Zuoz #2

Note Title 2/10/2014

Lecture 1 – Introduction to EFT in general
- General philosophy
- Explicit example
- Matching, power counting etc

Lecture 2 – Varieties of EFT and advanced techniques
- Survey and limits of EFT
- Techniques outside of textbooks

Lecture 3 - Gravity as an EFT
- GR as a gauge theory
- How to think about quantum gravity
- Reliable aspects of gravity and QM

John Donoghue's web pages

Research and teaching pages for John Donoghue

HOME WELCOME NEWS RESEARCH INTERESTS TEACHING PERSONAL LINKS DYNAMICS OF THE STANDARD MODEL

Teaching

Zuoz summer school – August 2014

Lecture 1: Introduction to EFT 1 NOTES
Why do quantum calculations work, QED example, EFT 1.0 Effective Lagrangians, EFT as a QFT, Example - the sigma model, power counting, matching at tree level, matching at one loop.

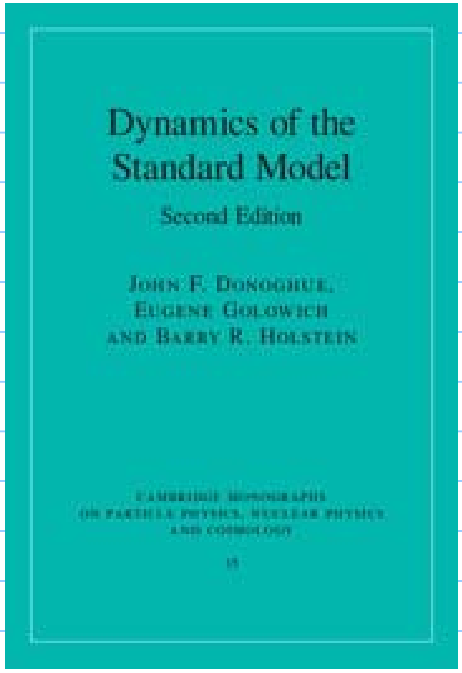
Lecture 2: Introduction to EFT 2 NOTES

Lecture 3: General Relativity as an Effective Field Theory NOTES

Supplementary material Excerpt on heat kernel methods from Dynamics of the Standard Model

RECENT CITATIONS

- Perturbative Quantum Gravity Comes of Age
- Inflation without quantum gravity
- Effective constraint algebras with structure functions
- The Schr\"odinger-Newton equation and its foundations
- Supersymmetry from Typicality
- Production and evaporation of higher dimensional black holes
- Rare top decay $t\rightarrow c\gamma$ in general THDM-III
- Full three-body problem in



Lecture

HE vs LE

HE $\Rightarrow$ LOCALITY $\Longrightarrow$ $\mathcal{L}_{eff}$

Expansion in Energy $\Rightarrow$ "power counting"

Construct $-\sigma$ model

Matching or measuring

EFT as QFT

Renormalized a "nonrenormalizeable" theory

- divergences local $\Rightarrow$ intr coeff
- coeff determined by full theory

Homework solution: $U(1)$

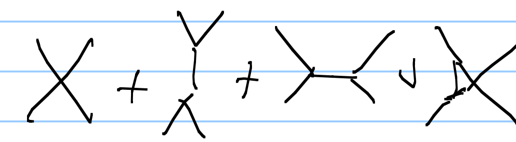
$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$\mathcal{L} = \partial_\nu\phi^*\partial^\nu\phi + \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

$$\langle\phi\rangle = v$$

$$\phi = \frac{1}{\sqrt{2}}(v + \sigma + i\chi)$$

$$\mathcal{L} = \partial_\nu\sigma\partial^\nu\sigma + \partial_\nu\chi\partial^\nu\chi - \frac{\mu^2}{2}\sigma^2 - \lambda v\sigma\chi^2 - \frac{\lambda}{4}(\sigma^2+\chi^2)^2$$

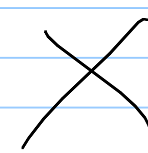


2 canc.

$$\phi = \frac{1}{\sqrt{2}}(v+\sigma)e^{i\chi/v}$$

$$\mathcal{L} = \left(\frac{v+\sigma}{v}\right)^2(\partial_\mu\chi\,\partial^\mu\chi)$$

$$\mathcal{L}_{eff} = \partial_\nu\chi\partial^\nu\chi - \frac{1}{\lambda v^4}\underline{(\partial_\nu\chi\partial^\nu\chi)^2}$$

 $\sim E^4$

Rules of EFT

1) What are Low E DOF & symmetries?
2) Write most general local $\mathcal{L}$, order in energy expansion
3) Start with lowest order $\mathcal{L}$, do QFT
4) Renormalize parameters
5) Match or measure

$\Longrightarrow$ predictions

Limits

$$\mathcal{M} = \frac{q^2}{v^2} + \frac{q^4}{v^4} l_1(\mu) + \frac{q^4}{16\pi^2 v^4} \ln q^2/\mu^2$$

$$\frac{q^4}{v^2 m_\sigma^2}$$

loops $\frac{q^2}{v^2}\left(1 + \frac{q^2}{16\pi^2 v^2} \ln q^2/\mu^2\right)$

Loops of $\mathcal{O}(1)$

$$= \frac{q^2}{v^2}\left(1 + \frac{q^2}{m_\sigma^2} + \dots\right)$$

fails when new DOF are dynamical

QCD Chiral Pert Theory

$\psi = \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} u \\ d \\ s \end{pmatrix}$

$m = 0$, $\mathcal{L} = \bar\psi_L \not{\partial} \psi_L + \bar\psi_R \not{\partial} \psi_R$

$\rightarrow$ $\psi_L \to L\,\psi_L$, $\psi_R \to R\,\psi_L$, $L, R \sim SU(2), SU(3)$

$\Longrightarrow \pi, K, \eta$ Goldstone boson $\Leftarrow$ L.O. DOF

$$\mathcal{L} = \frac{F^2}{4} \mathrm{Tr}\left(\partial_\mu U\, \partial^\mu U^\dagger\right)$$

$U = e^{i\vec{\tau}\cdot\vec{\pi}/F}$, $e^{i\lambda\cdot\phi/F}$

$U \to L\,U\,R^\dagger$

$$\rightarrow \mathcal{L} = \bar{\Psi} i \not{D} \Psi - \bar{\Psi}_L \mathcal{M} \Psi_R - \bar{\Psi}_R \mathcal{M}^\dagger \Psi_L$$

$\mathcal{M} \rightarrow L \mathcal{M} R^\dagger$ QCD when $\mathcal{M} = \begin{pmatrix} m_u & & 0 \\ 0 & m_d & \\ & & m_s \end{pmatrix}$

$$\mathcal{L} = \frac{F^2}{4} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{2B_0 F^2}{\cancel{4}} \mathrm{Tr}\left(\mathcal{M} U^\dagger + \mathcal{M}^\dagger U\right)$$

$\cancel{4}\begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \leftarrow$ $m_\pi^2 = B_0 (m_u + m_d)$

$m_K^2 = B_0 (m_s + m_u)$

Next order - Gasser Leutwyler

$$
\begin{aligned}
\mathcal{L}_4 &= \sum_{i=1}^{10} L_i O_i \\
&= L_1 \left[\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right)\right]^2 + L_2 \,\mathrm{Tr}\left(D_\mu U D_\nu U^\dagger\right) \cdot \mathrm{Tr}\left(D^\mu U D^\nu U^\dagger\right) \\
&+ L_3 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger\right) \\
&+ L_4 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right) \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \\
&+ L_5 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger \left(\chi U^\dagger + U \chi^\dagger\right)\right) + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)\right]^2 \\
&+ L_7 \left[\mathrm{Tr}\left(\chi^\dagger U - U \chi^\dagger\right)\right]^2 + L_8 \,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U \chi^\dagger U \chi^\dagger\right) \\
&+ i L_9 \,\mathrm{Tr}\left(L_{\mu\nu} D^\mu U D^\nu U^\dagger + R_{\mu\nu} D^\mu U^\dagger D^\nu U\right) + L_{10} \,\mathrm{Tr}\left(L_{\mu\nu} U R^{\mu\nu} U^\dagger\right)
\end{aligned}
$$

$\chi = 2B_0 M$

Table VII–1. Renormalized coefficients in the chiral lagrangian $\mathcal{L}_4$ given in units of 10^{-3} and evaluated at renormalization point $\mu = m_\rho$ [BiJ 12].

Coefficient	Value	Origin
L_1^r	1.12 ± 0.20	$\pi\pi$ scattering
L_2^r	2.23 ± 0.40	and
L_3^r	-3.98 ± 0.50	$K_{\ell 4}$ decay
L_4^r	1.50 ± 1.01	F_K/F_π
L_5^r	1.21 ± 0.08	F_K/F_π
L_6^r	1.17 ± 0.95	F_K/F_π
L_7^r	-0.36 ± 0.18	meson masses
L_8^r	0.62 ± 0.16	F_K/F_π
L_9^r	7.0 ± 0.2	rare pion
L_{10}^r	-5.6 ± 0.2	decays

[a]Determined only with the additional assumption of η-η' mixing.
[b]Vanishes in the $N_c \to \infty$ limit.

Table VII–2. The radiative complex of pion and kaon transitions.

Pions	Kaons
$\gamma \to \pi^+\pi^-$	$\gamma \to K^-K^+$
$\gamma\pi^+ \to \gamma\pi^+$	$\gamma K^+ \to \gamma K^+$
$\pi^+ \to e^+\nu_e\gamma$	$K^+ \to e^+\nu_e\gamma$
$\pi^+ \to \pi^0 e^+\nu_e$	$K \to \pi e^+\nu_e$
$\pi^+ \to e^+\nu_e e^+ e^-$	$K^+ \to e^+\nu_e e^+ e^-$
	$K^+ \to \pi^0 e^+\nu_e\gamma$

Table VII–3. Chiral predictions and data in the radiative complex of transitions.

Reaction	Quantity	Theory	Experiment
$\gamma \to \pi^+\pi^-$	$\langle r_\pi^2\rangle$ (fm^2)	0.45^a	0.45 ± 0.01
$\gamma \to K^+K^-$	$\langle r_K^2\rangle$ (fm^2)	0.45	0.31 ± 0.03
$\pi^+ \to e^+\nu_e\gamma$	$h_V\,(m_\pi^{-1})$	0.027	0.0254 ± 0.0017
	h_A/h_V	0.441^a	0.441 ± 0.004
$K^+ \to e^+\nu_e\gamma$	$(h_V + h_A)(m_K^{-1})$	0.136	0.133 ± 0.008
$\pi^+ \to e^+\nu_e e^+ e^-$	r_A/h_V	2.6	2.2 ± 0.3
$\gamma\pi^+ \to \gamma\pi^+$	$(\alpha_E + \beta_M)(10^{-4}\text{fm})$	0	0.17 ± 0.02
	$(\alpha_E - \beta_M)\,(10^{-4}\text{fm})$	5.6	13.6 ± 2.8
$K \to \pi e^+\nu_e$	$\xi = f_-(0)/f_+(0)$	-0.13	-0.17 ± 0.02
	λ_+ (fm^2)	0.067	0.0605 ± 0.001
	λ_0 (fm^2)	0.040	0.0400 ± 0.002

[a]Used as input.

Table VII–4. The pion scattering lengths and slopes.

	Experimental	Lowest Order[a]	First Two Orders[a]
a_0^0	0.220 ± 0.005	0.16	0.20
b_0^0	0.25 ± 0.03	0.18	0.26
a_0^2	-0.044 ± 0.001	-0.045	-0.041
b_0^2	-0.082 ± 0.008	-0.089	-0.070
a_1^1	0.038 ± 0.002	0.030	0.036
b_1^1	—	0	0.043
a_2^0	$(17 \pm 3) \times 10^{-4}$	0	20×10^{-4}
a_2^2	$(1.3 \pm 3) \times 10^{-4}$	0	3.5×10^{-4}

[a]Predictions of chiral symmetry.

Regions of a single field

- previous – heavy particle
- also – heavy regions of light field

– Wilson

Nonrelativistic EFT (H.Q.E.T), NRQCD ..

— anti particle heavy

$$\psi = \begin{pmatrix} u \\ \ell \end{pmatrix} e^{-imt} \qquad \ell = \frac{i\vec{\not{D}}}{2M_H} u$$

$$\mathcal{L} = u(x,t)\left[i\partial_0 - \frac{1}{2m}\left(\vec{\not{D}} + \frac{g_3}{4}\lambda^A \sigma^{\mu\nu} F^A_{\mu\nu} \right) \right] u$$

$\Longrightarrow$ full EFT

Operator Product expansion

$$\mathcal{L} = \sum_i C_i(\mu)\, O_i(\partial) \qquad \langle O(\partial)\rangle_\mu \leftarrow \text{all below } \mu$$

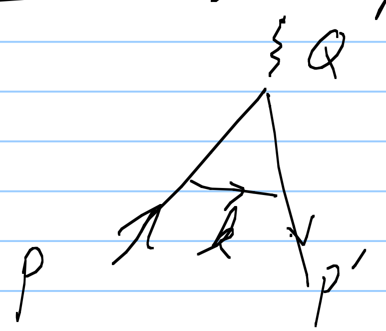
$C_i(\mu)$: calculated above μ — pert.?

O_i: local

$\langle O(\partial)\rangle_\mu$: lattice

$$H_W = \sum_i C_i \,(\bar{q}\, q)(\bar{q}\, q)$$

SCET — multiple low energy field (T. Becher, lectures)



$Q^2 \gg \Lambda_{QCD}^2$

$k^2 \sim Q^2$ Hard region (OPE)

$k^2 \sim 0$ $k \parallel p$ collinear region 1

$k^2 \sim 0$ $k \parallel p'$ " 2

$k^2 \sim 0$ all components small soft

$$\phi = \phi_1 + \phi_2 + \phi_3$$

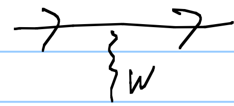
Double counting? Method of regions

↖ Overlap region — "zero bin subtraction"

Background Field Method

- keep fields explicit
- generate
  renormalize $\mathcal{L}_{eff}$ } then take matrix elements
- powerful construction

$\underline{\mathcal{H}_W}$



$\bar{\psi}\gamma_\mu(1+\gamma_5)\psi$

$\frac{g_2}{2\sqrt{2}} J_\mu W^\mu$

$$S = \left(\frac{g_2}{2\sqrt{2}}\right)^2 \int d^4x\, d^4y\, J_\mu(x)\, D_F(x-y)\, J^\mu(y)$$

$$-\frac{1}{M_W^2}\,\delta^4(x-y)$$

$$= -G_F \int d^4x\, J_\mu J^\mu(x)$$

QED massless scalar

$$\mathcal{L} = (D_\mu\phi)^* (D^\mu\phi) = -\underline{\phi^* D_\mu D^\mu \phi}$$

$$D_\mu = \partial_\mu + ie A_\mu$$
$$= \partial_\mu + i A_\mu$$

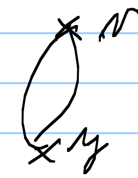
$$S = \mathrm{Tr}\ln D_\mu D^\mu = \mathrm{Tr}\ln\left(\Box + i\{\partial_\mu, A\} - A_\mu A^\mu\right)$$
$$= \mathrm{Tr}\ln(\Box + V)$$

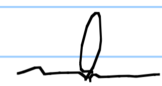
Pert theory

$$\mathrm{Tr}\ln\left(\Box\left(1 + \frac{1}{\Box}V\right)\right) = \mathrm{Tr}\ln\Box + \mathrm{Tr}\left(\frac{1}{\Box}V + \frac{1}{2}\frac{1}{\Box}V\frac{1}{\Box}V + \cdots\right)$$

$$\langle x|\frac{1}{\Box}|y\rangle = D_F(x-y)$$

$$\hookrightarrow = \operatorname{Tr} \cancel{\ln \Box} + \int d^4x \, D_F(x-x) V(x) + \frac{1}{2} \int d^4x \, d^4y \, D_F(x-y) V(y) D_F(y-x) V(x)$$





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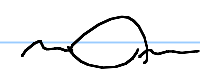


$$\mathcal{L} = D_F(0) A_\mu A^\mu + \frac{1}{-2} \int d^4y \, D_F(x-y) \{\partial, A_\mu(y)\} D_F(y-x) \{\partial_\mu, A\}_x$$

Derivatives onto fields

D_F

$$\begin{aligned}
\Delta_F(0) &= 0 \\
\Delta_F(x)\partial_\mu \Delta_F(x) &= \frac{1}{2}\partial_\mu \Delta_F^2(x) \\
\Delta_F(x)\partial_\mu\partial_\nu \Delta_F(x) &= [d\partial_\mu\partial_\nu - g_{\mu\nu}\Box]\,\frac{\Delta_F^2(x)}{4(d-1)} \\
\partial_\mu\Delta_F(x)\partial_\nu \Delta_F(x) &= [(d-2)\partial_\mu\partial_\nu + g_{\mu\nu}\Box]\,\frac{\Delta_F^2(x)}{4(d-1)} \quad .
\end{aligned}$$

$$S = \int d^4x \, \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \int d^4x\, d^4y \; F_{\mu\nu}(x) \frac{D_F^2(x-y)}{4(d-1)} F_{\mu\nu}(y)$$

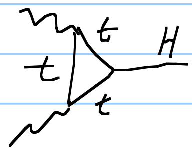
$$D_F^2(x-y) = \underline{\frac{\delta^4(x-y)}{16\pi^2}\left[\frac{2}{\varepsilon} \cdots\right]} + \frac{1}{16\pi^2} \underbrace{\int d^4q \; e^{iq\cdot(x-y)} \ln\left(q^2/\mu^2\right)}_{L(x-y)}$$

non local

$$S = \int d^4x \, F_{\mu\nu}\left[\frac{1}{4e^2(\mu)} - b \ln \Box/\mu^2\right] F^{\mu\nu}$$

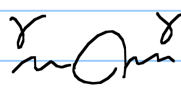
$$\langle x | \ln \Box/\mu^2 | y \rangle = L(x-y)$$

Higgs $GG \to H$, HH, HHH, $H \to \gamma\gamma$

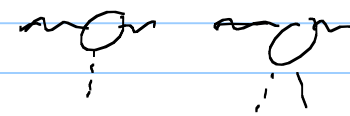


$m_t \gg m_H \leftarrow$

$$\mathcal{L} = m_t(1 + H/v)\bar{t}t = m_t(H)\bar{t}t$$

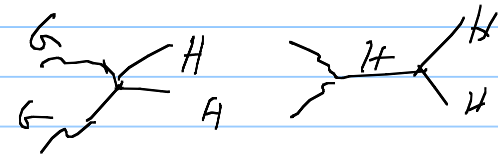


$$= -\frac{e^2}{12\pi^2}\ln\frac{m_t^2(H)}{\mu^2}\left(g_{\mu\nu}q^2 - q_\mu q_\nu\right)$$



$$\mathcal{L} = \frac{1}{4e^2}F^2 + \frac{\alpha}{48\pi}\ln\left(\frac{v+H}{v}\right)F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_s}{12\pi}\ln\left(1+H/v\right)F^A_{\mu\nu}F^{A\mu\nu}$$

HW Show



cancel at threshold

Example σ model at 1 loop

Logic $\pi = \bar{\pi} + \delta\pi$ $\Longrightarrow$ 1 loop of many processes

$\uparrow$ background, fluctuations

$$U = \bar{U} e^{i\Delta}$$

$$\mathcal{L} = \mathcal{L}(\bar{U}) + \Delta\left[d_\mu d^\mu + \sigma\right]\Delta$$

$$[\Box + \sigma]$$

Result

renorm.

$$\mathcal{L}_{1\,loop} = \frac{v^2}{4} Tr\left(\partial_\mu U\, \partial^\mu U^\dagger\right) + l_1 \left(Tr[\ \ ]\right)^2 + \cdots$$

$$+ \frac{1}{128\pi} Tr\left(\partial_\mu U\, \partial^\mu U^\dagger\right) \ln \frac{\square}{\mu^2}\, Tr\left(\partial_\mu U\, \partial^\mu U^\dagger\right)$$

$$+ \cdots$$

log for all processes!

Summary BFM

- keep fields explicit in calc.
- generating L_{eff}
- useful