Renewable Energy Technologies I
529 0193-00L
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Exercise: Wind and Ocean Energy (Answers/Questions)

Solutions will be put online: 15.11.2016
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1. Wind energy

1.1 If the wind speed is 11.5m/s and the speed after the turbine is 8m/s, what is the power extraction coefficient of this wind turbine? (air density 1.225 kg/m^3)

The kinetic energy per time available in wind

$$P = 0.5\rho_{air}AU^3$$

Extracted power

$$P' = 0.5\rho_{air}AU'(U^2 - U"^2)$$

The power extraction coefficient is

$$C_p = \frac{P'}{P} = \frac{U'(U^2 - U"^2)}{U^3} = 0.44$$

1.2 The rated output power for a turbine model at 15 m/s is 3 MW. The rotor diameter is 90m. The rotor rotates at a constant frequency of 0.198 Hz. Please calculate the tip to speed ratio and power conversion coefficient of this model.

The linear velocity of the tip:

$$v_t = \omega * R = 2\pi f * \frac{D}{2} = 2\pi * 0.198Hz * \frac{90m}{2} = 56m/s$$

The tip to speed ratio:

$$r = \frac{v_t}{U} = \frac{56m/s}{15m/s} = 3.7$$

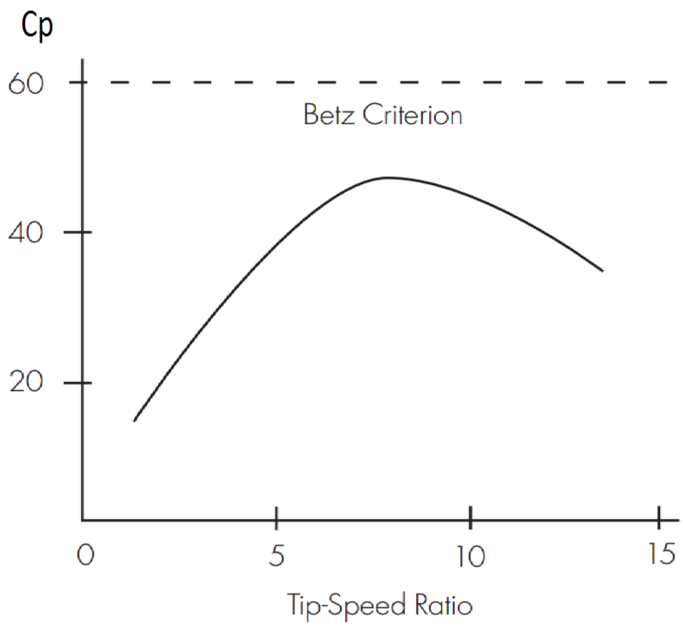
The wind power at 15m/s:

$$P = \frac{1}{2}\rho AU^3 = \frac{1}{2} * \frac{1.225kg}{m^3} * \pi * (45m)^2 * \left(\frac{15m}{s}\right)^3 = 13MW$$

The power conversion coefficient:

$$\varepsilon = \frac{3MW}{13MW} = 23\%$$

1.3 Based on the figure below, what do you expect about the power extraction coefficient of the model in question 1.2 when the wind speed is at 8 m/s? In addition, what do you expect about the output power at this wind speed? Is it more than the output power at 15m/s? The rotor rotates at the constant frequency. Give your arguments.



The power extraction coefficient goes up when the wind speed is 8m/s.

Tip to speed ratio at 8m/s:

$$r = \frac{56m/s}{8m/s} = 7$$

According to the figure, the C_p is at maximum with this tip to speed ratio.

The wind power at 8m/s:

$$P = \frac{1}{2}\rho A U^3 = \frac{1}{2} * \frac{1.225kg}{m^3} * \pi * (45m)^2 * \left(\frac{8m}{s}\right)^3 = 2MW$$

The estimated output power is
2MW*0.48=0.96MW, smaller than at 15m/s.

2. Tidal energy

The Bay of Funday is known for having the highest tidal range in the world. The tidal range could approach 17m in extremity. About 110 billion tons of water flow into and out of the bay in one cycle. Calculate the total potential tidal energy of the Bay of Funday in this extreme case in one year by using **bidirectional** turbines. (Gravity acceleration 9.8 m/s^2)

Number of tidal cycles per year

$$n_{cyc} = \frac{24h * 365}{12.4h} = 706.5$$

Number of times for the tide to drive turbines

$$n = 2n_{cyc} = 1413$$

Total tidal energy per year

$$E = \frac{nmgh}{2} = \frac{1413 * 1.1 * 10^{14}kg * 9.8\,m/s^2 * 17m}{2} = 1.3 * 10^{19}J$$

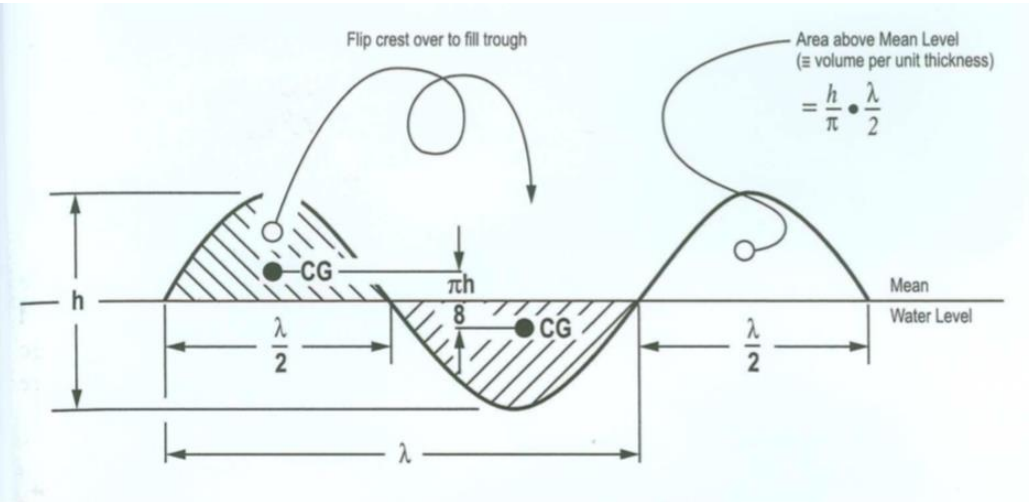
$$= 3.6 * 10^{12}kWh$$

3. Wave energy

Based on the figure below, deduce the the wave power per unit length.

Suppose crest-to-trough height of wave is h, wavelength is λ, wave period is T, and the wave shape follows the sine function.

Given: surface wavelength $\lambda = \frac{gT^2}{2\pi}$



There are two mistakes in the tutorial at the lecture. The corrections are made below.

Mass of a crest or trough per unit length:

$$m = \rho \cdot \int_0^{\frac{\lambda}{2}} \frac{h}{2} sin\frac{2\pi}{\lambda}x\,dx = \rho\frac{\lambda h}{2\pi}$$

(this calculation was wrong in the tutorial)

Next, we want to know the energy stored in a wavelength. The total energy of a crest is **not** the potential energy from crest to sea level, but it's the potential energy from a crest to a trough. (Think the other way round, we need the energy to raise the water from the trough below sea level to the crest, above sea level).

Energy stored in a crest per unit length: (energy stored in a half wavelength)

$$\Delta E = \mathrm{mg}\Delta h = \rho \frac{\lambda h}{2\pi} g \cdot \frac{\pi h}{8} = \rho\lambda g h^2/16$$

Energy stored in a full wavelength is then $2\Delta E = \rho\lambda g h^2/8$

This is the energy gain per unit length for a whole wavelength.

As a wavelength corresponds to a wave period,

the wave power per unit length is then:

$$P = \frac{2\Delta E}{T} = \frac{\rho\lambda g h^2/8}{T} = \frac{\rho g^2 T h^2}{16\pi}$$

In the tutorial, I mentioned the process for a quarter wave period or a half wave period. That is actually also wrong. In this calculation, we only calculate the energy stored in the wave. So the wave is not doing work to other media. The total amount of energy in the wave remains the same. Once we calculate the total energy in a wavelength, we don't have to relate the process to a half wave period or a quarter wavelength.

With ρ=1g/cm^3, g=9.8m/s, and π=3.14,

$$P = \frac{\frac{1g}{cm^3} * \left(\frac{9.8m}{s^2}\right)^2 * T * h^2}{16 * 3.14} \approx 1.91 \frac{kW}{sm^3} T(s) * h(m)^2$$