_\gamma \rightarrow \infty$ and $r_Z \leq 10^{-18} \text{ m}$
  $\Rightarrow$ for Z-exchange: $\mu^-$ has to be within nucleus! Probability?!

$\Rightarrow$ contributions need to be treated qualitatively differently!!
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- photonic contribution: "long range"
- non-photonic contribution: "short range"

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- reduced Bohr radius: $a_0 \frac{m_e}{m_\mu} \sim O(10^{-10} \text{ m})$
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\[ \begin{array}{c}
\mu^- \xrightarrow{\gamma} l^+ S^- S^- e^- \\
\mu^- \xrightarrow{Z, \gamma} q q \\
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\end{array} \]

\[ \begin{array}{c}
\text{photonic contribution:} \\
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\text{non-photonic contribution:} \\
\text{"short range"} \\
\Rightarrow \text{suppressed}
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\( \Rightarrow \) contributions need to be treated qualitatively differently!!
Photonic Contribution

\[ M \propto \int d^3 r \bar{\psi}_{jlm}^e(p_e, r) \Gamma^\nu \psi_{j\mu l\mu}^\mu(p_\mu, r) \langle N|\bar{q}\gamma_\nu q|N\rangle \]

\[ Z \rho^{(P)}(r) \delta_{\nu 0} \]

→ wave functions for $\mu^-$ and $e^-$ obtained by solving modified Dirac equation (+ Coulomb potential)

→ Most general (Lorentz-) invariant expression for $\Gamma^\nu$:

\[ \Gamma^\nu = \left( \gamma^\nu - \frac{q'^\nu}{q'^2} \right) F_1(q'^2) + \frac{i \sigma^{\nu \rho} q'^\rho}{m_\mu} F_2(q'^2) + \left( \gamma^\nu - \frac{q'^\nu}{q'^2} \right) \gamma_5 G_1(q'^2) + \frac{i \sigma^{\nu \rho} q'^\rho}{m_\mu} \gamma_5 G_2(q'^2) \]

with $q' = p_e - p_\mu$.

In non-relativistic limit:

$\Rightarrow \psi_{jlm}$ and $Z \rho^{(P)}(r)$ factorise from $\Gamma^0$ on matrix element level
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Photonic Contribution

Write **branching ratio** as product of **nuclear** and **particle physics parts**

\[
\text{BR}(\mu^- N \to e^- N) = \frac{8\alpha^5 m_\mu Z_{\text{eff}}^4 Z F_p^2}{\Gamma_{\text{capt}}} \Xi^2
\]

→ **factorisation** works perfectly for **photonic** contributions
→ \(\Xi\) has to be modified for **non-photonic** contributions to be a function of the nuclear characteristics \((A,Z)\)

**Particle physics information** absorbed into

\[
\Xi^2 = \left| - F_1(-m_\mu^2) + F_2(-m_\mu^2) \right|^2 + \left| G_1(-m_\mu^2) + G_2(-m_\mu^2) \right|^2
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see Kuno, Okada
Rev. Mod. Phys. 73 (2001) 151-202
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⇒ determine **form factors** from amputated diagrams with off-shell photon with help of Mathematica package Package–X (Patel,
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Photonic Contribution: Results

In good approximation (up to a few per cent), we use

\[
F_1(q'^2) = G_1(q'^2) = -f_{e\alpha} f_{\alpha \mu} \left[ \frac{2m_a^2 + m_\mu^2 \log (\frac{m_\mu}{M_S})}{12\pi^2 M_S^2} + \frac{\sqrt{m_\mu^2 + 4m_a^2} (m_\mu^2 - 2m_a^2)}{12\pi^2 m_\mu M_S^2} \text{Arctanh} \left( \frac{m_\mu}{\sqrt{m_\mu^2 + 4m_a^2}} \right) \right]
\]

\[
F_2(q'^2) = -G_2(q'^2) = f_{e\alpha} f_{\alpha \mu} \frac{m_\mu^2}{24\pi^2 M_S^2}
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with \(q'^2 = -m_\mu^2\) for the particle physics factor:

\[
\Xi_{\text{photonic}}^2 = \frac{1}{288 \pi^4 m_\mu^2 M_S^4} \left| \sum_{a=e, \mu, \tau} f_{e\alpha} f_{\alpha \mu} \left( 4m_a^2 m_\mu - m_\mu^3 + 2(-2m_a^2 + m_\mu^2) \sqrt{4m_a^2 + m_\mu^2} \right) \text{Arctanh} \left( \frac{m_\mu}{\sqrt{4m_a^2 + m_\mu^2}} \right) + m_\mu^3 \ln \left( \frac{m_a^2}{M_S^2} \right) \right|^2
\]

→ while \(F_2\) is independent of \(m_a\), \(|F_1|\) decreases with increasing \(m_a\)

→ hierarchy: \(|F_2| < |F_1|\) but for \(M_S \sim 10 \text{ GeV}\) of order 10 %

→ compare to \(\mu \rightarrow e\gamma\): \(F_1(q'^2 = 0) = G_1(q'^2 = 0) = 0\) and

\(F_2(q'^2 = 0) = -G_2(q'^2 = 0) = F_2(q'^2 = -m_\mu^2) \Rightarrow \mu^- \rightarrow e^- \) conversion enhanced by \(F_1\) contribution
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Non-Photonic Contribution

Short-range ↔ takes place inside the nucleus:

**EFT** treatment ⇒ **Integrating out** the Z-boson:

→ four-point vertices
→ consider operators up to **dimension six**
→ for the coherent $\mu^−− e^−$ conversion, the only vertex realised in this model is described by

\[
\mathcal{L}_{\text{short-range}} = -\frac{G_F}{\sqrt{2}} \left( 2(1 + k_q \sin^2 \theta_W) \cos \theta_W \right) \frac{A_R(q'^2)}{g} \frac{e_R \gamma_\nu \mu_R \bar{q} \gamma^\nu q}{g_{RV(q)}}
\]

in terms of the chiral form factor $A_R(q'^2)$.
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in terms of the chiral form factor $A_R(q'^2)$
Non-Photonic Contribution

We can write the branching ratio as

\[
BR(\mu^- N \to e^- N) = \frac{8\alpha^5 m_\mu Z_{\text{eff}}^4 Z F^2_{\text{p}}}{\Gamma_{\text{capt}}} \Xi^2_{\text{non-photonic}}(Z, N, A_R(q'^2))
\]

→ no perfect factorisation anymore: \(\Xi\) modified to be function of nuclear characteristics
→ instead of lines we do have bands with finite widths for \(\Xi\)
⇒ determine form factors from amputated diagrams with off-shell Z-Boson

Combining photonic and non-photonic contributions:

\(\Xi_{\text{particle}} \rightarrow \Xi_{\text{combined}}(Z, N) = \Xi_{\text{photonic}} + \Xi_{\text{non-photonic}}(Z, N)\)

→ dependence on nuclear characteristics
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→ dependence on nuclear characteristics
Combining the Contributions: Results
see TG, Merle Phys.Rev. D93 (2016) 055039

Benchmark Points:

$f_{ab}$ such that LFV/LNV bounds fulfilled + suitable neutrino mass matrix reproduced

- 'red': $f_{ee} \approx 0$ and $f_{e\tau} \approx 0$
- 'purple': $f_{ee} \approx 0$ and $f_{e\mu} \approx \frac{f_{\mu\tau}^*}{f_{\mu\mu}^*} f_{e\tau}$
- 'blue': $f_{e\mu} \approx \frac{f_{\mu\tau}^*}{f_{\mu\mu}^*} f_{e\tau}$

choose representative 'average' set for each scenario to display $M_S$ dependence
Combining the Contributions: Results
see TG, Merle Phys.Rev. D93 (2016) 055039

→ widths of the bands so small that appear as lines
→ non-photonic (DASHED) contributions negligibly small
→ approximate process by its purely photonic (SOLID) contribution
→ factorisation: dependence on isotope only in width of limit
Results: Photonic Contribution vs $\mu \rightarrow e\gamma$

For $\mu^+ \rightarrow e^+ \gamma$:
strongest bound for red, weakest for blue points

$$A \propto \left| f_{ee} f_{e\mu}^* + f_{e\mu} f_{\mu\mu}^* + f_{e\tau} f_{\tau\mu}^* \right| \cdot C$$

$\rightarrow$ some amount of cancellation

For $\mu^- \rightarrow e^- \gamma$ conversion:
!! other way around !!

$$A \propto \left| C_e f_{ee} f_{e\mu} + C_\mu f_{e\mu} f_{\mu\mu} + C_\tau f_{e\tau} f_{\tau\mu} \right|$$

$\rightarrow$ flavour-dependent coefficients:
prevent similar cancellations
shape of amplitude leads to drastical change (not mainly off-shell contributions)
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Results: Complementarity

see TG, King, Merle, No, Panizzi Phys.Rev. D93 (2016) 073007

From ‘average scenarios’ (depicted by lines), we can estimate the lower limits on $M_S$ resulting from $\mu$-e conversion:

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How to tackle $\mu^- - e^+$ conversion (using the example of a realisation via doubly charged scalars)?
Motivation

In the following, we perform the computation for the decay rate for one particular short-range operator $\epsilon_{LLz}^3$. But why?!

- There are a few earlier references available focusing on $\mu^{-} \rightarrow e^{+}$ conversion from Majorana neutrinos but no uniform formalism is used:
  $\rightarrow$ has the nuclear matrix elements (for $^{48}$Ti) that we use: $\epsilon_{LLz}^3$
  $\rightarrow$ explicit computation focusing on the nuclear physics
  $\Rightarrow$ includes the formalism that we want make accessible to the particle physics community

- many aspects do not change if another operator was realised

$\rightarrow$ guideline how to use existing results and establish a general formalism to replicate such a computation for different scenarios
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\( \mu^- \rightarrow e^+ \) Conversion from doubly charged scalars

- formalism to describe \( \mu^- \rightarrow e^+ \) conversions within general framework
- use EFT to neatly separate the nuclear physics from the respective particle physics realisation of the conversion \( \rightarrow \) factorisation

\( \rightarrow \) map the model onto short-range operators
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\[ \xi \ AU \]

\[ \mu^- \to e^+ \]

\[ \frac{Z}{A} \]

\[ \xi \ LLR \]

\[ \map \] map the model onto short-range operators
\( \mu^- \rightarrow e^+ \) Conversion from doubly charged scalars

- formalism to describe \( \mu^- \rightarrow e^+ \) conversions within **general framework**
- use **EFT** to neatly separate the **nuclear** physics from the respective **particle** physics realisation of the conversion \( \rightarrow \) **factorisation**

\[ \xi f_{\mu e}^* \rightarrow \rightarrow \mu^- e^+ \]

\[ W^- W^- \downarrow \text{nucleus} \]

\[ \Leftrightarrow \]

\[ \mu^- \rightarrow e^+ \]

\[ (Z, A) \quad (Z-2, A) \quad \varepsilon_{LLR}^3 \rightarrow \rightarrow \]

\[ \text{\rightarrow map the model onto short-range operators} \]

Employ **EFT formalism** to generally describe $\mu^– e^+$ conversion $\Rightarrow$ dim-9 short-range operators:

$$
\mathcal{L}_{\text{short-range}}^{\mu e} = \frac{G^2_F}{2m_p} \sum_{x,y,z=L,R} \left[ \epsilon_1^{xyz} J_x J_y j_z + \epsilon_2^{xyz} J_x J_y,\nu j_z + \epsilon_3^{xyz} J_x J_y,\nu j_z + \epsilon_4^{xyz} J_x J_y,\nu_j^\rho j_z^\sigma + \epsilon_5^{xyz} J_x j_y,\nu_j^\nu + \epsilon_6^{xyz} J_x j_y,\nu j_z^\rho + \epsilon_7^{xyz} J_x j_y j_z,\nu_j^\nu + \epsilon_8^{xyz} J_x,\nu_j^\nu j_y,\nu_j^\rho \right]
$$

using the hadronic currents:

$$
J_{R,L} = \bar{d}(1 \pm \gamma_5)u, \quad J_{R,L}^{\nu} = \bar{d} \gamma^\nu (1 \pm \gamma_5)u, \quad J_{R,L}^{\nu,\rho} = \bar{d} \sigma^{\nu,\rho} (1 \pm \gamma_5)u,
$$

and the leptonic currents:

$$
J_{R,L} = \bar{e^c}(1 \pm \gamma_5)\mu = 2(\bar{e_{R,L}})^c \mu_{R,L}, \quad J_{R,L}^{\nu} = \bar{e^c} \gamma^\nu (1 \pm \gamma_5)\mu = 2(\bar{e_{L,R}})^c \gamma^\nu \mu_{R,L},
$$

and

$$
J_{R,L}^{\nu,\rho} = \bar{e^c} \sigma^{\nu,\rho} (1 \pm \gamma_5)\mu = 2(\bar{e_{R,L}})^c \sigma^{\nu,\rho} \mu_{R,L}.
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$\Rightarrow$ derive the decay rate using the example of doubly charged scalars
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and the leptonic currents:

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\]

$\Rightarrow$ derive the **decay rate** using the example of doubly charged scalars.
Deriving the Decay Rate for $\epsilon_3$ based on TG, Merle arXiv:1612.00452

Start with the **amplitude** obtained from EFT diagram

\[
\langle N', f | S_{\text{eff}}^{(1)} | N, i \rangle = -i \langle N', f | \int d^4x \hat{T}\{\mathcal{L}_{\text{eff}}(x)\} | N, i \rangle \\
= -i \frac{G_F^2}{2m_p} \epsilon_3^{\text{LLR}} \int d^4x \langle N', f | \hat{T}\{J_{L,\nu}(x)J_{L}^{\nu}(x)j_{R}(x)\} | N, i \rangle
\]
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\[ \langle N', f \mid S^{(1)}_{\text{eff}} \mid N, i \rangle = -i \langle N', f \mid \int d^4x \, \hat{T}\{ \mathcal{L}_{\text{eff}}(x) \} \mid N, i \rangle \]

\[ = -i \frac{G_F^2}{2m_p} \epsilon_{3}^{LLR} \int d^4x \langle N', f \mid \hat{T}\{ J_{L\nu}(x) J'_{L}(x) j_R(x) \} \mid N, i \rangle \]

which is
Deriving the Decay Rate for $\epsilon_3$ based on TG, Merle arXiv:1612.00452

Structure can be split into hadronic and leptonic parts:

$$\langle N', f|\hat{T}\{J_{L,\nu}(x)J'_{L}(x)j_{R}(x)\}|N, i\rangle = \langle N'|\hat{T}\{J_{L,\nu}(x)J'_{L}(x)\}|N\rangle\langle f|j_{R}(x)|i\rangle$$

**Leptonic part:**

- muon is bound in 1s state
- positron propagates freely under the influence of the nucleus’ Coulomb potential

$\Rightarrow$ need to modify the free spinors $u$ and $\nu$ respectively

$$\langle f|j_{R}(x)|i\rangle = 2e^{ik_{e}\cdot x}e^{-iE_{\mu}\cdot x^0}\sqrt{F(Z-2,E_{e})}\phi_{\mu}(\vec{x})\overline{\nu_{e}}(k_{e})P_{R}u_{\mu}(k_{\mu})$$

with bound muon wave function $\phi_{\mu}(\vec{x})$ and the Fermi function $F(Z, E)$
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\]

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\[
\langle f \mid j_{R}(x) \mid i \rangle = 2 e^{i k_{e} \cdot x} e^{-i E_{\mu} \cdot x^0} \sqrt{F(Z - 2, E_{e})} \phi_{\mu}(\vec{x}) \bar{v}_{e}(k_{e}) P_{R} u_{\mu}(k_{\mu})
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Deriving the Decay Rate for $\epsilon_3$ based on TG, Merle arXiv:1612.00452

Structure can be split into hadronic and leptonic parts:

$$\langle N', f | \hat{T} \{ J_{L,\nu}(x) J_{L'}(x) j_{R}(x) \} | N, i \rangle = \langle N' | \hat{T} \{ J_{L,\nu}(x) J_{L'}(x) \} | N \rangle \langle f | j_{R}(x) | i \rangle$$

Leptonic part:

- muon is bound in $1s$ state
- positron propagates freely under the influence of the nucleus’ Coulomb potential

$\Rightarrow$ need to modify the free spinors $u$ and $v$ respectively

$$\langle f | j_{R}(x) | i \rangle = 2 e^{ik_{e} \cdot x} e^{-iE_{\mu} \cdot x^0} \sqrt{F(Z - 2, E_{e})} \phi_{\mu}(\vec{x}) \overline{\nu_{e}}(k_{e}) P_{R} u_{\mu}(k_{\mu})$$

with bound muon wave function $\phi_{\mu}(\vec{x})$ and the Fermi function $F(Z, E)$
Deriving the Decay Rate for $\epsilon_3$ based on TG, Merle arXiv:1612.00452

Hadronic part:

- hadronic currents can be approximated by their non-relativistic versions $J_{\nu}(\vec{x})$

- need to account for quarks’ distribution within the nucleus
  → dipole parametrisation factor $\tilde{F}(k^2, \Lambda_i)$

- two nucleon interactions → take place with finite distance
  → introduce second location $\tilde{x}$ over which we also "sum" $\int d^3\tilde{x}$

$\Rightarrow$ need to modify hadronic currents $J_{\nu}$ respectively

$$\langle N' | \hat{T} \{ J_{L,\nu}(x) J_L^{\nu}(x) \} | N \rangle \rightarrow \int d^3\tilde{x} \int \frac{d^3k}{(2\pi)^3} \langle N' | e^{i\vec{k} \cdot (\vec{x} - \tilde{x})} \tilde{F}^2(k^2, \Lambda_i) J_{L,\nu}(\tilde{x}) J_L^{\nu}(\tilde{x}) | N \rangle$$
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Next:

- perform $x^0$ integration
  \[ \rightarrow \text{conservation of external energies} \quad 2\pi \delta(E_i + E_{\mu} - E_f - E_e) \]
- write non-relativistic currents in term of effective transition operators:

\[
\tilde{F}(\vec{k}^2, \Lambda_i) J_{\nu}(\vec{x}) = \sum_m \tau^m_{-} \left( g_{\nu} \tilde{F}(\vec{k}^2, \Lambda_{\nu}) g_{\nu 0} + g_{A} \tilde{F}(\vec{k}^2, \Lambda_{A}) g_{\nu j} \sigma^j_m \right) \delta^{(3)}(\vec{x} - \vec{r}_m)
\]

with nuclear isospin raising operator $\tau^m_{-}$ and the dominant spin structures given by the Fermi operator and the Gamow-Teller operator.

\[ \Rightarrow \text{allows for factorisation of nuclear physics from respective particle physics model:} \]

\[
\mathcal{M} = \frac{G^2_F \epsilon^{LLR}_3 g^2_A m_e}{2R} \sqrt{F(Z - 2, E_e)} \delta(E_f - E_i + E_e - E_{\mu}) \overline{v}_e(k_e) P_R u_\mu(k_\mu) \mathcal{M}(\mu^-, e^+) \phi
\]

with $\mathcal{M}(\mu^-, e^+) \phi$ being the nuclear matrix element.
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From amplitude to decay rate using **Fermi’s Golden rule**:

$$\Gamma = 2\pi \frac{1/T}{(2\pi)^3} \int d^3 k_e |M|^2$$

So, we need to

- spin sum/average $\rightarrow 1/4$
- rewrite *nuclear matrix element* using that the muon wave function varies only slowly within nucleus: $|M(\mu^-,e^+)\phi|^2 = \langle \phi_\mu \rangle^2 |M(\mu^-,e^+)|^2$
- square delta-function: “$\delta(E_f - E_i + E_e - E\mu)^2$” $= \frac{T}{2\pi} \delta(E_f - E_i + E_e - E\mu)$

and obtain the **decay rate**:

$$\Gamma = \frac{g_A^4 G_F^4 m_e^2 m_\mu^2 |\epsilon_3^{LLR}|^2}{32\pi^2 R^2} |F(Z-2,E_e)| \langle \phi_\mu \rangle^2 |M(\mu^-,e^+)|^2$$

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Further Realisations of $\epsilon_3$

**Cheng-Geng-Ng model**
Cheng, Geng, Ng Phys.Rev. D75 (2007) 053004

**EFT with doubly charged scalar**
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Reach of Future Experiments for $\epsilon_3$

- obvious: limits on $0\nu\beta\beta$ are superior to those of $\mu^- - e^+$ conversion by orders of magnitude
- but also apparent: there are models where LNV is much more prominent in $e\mu$ instead of $ee$ sector
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However: several **key pieces of information are missing!!**

We are in dire need of **improvements** from **different areas**:

- **Experiment**: more detailed sensitivity studies for $\mu^- - e^+$ conversion
- **Nuclear Matrix Elements**:
  - Detailed study on percentage of process that is "coherent"
  - Hardly any **nuclear matrix elements** (NMEs) are available
  - $\rightarrow$ need for NMEs for further element, e.g. $^{27}$Al, and for other operators like $\epsilon_{1,2}$
  - $\Rightarrow$ there are promising models but we cannot judge them properly
- **Particle Physics**: for many models there are no (detailed) studies on LNV in the $e\mu$ sector and no information on which effective operators are realised

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Open Issues of $\mu^{-} \rightarrow e^{+}$ based on TG, Merle, Zuber Phys.Lett. B764 (2017) 157

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Summary and Outlook

- **orders of magnitude** improvement of sensitivities in near-future experiments

- $\mu^-$–$e^-$ conversion:
  - **FIRST work** that treats $\mu^-$–$e^-$ conversion in such detail, i.e. beyond previous EFT treatment/approximations → analytic expression for $\Xi_{\text{particle}}$
  - **complementarity**: rich phenomenology of loop models → high- and low-energy processes → $\mu^-$–$e^-$ conversion important part of study

- $\mu^-$–$e^+$ conversion:
  - complete computation of the rate for the lepton flavour and number violating conversion process, mediated by the effective operator $\epsilon_3$
  - pointed out open issues and further models/operators
  - LNV possibly more prominent in $e\mu$ sector → experiments could make a countable physics impact
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Thank you for your attention!!

Any questions?
Backup Slides
Generating the Neutrino Mass

The mass is generated at two-loop level via the diagram

which leads to the neutrino mass

\[ \mathcal{M}_{\nu,ab}^{2\text{-loop}} = \frac{2 \xi m_a m_b M_S^2 g_{ab}(1+\delta_{ab})}{\Lambda^3} \mathcal{I}[M_W, M_S, \mu] \]

\[ \rightarrow \text{ Majorana mass term} \]
\[ \rightarrow \text{ further LNV processes} \]
Selection of interesting processes: **low energy physics**

- neutrinoless double beta decay:
  \[
  \frac{\xi f_{ee}}{M_S^2 \Lambda^3} < 4.0 \cdot 10^{-3} \frac{\text{TeV}^5}{M_S^2 \Lambda^3}
  \]

- \( \mu^- \rightarrow e^- \gamma \):
  \[
  |f_{ee}^* f_{e\mu} + f_{e\mu}^* f_{\mu\mu} + f_{e\tau}^* f_{\mu\tau}| < 3.2 \cdot 10^{-4} M_S^2 \text{[TeV]}
  \]
Testing the Model
based on King, Merle, Panizzi arXiv:1406.4137

benchmark points:

\[ f_{ab} \text{ such that bounds fulfilled } + \text{ suitable light neutrino mass matrix reproduced} \]

- 'red': \( f_{ee} \approx 0 \) and \( f_{e\tau} \approx 0 \)
- 'purple': \( f_{ee} \approx 0 \) and \( f_{e\mu} \approx \frac{f_{\mu\tau}^{*}}{f_{\mu\mu}^{*}} f_{e\tau} \)
- 'blue': \( f_{e\mu} \approx \frac{f_{\mu\tau}^{*}}{f_{\mu\mu}^{*}} f_{e\tau} \)

\[ \downarrow \]

complementary check with high energy experiments:
compute cross sections for e.g.

- \( S^{\pm\pm} \rightarrow W^{\pm\pm} \)
- \( S^{\pm\pm} \rightarrow l_a^{\pm\pm} l_b^{\pm\pm} \)
- ...

\[ \rightarrow \text{ some of the benchmark points already excluded by LHC data (7 TeV run) } \]
Testing the Model
based on King, Merle, Panizzi arXiv:1406.4137

benchmark points:

$f_{ab}$ such that bounds fulfilled + suitable light neutrino mass matrix reproduced

- 'red': $f_{ee} \simeq 0$ and $f_{e\tau} \simeq 0$
- 'purple': $f_{ee} \simeq 0$ and $f_{e\mu} \simeq \frac{f_{\mu\tau}^*}{f_{\mu\mu}^*} f_{e\tau}$
- 'blue': $f_{e\mu} \simeq \frac{f_{\mu\tau}^*}{f_{\mu\mu}^*} f_{e\tau}$

↓

complementary check with **high energy experiments**:
compute cross sections for e.g.

- $S^{\pm\pm} \rightarrow W^{\pm\pm}$
- $S^{\pm\pm} \rightarrow l_a^{\pm\pm} l_b^{\pm\pm}$
- ...

→ some of the benchmark points already excluded by LHC data (7 TeV run)
Photonic Contribution: Cross Check via UV Divergences

In form of \( i \mathcal{M} = e f_{ea}^* f_{a\mu} A_\nu(q') \overline{u}_e(p_e) \mathcal{I}_\nu u_\mu(p_\mu) \):

\[
-4 Q_S \int \frac{d^d k}{(2\pi)^d} \frac{P_L \gamma^\nu (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} 2i \frac{Q_S P_L \gamma^\nu}{(4\pi)^2 \epsilon}
\]

\[
-4 Q_+ \int \frac{d^d k}{(2\pi)^d} \frac{P_L (k + q' + m_a) \gamma^\nu (k + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} -i \frac{Q_+ P_L \gamma^\nu \gamma^\rho P_R}{(4\pi)^2 \epsilon}
\]

\[
-4 Q_- \int \frac{d^d k}{(2\pi)^d} \frac{P_L (\phi_e + m_\mu) \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} 2i \frac{Q_-}{(4\pi)^2 \epsilon} P_L \phi_e (\phi_e + m_\mu) \gamma^\nu
\]

\[
4 Q_e^- \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu \phi_\mu}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} -2i \frac{Q_e^-}{(4\pi)^2 \epsilon} \gamma^\nu \phi_\mu P_L \phi_\mu
\]

\[
\Rightarrow \Sigma \mathcal{I}^\nu = \frac{i}{(4\pi)^2 \epsilon} \left[ (2 Q_S + 2 Q_+ - Q_e^- - Q_-) P_L \gamma^\nu \right] = 0
\]
Photonic Contribution: Cross Check via UV Divergences

In form of

\[ i \mathcal{M} = e f_{ea}^* f_{a\mu} A_\nu(q') \bar{u}_e(p_e) \mathcal{I}^\nu u_\mu(p_\mu) : \]

\[-4Q_S \int \frac{d^d k}{(2\pi)^d} \frac{P_L k (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \quad \text{div} \rightarrow \frac{2i}{(4\pi)^2 \varepsilon} Q_S P_L \gamma^\nu \]

\[-4Q_I+ \int \frac{d^d k}{(2\pi)^d} \frac{P_L (k + q' + m_\rho) \gamma^\nu (k + m_\rho) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \quad \text{div} \rightarrow -\frac{i}{(4\pi)^2 \varepsilon} Q_I+ P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R \]

\[-4Q_I- \int \frac{d^d k}{(2\pi)^d} \frac{P_L (p_e + m_\mu) \gamma^\nu (p_e + m_\mu) P_R}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \quad \text{div} \rightarrow \frac{2i}{(4\pi)^2 \varepsilon} \frac{\mu}{m_\mu^2} P_L p_e (p_e + m_\mu) \gamma^\nu \]

\[4Q_e- \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu \phi_\mu P_L k}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \quad \text{div} \rightarrow -\frac{2i}{(4\pi)^2 \varepsilon} \frac{e}{m_\mu^2} \gamma^\nu \phi_\mu P_L \phi_\mu \]

\[\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2Q_S + 2Q_{I+} - Q_{e-} - Q_{\mu-}) P_L \gamma^\nu] = 0 \quad \checkmark\]
Photonic Contribution: Cross Check via UV Divergences

In form of \( i\mathcal{M} = e f_{ea}^* f_{a\mu} A_\nu(q') \bar{u}_e(p_\mu) \mathcal{I}_\nu u_\mu(p_\mu) \):

\[
-4QS \int \frac{d^dk}{(2\pi)^d} \frac{P_L \bar{k}(2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M^2_S][(p_\mu - k)^2 - M^2_S]} \text{ div} \frac{2i}{(4\pi)^2 \varepsilon} QSP_L \gamma^\nu
\]

\[
-4Q I_+ \int \frac{d^dk}{(2\pi)^d} \frac{P_L (\bar{k} + q' + ma)^\nu (k + ma)p_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M^2_S][(k + q')^2 - m_a^2]} \text{ div} \frac{-i}{(4\pi)^2 \varepsilon} QI_+ P_L \gamma^\rho \gamma^\nu \gamma^\rho P_R
\]

\[
-4Q_{\mu -} \int \frac{d^dk}{(2\pi)^d} \frac{P_L \bar{k} (\bar{p}_e + m_\mu)^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M^2_S][k^2 - m_a^2]} \text{ div} \frac{2i}{(4\pi)^2 \varepsilon} \frac{\mu -}{m_\mu} P_L \bar{p}_e (\bar{p}_e + m_\mu) \gamma^\nu
\]

\[
4Q_{e -} \int \frac{d^dk}{(2\pi)^d} \frac{\gamma^\nu \bar{p}_\mu P_L \bar{k}}{[p_\mu^2][(p_\mu - k)^2 - M^2_S][k^2 - m_a^2]} \text{ div} \frac{-2i}{(4\pi)^2 \varepsilon} \frac{e -}{m_\mu} \gamma^\nu \bar{p}_\mu P_L \bar{p}_\mu
\]

\[
\Rightarrow \sum \mathcal{I}_\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2QS + 2Q I_+ - Q_{e -} - Q_{\mu -}) P_L \gamma^\nu] = 0 \quad \checkmark
\]
Photonic Contribution: Cross Check via UV Divergences

In form of \( i \mathcal{M} = e f_{ea}^* f_{a\mu} A_\nu(q') \overline{u}_e(p_e) T^\nu u_\mu(p_\mu) : \)

\[
-4QS \int \frac{d^d k}{(2\pi)^d} \frac{P_L k(2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \quad \text{div} \rightarrow \frac{2i}{(4\pi)^2 \varepsilon} QS P_L \gamma^\nu
\]

\[
-4Q_I \int \frac{d^d k}{(2\pi)^d} \frac{P_L (k + q' + m_\alpha) \gamma^\nu (k + m_\alpha) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \quad \text{div} \rightarrow -i \frac{Q_I + P_L \gamma^\rho \gamma^\nu \gamma^\rho P_R}{(4\pi)^2 \varepsilon}
\]

\[
-4Q_{\mu -} \int \frac{d^d k}{(2\pi)^d} \frac{P_L \tilde{k} \tilde{(p_e + m_\mu)} \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \quad \text{div} \rightarrow \frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_{\mu -}}{m_\mu^2} P_L p_e (p_e + m_\mu) \gamma^\nu
\]

\[
4Q_e^- \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu p_{\mu -} P_L \tilde{k}}{[p_{\mu -}^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \quad \text{div} \rightarrow -2i \frac{Q_e^-}{(4\pi)^2 \varepsilon} \frac{\gamma^\nu p_{\mu -} P_L \tilde{p}_{\mu -}}{m_\mu^2}
\]

\[\Rightarrow \sum T^\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2QS + 2Q_I - Q_e^- - Q_{\mu -}) P_L \gamma^\nu] = 0 \quad \checkmark\]
Photonic Contribution: Cross Check via UV Divergences

In form of \( i \mathcal{M} = e f_{ea}^* f_{a\mu} A_\nu(q') \bar{u}_e(p_e) \mathcal{T}^\nu u_\mu(p_\mu) \):

\[
-4Q_S \int \frac{d^d k}{(2\pi)^d} \frac{P_L k (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} Q_S P_L \gamma^\nu
\]

\[
-4Q_I+ \int \frac{d^d k}{(2\pi)^d} \frac{P_L (k + q' + m_a) \gamma^\nu (k + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2 \varepsilon} Q_I+ P_L \gamma^\rho \gamma^\nu \gamma^\rho P_R
\]

\[
-4Q_\mu- \int \frac{d^d k}{(2\pi)^d} \frac{P_L (\phi_e + m_\mu) \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_\mu}{m_\mu^2} P_L \phi_e (\phi_e + m_\mu) \gamma^\nu
\]

\[
4Q_e- \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu \phi_\mu P_L k}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2 \varepsilon} \frac{Q_e}{m_\mu^2} \gamma^\nu \phi_\mu P_L \phi_\mu
\]

\[
\Rightarrow \sum \mathcal{T}^\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2Q_S + 2Q_I^+ - Q_e^- - Q_\mu^-) P_L \gamma^\nu] = 0 \quad \checkmark
\]
Photonic Contribution: Cross Check via UV Divergences

In form of \[ i\mathcal{M} = e f_{ea}^* f_{a\mu} A_\nu(q') \bar{u}_e(p_e) \mathcal{I}^\nu u_{\mu}(p_{\mu}) : \]

\[-4Q_S \int \frac{d^d k}{(2\pi)^d} \frac{P_L k(2p_{\mu} - 2k + q')^\nu}{[k^2 - m_a^2][(p_{\mu} - k + q')^2 - M_S^2][(p_{\mu} - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} Q_S P_L \gamma^\nu \]

\[-4Q_I^+ \int \frac{d^d k}{(2\pi)^d} \frac{P_L (k + q' - m_a) \gamma^\nu (k + m_a) P_R}{[k^2 - m_a^2][(p_{\mu} - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} -\frac{i}{(4\pi)^2 \varepsilon} Q_I^+ P_L \gamma^\rho \gamma^\nu \gamma^\rho P_R \]

\[-4Q_\mu^- \int \frac{d^d k}{(2\pi)^d} \frac{P_L \phi_{pe}^\nu + m_{\mu} \gamma^\nu}{[p_{pe}^2 - m_{\mu}^2][(p_{fe} - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_\mu^-}{m_{\mu}^2} P_L \phi_{pe}(\phi_{pe} + m_{\mu}) \gamma^\nu \]

\[4Q_e^- \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu \phi_{\mu} P_L k}{[p_{\mu}^2][(p_{\mu} - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} -\frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_e^-}{m_{\mu}^2} \gamma^\nu \phi_{\mu} P_L \phi_{\mu} \]

\[\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2Q_S + 2Q_I^+ - Q_e^- - Q_\mu^-)P_L \gamma^\nu] = 0 \]
Photonic Contribution: Cross Check via UV Divergences

In form of \( i\mathcal{M} = e f^*_e f_{a\mu} A_{\nu}(q') \bar{u}_e(p_e) T^\nu u_\mu(p_\mu) \):
Photonic Contribution: Results I


\[
F_1(-m^2_\mu) = G_1(-m^2_\mu) =
\]

\[
= -\frac{1}{128\,\pi^2m^4_\mu} \sum_{a=e,\mu,\tau} f^*_{ea} f_{a\mu} \left[ 2 m^2_\mu \left( -5m^2_a + 6m^2_\mu + 5M^2_S \right) - 2 S_a m^2_\mu \left( m^2_a + 3m^2_\mu - M^2_S \right) \right]
\]

\[
\ln \left[ \frac{2m^2_a}{2m^2_a + m^2_\mu(1+S_a)} \right] + 4 S_S m^2_\mu \left( m^2_a + m^2_\mu - M^2_S \right) \ln \left[ \frac{2M^2_S}{2M^2_S + m^2_\mu(1+S_S)} \right] + \left( 3m^2_a \left( 2m^2_a - m^2_\mu \right) - 4M^2_S \right) + 5m^4_\mu - 7m^2_\mu M^2_S + 6M^4_S \right) \ln \left[ \frac{m^2_a}{M^2_S} \right] + 2 T_a \left( -6m^2_a + m^2_\mu + 6M^2_S \right) \ln \left[ \frac{2m^2_a M_S}{m^2_a - m^2_\mu + M^2_S - T_a} \right]
\]

\[
+ 2 m^2_\mu \left[ \left( m^4_a + 8m^2_a m^2_\mu + M^4_S - 2M^2_S \left( m^2_a + 2m^2_\mu \right) \right) C_0 \left[ 0, -m^2_\mu, m^2_\mu; m_a, M_S, m_a \right] \right]
\]

\[
+ 2 \left( m^4_a - 2M^2_S \left( m^2_a - 2m^2_\mu \right) + M^4_S \right) C_0 \left[ 0, -m^2_\mu, m^2_\mu; M_S, m_a, M_S \right] \right]
\]

\[
\frac{M_S \gg m_a}{\to} -f^*_{ea} f_{a\mu} \left[ \frac{2m^2_a + m^2_\mu \ln \left( \frac{m_a}{M_S} \right)}{12\pi^2M^2_S} + \frac{\sqrt{m^2_\mu + 4m^2_a}(m^2_\mu - 2m^2_a)}{12\pi^2m_\mu M^2_S} \arctanh \left( \frac{m_\mu}{\sqrt{m^2_\mu + 4m^2_a}} \right) \right] + O(M^{-4}_S)
\]

Note: \( O(M^{-4}_S) \) gives corrections of up to a **few per cent**
Photonic Contribution: Results I


\[
F_1(-m^2_\mu) = G_1(-m^2_\mu) = \\
= -\frac{1}{128 \pi^2 m^4_\mu} \sum_{a=e, \mu, \tau} f^*_{ea} f_{a\mu} \left[ 2 m^2_\mu \left( -5m^2_a + 6m^2_\mu + 5M^2_S \right) - 2 S_a m^2_\mu \left( m^2_a + 3m^2_\mu - M^2_S \right) \right. \\
\ln \left[ \frac{2m^2_a}{2m^2_a + m^2_\mu (1+S_a)} \right] + 4 S_S m^2_\mu \left( m^2_a + m^2_\mu - M^2_S \right) \ln \left[ \frac{2M^2_S}{2M^2_S + m^2_\mu (1+S_S)} \right] + \left( 3m^2_a \left( 2m^2_a - m^2_\mu - 4M^2_S \right) + 5m^4_\mu - 7m^2_\mu M^2_S + 6M^4_S \right) \ln \left[ \frac{m^2_a}{M^2_S} \right] + 2 T_a \left( -6m^2_a + m^2_\mu + 6M^2_S \right) \ln \left[ \frac{2m^2_a M_S}{m^2_a - m^2_\mu + M^2_S - T_a} \right] \\
+ 2 m^2_\mu \left[ \left( m^4_a + 8m^2_a m^2_\mu + M^4_S - 2M^2_S \left( m^2_a + 2m^2_\mu \right) \right) C_0 \left[ 0, -m^2_\mu, m^2_\mu; m_a, M_S, m_a \right] \\
+ 2 \left( m^4_a - 2M^2_S \left( m^2_a - 2m^2_\mu \right) + M^4_S \right) C_0 \left[ 0, -m^2_\mu, m^2_\mu; M_S, m_a, M_S \right] \right] \\
\]

\[
\frac{M_S \gg m_a}{\rightarrow} - f^*_{ea} f_{a\mu} \left[ \frac{2m^2_a + m^2_\mu}{12\pi^2 M^2_S} \log \left( \frac{m^2_a}{M^2_S} \right) + \frac{\sqrt{m^2_\mu + 4m^2_a \left( m^2_\mu - 2m^2_a \right)}}{12\pi^2 m_\mu M^2_S} \operatorname{Arctanh} \left( \frac{m^2_\mu}{\sqrt{m^2_\mu + 4m^2_a}} \right) \right] + O(M_S^{-4})
\]

Note: $O(M_S^{-4})$ gives corrections of up to a few per cent
Photonic Contribution: Results I


\[
F_1(-m^2_\mu) = G_1(-m^2_\mu) = \\
= -\frac{1}{128}\frac{2m^2_a}{\pi^2m^4_\mu}\sum_{a=e,\mu,\tau} f^*_{ea} f_{a\mu}\left[2m^2_\mu\left(-5m^2_a + 6m^2_\mu + 5M^2_S\right) - 2S_a m^2_\mu\left(m^2_\mu + 3m^2_a - M^2_S\right)\right] \\
\ln\left[\frac{2m^2_a}{2m^2_a + m^2_\mu(1+S_a)}\right] + 4S_a m^2_\mu\left(m^2_a + m^2_\mu - M^2_S\right) \ln\left[\frac{2M^2_S}{2M^2_S + m^2_\mu(1+S_a)}\right] + \left(3m^2_a\left(2m^2_a - m^2_\mu - 4M^2_S\right) + 5m^4_\mu - 7m^2_\mu M^2_S + 6M^4_S\right) \ln\left[\frac{m^2_a}{M^2_S}\right] + 2T_a\left(-6m^2_a + m^2_\mu + 6M^2_S\right) \ln\left[\frac{2m_a M_S}{m^2_a - m^2_\mu + M^2_S - T_a}\right] \\
+ 2m^2_\mu\left[\left(m^4_a + 8m^2_a m^2_\mu + M^4_S - 2M^2_S\left(m^2_a + 2m^2_\mu\right)\right) C_0\left[0, -m^2_\mu, m^2_\mu; m_a, M_S, m_a\right] \\
+ 2\left(m^4_a - 2M^2_S\left(m^2_a - 2m^2_\mu\right) + M^4_S\right) C_0\left[0, -m^2_\mu, m^2_\mu; M_S, m_a, M_S\right]\right] \\
M_S \gg m_a \\
\xrightarrow{M_S \gg m_a} -f^*_{ea} f_{a\mu}\left[\frac{2m^2_a + m^2_\mu}{12\pi^2 M^2_S}\log\left(\frac{m_a}{M^2_S}\right) + \sqrt{m^2_\mu + 4m^2_a\left(m^2_\mu - 2m^2_a\right)} \frac{\text{Arctanh}\left(\frac{m_\mu}{\sqrt{m^2_\mu + 4m^2_a}}\right)}{12\pi^2 m_\mu M^2_S}\right] + \mathcal{O}(M^{-4}_S)
\]

Note: \(\mathcal{O}(M^{-4}_S)\) gives corrections of up to a **few per cent**

\[
F_2(-m_{\mu}^2) = -G_2(-m_{\mu}^2) = \\
= -\frac{1}{128 \pi^2 m_\mu^4} \sum_{a=e, \mu, \tau} f^*_e f_a \mu \left[ 2 m_\mu^2 \left( -m_a^2 + 6 m_\mu^2 + M_S^2 \right) + 2 S_a m_\mu^2 \left( 3 m_a^2 + m_\mu^2 - 3 M_S^2 \right) \right] \\
\ln \left[ \frac{2 m_a^2}{2 m_a^2 + m_\mu^2 (1 + S_a)} \right] + 4 S_S m_\mu^2 \left( -3 m_a^2 + m_\mu^2 + 3 M_S^2 \right) \ln \left[ \frac{2 M_S^2}{2 M_S^2 + m_\mu^2 (1 + S_S)} \right] \\
+ \left( m_a^2 \left( -2 m_a^2 - 7 m_\mu^2 + 4 M_S^2 \right) + m_\mu^4 + 5 m_\mu^2 M_S^2 - 2 M_S^4 \right) \ln \left[ \frac{m_a^2}{M_S^2} \right] + 2 T_a \left( 2 m_a^2 - 3 m_\mu^2 - 2 M_S^2 \right) \\
\ln \left[ \frac{2 m_a M_S}{m_a^2 - m_\mu^2 + M_S^2 - T_a} \right] + 2 m_\mu^2 \left( -3 m_a^4 - 3 M_S^4 + 2 M_S^2 \left( 3 m_a^2 + 2 m_\mu^2 \right) \right) C_0 \left[ 0, -m_\mu^2, m_\mu^2; m_a, M_S, m_a \right] \\
+ 2 \left( -3 m_a^4 + 2 m_a^2 \left( 3 M_S^2 + 2 m_\mu^2 \right) - 3 M_S^4 \right) C_0 \left[ 0, -m_\mu^2, m_\mu^2; M_S, m_a, M_S \right] \right]
\]

\[
\frac{M_S \gg m_a}{\rightarrow} f^*_e f_a \mu \frac{m_\mu^2}{24 \pi^2 M_S^2} + \mathcal{O}(M_S^{-4})
\]

Note: \( \mathcal{O}(M_S^{-4}) \) gives corrections of up to a few per cent
Photonic Contribution: Results I


\[
F_2(-m^2_\mu) = -G_2(-m^2_\mu) =
\]

\[
= - \frac{1}{128 \pi^2 m^4_\mu} \sum_{a=e, \mu, \tau} f^*_a f_{a\mu} \left[ 2 m^2_\mu \left( - m^2_a + 6 m^2_\mu + M^2_S \right) + 2 S_a m^2_\mu \left( 3 m^2_a + m^2_\mu - 3 M^2_S \right) \right]
\]

\[
\ln \left[ \frac{2 m^2_a}{2 m^2_a + m^2_\mu (1 + S_a)} \right] + 4 S_S m^2_\mu \left( - 3 m^2_a + m^2_\mu + 3 M^2_S \right) \ln \left[ \frac{2 M^2_S}{2 M^2_S + m^2_\mu (1 + S_S)} \right]
\]

\[
+ \left( m^2_a \left( - 2 m^2_a - 7 m^2_\mu + 4 M^2_S \right) + m^4_\mu + 5 m^2_\mu M^2_S - 2 M^4_S \right) \ln \left[ \frac{m^2_a}{M^2_S} \right] + 2 T_a \left( 2 m^2_a - 3 m^2_\mu - 2 M^2_S \right)
\]

\[
\ln \left[ \frac{2 m a M_S}{m^2_a - m^2_\mu + M^2_S - T_a} \right] + 2 m^2_\mu \left[ \left( - 3 m^4_a - 3 M^4_S + 2 M^2_S (3 m^2_a + 2 m^2_\mu) \right) C_0 \left[ 0, - m^2_\mu, m^2_\mu; m_a, M_S, m_a \right]
\]

\[
+ 2 \left( - 3 m^4_a + 2 m^2_a (3 M^2_S + 2 m^2_\mu) - 3 M^4_S \right) C_0 \left[ 0, - m^2_\mu, m^2_\mu; M_S, m_a, M_S \right] \right]
\]

\[
M_S \gg m_a \rightarrow f^*_a f_{a\mu} \frac{m^2_\mu}{24 \pi^2 M^2_S} + \mathcal{O}(M^{-4}_S)
\]

Note: \( \mathcal{O}(M^{-4}_S) \) gives corrections of up to a few per cent

\[
F_2(-m^2_\mu) = -G_2(-m^2_\mu) = \frac{1}{128 \pi^2 m^4_\mu} \sum_{a=e, \mu, \tau} f^*_{ea} f_{a\mu} \left[ 2 m^2_\mu \left( - m^2_a + 6 m^2_\mu + M^2_S \right) + 2 S_a m^2_\mu \left( 3 m^2_a + m^2_\mu - 3 M^2_S \right) \right. \\
\left. \ln \left( \frac{2 m^2_a}{2 m^2_a + m^2_\mu (1 + S_a)} \right) \right] + 4 S_S m^2_\mu \left( - 3 m^2_a + m^2_\mu + 3 M^2_S \right) \ln \left[ \frac{2 M^2_S}{2 M^2_S + m^2_\mu (1 + S_S)} \right] \\
+ \left( m^2_a \left( - 2 m^2_a - 7 m^2_\mu + 4 M^2_S \right) + m^4_\mu + 5 m^2_\mu M^2_S - 2 M^4_S \right) \ln \left[ \frac{m^2_a}{M^2_S} \right] + 2 T_a \left( 2 m^2_a - 3 m^2_\mu - 2 M^2_S \right) \\
\ln \left[ \frac{2 m_a M_S}{m^2_a - m^2_\mu + M^2_S - T_a} \right] + 2 m^2_\mu \left[ \left( - 3 m^4_a - 3 M^4_S + 2 M^2_S \left( 3 m^2_a + 2 m^2_\mu \right) \right) C_0 [0, -m^2_\mu, m^2_\mu; m_a, M_S, m_a] \\
+ 2 \left( - 3 m^4_a + 2 m^2_a \left( 3 M^2_S + 2 m^2_\mu \right) - 3 M^4_S \right) C_0 [0, -m^2_\mu, m^2_\mu; M_S, m_a, M_S] \right] \\

\frac{M_S \gg m_a}{\Rightarrow} f^*_{ea} f_{a\mu} \frac{m^2_\mu}{24 \pi^2 M^2_S} + \mathcal{O}(M^4_S)
\]

Note: \( \mathcal{O}(M^{-4}_S) \) gives corrections of up to a few per cent
'Average Scenario’ Couplings

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**Table:** First part: 'average scenario’ couplings for the benchmark points as extracted from Tab. 7 in *King, Merle, Panizzi: arXiv:1406.4137*. Second part: combination of couplings that enter the $\mu$–$e$ conversion amplitude. The bold values indicate the dominant photonic contribution.
Non-Photonic Bands

- The amplitude that enters the non-photonic $\Xi$ takes the form

$$A \propto |f_{ee}^* f_{e\mu} D(m_e) + f_{e\mu}^* f_{\mu\mu} D(m_\mu) + f_{e\tau}^* f_{\tau\mu} D(m_\tau)|.$$  

- The function $D(m_a)$ strongly varies with $m_a$.
  - $\rightarrow$ dominant term stems from the tau propagating within the loop, i.e. $D(m_\tau)$
  - $\rightarrow$ exceeds the muon and electron contribution by three to four orders of magnitude

- blue/purple scenario: neither $f_{ee}^* f_{e\mu}$ nor $f_{e\mu}^* f_{\mu\mu}$ bypasses this difference
  + identical $f_{e\tau}^* f_{\tau\mu}$ in both scenarios
  - $\rightarrow$ indistinguishable curves

- red/grey scenario:
  dominant contributions: $f_{e\mu}^* f_{\mu\mu} D(m_\mu) \sim f_{e\tau}^* f_{\tau\mu} D(m_\tau)$
  - $\rightarrow$ same order of magnitude, i.e. comparable values of non-photonic contribution