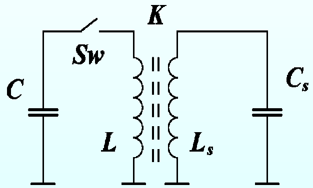
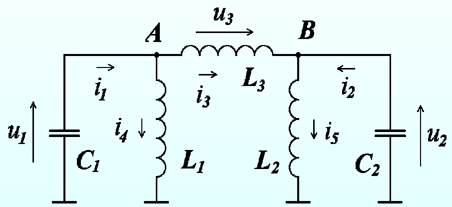
# Air-core transformer electrical modeling

## Two ideal coupled resonant circuits




Simplified Circuit




Equivalent Circuit

$$\begin{cases} \ddot{u}_1 + k_1 u_1 - k_2 u_2 = 0 \\ \ddot{u}_2 + k_1 u_2 - k_2 u_1 = 0 \end{cases}$$

$$k_1 = \frac{1}{LC(1-K^2)}$$

$$k_2 = \frac{K}{LC(1-K^2)}$$

Differential system

$$u_1 = \frac{U}{2}\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$$

$$u_2 = \frac{U}{2}\left[\cos(\omega_1 t) - \cos(\omega_2 t)\right]$$

$$\omega_1 = \sqrt{\frac{1}{LC(1+K)}}$$

$$\omega_2 = \sqrt{\frac{1}{LC(1-K)}}$$

Solution

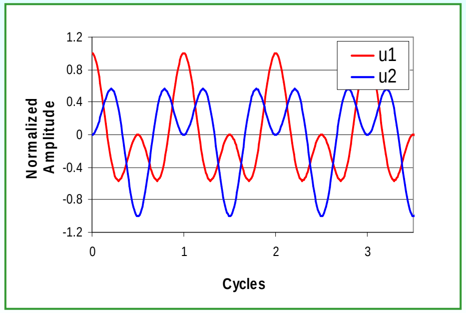
The differential system has no general analytic solution except for $LC = L_sC_s$. The oscillations of the system are not necessarily periodic.

$$2\omega_1 = \omega_2$$

$$2\sqrt{\frac{1}{LC(1+K_c)}} = \sqrt{\frac{1}{LC(1-K_c)}}$$

$$K_c = 0.6$$

Critical coupling

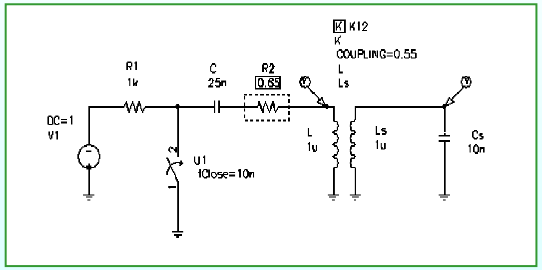



Wave forms

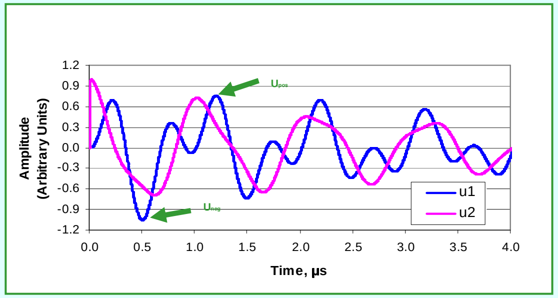
For critical coupling ($K_c$=0.6) the oscillation consist of the fundamental and the second harmonic. The signal is not symmetric with respect to zero. The ratio between pos. and neg. maximum is 1.78.

# Air-core transformer electrical modeling

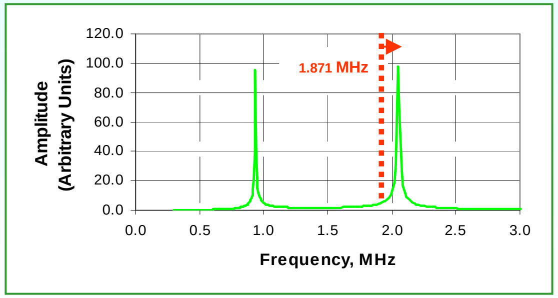
## Numeric simulations of lossy air-core resonant transformer




Equivalent PSpice Circuit




Time domain




Frequency domain

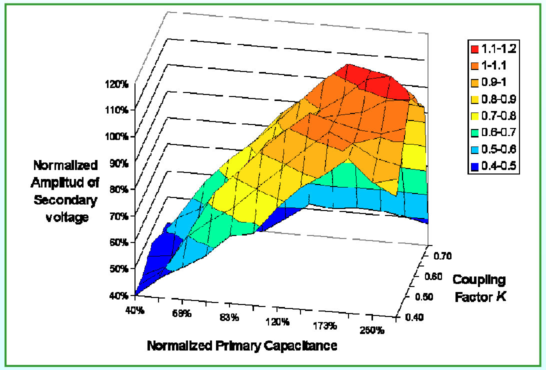
System sensitivity due to components value variation was examined using numeric simulations.

Small coupling factor deviations from Kc = 0.6 can be compensated by changing the tuning of the resonators.

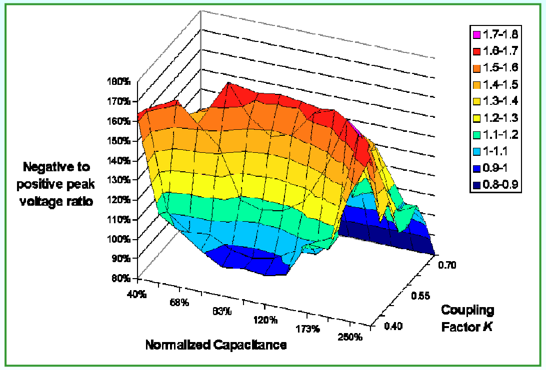
Design and construction of a transformer with a precise coupling factor is a challenge.

# Air-core transformer electrical modeling

## Parametric study of lossy air-core resonant transformer




Normalized secondary peak voltage amplitude as function of coupling and primary capacitance with 30% loss factor




Negative to positive peak voltage ratio of secondary voltage as function of coupling and primary capacitance with 30% loss factor

The study used two criteria:

- Maximum output amplitude
- Maximum positive to negative peak voltage ratio

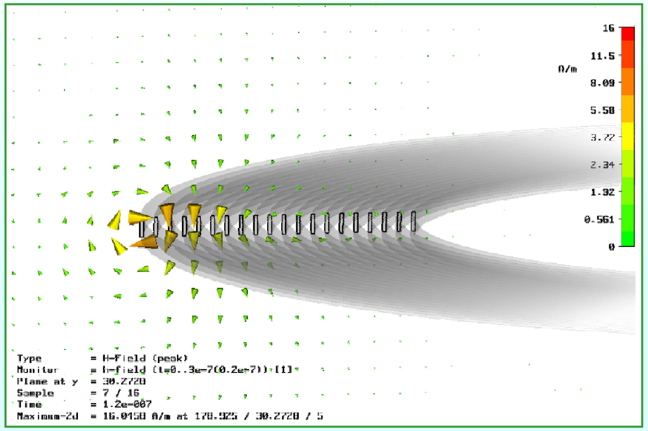
The varied parameters were **coupling factor** and **primary resonator tuning** (varying the primary capacitance).

The value of the resistor in primary side was iteratively determined to ensure that the loss factor is kept constant (~30% per cycle)

The determination of negative to positive peak voltage ratio was difficult because, if there is no significant loss in the system, after many cycles the negative peak could appear as positive one.

# Air-core transformer magnetic modeling

## Numerical and physical modeling of the transformer




Electromagnetic field distribution simulation

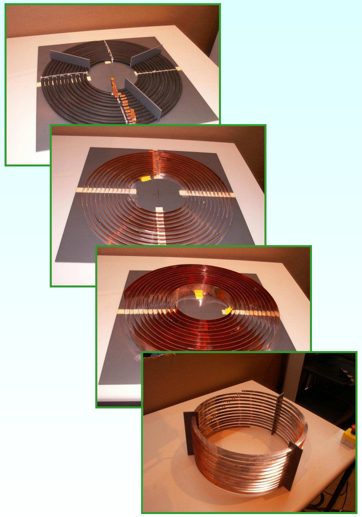
The full 3D numerical simulation is limited by:

- Ø **enormous aspect ratio;**
- Ø **excitation pulse length limitations**

Simplified geometry was a necessity.

Measurements on scaled physical models:

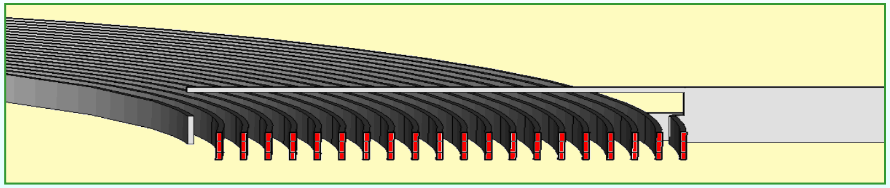
- Ø confirmed 3D numeric simulations
- Ø confirmed scalability of the air-core coils / transformers
- Ø defined the best geometry of the air-core transformer
- Ø showed the sensitivity of the transformer to surrounding space and close metal objects



ç Used physical models

# Air-core transformer magnetic modeling

## Physical models vs. 3D numeric simulations



An additional conductor is placed in 3D model in order to shorten the secondary.

| Number of turns | | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|---|
| Measured | L, nH | 1090 | 1090 | 1090 | 1090 | 1090 |
| | Lsh, nH | 276 | 451 | 553 | 619 | 666 |
| | coupling, - | 0.864 | 0.766 | 0.702 | 0.657 | 0.624 |
| Simulated | L, nH | 1101 | 1101 | 1101 | 1101 | 1101 |
| | Lsh, nH | 290 | 436 | 563 | 625 | 678 |
| | coupling, - | 0.858 | 0.761 | 0.699 | 0.658 | 0.620 |
| L relative error, % | | 1.01% | 1.01% | 1.01% | 1.01% | 1.01% |
| Lsh relative error, % | | 5.07% | 2.66% | 1.81% | 0.97% | 1.80% |
| Coupling factor error, % | | 0.68% | 0.58% | 0.41% | 0.03% | 0.62% |

Measured values vs. simulated values and the relative errors between them.

**Coupling factor** $K$ is given by the formula:

$$K = \sqrt{\frac{L - L_{sh}}{L}}$$

where $L$ and $Lsh$ are the values of primary inductance when the secondary is open and shorted. The coils are used in autotransformer mode and one turn primary.

The relative error between the simulated and measured inductance values was <6% and coupling values <1%. Simulation conditions were kept strictly unchanged for $L$ and $Lsh$ (meshing, coil geometry, etc.).

Insensitivity to coupling errors because meshing errors tend to cancel.

# Air-core transformer magnetic modeling

## Air-core transformers scalability

$$\vec{B} = \frac{\mu . i}{4\pi} \int \frac{d\vec{L} \times \vec{r}}{r^3} \qquad B \sim \frac{1}{r}$$

$$\Phi = \int_S \vec{B} \bullet d\vec{s} \qquad \Phi \sim r$$

$$L = \frac{\Phi}{i} \qquad L \sim r$$

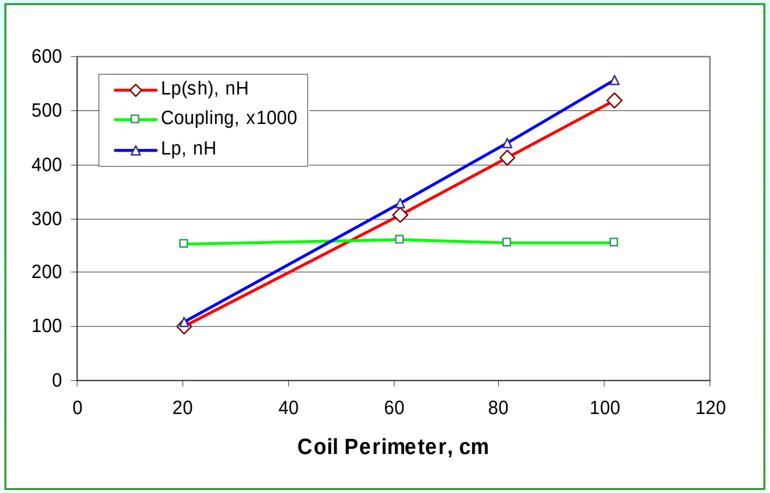
$$M = \frac{\Phi_{21}}{i_1} \qquad M \sim r$$

$$K = \frac{M}{\sqrt{L_P L_S}} \qquad K = const$$

Basic relations

Mutual inductance of air-core transformers depends only on geometry.

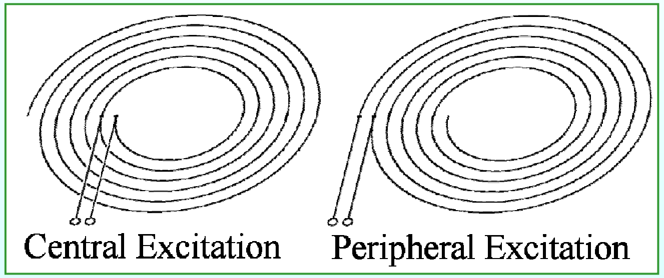
Using the definitions of magnetic flux density B, magnetic flux Φ, self inductance L, mutual inductance M and coupling factor K, **inductances scale linearly** with the geometrical dimensions and the **coupling factor stays constant**.



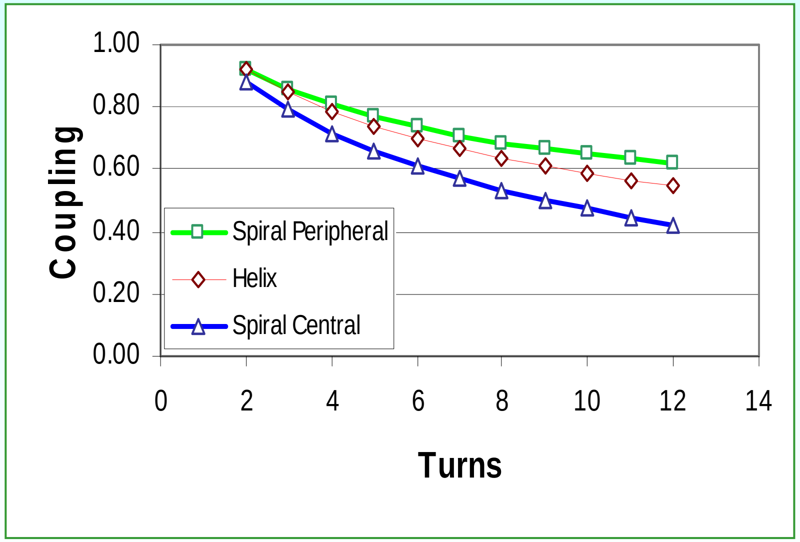

Measured results

# Air-core transformer magnetic modeling

## Optimal air-core transformer geometry – coil shape and excitation




Excitation scheme




Coupling factor vs. number secondary turns for three different configurations

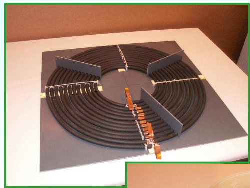
To optimize the coupling factor of air-core transformer the influence of the coils shape was studied. The model coils were used in autotransformer mode.

Three basic configurations were compared:

- Spiral shape with peripheral excitation
- Spiral shape with central excitation
- Helical shape

Highest coupling factor is obtained for **peripherally excited spiral secondary coil**.

The higher the number of turns (higher step up ratio) the lower the coupling.

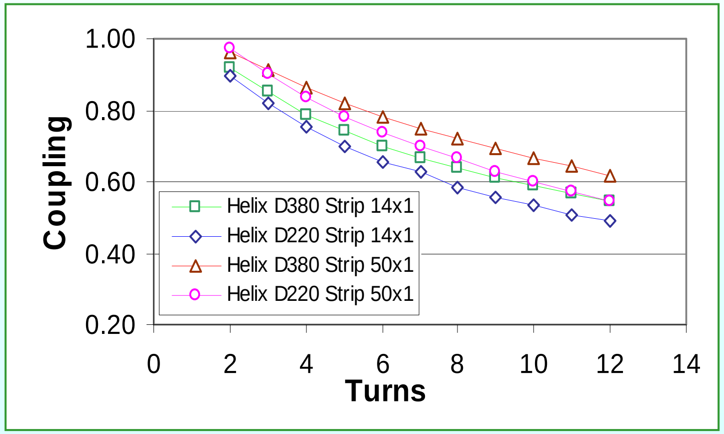


ç Spiral coil model

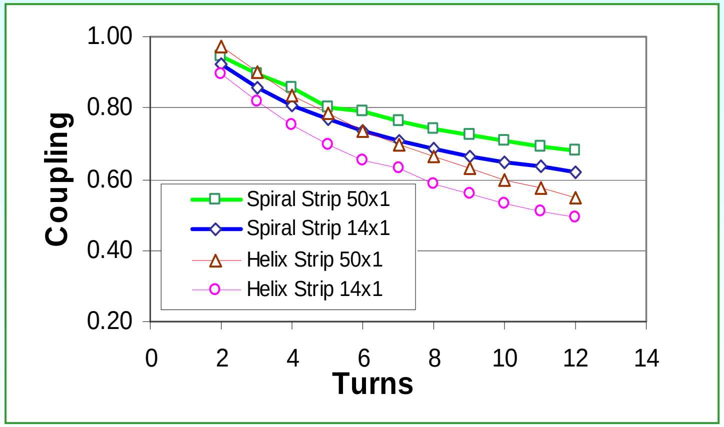
ç Helical coil model

# Air-core transformer magnetic modeling

## Optimal air-core transformer geometry – coil dimensions and conductor cross-section




Influence of coil diameter

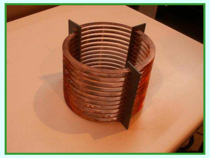
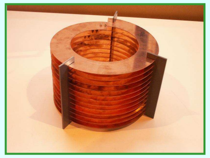
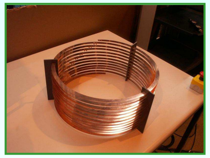
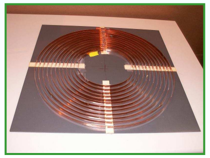



Influence of conductor cross section dimensions

Another factor that influences the coupling is the transformer diameter and the conductor cross-section.

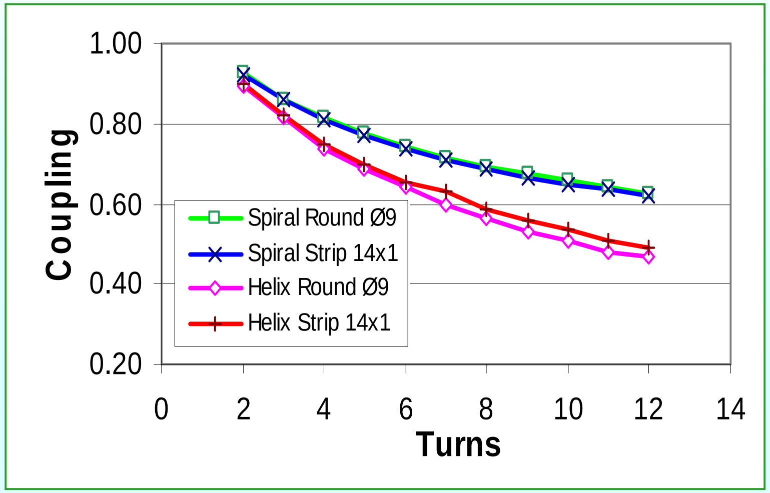
The larger the transformer diameter and the wider the strip the higher the coupling.

Conductors span is kept 15mm for all coils.

| Conductor / Shape | Strip 14x1 mm | Strip 50x1 mm |
| --- | --- | --- |
| Helix D220 mm |  |  |
| Helix D380 mm |  |  |
| Spiral Dm380 mm |  |  |

# Air-core transformer magnetic modeling

## Optimal air-core transformer geometry – conductor cross section




Influence of coil cross-section shape

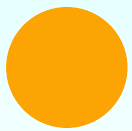
The conductor cross-section shape was studied. The two extreme cases was investigated. Conductors span was kept constant.

- **Round conductor**
- **Strip conductor (width >> thickness)**

Coupling factor depends on the **perimeter** of the conductor cross-section and it is relatively insensitive to the conductor cross-section shape.

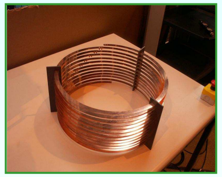
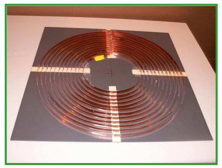
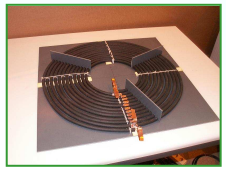
Strip 14x1 mm

Perimeter 30 mm



Round D9 mm

Perimeter 28.3 mm

| Conductor / Shape | Strip 14x1 mm | Round D9 mm |
|---|---|---|
| Helix D380 mm |  |  |
| Spiral Dm380 mm |  |  |

# Air-core transformer magnetic modeling

## Optimal air-core transformer geometry – proximity effects

There was a concern about the deterioration of the transformer performance due to limited surrounding space and near large metal objects.

| Surrounding space influence on air-core transformer model - Scaling coefficient 3.64 (04 Oct 2004) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Primary Loop type | Free space | | | In the tank | | | | | |
| | | | | Without central piece | | | With central piece | | |
| | L, nH | Lsh, nH | K | L, nH | Lsh, nH | K | L, nH | Lsh, nH | K |
| D165/14/0.5mm (1MHz) | 330 | 210 | 0.60 | 250 | 185 | 0.51 | 248 | 184 | 0.51 |
| D165/28/0.5mm (1MHz) | 270 | 170 | 0.61 | 193 | 140 | 0.52 | 192 | 140 | 0.52 |

Coupling factor deteriorated by 15% due to limited surrounding space. The metal stalk in the center of the transformer did not show any significant further deterioration.

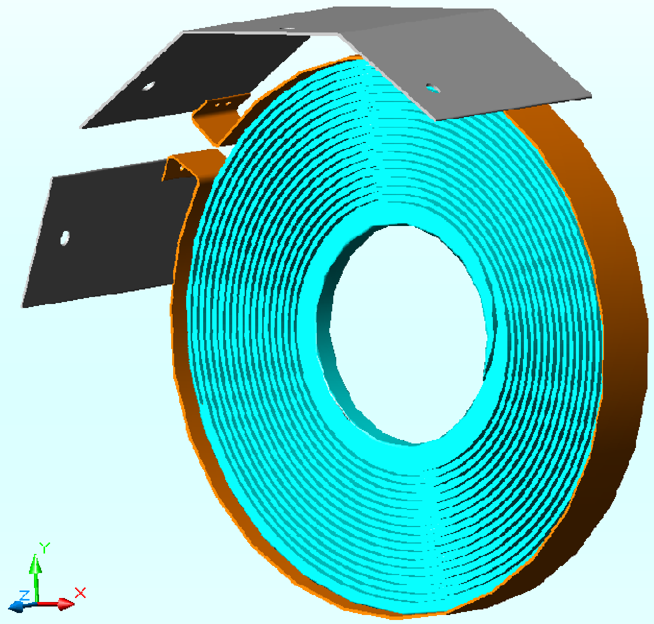
| Primary and secondary estimation | | | | | |
|---|---|---|---|---|---|
| Coil | Model | Estimation | Measured | Error | Note |
| Primary (Oct 2004) | **248 nH** | **903 nH** | **1078 nH** | 19% | Since the final shape of the coil was not known the model was not exact |
| Primary (Dec 2006) | **150 nH** | **1088 nH** | **1078 nH** | 1% | |
| Secondary (Oct 2004) | **290 uH** | **141 uH** | **133 uH** | 6% | Since the final shape of the coil was not known the model was not exact |

The study gave an input for the electrical simulations.

Later on, the comparison between the estimated values based, on the scaled models, and the real transformer values showed good agreement.

# 500kV Pulser design

## Air-core transformer




Air-core transformer (3D model)

Single turn Cu strip primary, transformer base with spiral channel for 16.5 turn secondary winding and connecting terminals (secondary conductor is not shown)

The shortest possible secondary length gives the shortest pulse length. A planar spiral secondary coil with peripheral single-turn primary was chosen.

The secondary consist of 16.5 turns of 4mm diameter copper tube (compatible with 3.6 mm semi-rigid coax cable)

To prevent voltage breakdown the transformer is operated in sulfur hexafluoride ($SF_6$) gas at up to 5 bar.

The transformer base is made out of Acrylglas (Plexiglas). OD 600 mm, ID 240 mm. On one face is a spiral channel for the conductor and on the other corrugations to prevent surface discharges.