Exercise 12: Methanation<br>Julia Witte, (julia.witte@psi.ch)<br>A detailed solution is provided at the $\mathbf{1 3}^{\mathbf{t h}}$ of December.<br>Please see: http://www.psi.ch/ene/ret1

## Problem: Direct Methanation of Carbon Dioxide from Biogas



Figure 1: Flowsheet of a direct methanation

Biogas is produced via fermentation of biomass, deactivation of the catalyst due to carbon deposition. The mixture (4) enters the methanation reactor, where carbon dioxide and hydrogen convert to methane and water:

$$
\begin{equation*}
\mathrm{CO}_{2}+4 \mathrm{H}_{2} \leftrightarrow \mathrm{CH}_{4}+2 \mathrm{H}_{2} \mathrm{O} \quad \Delta H_{R}^{0}=-164.7 \mathrm{~kJ} / \mathrm{mol} \tag{I}
\end{equation*}
$$

The reactor is cooled by an internal heat exchanger. The outlet stream of the reactor (5) contains mainly methane and water.

## Tasks:

i. Methanation without steam: Calculate the molar flows of all components in stream $5 \dot{n}_{5, i}[\mathrm{~mol} / \mathrm{s}]$ and the heat stream $\dot{Q}_{\text {out }, M}[\mathrm{~kW}]$, which must be dissipated in order to maintain a temperature of $\mathrm{T}_{\text {meth }}=320^{\circ} \mathrm{C}$. A steady state process is
assumed. Stream 4 has a temperature of $\mathrm{T}_{4}=270^{\circ} \mathrm{C}$. The conversion of $\mathrm{CO}_{2}$ in the methanation unit reaches $\mathrm{C}_{\mathrm{CO}}=95 \%$.

Further values:

- Biogas stream: $\dot{n}_{1}=4 \frac{\mathrm{~mol}}{\mathrm{~s}}, x_{1, C H_{4}}=0.55, x_{1, C O_{2}}=0.45$
- In this case: No steam is added, $\dot{n}_{3}=0 \frac{\mathrm{~mol}}{\mathrm{~s}}$,
- The ratio of $\mathrm{H}_{2}$ to $\mathrm{CO}_{2}$ in stream 4 is stoichiometric
- $p=5$ bar
- Specific heat capacities: $c_{p, 4 m i x}=31.7 \frac{\mathrm{~J}}{\operatorname{mol~K}}, c_{p, 5 \operatorname{mix}}=34.4 \frac{\mathrm{~J}}{\mathrm{~mol} \mathrm{~K}}$

1. Determine the mole flows of $\dot{n}_{4}$ :
$\dot{n}_{1}=4.0 \mathrm{~mol} / \mathrm{s} \rightarrow \dot{n}_{1, C H_{4}}=x_{1, C H_{4}} \cdot \dot{n}_{1}=2.2 \mathrm{~mol} / \mathrm{s} ; \dot{n}_{1, C O_{2}}=x_{1, C O_{2}} \cdot \dot{n}_{1}=1.8 \mathrm{~mol} / \mathrm{s}$ $\dot{n}_{2}=4 \cdot \dot{n}_{1, C O_{2}}=7.2 \mathrm{~mol} / \mathrm{s}$
$\rightarrow \dot{n}_{4, H_{2}}=7.2 \mathrm{~mol} / \mathrm{s} ; \dot{n}_{4, C H_{4}}=2.2 \mathrm{~mol} / \mathrm{s} ; \dot{n}_{4, C O_{2}}=1.8 \mathrm{~mol} / \mathrm{s} ; \dot{n}_{4, \mathrm{H}_{2} \mathrm{O}}=0 \mathrm{~mol} / \mathrm{s}$
2. Determine the mole flows of $\dot{n}_{5}$ :
$\dot{n}_{5, C O_{2}}=\dot{n}_{4, C O_{2}} *\left(1-C_{C O_{2}}\right)=0.09 \mathrm{~mol} / \mathrm{s}$
$\dot{n}_{5, \mathrm{CH}_{4}}=\dot{n}_{4, \mathrm{CH}_{4}}+\dot{n}_{4, \mathrm{CO}_{2}} * C_{\mathrm{CO}_{2}}=3.91 \mathrm{~mol} / \mathrm{s}$
$\dot{n}_{5, H_{2}}=4 * \dot{n}_{4, C O_{2}} *\left(1-C_{C O_{2}}\right)=0.36 \mathrm{~mol} / \mathrm{s}$
$\dot{n}_{5, \mathrm{H}_{2} \mathrm{O}}=2 * \dot{n}_{4, \mathrm{CO}_{2}} * C_{\mathrm{CO}_{2}}=3.42 \mathrm{~mol} / \mathrm{s}$
3. Determine the heat stream, which must be dissipated to reach $\mathrm{T}_{5}=320^{\circ} \mathrm{C}$ in the reactor:

$\dot{Q}_{\text {out }, M}=\dot{H}_{5}-\dot{H}_{4}+\dot{Q}_{\text {reac }}$
$\dot{H}_{5}=\dot{n}_{5} \cdot c_{p, 5 m i x} \cdot T_{5}=158.7 \mathrm{~kW}$
$\dot{H}_{4}=\dot{n}_{4} \cdot c_{p, 4 m i x} \cdot T_{4}=192.8 \mathrm{~kW}$
(Annotation: $\dot{H}_{5}<\dot{H}_{4}$, although $T_{5}>T_{4}$ because of the mole decreasing reaction: $\dot{n}_{5}<\dot{n}_{4}$ )

$$
\begin{gathered}
\dot{Q}_{\text {reac }}=C_{C O_{2}} \cdot \dot{n}_{4, C O_{2}} \cdot \Delta H_{R}^{0}=-281.6 \mathrm{~kW} \\
\rightarrow \quad \dot{Q}_{\text {out }, M}=158.7 \mathrm{~kW}-192.8 \mathrm{~kW}-281.6 \mathrm{~kW}=
\end{gathered}
$$

$$
=-315.7 \mathrm{~kW}
$$

Thus a heat stream of 326.8 kW must be dissipated in order to reach a reactor temperature of $320^{\circ} \mathrm{C}$.

## ii. Equilibrium Constant

Calculate the equilibrium constant $\mathrm{K}_{\text {eq }}$ with thermodynamic data and check if the reaction reaches its equilibrium state. Use the NIST database in the appendix for standard enthalpy and entropy calculations at $\mathrm{T}_{\text {meth }}$.

1. Calculate Standard Enthalpy of formation and Entropy for each component.

Correlation used by NIST database:
$\Delta H_{f}^{0}(T)=\mathrm{A} * \mathrm{t}+\mathrm{B}^{*} \mathrm{t}^{2} / 2+\mathrm{C}^{*} \mathrm{t}^{3} / 3+\mathrm{D}^{*} \mathrm{t}^{4} / 4-\mathrm{E} / \mathrm{t}+\mathrm{F}-\mathrm{H}+\Delta H_{f}^{0}\left(T^{0}\right)$ $\Delta S_{-}^{0}(T)=\mathrm{A} * \ln (\mathrm{t})+\mathrm{B} * \mathrm{t}+\mathrm{C}^{*} \mathrm{t}^{2} / 2+\mathrm{D}^{*} \mathrm{t}^{3} / 3-\mathrm{E} /\left(2 * \mathrm{t}^{2}\right)+\mathrm{G}$

Coefficients A to H can be found in the appendix for each component.
The Standard Enthalpy of formation $\Delta H_{f, i}^{0}(T)$ and Entropy $\Delta S_{i}^{0}(T)$ for each component at T= 593.15 K are:

| Enthalpy, kJ/mol |  |  |  |
| :--- | :--- | :--- | :--- |
| H2 | CO2 | CH4 | H20 |
| 8.61 | -380.94 | -62.13 | -231.75 |


| Entropy, J/(mol*K) |  |  |  |
| :--- | :--- | :--- | :--- |
| H2 | C02 | CH4 | H20 |
| 150.74 | 242.74 | 215.39 | 212.63 |

The total Standard Enthalpy of formation and Entropy is: $v_{i}>0$ as product and $v_{i}<0$ as educt for component $i$
$\Delta H_{R}^{0}(T)=\sum_{i} v_{i} \cdot \Delta H_{f, i}^{0}(T)=-179.13 \mathrm{~kJ} / \mathrm{mol}$
$\Delta S_{R}^{0}(T)=\sum_{i}^{i} v_{i} \cdot \Delta S_{i}^{0}(T)=-205.05 \mathrm{~J} /(\mathrm{mol} \mathrm{K})$
2. Calculate Gibb's Enthalpy $\Delta G_{R}^{0}(T)$ :
$\Delta G_{R}^{0}(T)=\Delta H_{R}^{0}(T)-T \cdot \Delta S_{R}^{0}(T)=-57.5 \mathrm{~kJ} / \mathrm{mol}$
3. Calculate the equilibrium constant:
$K_{\text {eq }}=e^{-\Delta G_{R}^{0}(T) / R T}\left\{p_{0}^{\sum v_{i}}\right\}=1.16 \cdot 10^{5} 1 /$ bar $^{2}$
The term in the braces is ' 1 ' with $p_{0} \approx 1$ bar, but is retained to keep the equation dimensionally correct.
$K_{e q} \gg 1$, equilibrium of reaction is on the product side
4. Reached the reaction its equilibrium state? In equilibrium $K_{p}=K_{e q}$.
$K_{p}=\prod_{i} p_{i}^{v_{i}}=\frac{p_{C H_{4}} \cdot p_{H_{2} \mathrm{O}}^{2}}{p_{\mathrm{CO}_{2}} \cdot p_{\mathrm{H}_{2}}^{4}}=\frac{2.513 \mathrm{bar} \cdot 2.198^{2} \mathrm{bar}^{2}}{0.058 \mathrm{bar}^{2} \cdot 0.231^{4} \mathrm{bar}^{4}}=7.35 \cdot 10^{4} 1 / \mathrm{bar}^{2}$
$K<K_{e q} \rightarrow$ reaction reached not equilibrium (due to the low temperature of $320^{\circ} \mathrm{C}$, where the catalyst do not show a high activity).
iii. Equilibrium constant and methanation with steam: The catalyst starts to deactivate due to carbon deposition. A countermeasure is the injection of steam into the reactor. How does the steam affect the maximum conversion rate of carbon dioxide? Show it in a qualitative way with the equation $K_{p}=\Pi_{i} p_{i}^{v_{i}}$ for component i.
$\mathrm{K}_{\mathrm{eq}}$ is the maximum achievable value of K at given temperature T and for the given methanation reaction. If steam is added, the partial pressure of water $p_{H_{2} O}^{2}$ in the $\mathrm{K}_{\mathrm{p}}$ - equation is increasing and with that the K -value for the same conversion rate of carbon dioxide. Thus the maximum Kvalue $K_{\text {eq }}$ is reached with lower conversion rate.
$K_{p}=\prod_{i} p_{i}^{v_{i}}=\frac{p_{\mathrm{CH}_{4}} \cdot p_{\mathrm{H}_{2} \mathrm{O}}^{2}}{p_{\mathrm{CO}_{2}} \cdot p_{\mathrm{H}_{2}}^{4}}$; for
$C_{\mathrm{CO}_{2}}=$ const. and $p_{\mathrm{H}_{2} \mathrm{O}}^{2} \uparrow \rightarrow K_{p} \uparrow$, while $K_{\text {eq }}=$ const. (at fixed temp.)

For increasing $\mathrm{CO}_{2}$ conversions the K -value reaches faster the value of $\mathrm{K}_{\text {eq }}$, where the conversion reaches its thermodynamic limit $C_{\mathrm{CO}_{2}}^{e q}$. Thus:
$C_{C O_{2}}^{e q, \text { steam }}<C_{\mathrm{CO}_{2}}^{e q}$
The maximum possible conversion rate of carbon dioxide is decreasing with the addition of steam. Hence the addition of steam in order to protect the catalyst against deactivation should be as low as possible.
iv. Equilibrium constant and operational conditions in the reactor: How can the effect of steam addition be compensated? Show it in a qualitative way with the equation $K=\Pi_{i} p_{i}^{v_{i}}$ for components $i$ at constant temperature $\mathrm{T}_{\text {meth }}$.

The decreasing conversion rate of $\mathrm{CO}_{2}$, due to steam addition can be compensated by the increase of pressure in the reactor ( $\mathrm{p}^{\text {low }}<\mathrm{p}^{\text {high }}$ ):
$K=\prod_{i} p_{i}^{v_{i}}=\frac{p_{\mathrm{CH}_{4}} \cdot p_{\mathrm{H}_{2} \mathrm{O}}^{2}}{p_{\mathrm{CO}_{2}} \cdot p_{\mathrm{H}_{2}}^{4}}=\frac{p x_{\mathrm{CH}_{4}} \cdot p^{2} x_{\mathrm{H}_{2} \mathrm{O}}^{2}}{p x_{\mathrm{CO}_{2}} \cdot p^{4} x_{\mathrm{H}_{2}}^{4}}$
Due to the higher degree of the reactant hydrogen ( $44^{\text {th }}$ degree) in comparison with the other components, the denominator of K is increasing stronger than the numerator for constant carbon dioxide conversion. Thus K is decreasing for the same $\mathrm{CO}_{2}$ Conversion.
$K_{p}^{\text {low }}>K_{p}^{\text {high }}$
This leads to a higher maximum $\mathrm{CO}_{2}$ Conversion $C_{\mathrm{CO}_{2}}^{e q}$ (explained in task iii).
$C_{\mathrm{CO}_{2}}^{\text {eq,pow }}<\mathrm{C}_{\mathrm{CO}_{2}}^{\text {eq,phigh }}$

## Appendix:

## NIST - Database

## Correlations:

$\mathrm{H}^{\circ}-\mathrm{H}^{\circ}{ }_{298.15}=\mathrm{A} * \mathrm{t}+\mathrm{B}^{*} \mathrm{t}^{2} / 2+\mathrm{C} * \mathrm{t}^{3} / 3+\mathrm{D}^{*} \mathrm{t}^{4} / 4-\mathrm{E} / \mathrm{t}+\mathrm{F}-\mathrm{H}$
$\mathrm{S}^{\circ}=\mathrm{A} * \ln (\mathrm{t})+\mathrm{B}^{*} \mathrm{t}+\mathrm{C}^{*} \mathrm{t}^{2} / 2+\mathrm{D}^{*} \mathrm{t}^{3} / 3-\mathrm{E} /\left(2^{*} \mathrm{t}^{2}\right)+\mathrm{G}$
$\mathrm{H}^{\circ}=$ standard enthalpy (kJ/mol)
$\mathrm{S}^{\circ}=$ standard entropy $(\mathrm{J} / \mathrm{mol} * \mathrm{~K})$
$\mathrm{t}=$ temperature $(\mathrm{K}) / 1000$.
$\mathrm{T} 0=298.15 \mathrm{~K}$

## Hydrogen

$\Delta_{\mathrm{f}} \mathrm{H}^{\circ}{ }_{\text {gas }}(\mathrm{T} 0)=0.0 \mathrm{~kJ} / \mathrm{mol}$

| Temperature <br> (K) | 298.- 1000. | $\mathbf{1 0 0 0 . - 2 5 0 0 .}$ | $\mathbf{2 5 0 0} \mathbf{- 6 0 0 0 .}$ |
| :--- | :--- | :--- | :--- |
| A | 33.066178 | 18.563083 | 43.413560 |
| B | -11.363417 | 12.257357 | -4.293079 |
| C | 11.432816 | -2.859786 | 1.272428 |
| D | -2.772874 | 0.268238 | -0.096876 |
| E | -0.158558 | 1.977990 | -20.533862 |
| F | -9.980797 | -1.147438 | -38.515158 |
| G | 172.707974 | 156.288133 | 162.081354 |
| H | 0.0 | 0.0 | 0.0 |
| Reference | Chase, 1998 | Chase, 1998 | Chase, 1998 |
| Comment | Data last reviewed in <br> March, 1977; New <br> parameter fit October <br> 2001 | Data last reviewed in <br> March, 1977; New <br> parameter fit October <br> 2001 | Data last reviewed in <br> March, 1977; New <br> parameter fit October <br> 2001 |

Carbon dioxide
$\Delta_{\mathrm{f}} \mathrm{H}^{\circ}{ }_{\text {gas }}(\mathrm{T} 0)=-393.52 \mathrm{~kJ} / \mathrm{mol}$

| Temperature <br> $(\mathrm{K})$ | 298. - 1200. | $\mathbf{1 2 0 0 .} \mathbf{- 6 0 0 0 .}$ |
| :--- | :--- | :--- |
| A | 24.99735 | 58.16639 |
| B | 55.18696 | 2.720074 |
| C | -33.69137 | -0.492289 |
| D | 7.948387 | 0.038844 |
| E | -0.136638 | -6.447293 |
| F | -403.6075 | -425.9186 |
| G | 228.2431 | 263.6125 |
| H | -393.5224 | -393.5224 |
| Reference | $\underline{\text { Chase, 1998 }}$ | Chase, 1998 |
| Comment | Data last reviewed in September, <br> 1965 | Data last reviewed in September, <br> 1965 |

## Methane

$\Delta_{\mathrm{f}} \mathrm{H}^{\circ}{ }_{\text {gas }}(\mathrm{T} 0)=-74.87 \mathrm{~kJ} / \mathrm{mol}$

| Temperature (K) | 298. - 1300. | $\mathbf{1 3 0 0 .} \mathbf{- 6 0 0 0 .}$ |
| :--- | :--- | :--- |
| A | -0.703029 | 85.81217 |
| B | 108.4773 | 11.26467 |
| C | -42.52157 | -2.114146 |
| D | 5.862788 | 0.138190 |
| E | 0.678565 | -26.42221 |
| F | -76.84376 | -153.5327 |
| G | 158.7163 | 224.4143 |
| H | -74.87310 | -74.87310 |
| Reference | Chase, 1998 | Chase, 1998 |
| Comment | Data last reviewed in March, 1961 | Data last reviewed in March, 1961 |

## Water (gaseous)

$\Delta_{\mathrm{f}} \mathrm{H}^{\circ}{ }_{\text {gas }}(\mathrm{T} 0)=-241.83 \mathrm{~kJ} / \mathrm{mol}$

| Temperature (K) | $\mathbf{5 0 0 .} \mathbf{- 1 7 0 0 .}$ | $\mathbf{1 7 0 0 .} \mathbf{- 6 0 0 0 .}$ |
| :--- | :--- | :--- |
| A | 30.09200 | 41.96426 |
| B | 6.832514 | 8.622053 |
| C | 6.793435 | -1.499780 |
| D | -2.534480 | 0.098119 |
| E | 0.082139 | -11.15764 |
| F | -250.8810 | -272.1797 |
| G | 223.3967 | 219.7809 |
| H | -241.8264 | -241.8264 |
| Reference | Chase, 1998 | Chase, 1998 |
| Comment | Data last reviewed in March, 1979 | Data last reviewed in March, 1979 |

