Exploring the limits of the Standard Model

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LTP/PSI Thursday Colloquium
Paul Scherrer Institut
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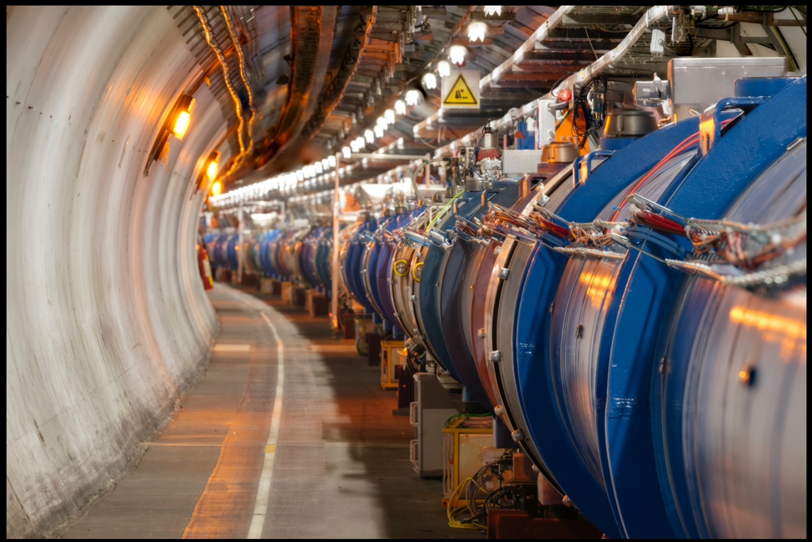
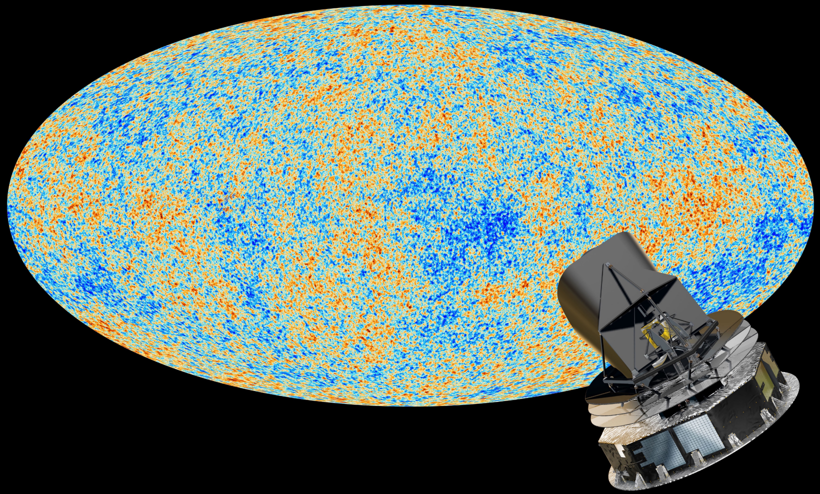
HIM
Helmholtz-Institut Mainz

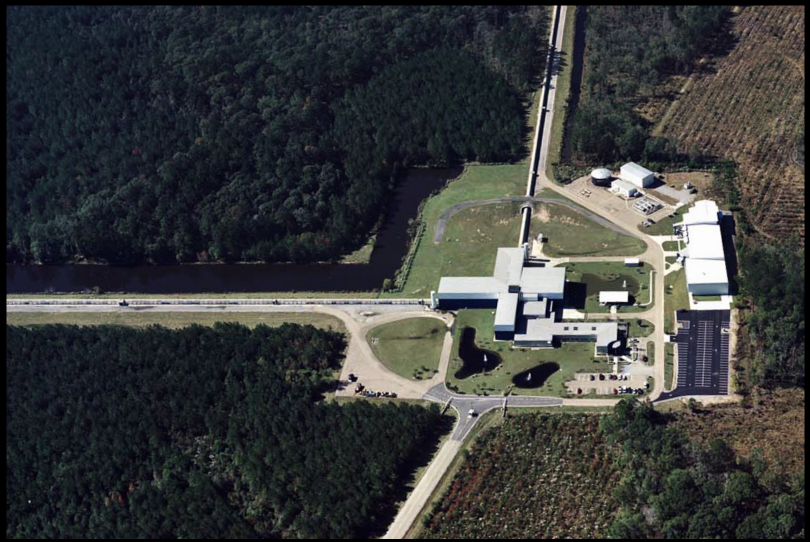
PRISMA

SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

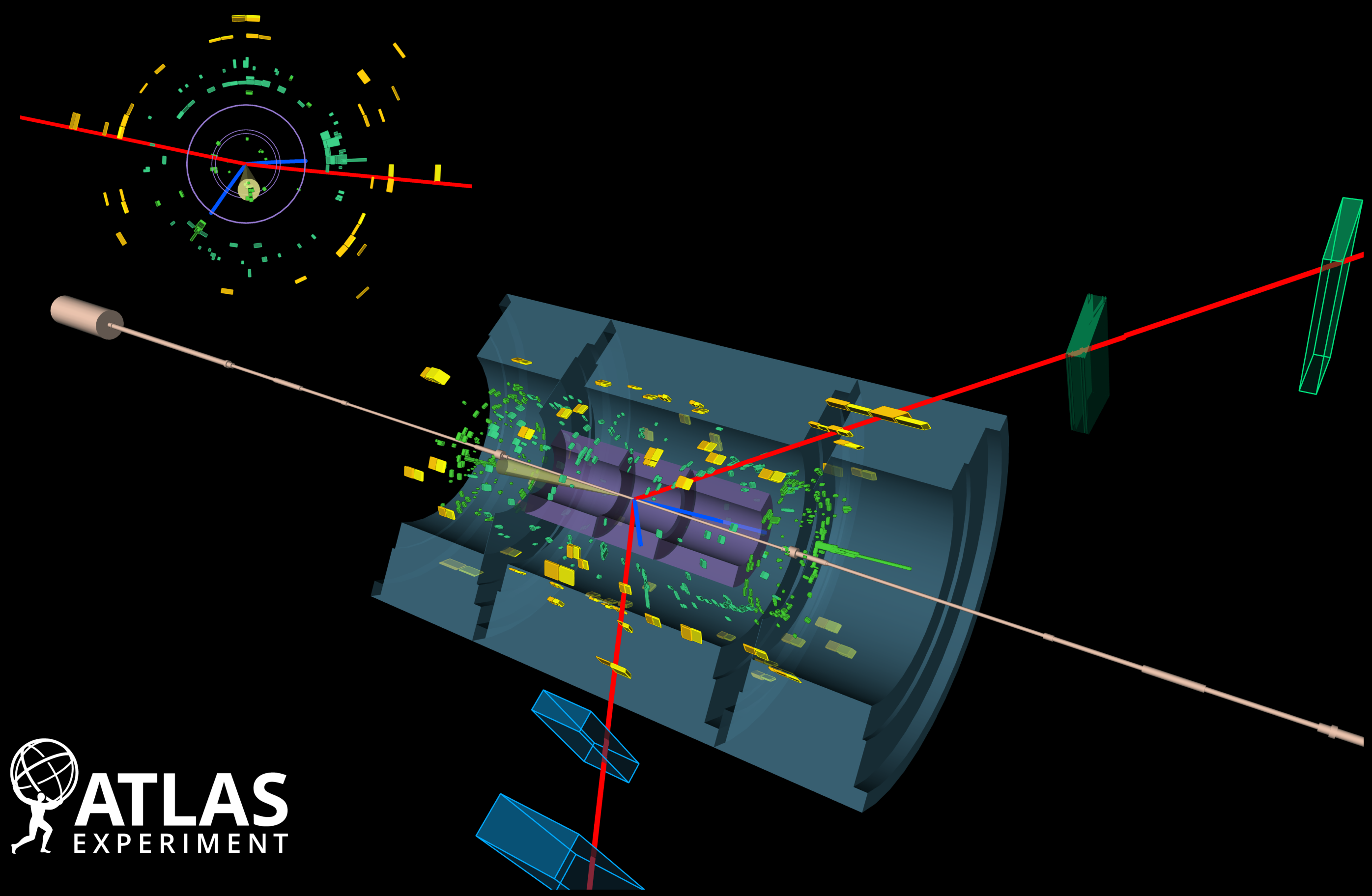
The Quest for New Physics

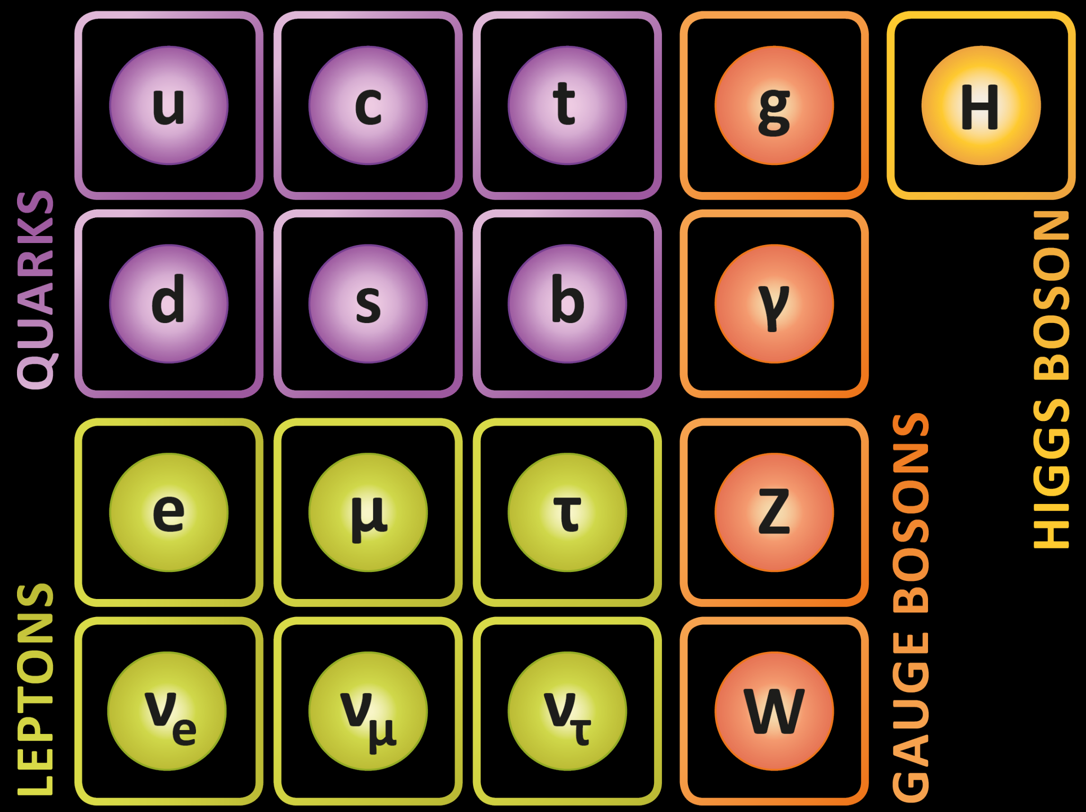






The Quest for New Physics

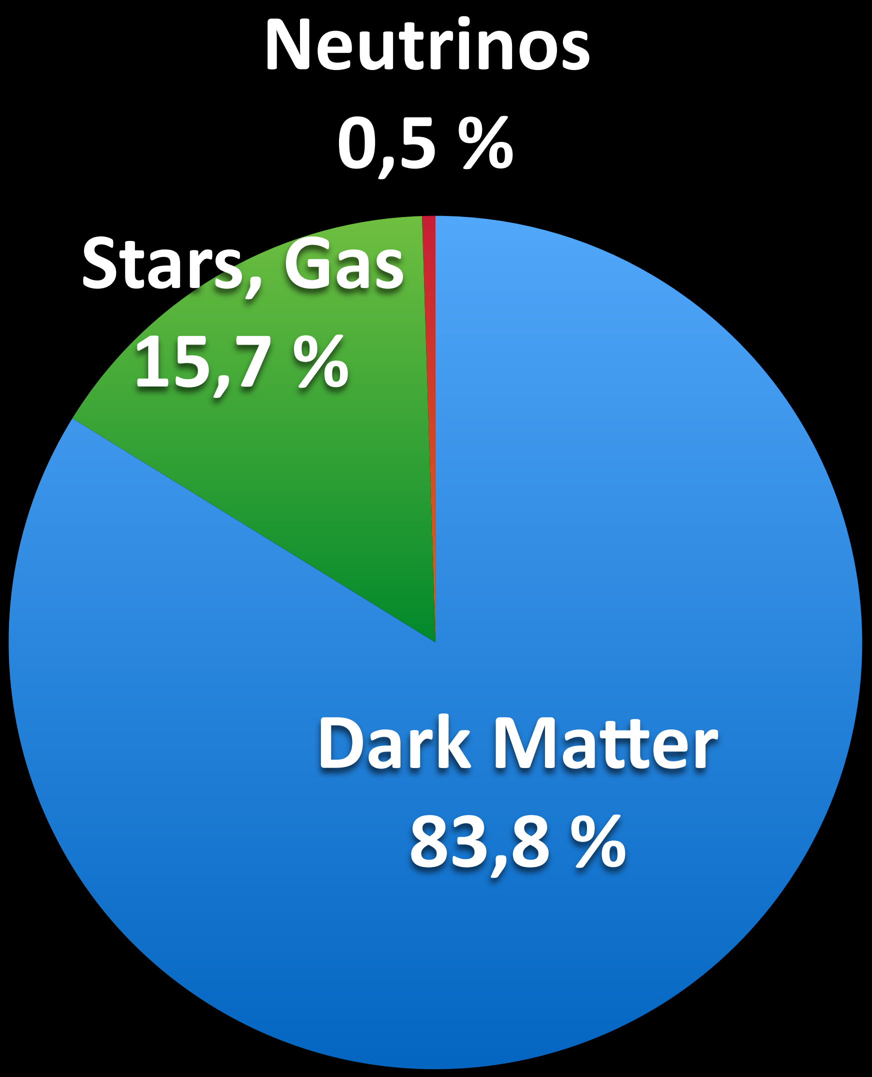




The Quest for New Physics

Standard Model does not explain

- Baryon asymmetry
- Mass and scale hierarchies
- Existence of dark matter



Standard Model does not provide a complete description of Nature

The Quest for New Physics

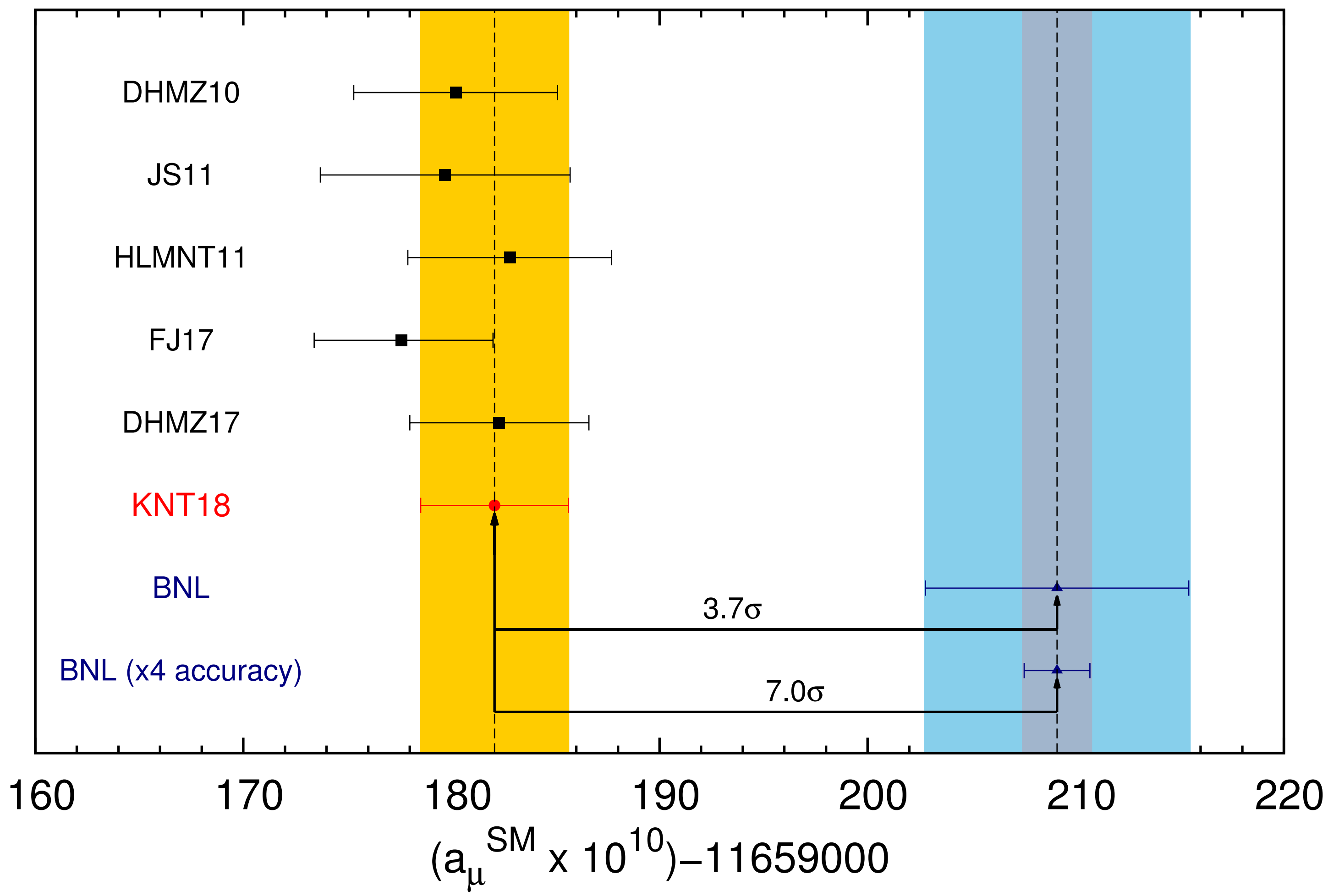
* Explore the limits of the Standard Model
  - Search for new particles and phenomena at high energies
  - Search for enhancement of rare phenomena
  - Compare precision measurements to SM predictions

* Realise extreme levels of experimental sensitivity, matched by equally precise theoretical calculations

* Control over "hadronic uncertainties" — effects arising from the strong interaction

Precision observables as probes of the SM

* Muon anomalous magnetic moment:

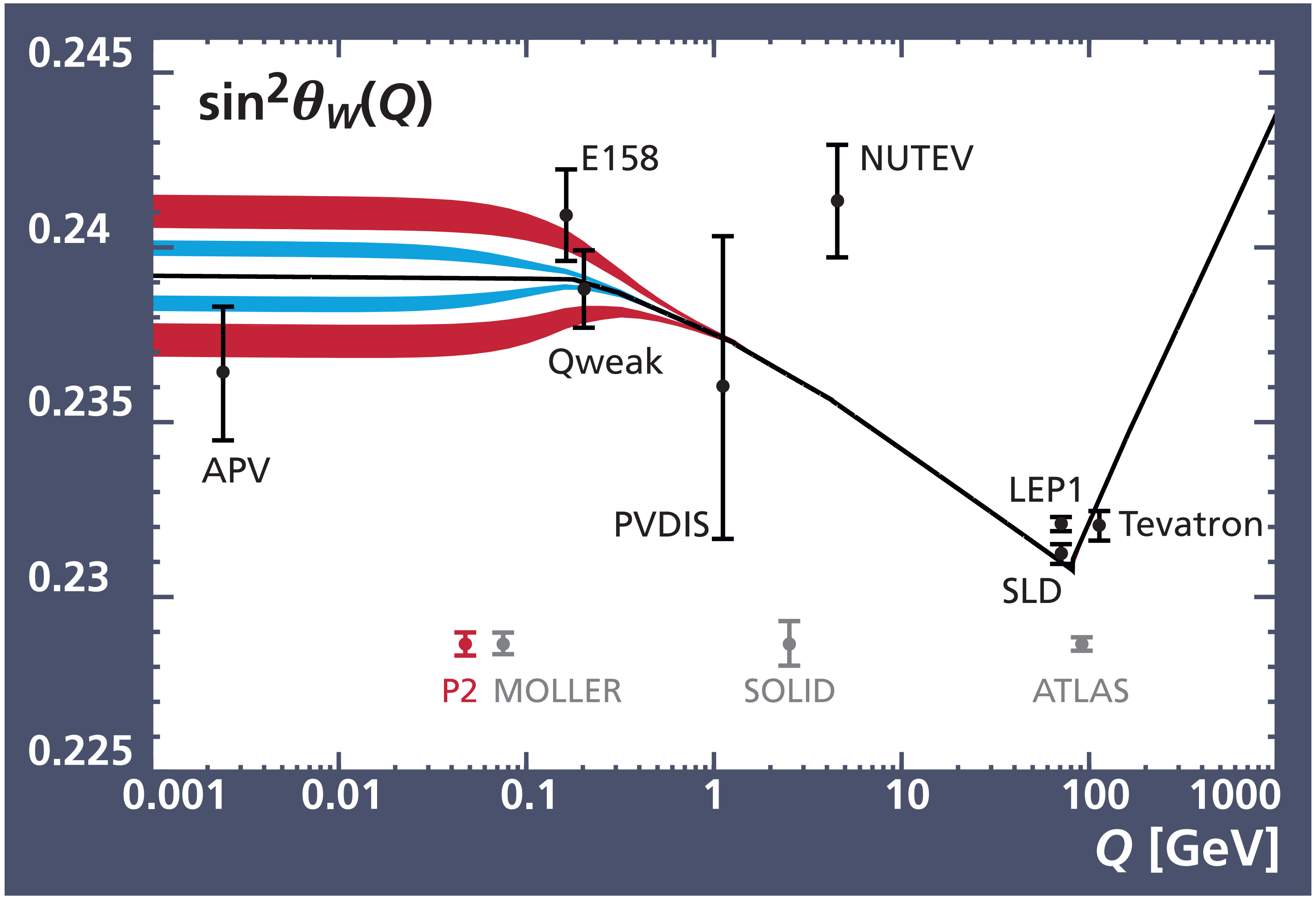


[Keshavarzi et al., arXiv:1802.02995]

* Theoretical error dominated by strong interaction contributions

Precision observables as probes of the SM

* Running of the weak mixing angle: $\sin^2\theta_W$

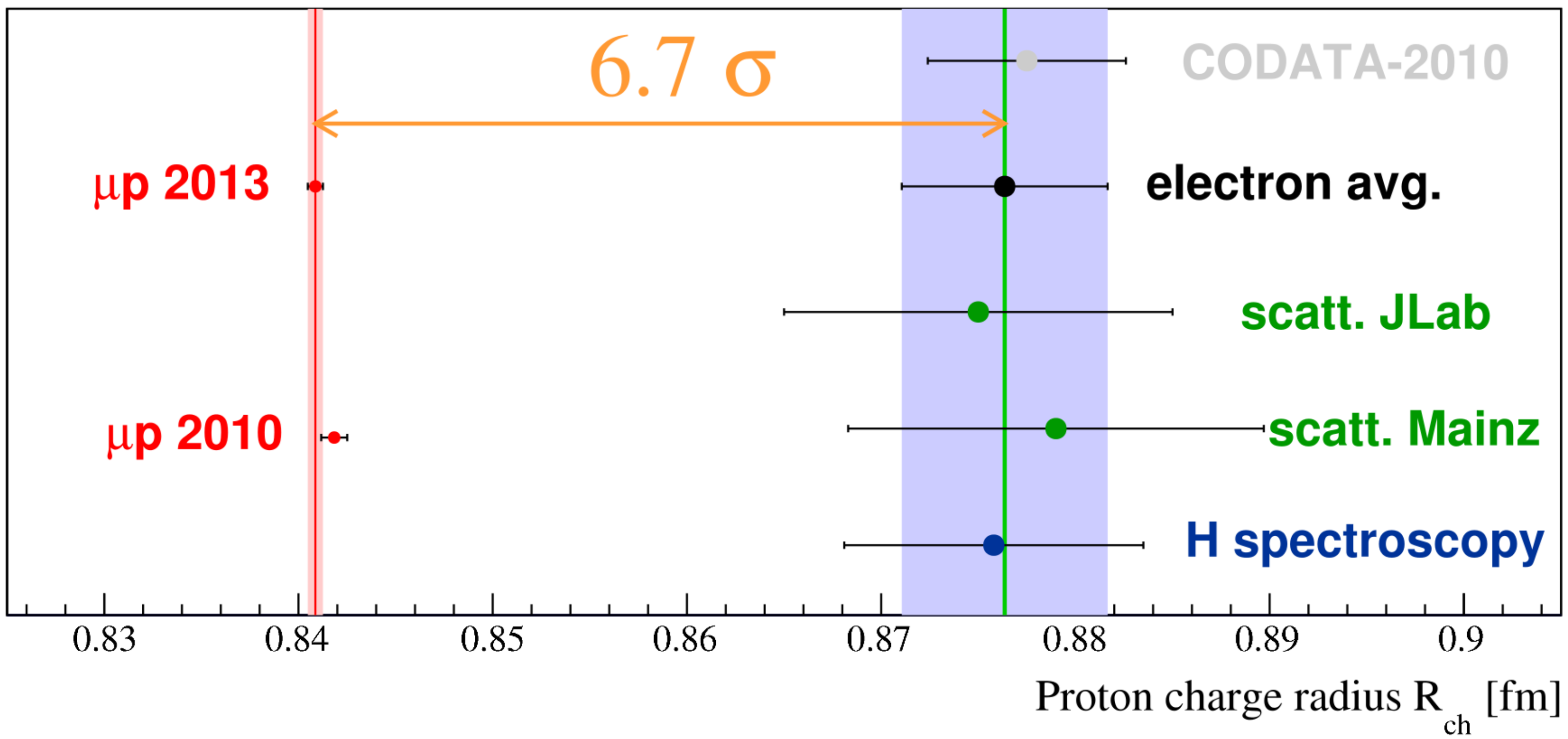


* Running at low energy discriminates between different scenarios for "New Physics"

Precision observables as probes of the SM

- Proton Radius Puzzle

[Arrington 2015]



- Signal for New Physics?
- Unknown systematic effects?
- Uncontrolled hadronic uncertainties?

Outline

Low-energy precision experiments at Mainz

The muon anomalous magnetic moment

The muon $g - 2$ in lattice QCD

Summary & Outlook

Low-energy precision experiments at Mainz

Mainz Microtron
MAMI

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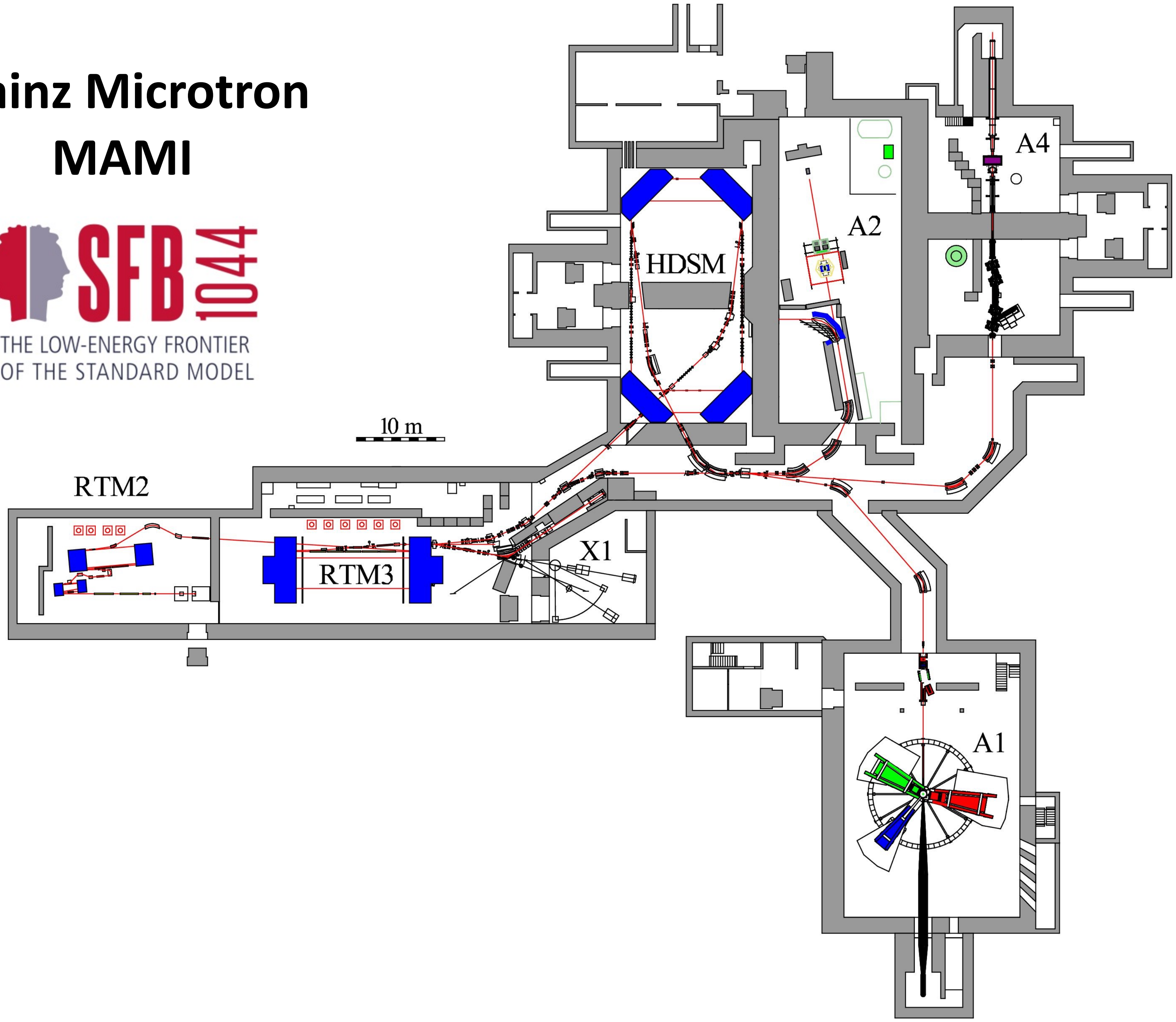
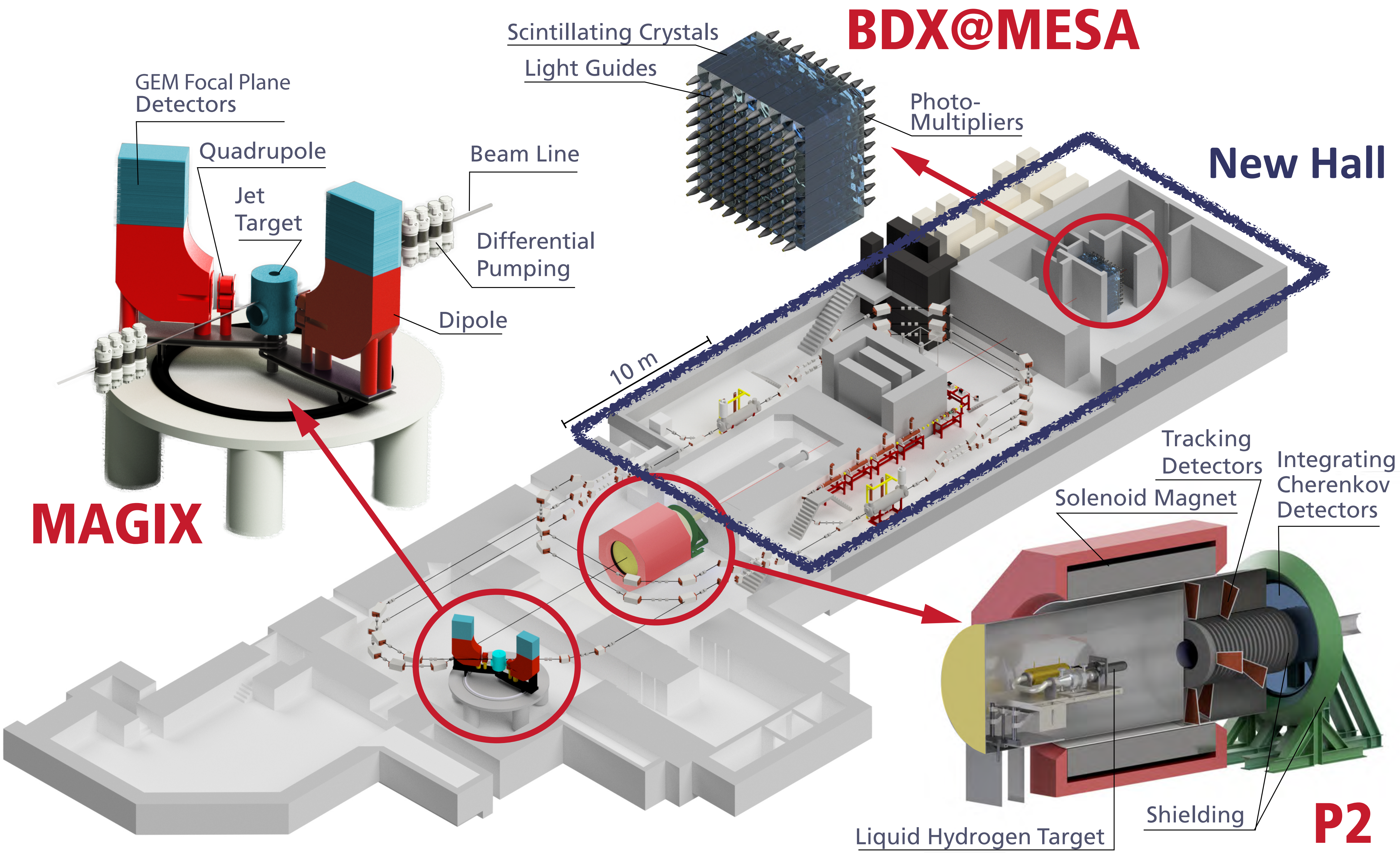
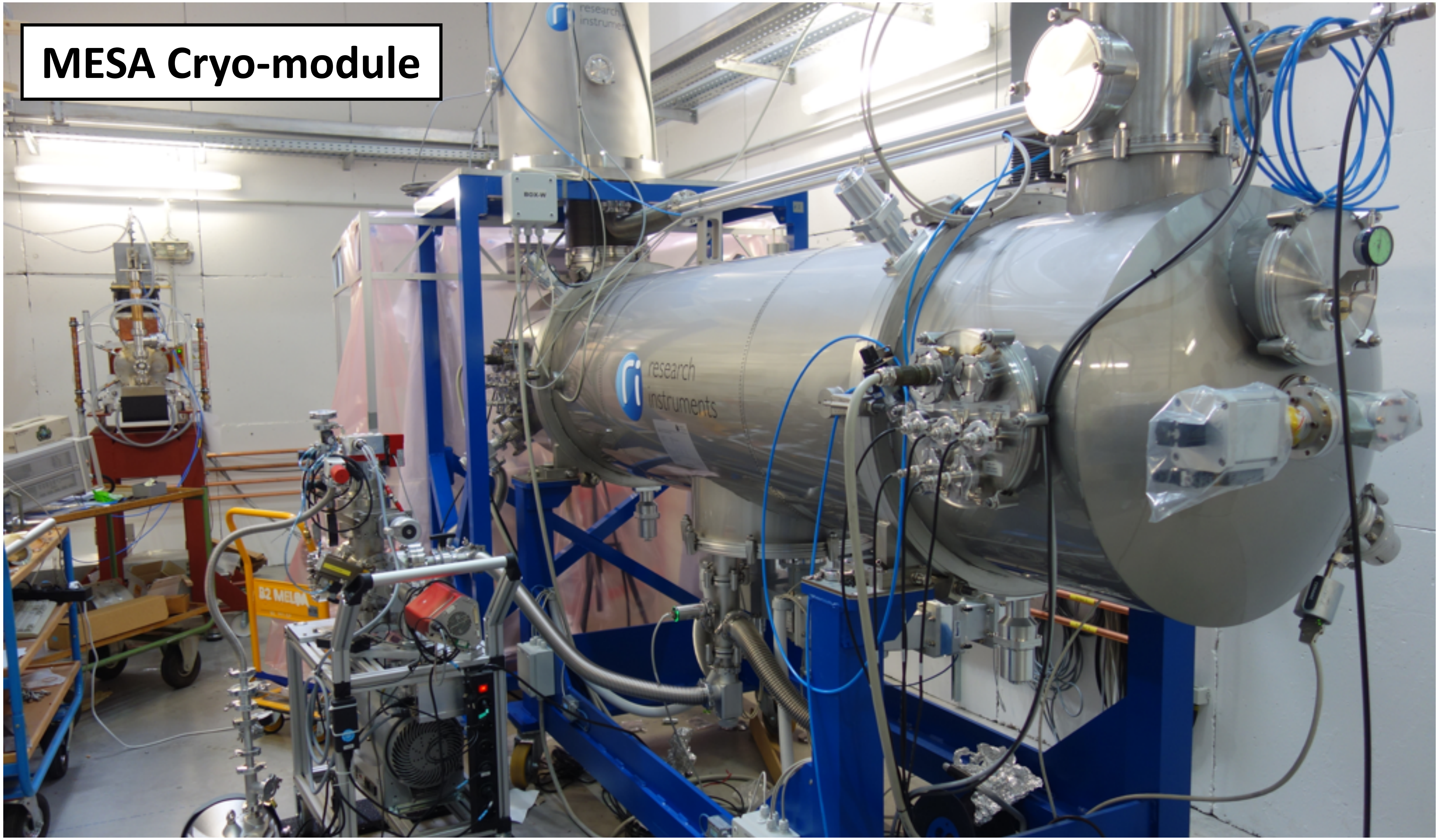


The MESA Facility



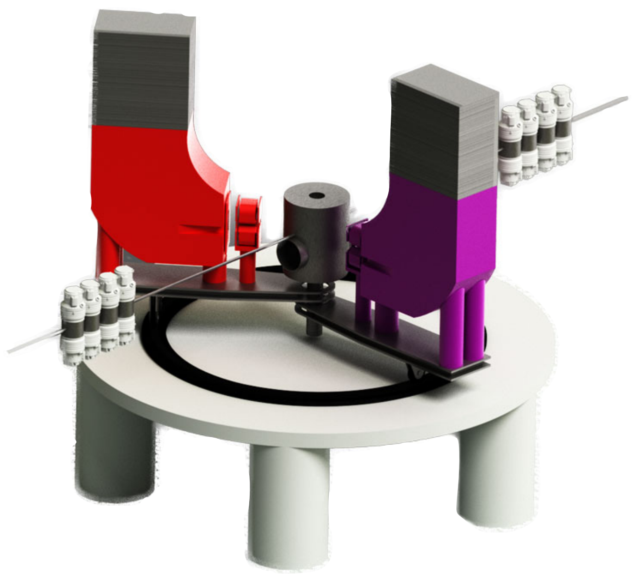

MESA Cryo-module
research instruments
BOX-W
B2 MELBA

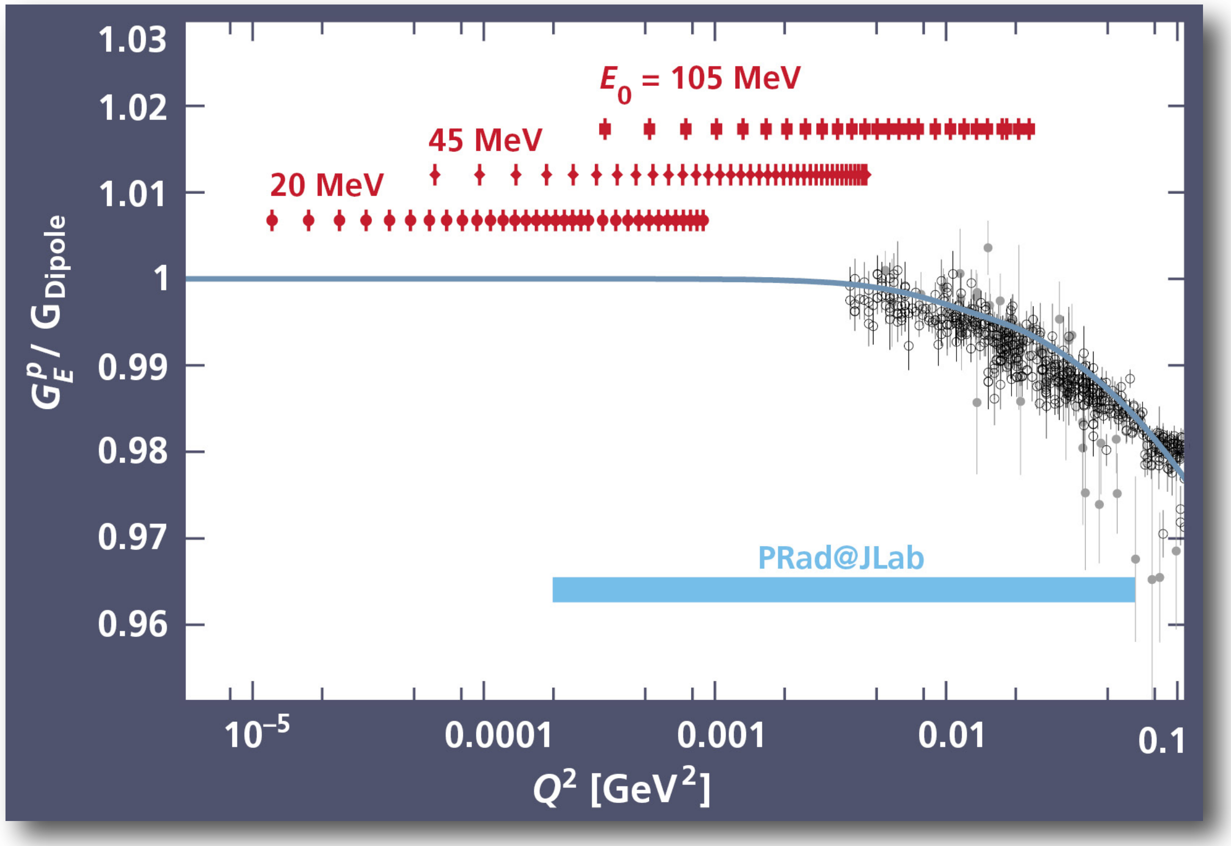
The MAGIX spectrometer

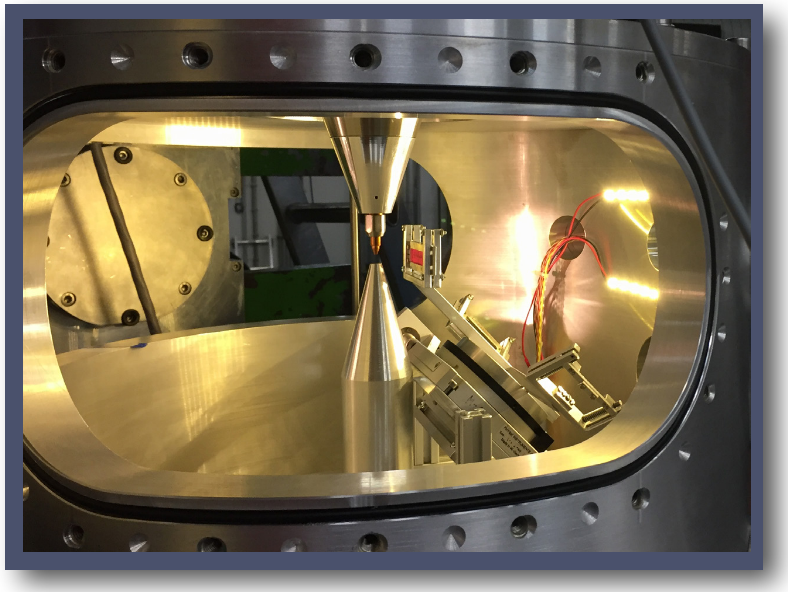
Double arm spectrometer

Internal gas target

Momentum resolution: $\Delta p/p < 10^{-4}$







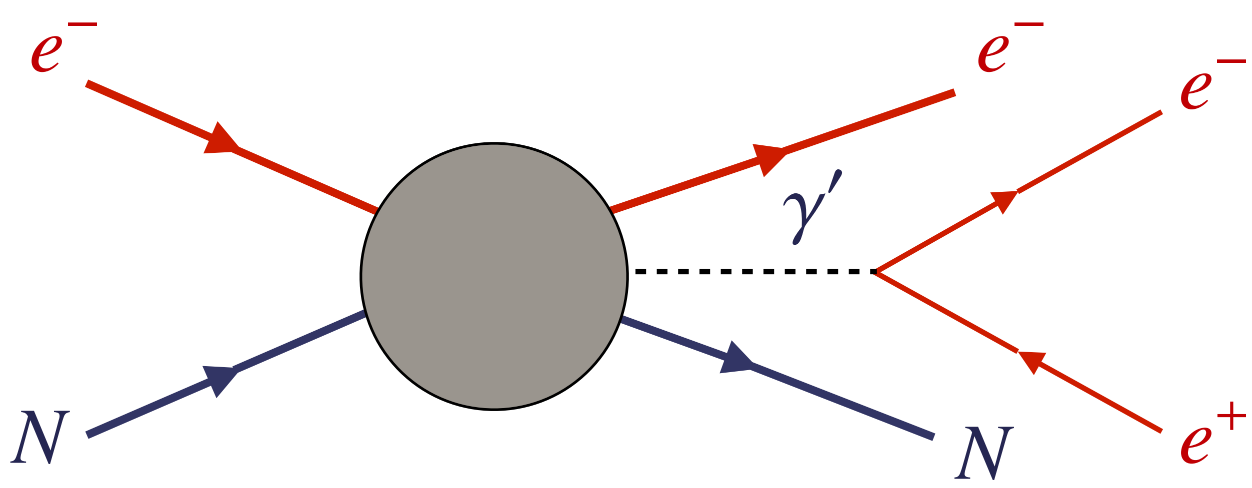
⇒ Proton radius puzzle: form factor measurements for $Q^2 \gtrsim 10^{-5}\,\mathrm{GeV}^2$

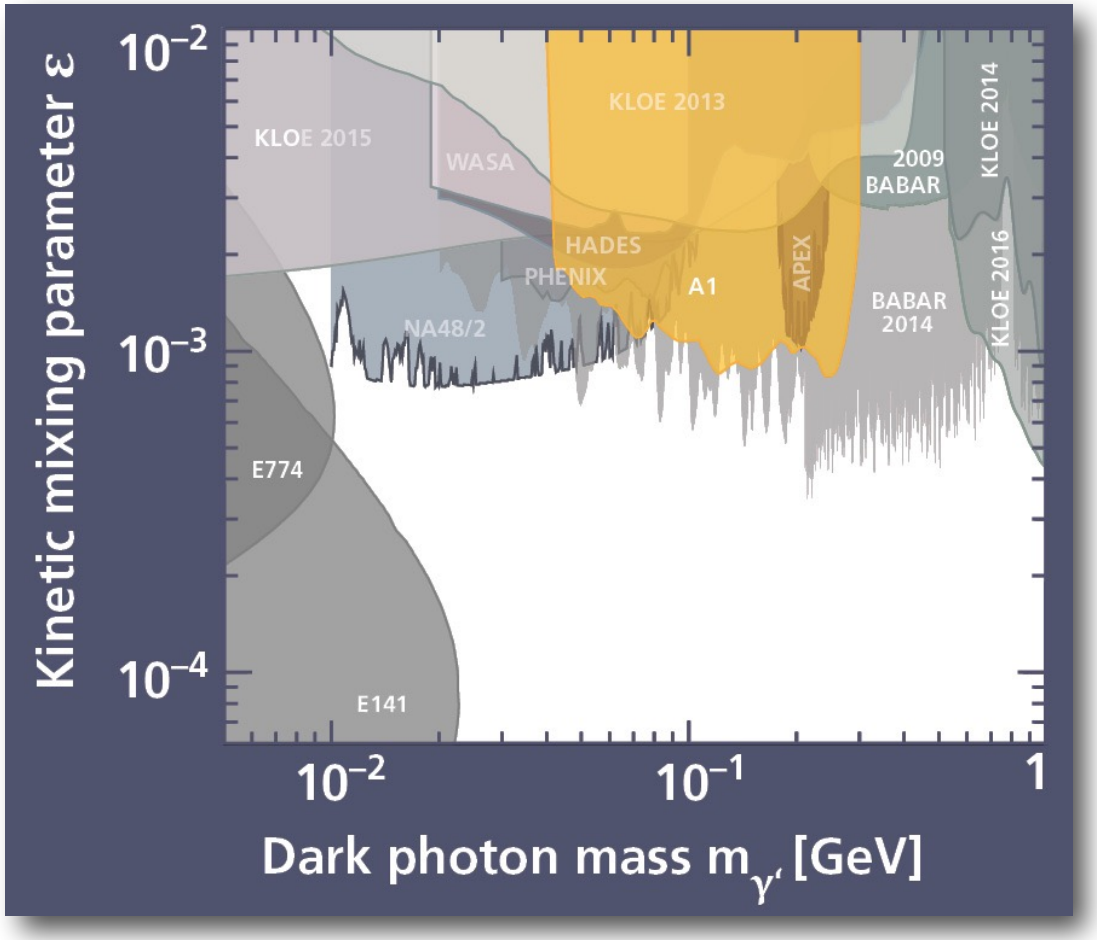
The MAGIX spectrometer

* Searching for "dark photons": Messengers to the dark sector

$$G_{\mathrm{BSM}} = G_{\mathrm{SM}} \otimes U(1)^n, \quad n \geq 1$$

* Dark photon production in *ep* scattering:





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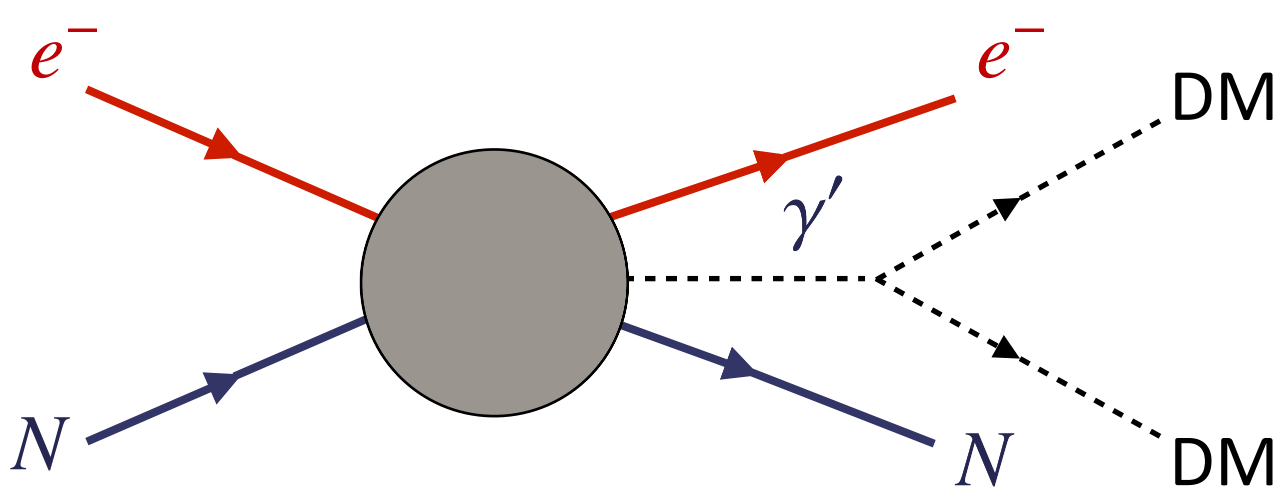
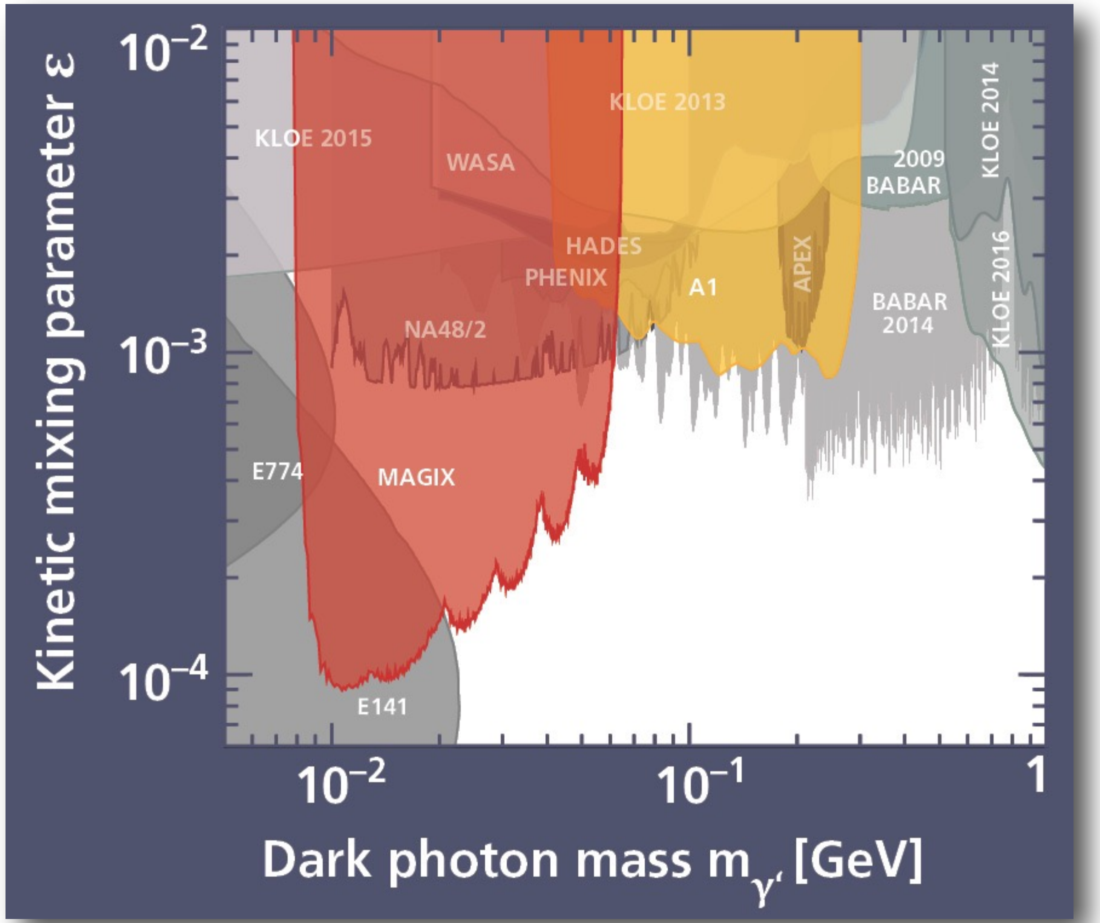




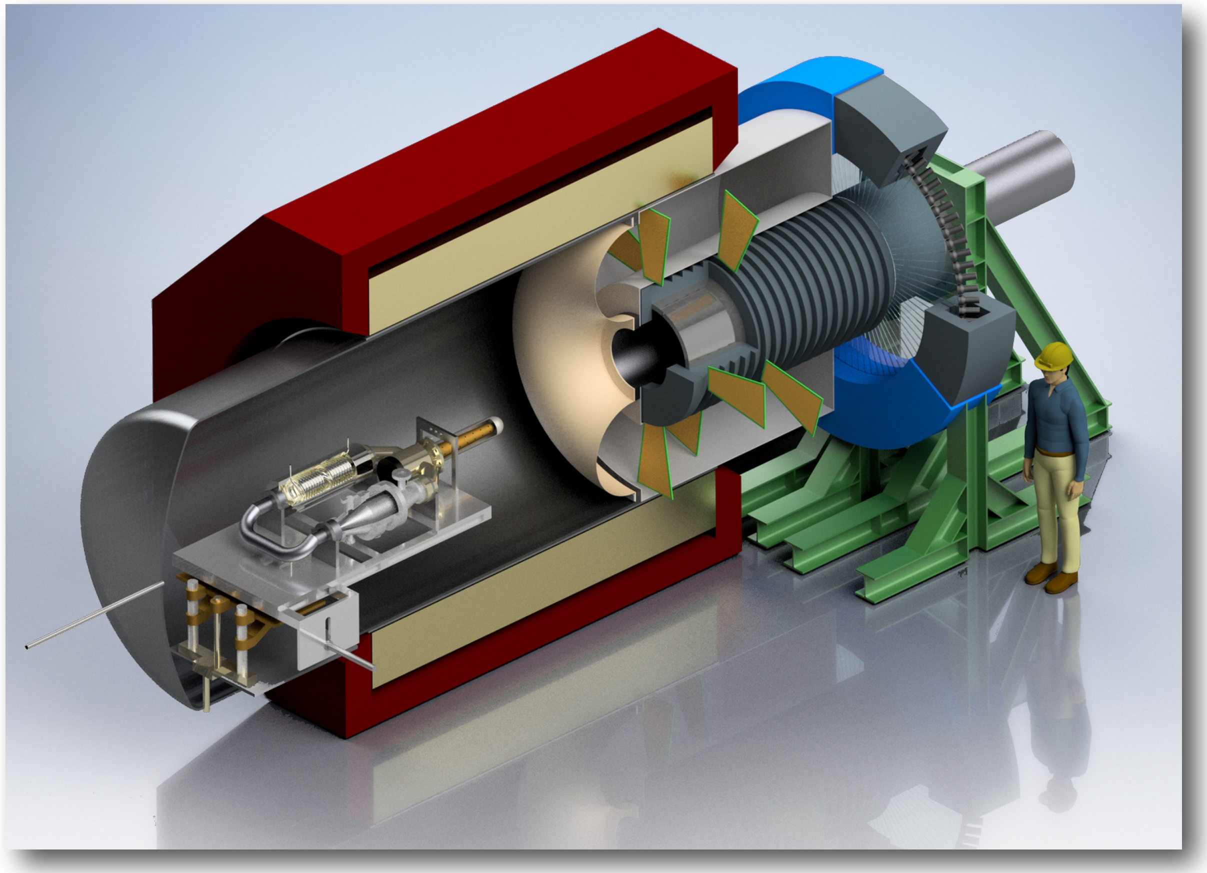
* MAGIX: sensitive to invisible dark photon decays

P2 — Parity violation at low energies

- Left-right asymmetry in polarised *ep*-scattering:

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\frac{G_F\, Q^2}{4\sqrt{2}\pi\alpha}\left(Q_W^p + F^p\right)$$

- Weak charge of the proton: $Q_W^p = 1 - 4\sin^2\theta_W$ (tree level)



Magnetic spectrometer

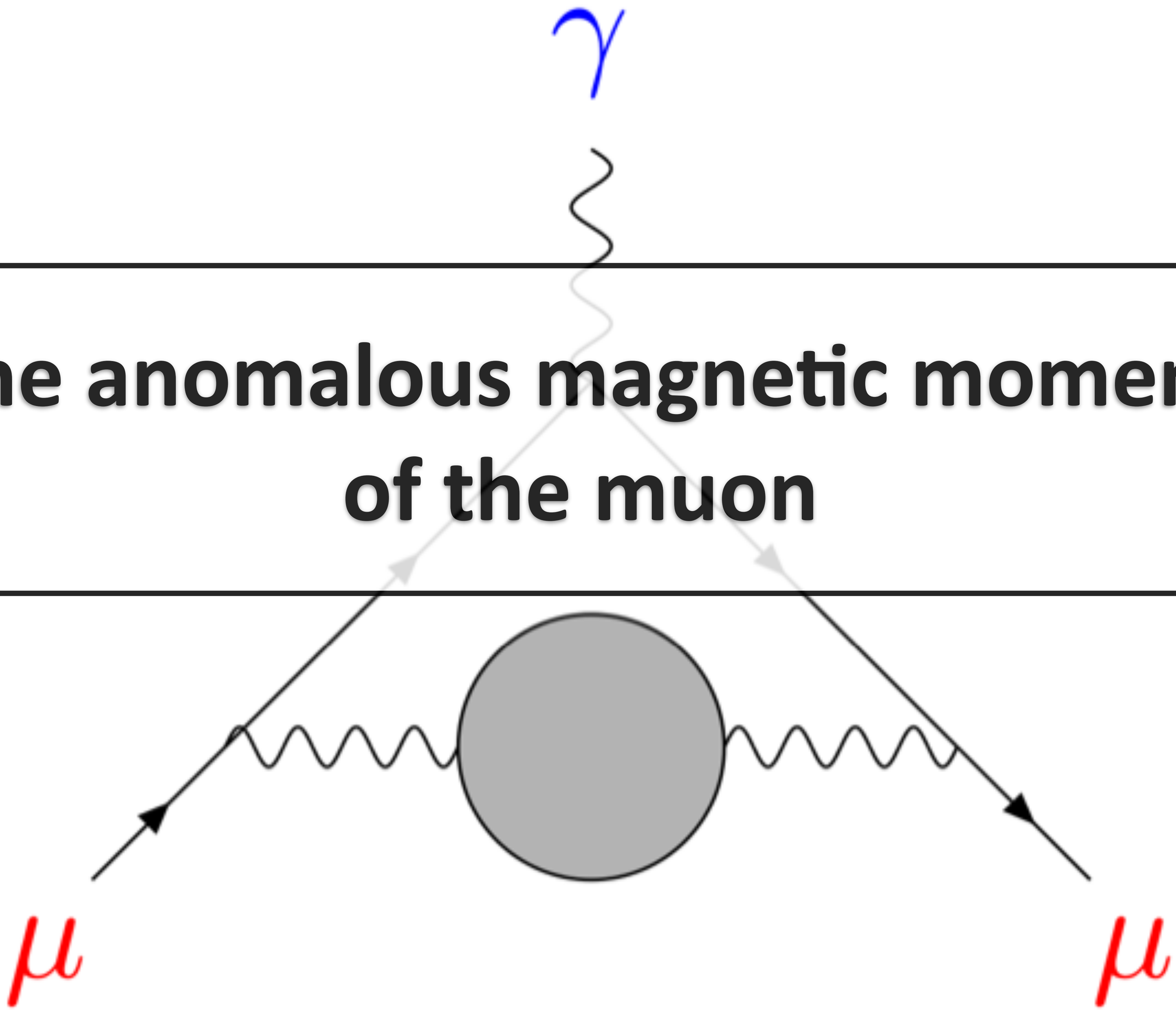
Liquid hydrogen target

Expected precision:

$$\frac{\delta(\sin^2\theta_W)}{\sin^2\theta_W} = 0.15\%$$

[Becker et al., arXiv:1802.04759]

The anomalous magnetic moment of the muon



Anomalous magnetic moment

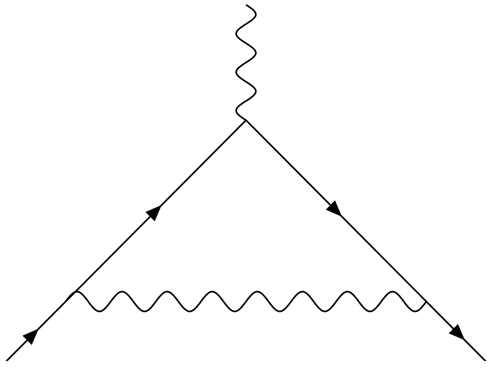
* Particle with charge e and mass m:

$$\boldsymbol{\mu} = g\,\frac{e\hbar}{2m}\,\boldsymbol{S}, \qquad \boldsymbol{S} = \frac{\boldsymbol{\sigma}}{2}$$

* Dirac value of $g = 2$ modified by quantum corrections

$$g = 2(1+a) \quad \Rightarrow \quad a = \tfrac{1}{2}(g-2)$$

$$a_e^{\rm exp} = 0.001\,159\,652\,181\,643(764)$$



$$a^{(2)} = \frac{\alpha}{2\pi} = 0.001\,161\,40\ldots$$

[J. Schwinger, Phys Rev 73 (1948) 416]

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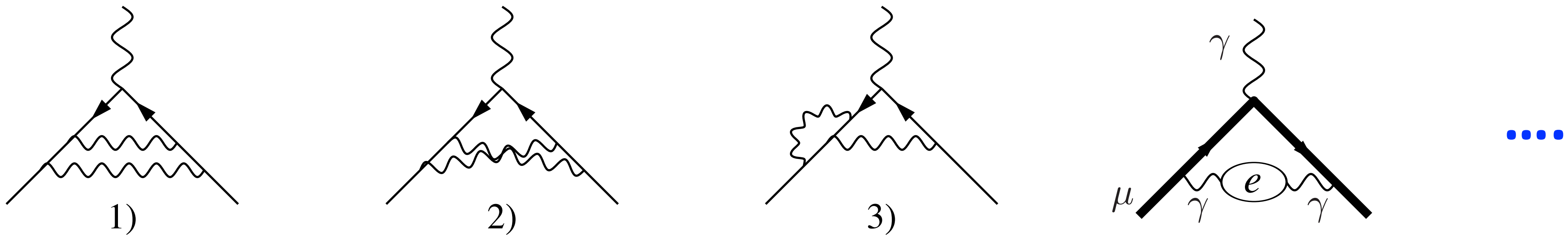
$$a_\mu^{\text{exp}} = 0.001\,165\,920\,9(6)$$

Higher-order corrections

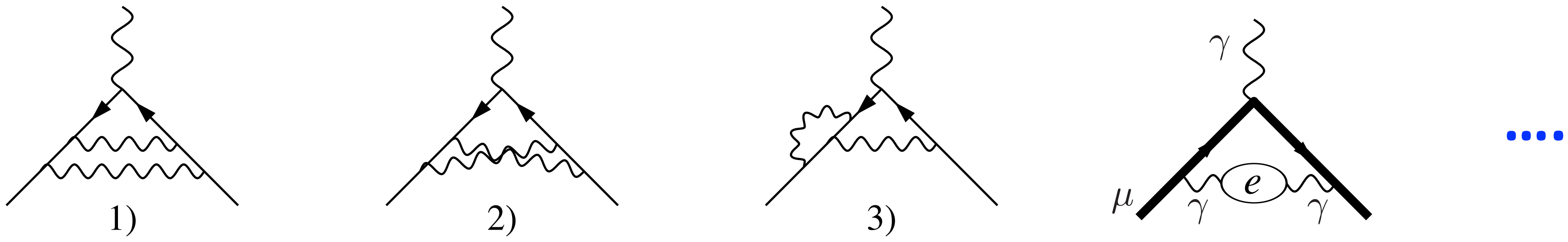
* QED corrections:



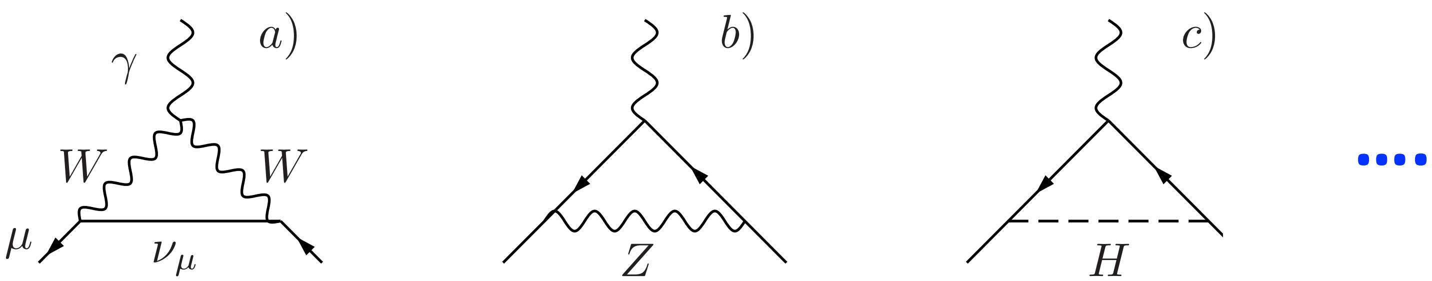
SM	116 591 776.000	100 %	#diagrams
QED, tot	116 584 718.951	99,9939 %	
2	116 140 973.318	99,6133 %	1
4	413 217.629	0,3544 %	9
6	30 141.902	0,0259 %	72
8	381.008	0,0003 %	891
10	5.094	$4\cdot10^{-6}$ %	12672

Higher-order corrections

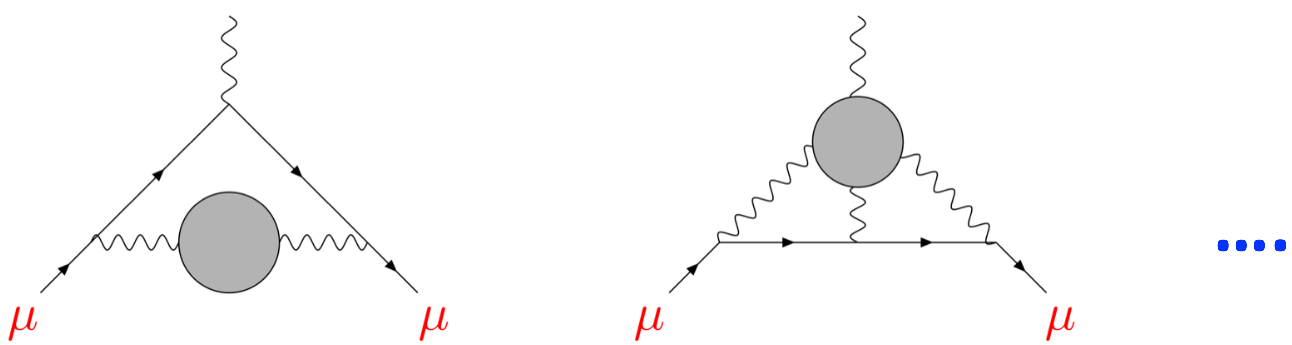
* QED corrections:



* Weak corrections:



* Strong corrections:

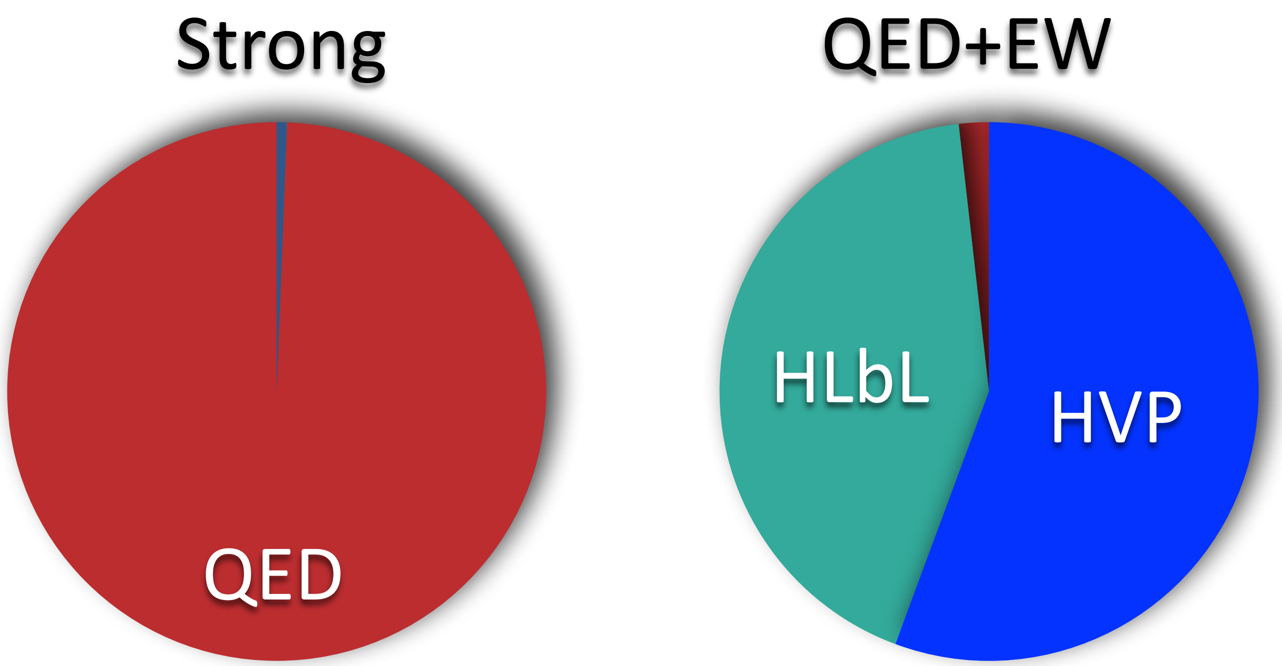


Current status of the muon g – 2

$$a_\mu \equiv \tfrac{1}{2}(g-2)_\mu = \begin{cases} 116\,592\,080(54)(33)\cdot 10^{-11} & \text{E821 @ BNL} \\ 116\,591\,825(34)(26)(1)\cdot 10^{-11} & \text{SM prediction} \end{cases}$$

* SM estimate dominated by QED; error dominated by QCD

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{strong}}$$



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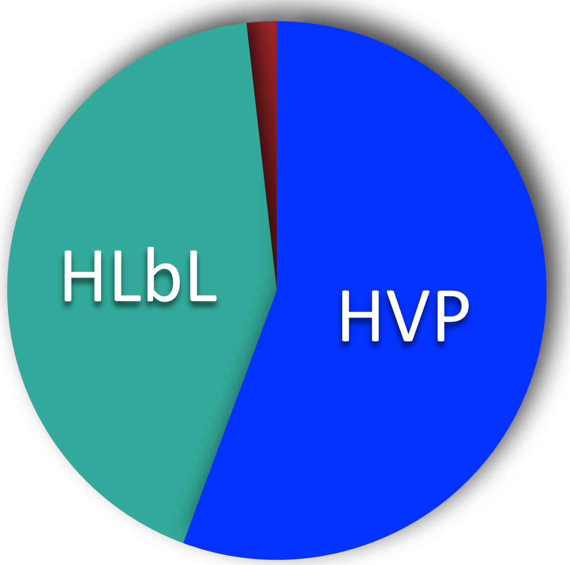
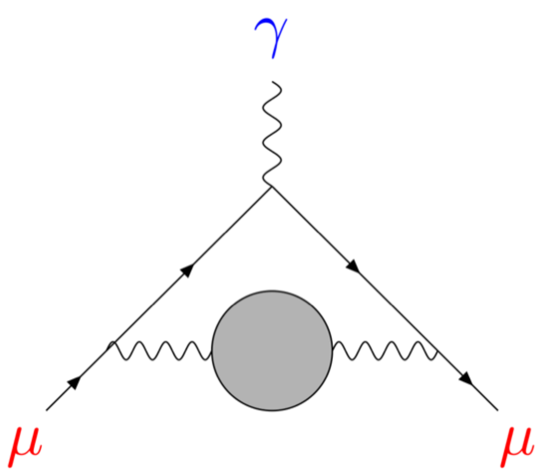
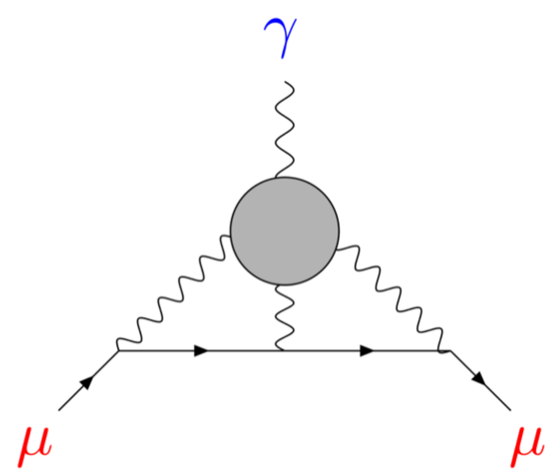
QED+EW



Hadronic vacuum polarisation:



Hadronic light-by-light scattering:



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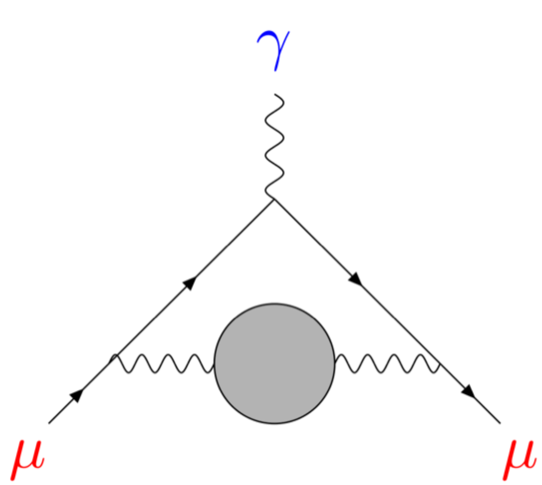
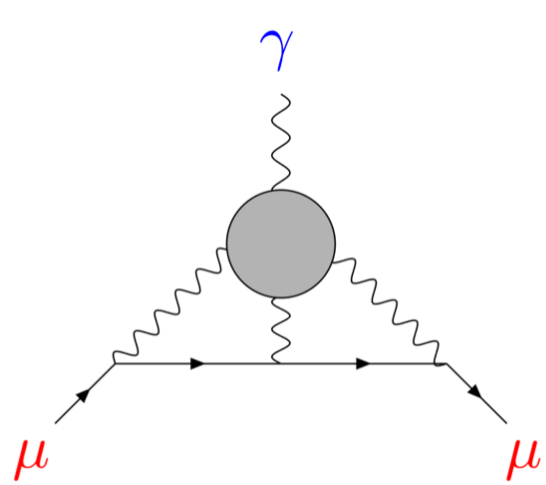
* SM estimate dominated by QED; error dominated by QCD

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm strong} + a_\mu^{\rm NP?}$$

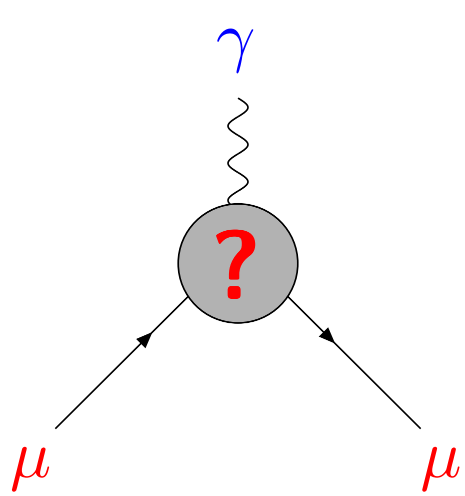
Hadronic
vacuum polarisation:



Hadronic
light-by-light scattering:

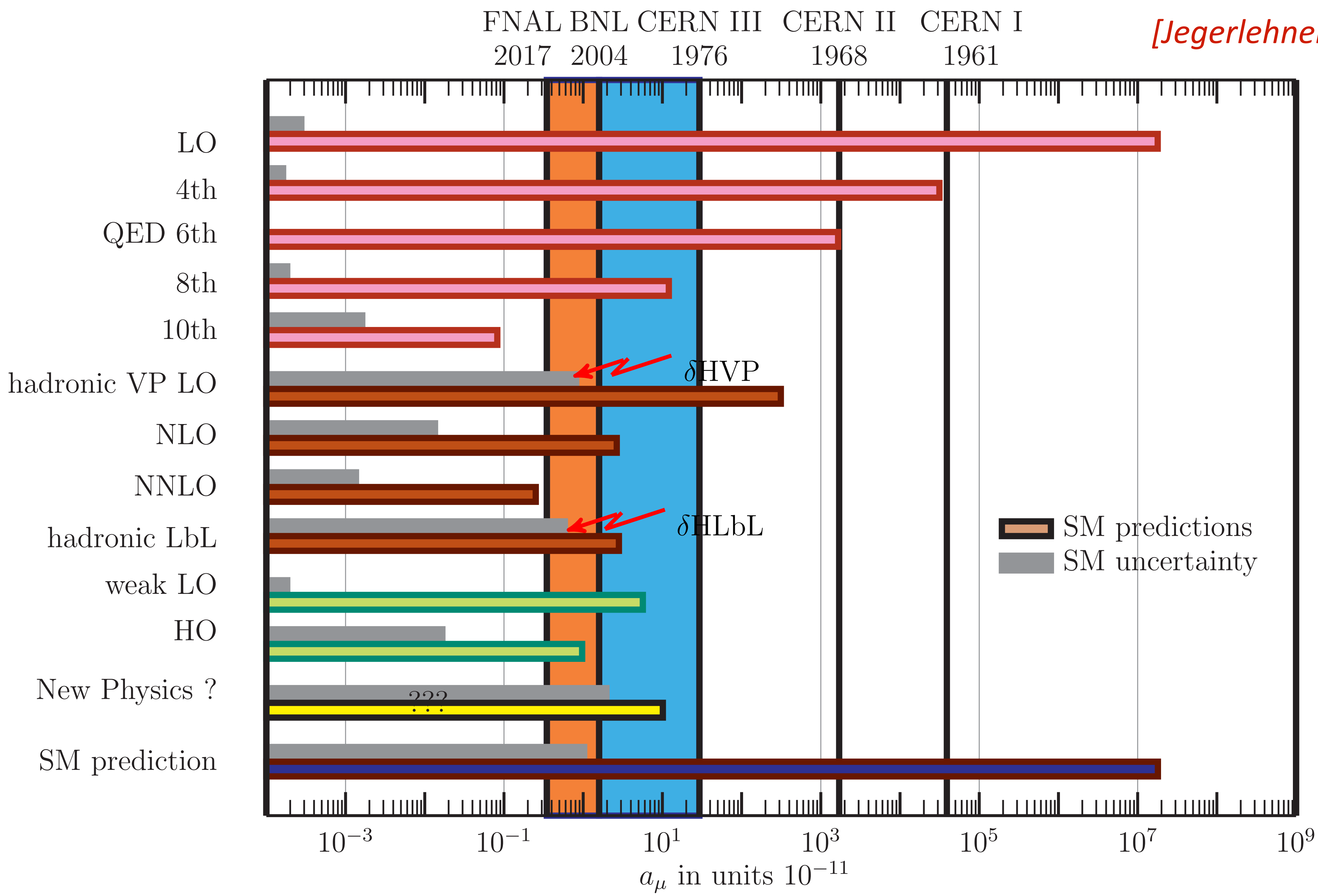


Contribution from
"New Physics"?



Theory confronts experiment

[Jegerlehner, arXiv:1705.00263]



* Reduce hadronic uncertainties to compete with experimental sensitivity

From BNL E821 to Fermilab E989

$$a_\mu^{\rm exp} = 116\,592\,089\,(54)_{\rm stat}\,(33)_{\rm syst}\cdot 10^{-11}$$

* Total precision of 0.54 ppm, dominated by statistics
* Use hotter beam of Fermilab proton booster: 8 GeV/c
* Suppress pion background — longer pion decay channel

  BNL: 80 m —> Fermilab: 2 km

* Aim for 100 ppb statistical and 100 ppb systematic error

  —> 0.14 ppm total error

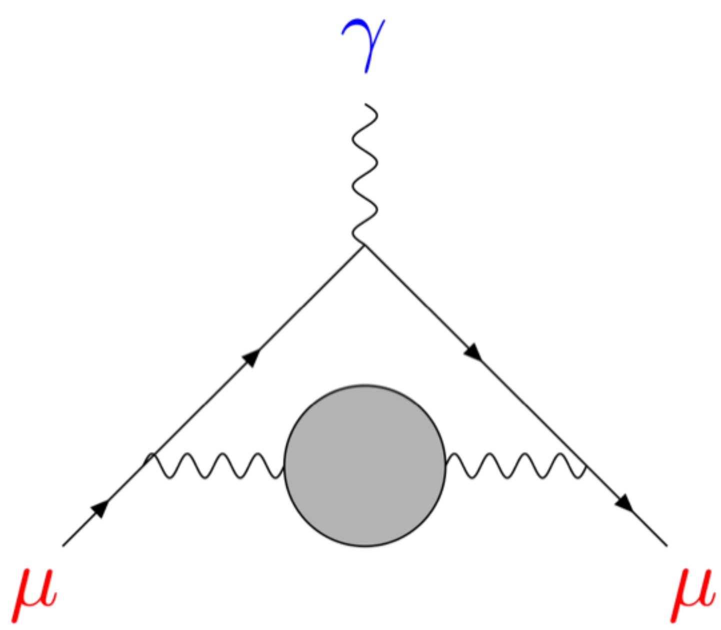
* Transport BNL storage ring to Fermilab

Re-assembly of the BNL storage ring

[©B. Lee Roberts]

Hadronic contributions to a_μ

Hadronic vacuum polarisation:

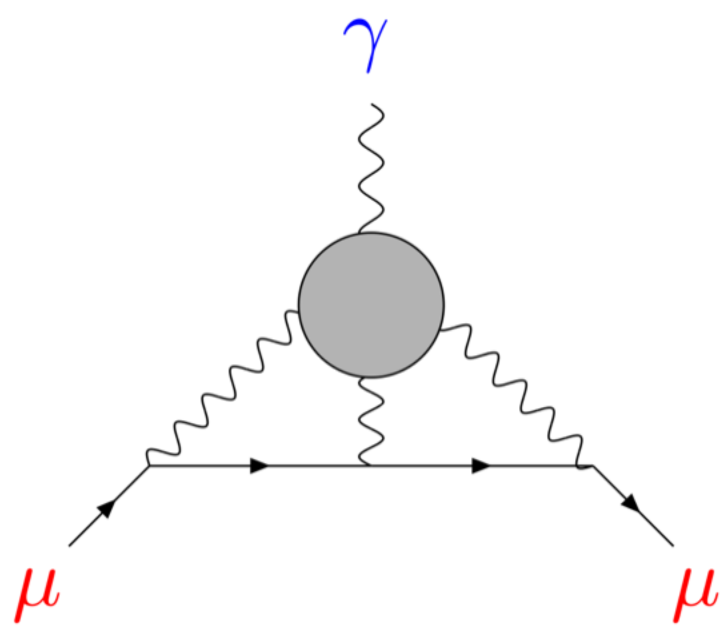


Dispersion theory:

$$a_\mu^{\rm hvp} = (6888 \pm 34) \cdot 10^{-11}$$

(combined e^+e^- and τ data)

Hadronic light-by-light scattering:

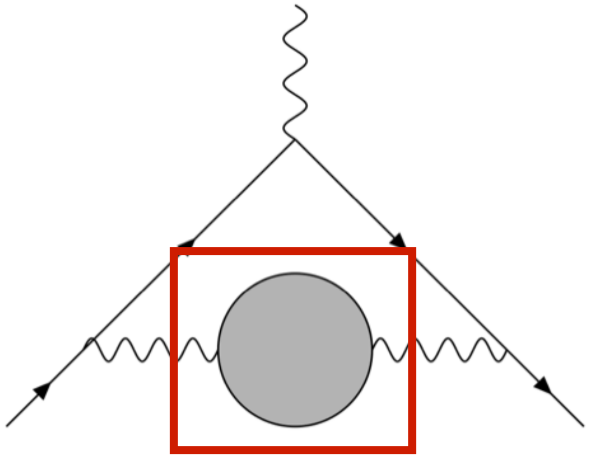


Model estimates:

$$a_\mu^{\rm hlbl} = (105 \pm 26) \cdot 10^{-11}$$

"Glasgow consensus"

Hadronic vacuum polarisation

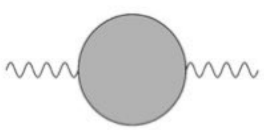
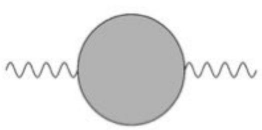


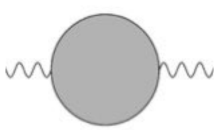
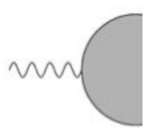
* Hadronic electromagnetic current:

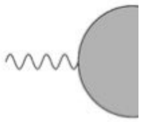
$$J^\mu(x) = \tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d - \tfrac{1}{3}\bar{s}\gamma^\mu s + \tfrac{2}{3}\bar{c}\gamma^\mu c + \ldots$$

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\,\Pi(q^2) = ie^2 \int d^4x\, \mathrm{e}^{iq\cdot x}\, \langle 0|T\, J^\mu(x) J^\nu(0)|0\rangle$$

* **Optical theorem:**

 $= \displaystyle\int \frac{ds}{\pi(s-q^2)}\,\mathrm{Im}$ 

$2\,\mathrm{Im}$  $= \displaystyle\sum_{\text{had}} \int d\Phi\, \Big|$  $\Big|^2$

$\Big|$  $\Big|^2 \propto \sigma(e^+e^- \to \text{hadrons})$

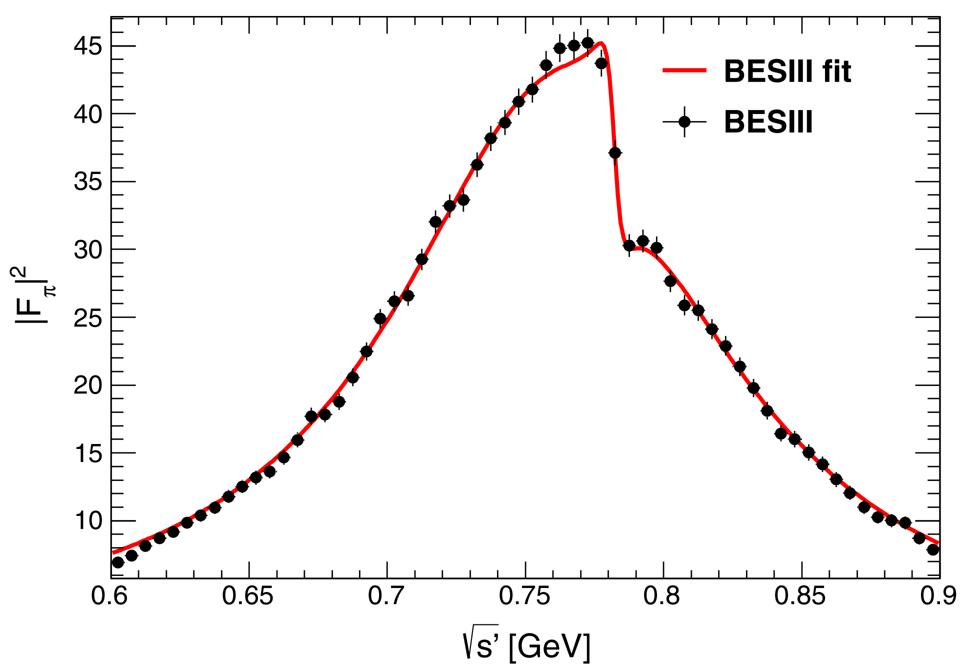
$$a_\mu^{\text{hvp}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^\infty ds\, \frac{R_{\text{had}}(s)\,\hat{K}(s)}{s^2}, \qquad R_{\text{had}}(s) = \sigma(e^+e^- \to \text{hadrons}) \Big/ \frac{4\pi\alpha(s)}{(3s)}$$

HVP contribution from dispersion relations

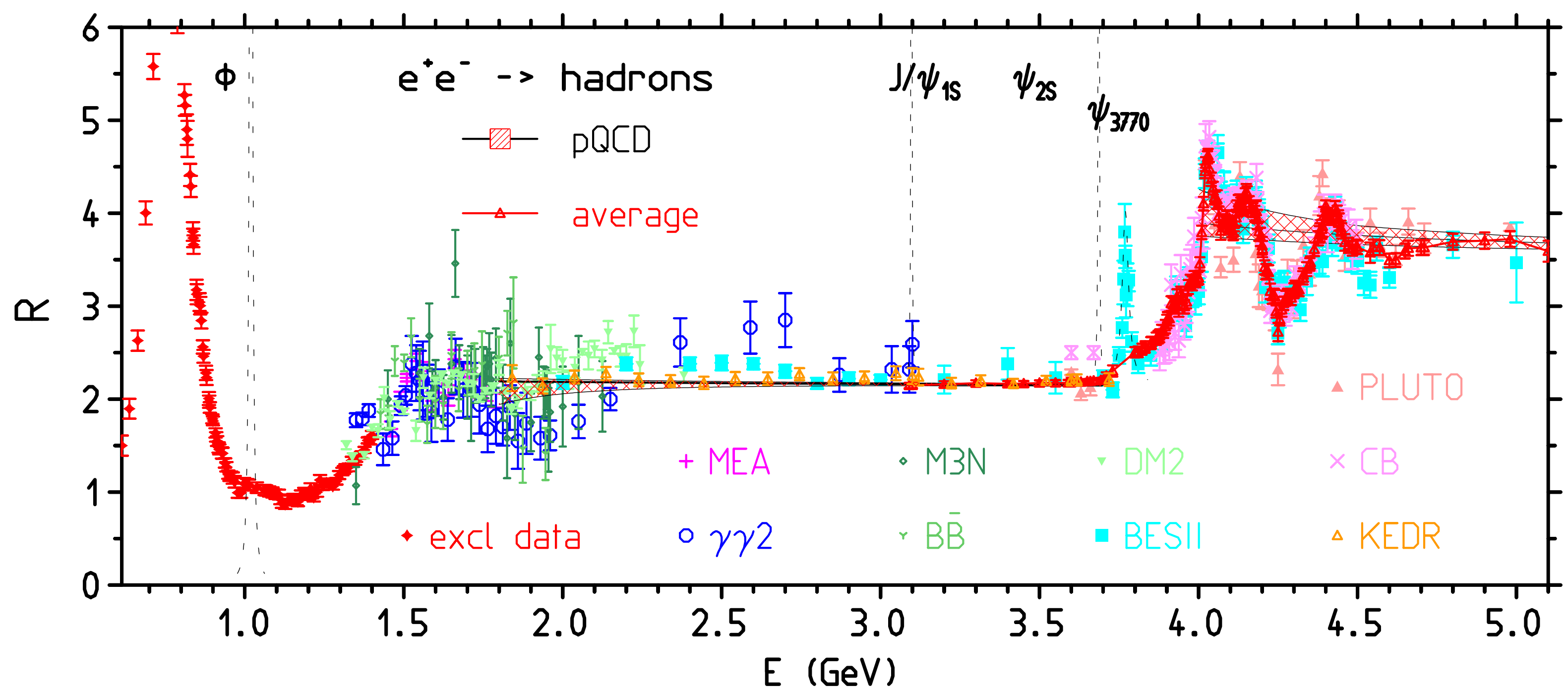
* Knowledge of $R_{\rm had}(s)$ required down to pion threshold

$$a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left\{ \int_{m_{\pi^0}^2}^{E_{\rm cut}^2} ds\, \frac{R_{\rm had}^{\rm data}(s)\,\hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds\, \frac{R_{\rm had}^{\rm pQCD}(s)\,\hat{K}(s)}{s^2} \right\}$$

⇒ Use experimental data for cross section ratio $R_{\rm had}(s)$

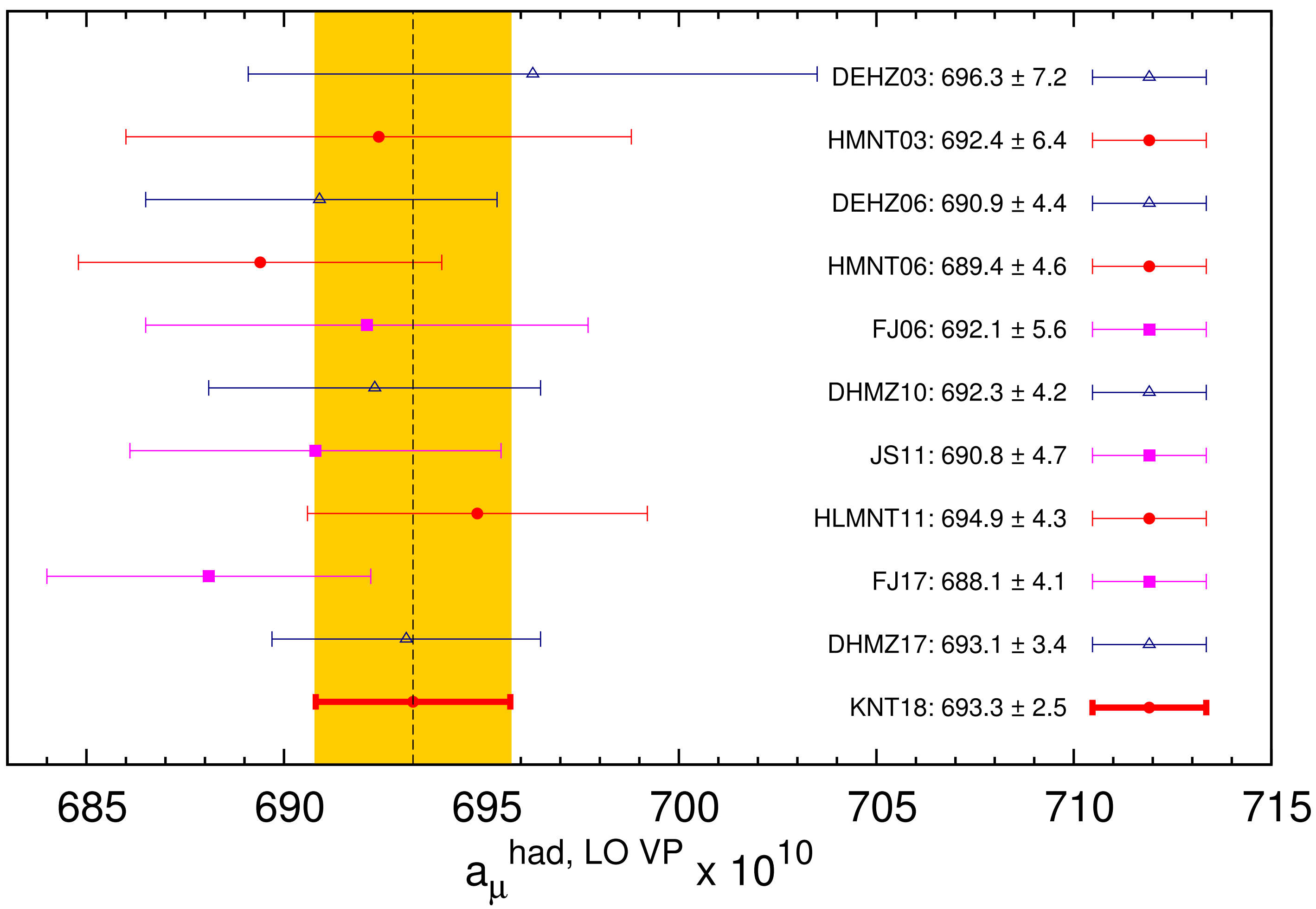


[BESIII Collaboration, 2016]



[Jegerlehner, arXiv:1705.00263]

HVP contribution from dispersion relations

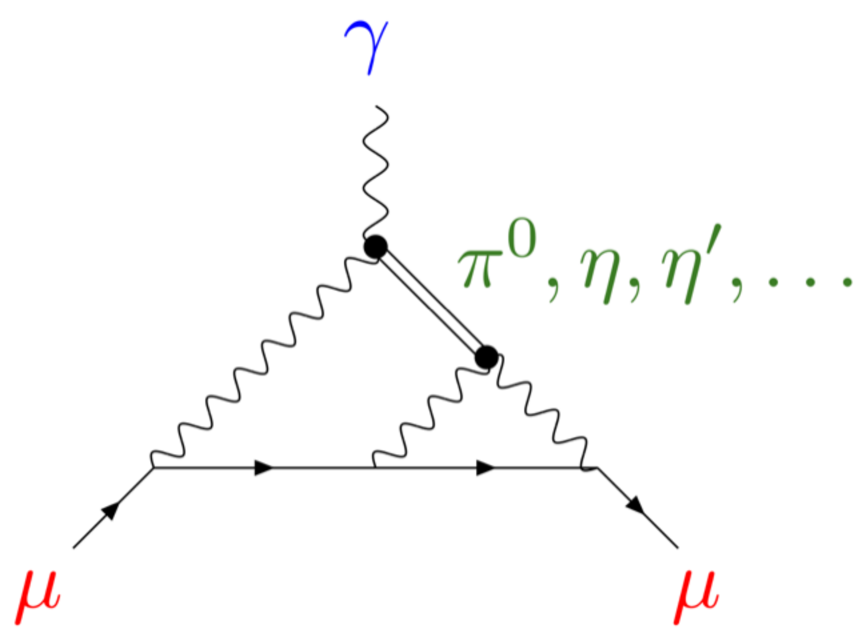


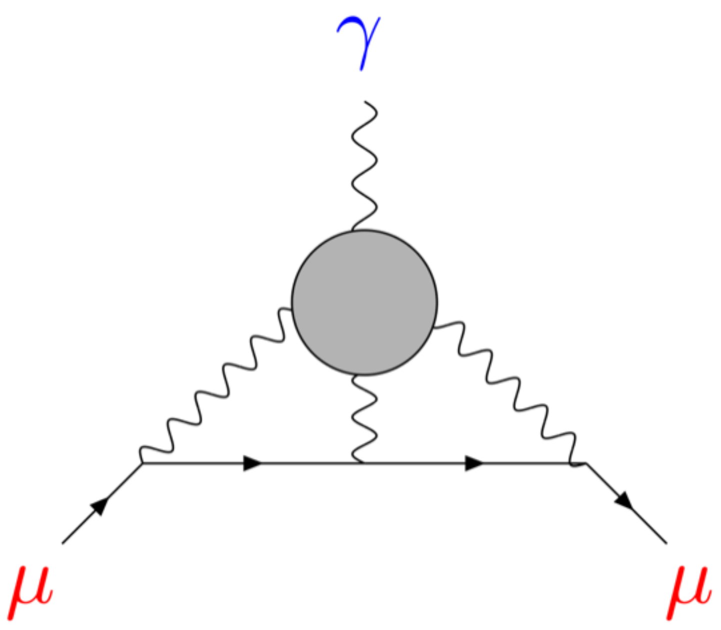
[Keshavarzi et al., arXiv:1802.02995]

- Stable deviation of ≈ 4 standard deviations between SM and experiment
- Overall precision of HVP estimate: $\approx 0.4\%$
- Theory estimate subject to experimental uncertainties
- Disagreement over individual hadronic channels

Hadronic Light-by-Light scattering

- No simple dispersive framework
- Identify dominant sub-processes, e.g.

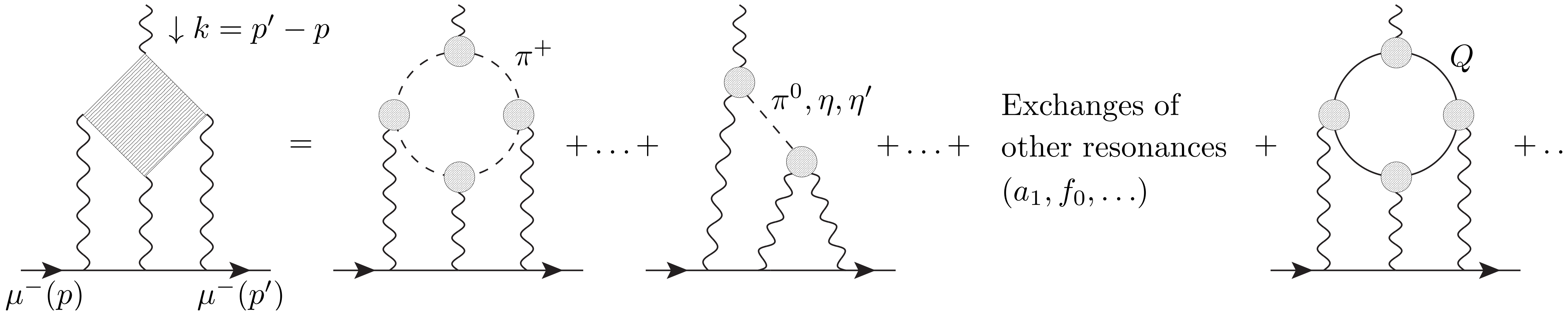




- Individual contributions estimated using model calculations
- Dispersive formalism set up for various subprocesses *[Colangelo et al., 2014 ff]*
- Lattice QCD calculations

Hadronic Light-by-Light scattering

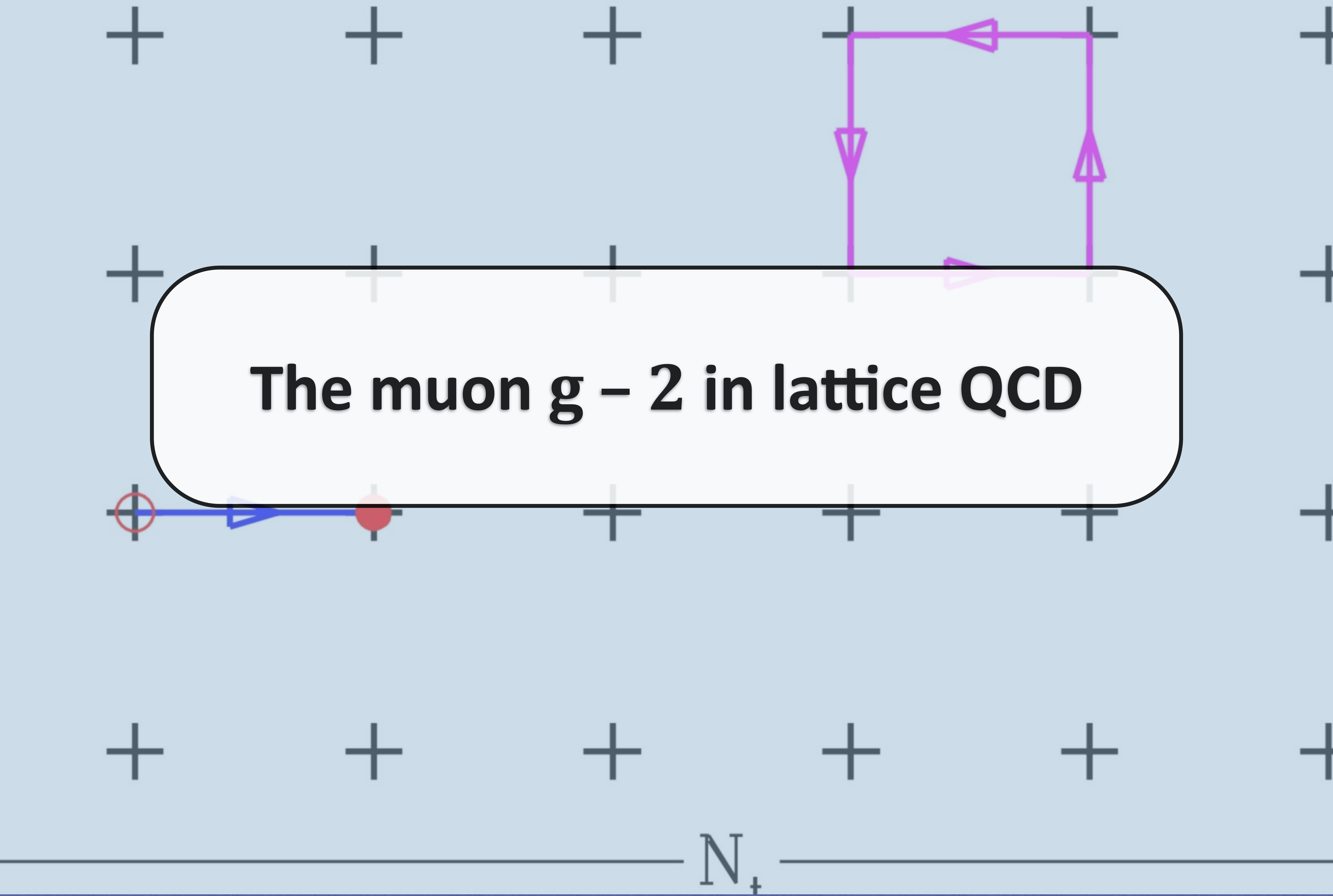
* Dominant hadronic contributions to a_μ^{hlbl} *[Nyffeler, arXiv:1710.09742]*



"Glasgow consensus"

Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99 ± 16
axial vectors	2.5±1.0	1.7±1.7	–	22±5	–	15 ± 10	22 ± 5
scalars	−6.8±2.0	–	–	–	–	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops +subl. N_C	–	–	–	0±10	–	–	–
quark loops	21±3	9.7±11.1	–	–	–	2.3 (c-quark)	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105 ± 26	116 ± 39

BPP = Bijnens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijnens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09



The muon g – 2 in lattice QCD

The muon g – 2 in lattice QCD

Motivation for first-principles approach:

* No reliance on experimental data

  — except for simple hadronic quantities to fix bare parameters

* No model dependence

  — except for chiral extrapolation and constraining the IR regime

* Can lattice QCD deliver estimates with sufficient accuracy in the coming years?

$$\delta a_\mu^{\rm hvp}/a_\mu^{\rm hvp} < \ 0.5\%, \qquad \delta a_\mu^{\rm hlbl}/a_\mu^{\rm hlbl} \lesssim \ 10\%$$

The muon g – 2 in lattice QCD

ELSEVIER

Progress in Particle and Nuclear Physics

Volume 104, January 2019, Pages 46-96

Review

Lattice QCD and the anomalous magnetic moment of the muon

Harvey B. Meyer, Hartmut Wittig

⊞ Show more

https://doi.org/10.1016/j.ppnp.2018.09.001

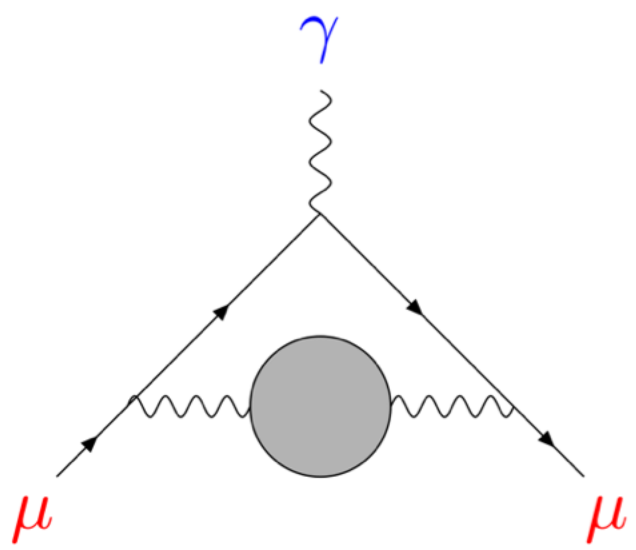
arXiv:1807.09370

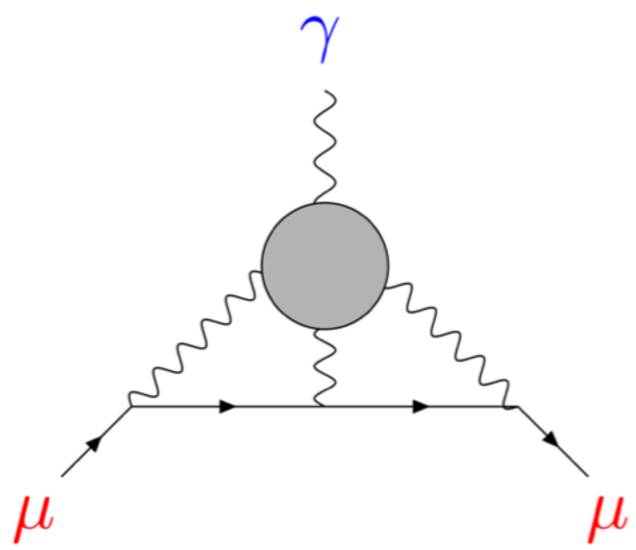
The Mainz $(g-2)_\mu$ project

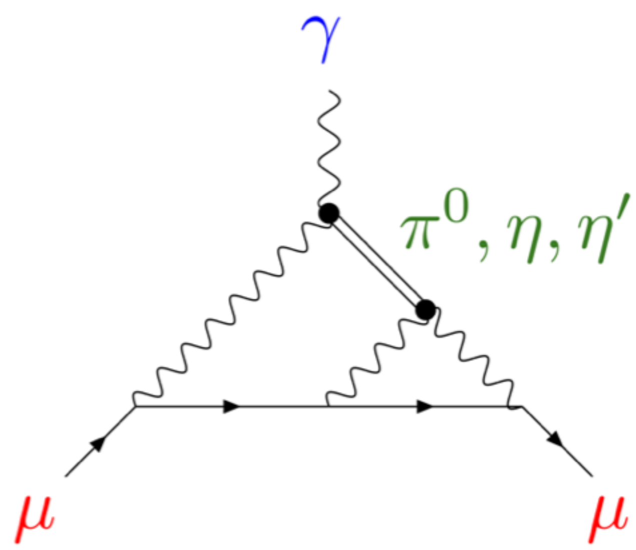
Collaborators:

M. Cè, A. Gérardin, O. Gryniuk, G. von Hippel, B. Hörz, H.B. Meyer, K. Miura, A. Nyffeler, K. Ottnad, V. Pascalutsa, A. Risch, T. San José Perez, HW

N. Asmussen, J. Green, B. Jäger, G. Herdoíza







- Direct determinations of LO a_μ^{hvp}
- Running of α and $\sin^2\theta_{\mathrm{W}}$

- Exact QED kernel
- Forward scattering amplitude

- Transition form factor for $\pi^0 \to \gamma^*\gamma^*$

Lattice QCD approach to HVP

* Convolution integral over Euclidean momenta: *[Lautrup & de Rafael; Blum]*

$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, f(Q^2)\hat{\Pi}(Q^2), \quad \hat{\Pi}(Q^2) = 4\pi^2\left(\Pi(Q^2) - \Pi(0)\right)$$

* Vacuum polarisation tensor:

$$\Pi_{\mu\nu}(Q) = i\int d^4x\, {\rm e}^{iQ\cdot(x-y)}\left\langle J_\mu(x)J_\nu(y)\right\rangle \equiv (Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)\Pi(Q^2)$$

* Electromagnetic current:

$$J_\mu = \tfrac{2}{3}\overline{u}\gamma_\mu u - \tfrac{1}{3}\overline{d}\gamma_\mu d - \tfrac{1}{3}\overline{s}\gamma_\mu s + \ldots$$

* Weight function $f(Q^2)$ strongly peaked near muon mass

Lattice QCD approach to HVP

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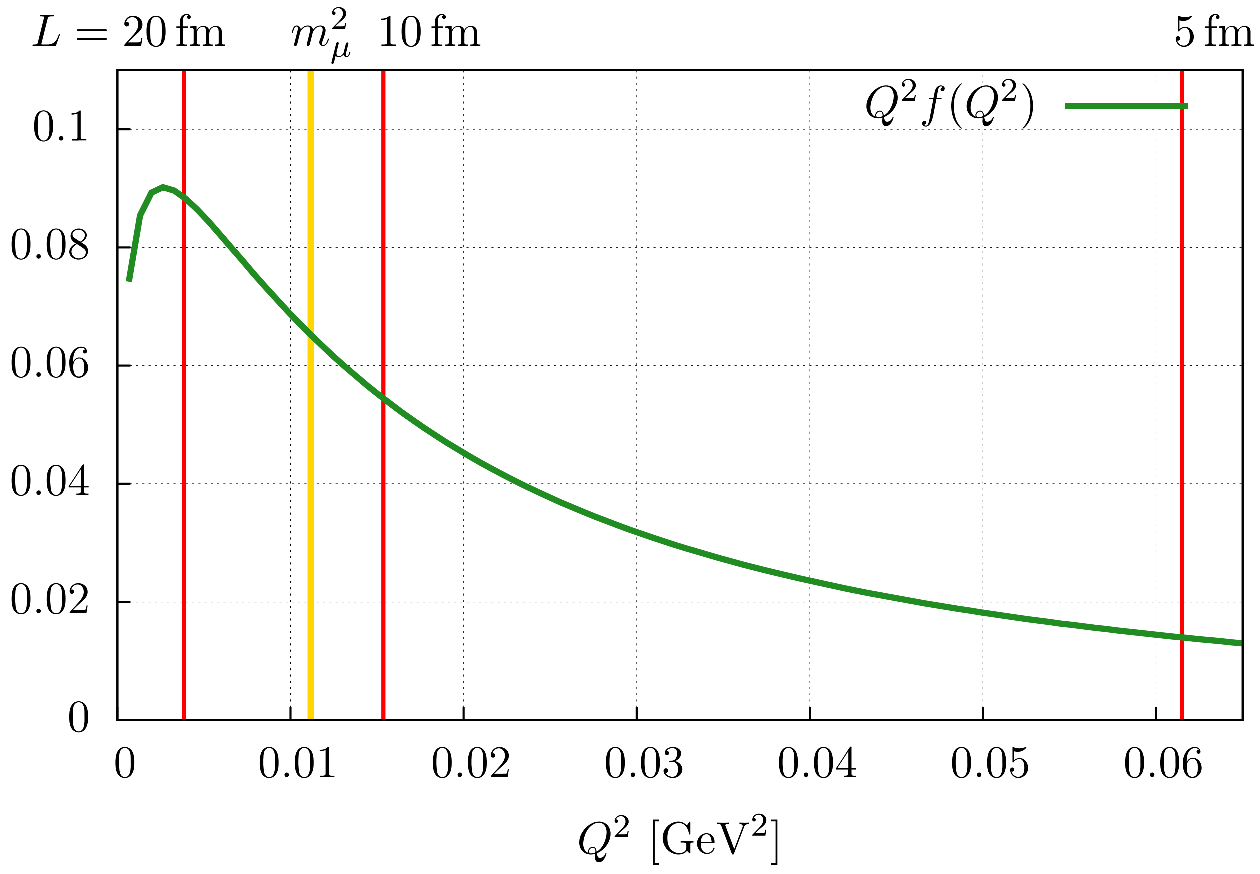
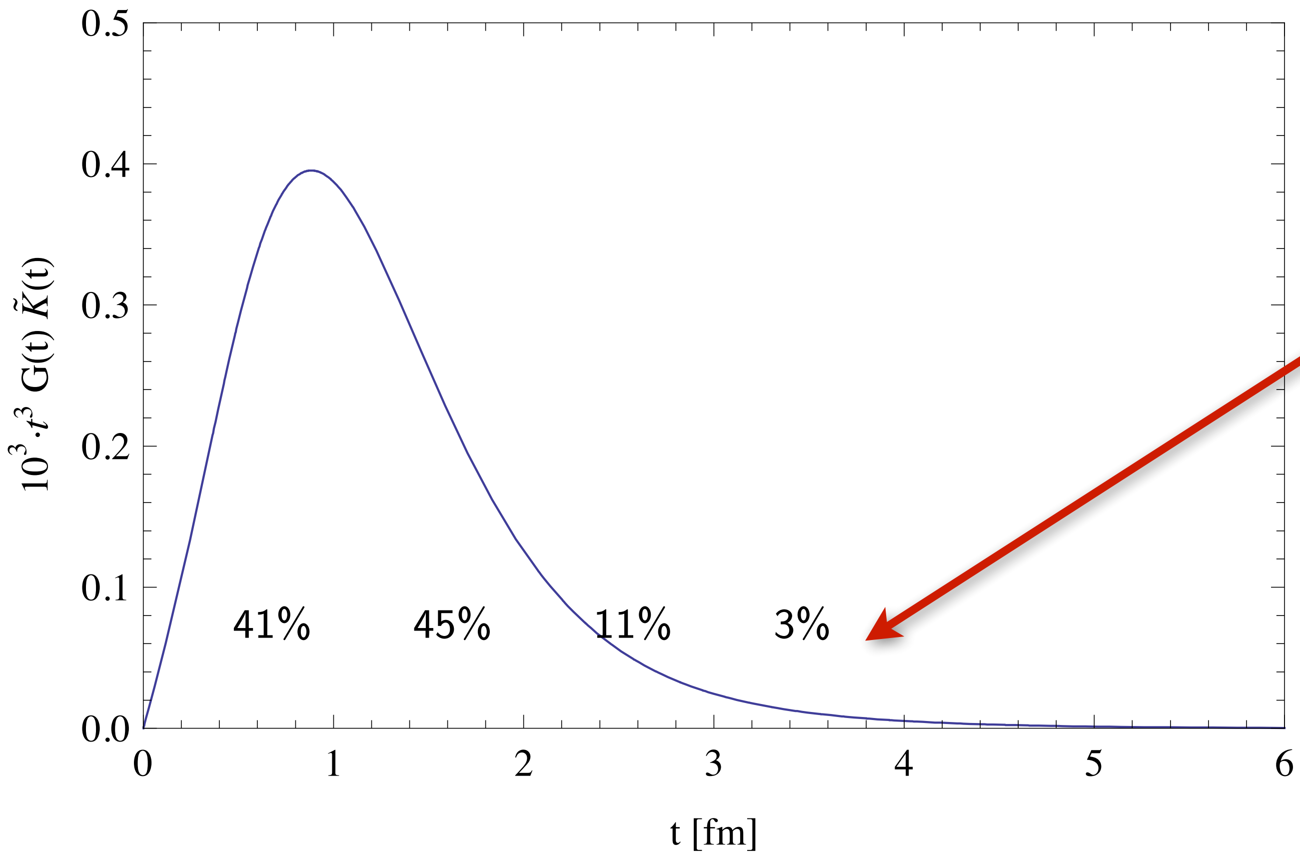


Lattice QCD approach to HVP

* **Time-momentum representation:** *[Bernecker & Meyer]*

$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, \tilde{K}(x_0)\, G(x_0), \qquad G(x_0) = -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

$$\tilde{K}(x_0) = 4\pi^2 \int_0^\infty dQ^2\, f(Q^2) \left[x_0^2 - \frac{4}{Q^2} \sin^2\left(\tfrac{1}{2} Q x_0\right) \right]$$

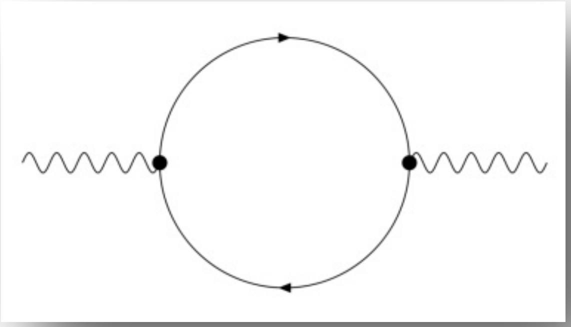


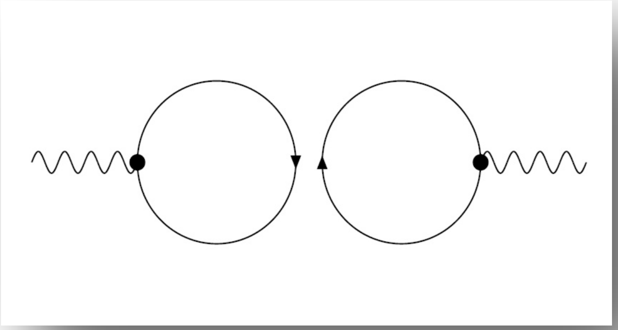
Significant contribution from tail of $G(x_0)$

Lattice QCD approach to HVP

Challenges:

- ✱ Statistical accuracy at the sub-percent level required
- ✱ Control infrared regime of vector correlator: $G(x_0)$ at large x_0
- ✱ Perform comprehensive study of finite-volume effects
- ✱ Include quark-disconnected diagrams





- ✱ Include isospin breaking: $m_u \neq m_d$, QED corrections

Current data sets

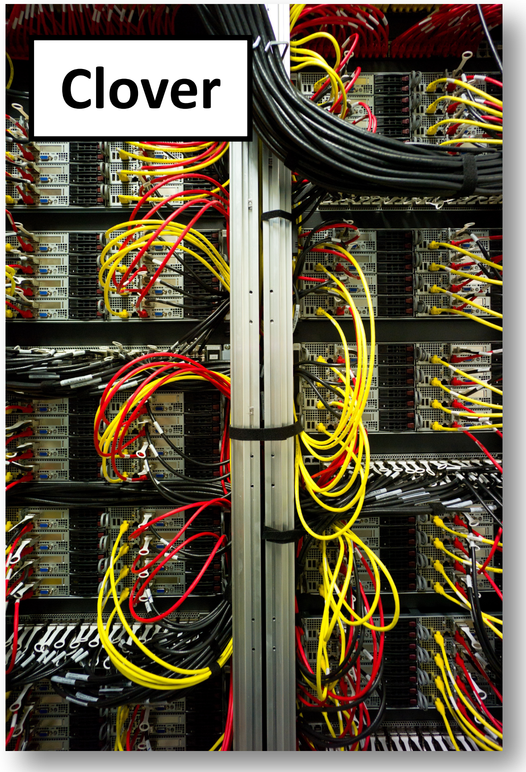
CLS consortium — "Coordinated Lattice Simulations"

* $N_f = 2$ flavours of $O(a)$ improved Wilson fermions
* Three values of the lattice spacing: $a = 0.076, 0.066, 0.049\,\mathrm{fm}$
* Pion masses and volumes: $m_\pi^{\min} = 185\,\mathrm{MeV}, \quad m_\pi L > 4$

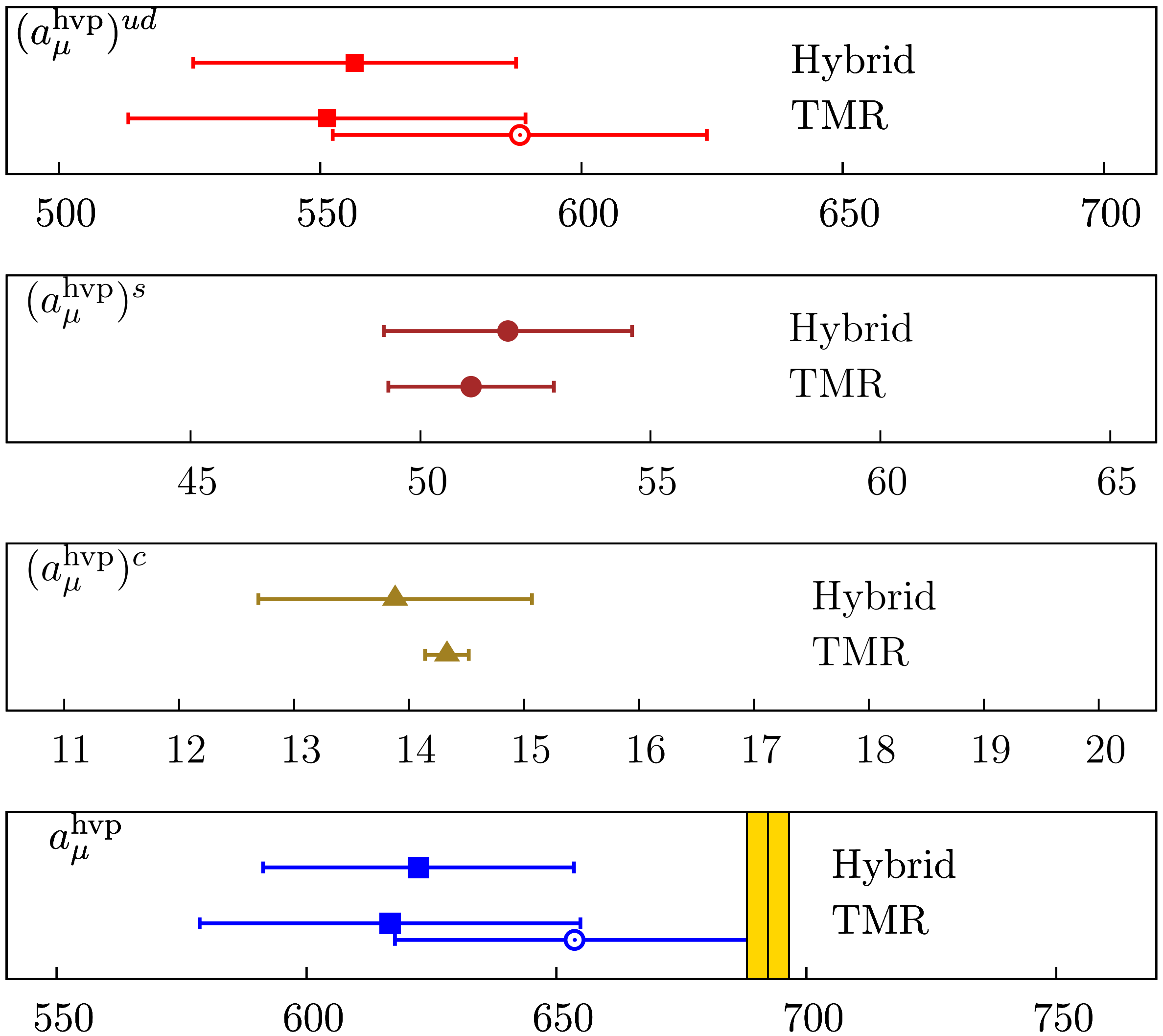
---

* $N_f = 2{+}1$ flavours of $O(a)$ improved Wilson fermions;
  tree-level Symanzik gauge action
* Four values of the lattice spacing: $a = 0.085, 0.077, 0.065, 0.050\,\mathrm{fm}$
* Pion masses and volumes: $m_\pi^{\min} \approx 135\,\mathrm{MeV}, \quad m_\pi L > 4$

Simulations and Machines



Results in two-flavour QCD



- Compare different methods to constrain infrared regime
- Finite-volume corrections sizeable
- Quark-disconnected diagrams contribute < 2%

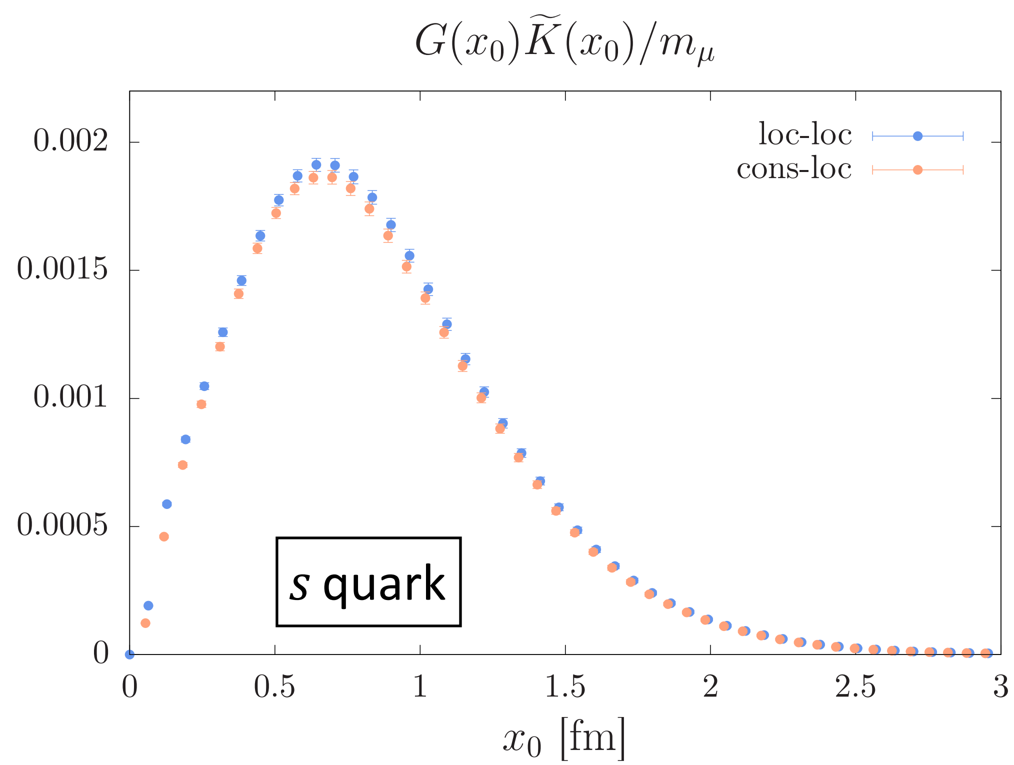
$$a_\mu^{\rm hvp} = (654 \pm 32_{\rm stat} \pm 17_{\rm syst} \pm 10_{\rm scale} \pm 7_{\rm FV}\, {}^{+\ 0}_{-10}{}_{\rm disc}) \cdot 10^{-10}$$

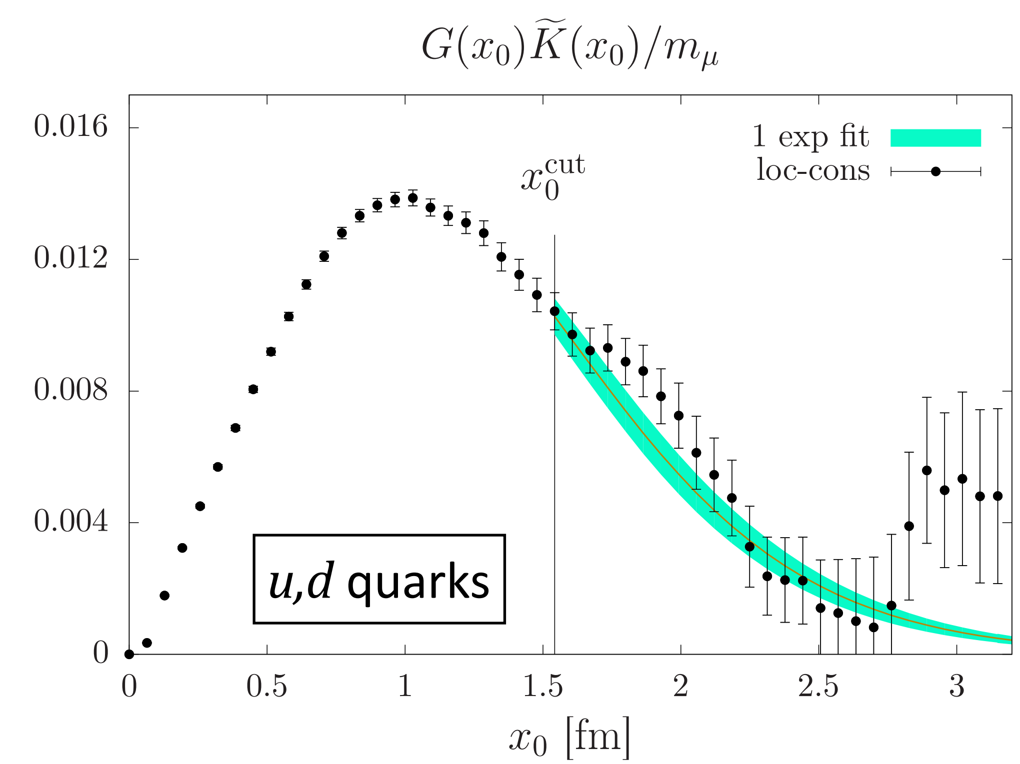
[Della Morte et al., JHEP 10 (2017) 020]

Preliminary results for $N_f = 2+1$

- TMR integrand and its long-distance behaviour: *[Antoine Gérardin]*

  Physical pion mass:





- Estimate at $m_\pi = m_\pi^{\rm phys}$ still statistics-limited

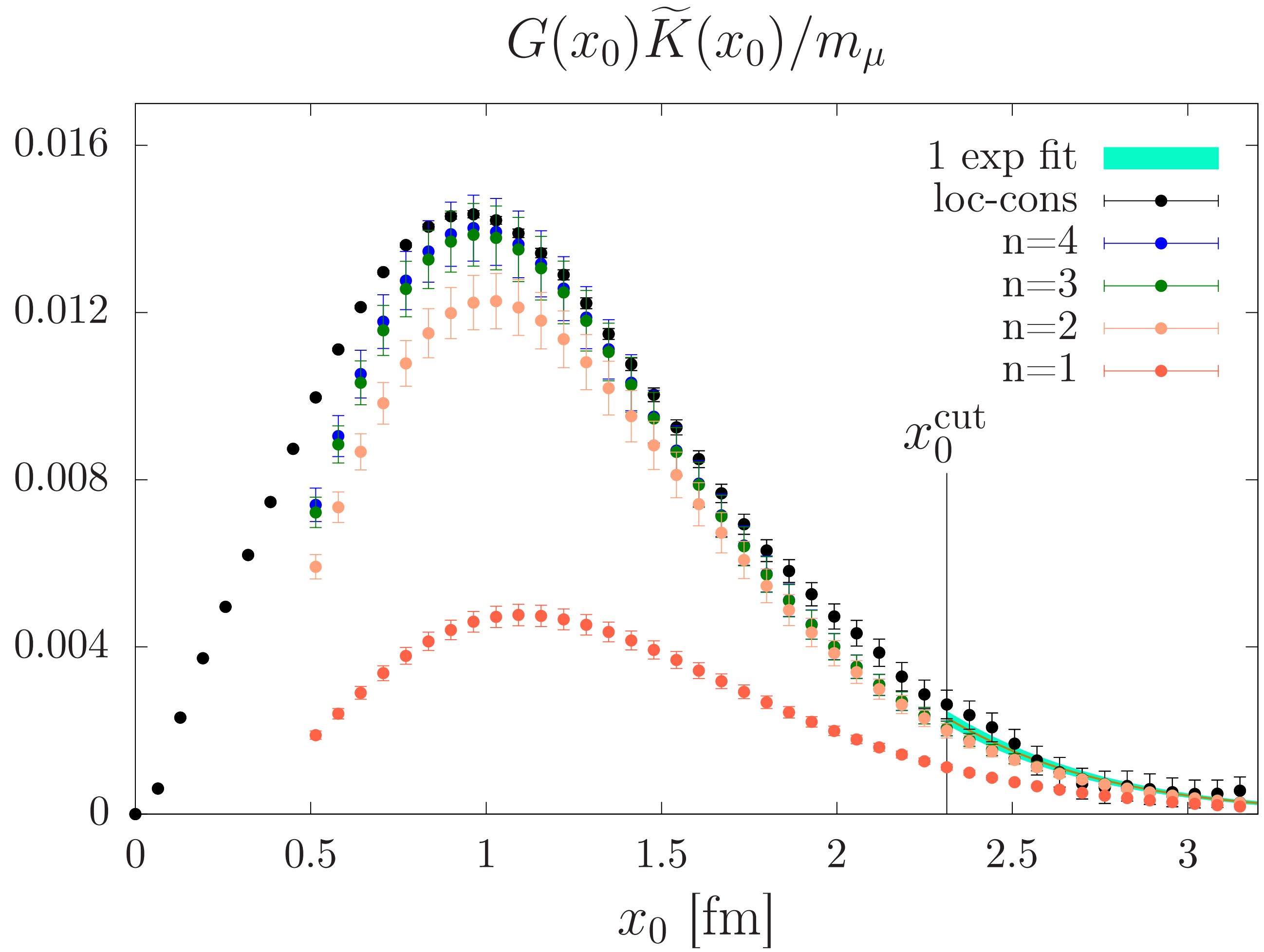
- Large- x_0 regime requires modelling for $x_0 > 1.5$ fm

Preliminary results for $N_f = 2+1$

* Saturation of the correlation by low-lying states

[Antoine Gérardin]

$$G(x_0) = \sum_{n=1}^{\infty} A_n \, \mathrm{e}^{-\omega_n x_0} \qquad \text{as } x_0 \to \infty$$



Pion mass: $m_\pi = 200\,\mathrm{MeV}$

[Update of Wittig et al., arXiv:1710.10072]

Preliminary results for $N_f = 2+1$

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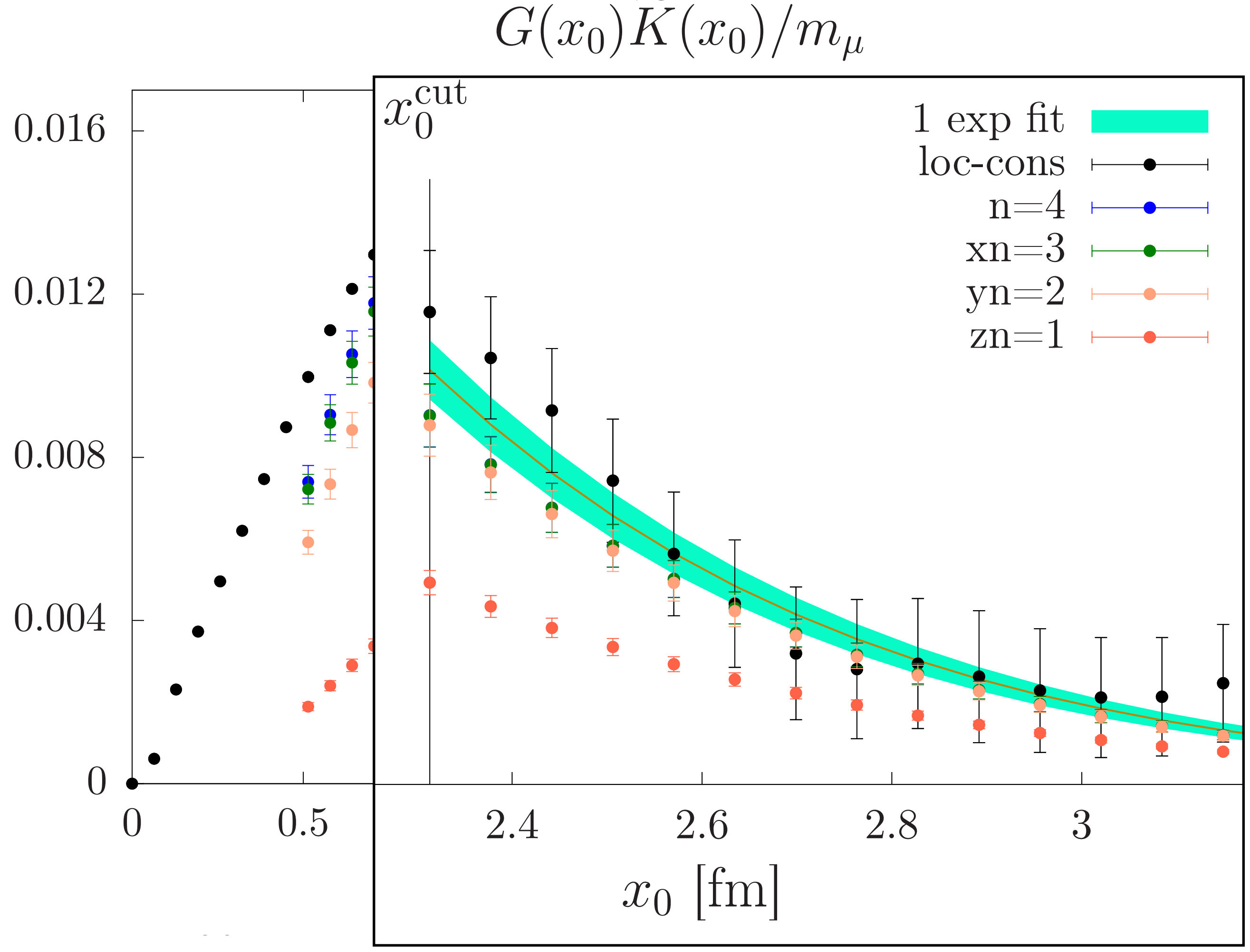


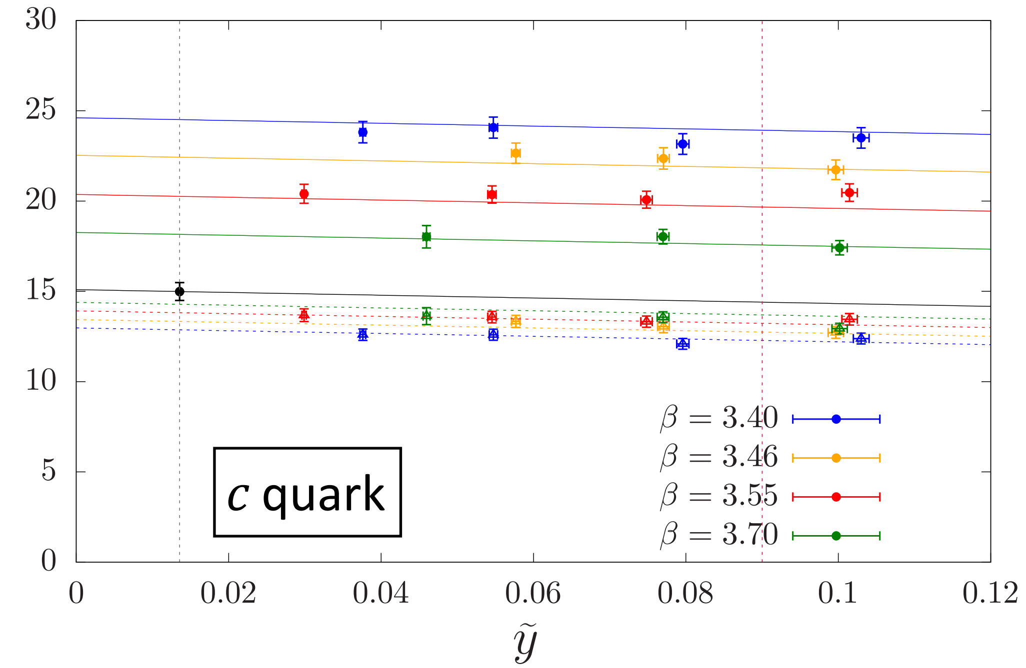
Pion mass: $m_\pi = 200\,\mathrm{MeV}$

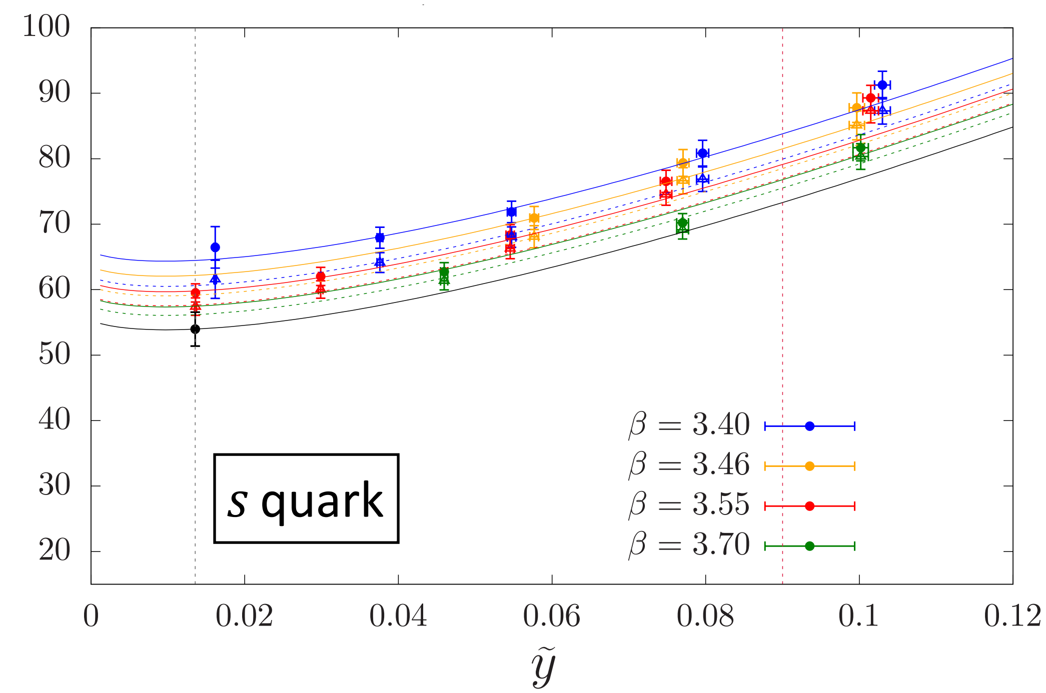
[Update of Wittig et al., arXiv:1710.10072]

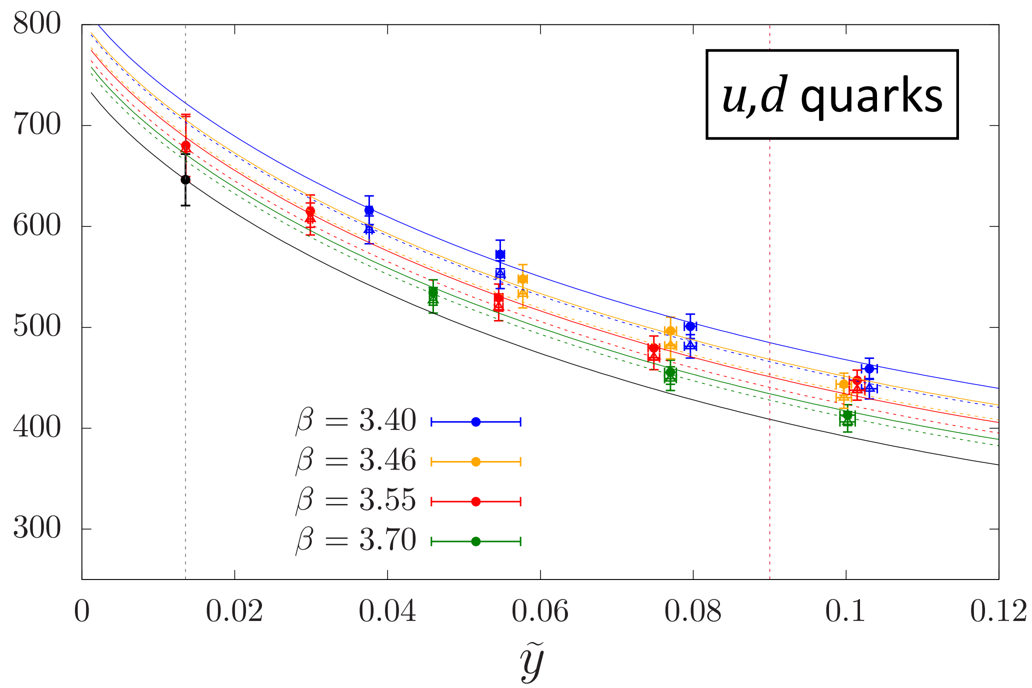
Preliminary results for $N_f = 2+1$

* Extrapolation to the physical point

[Antoine Gérardin]







$$(a_\mu^{\rm hvp})^c = 14.95(47)(11) \times 10^{-10}$$

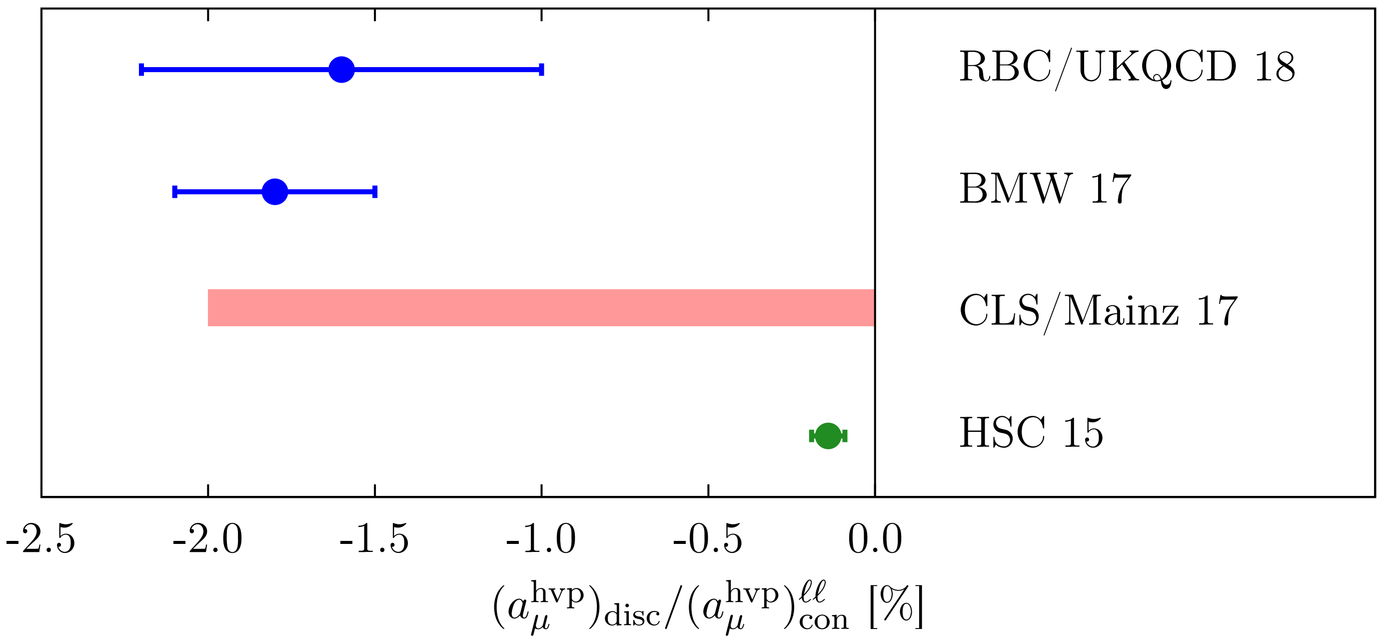
$$(a_\mu^{\rm hvp})^s = 53.6(2.5)(0.8) \times 10^{-10}$$

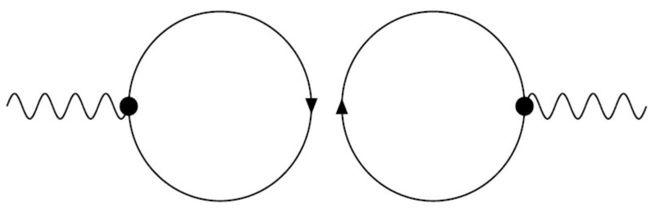
$$(a_\mu^{\rm hvp})^{ud} = 643(21)(xx) \times 10^{-10}$$

Error limited by lattice scale

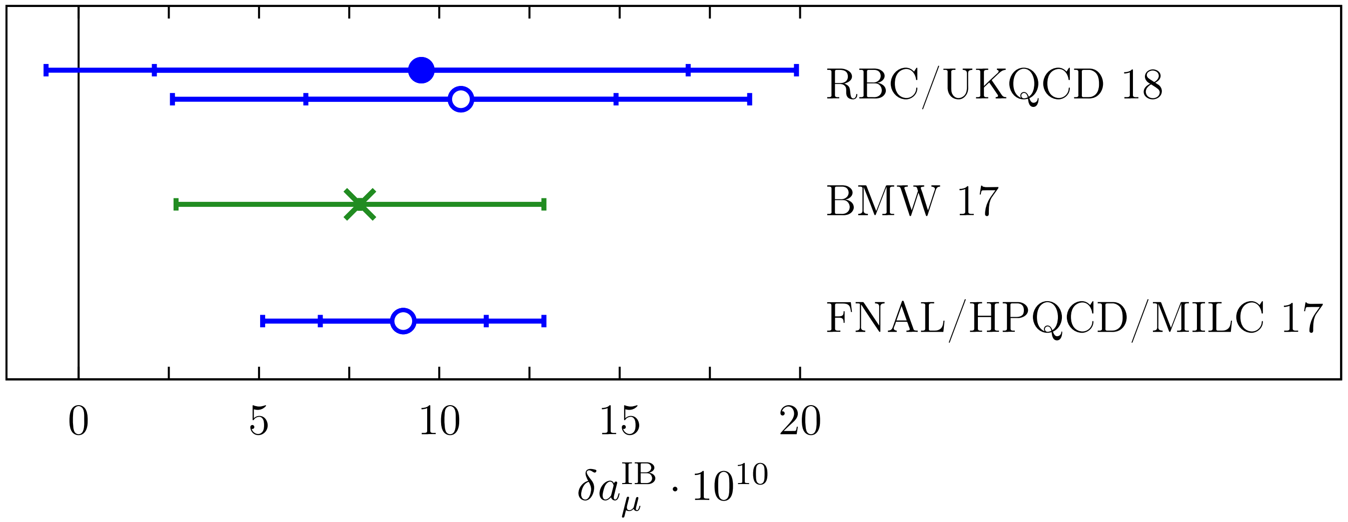
Towards sub-percent accuracy

* Quark-disconnected diagrams





* Iso-spin breaking corrections

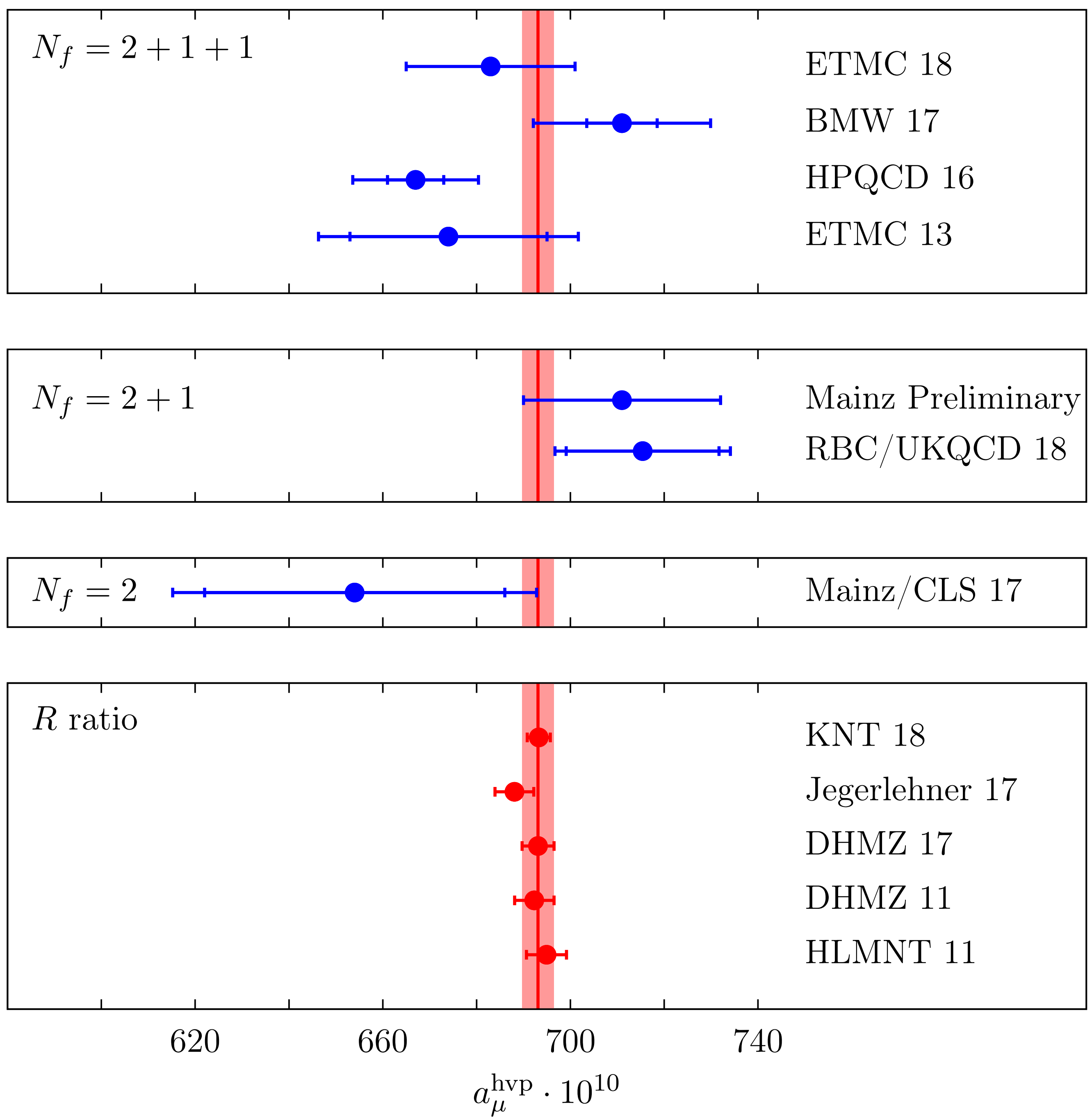


$m_u/m_d = 0.46(2)(2)$

$q_u = 2/3, \quad q_d = -1/3$

Compilation & comparison

* **Lattice QCD** vs. **dispersion theory**:

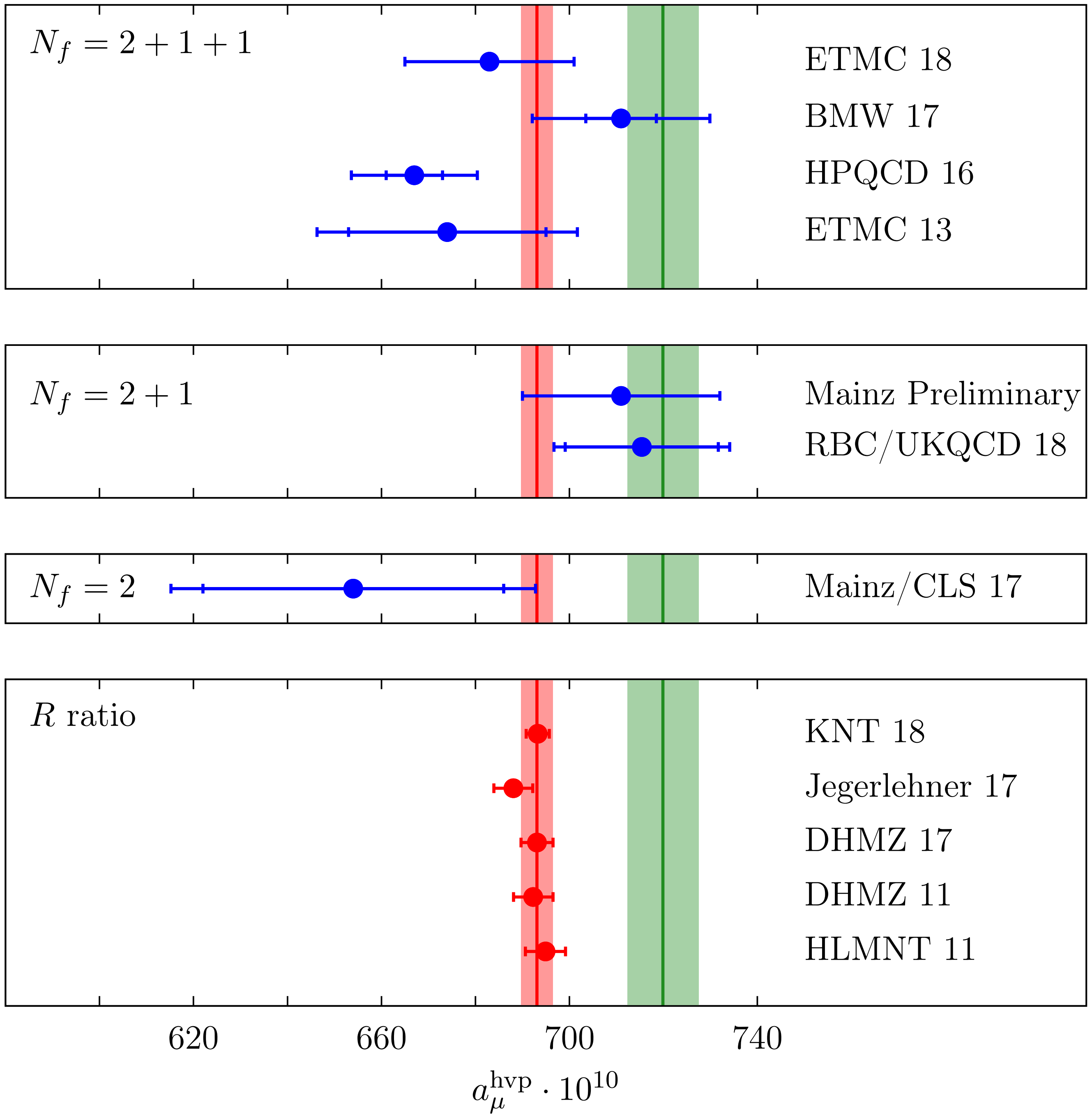


Our preliminary result:

$$a_\mu^{\rm hvp} = 711 \cdot 10^{-10} \pm 3\%$$

Compilation & comparison

* **Lattice QCD** vs. **dispersion theory**:



Our preliminary result:

$$a_\mu^{\rm hvp} = 711 \cdot 10^{-10} \pm 3\%$$

"No New Physics":

$$(a_\mu^{\rm hvp})_{\rm NNP} = (a_\mu^{\rm hvp})_{\rm disp} + (a_\mu^{\rm exp} - a_\mu^{\rm SM})$$

Lattice QCD approaches to HLbL

* Matrix element of e.m. current between muon initial and final states:

$$\left\langle \mu(\boldsymbol{p}', s') \left| J_\mu(0) \right| \mu(\boldsymbol{p}, s) \right\rangle = -e\, \bar{u}(\boldsymbol{p}', s') \left(F_1(Q^2)\gamma_\mu + \frac{F_2(Q^2)}{2m} \sigma_{\mu\nu} Q_\nu \right) u(\boldsymbol{p}, s)$$

$$a_\mu^{\mathrm{hlbl}} = F_2(0)$$

RBC/UKQCD:

* QCD + QED simulations *[Hayakawa et al. 2005; Blum et al. 2015]*
* QCD + stochastic QED *[Blum et al. 2016, 2017]*

Mainz group:

* Exact QED kernel in position space *[Asmussen et al. 2015, 2016, and in prep.]*
* Transition form factors of sub-processes *[Gérardin, Meyer, Nyffeler 2016]*
* Forward scattering amplitude *[Green et al. 2015, 2017]*

QCD + Stochastic QED

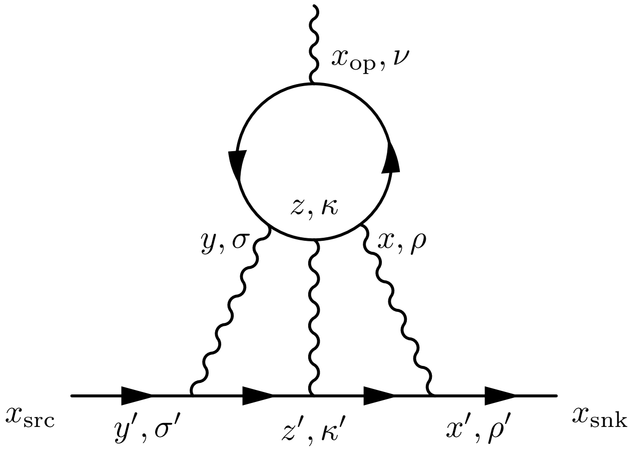
* Stochastic treatment of QED contribution:
  ⇒ insertion of three exact Feynman gauge photon propagators

$$G_{\mu\nu}(x,y) = \frac{1}{VT}\delta_{\mu\nu} \sum_{k,\,|\vec{k}|\neq 0} \frac{e^{ik\cdot(x-y)}}{\hat{k}^2}$$

[Blum et al., Phys Rev D93 (2016) 014503]

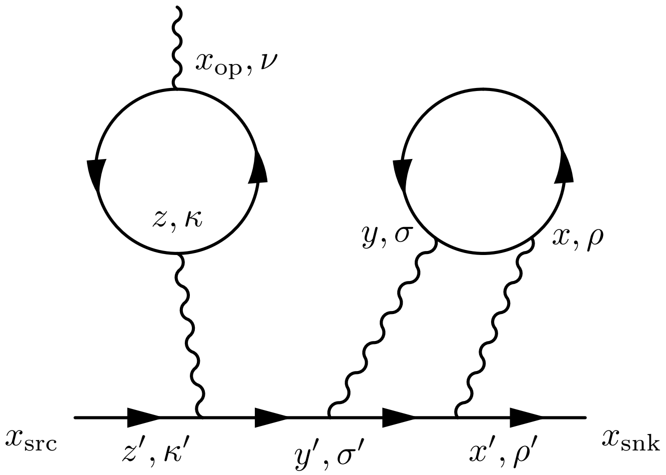
* Connected contribution:

$$(a_\mu^{\rm hlbl})_{\rm con} = (116.0 \pm 9.6) \cdot 10^{-11}$$



* Leading disconnected contribution:

$$(a_\mu^{\rm hlbl})_{\rm disc} = (-62.5 \pm 8.0) \cdot 10^{-11}$$



* Compute sub-leading disconnected diagrams

[Blum et al., Phys Rev Lett 118 (2017) 022005]

Exact QED kernel in position space

* Determine QED part perturbatively in the continuum in infinite volume

⇒ no power-law volume effects

$$a_\mu^{\rm hlbl} = F_2(0) = \frac{me^6}{3}\int d^4y \int d^4x\, \overline{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\; i\Pi_{\rho;\mu\nu\lambda\sigma}(x,y)$$

* QCD four-point function: $i\Pi_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z\, z_\rho \left\langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0)\right\rangle$

* QED kernel function: $\overline{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ *[Asmussen, Green, Meyer, Nyffeler, in prep.]*
    - Infra-red finite; can be computed semi-analytically
    - Admits a tensor decomposition in terms of six weight functions which depend on $x^2, \quad y^2, \quad x\cdot y$

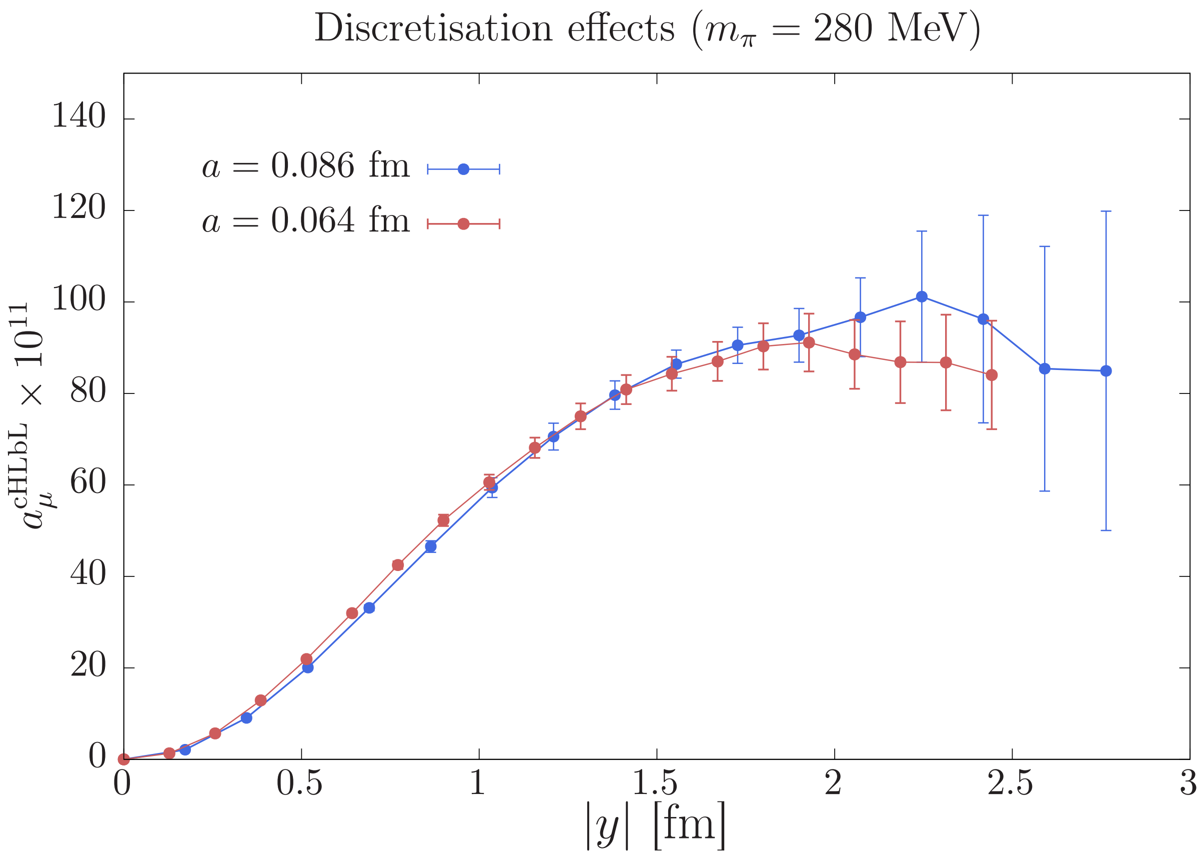
  ⇒ 3D integration instead of $\int d^4x \int d^4y$

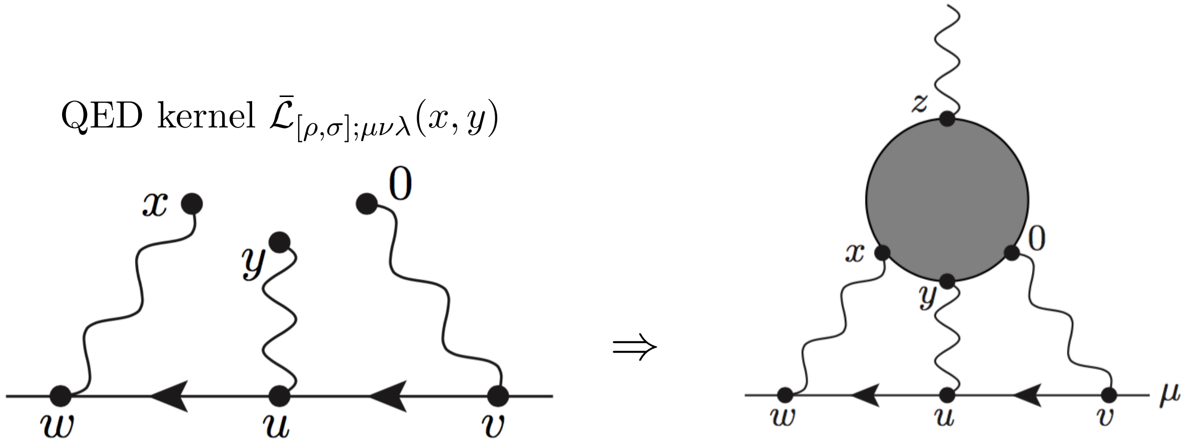
* Weight functions computed and stored on disk

Preliminary results

* Accumulated connected contribution

$$a_\mu^{\mathrm{chlbl}} = \frac{me^6}{3} 2\pi^2 \int_0^{y_{\max}} |y|^3\, d|y| \int d^4x\, \overline{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\; i\Pi_{\rho;\mu\nu\lambda\sigma}(x,y)$$

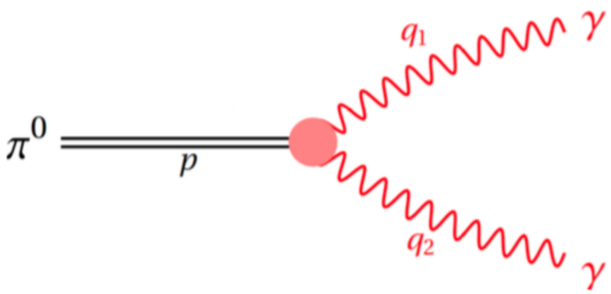




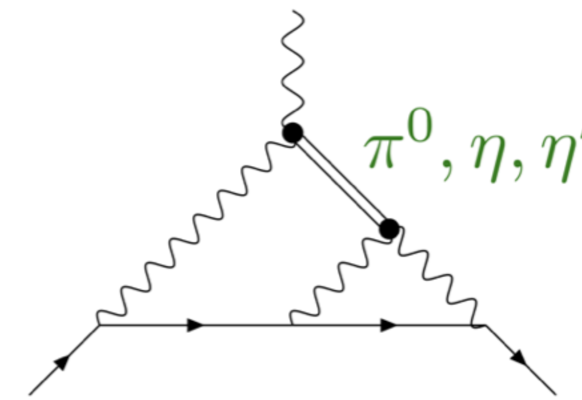
[Asmussen, Gérardin, Nyffeler, Meyer, arXiv:1811.08320]

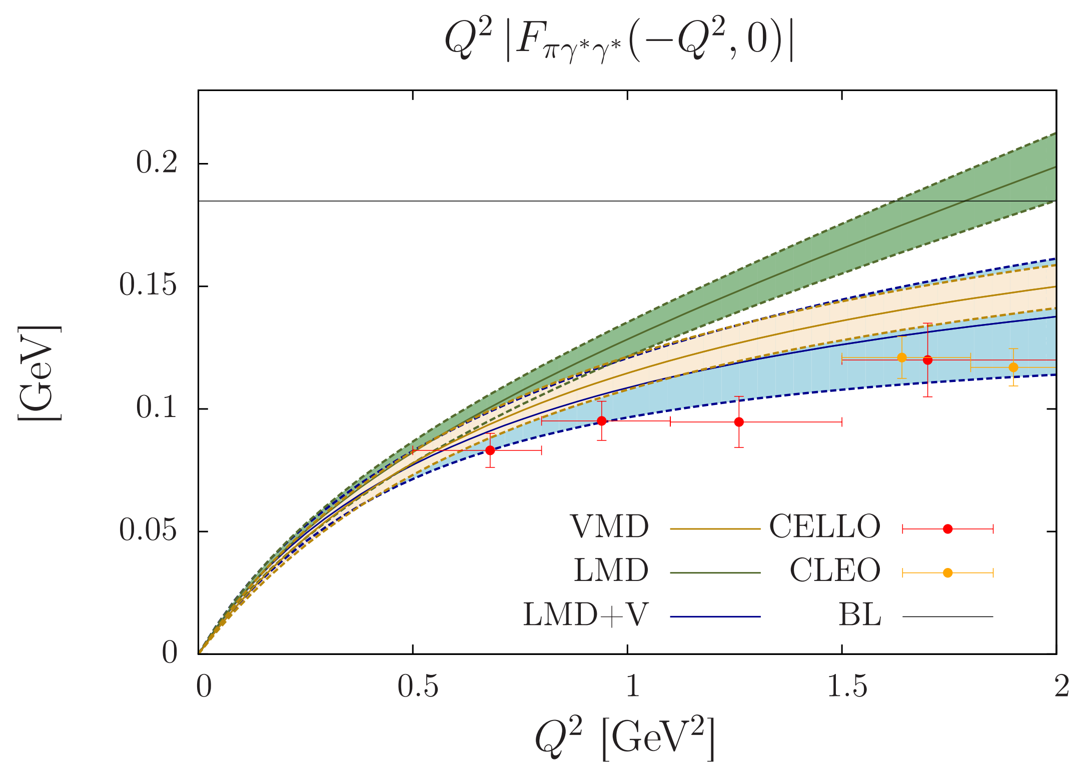
* Controlled discretisation and finite-volume effects

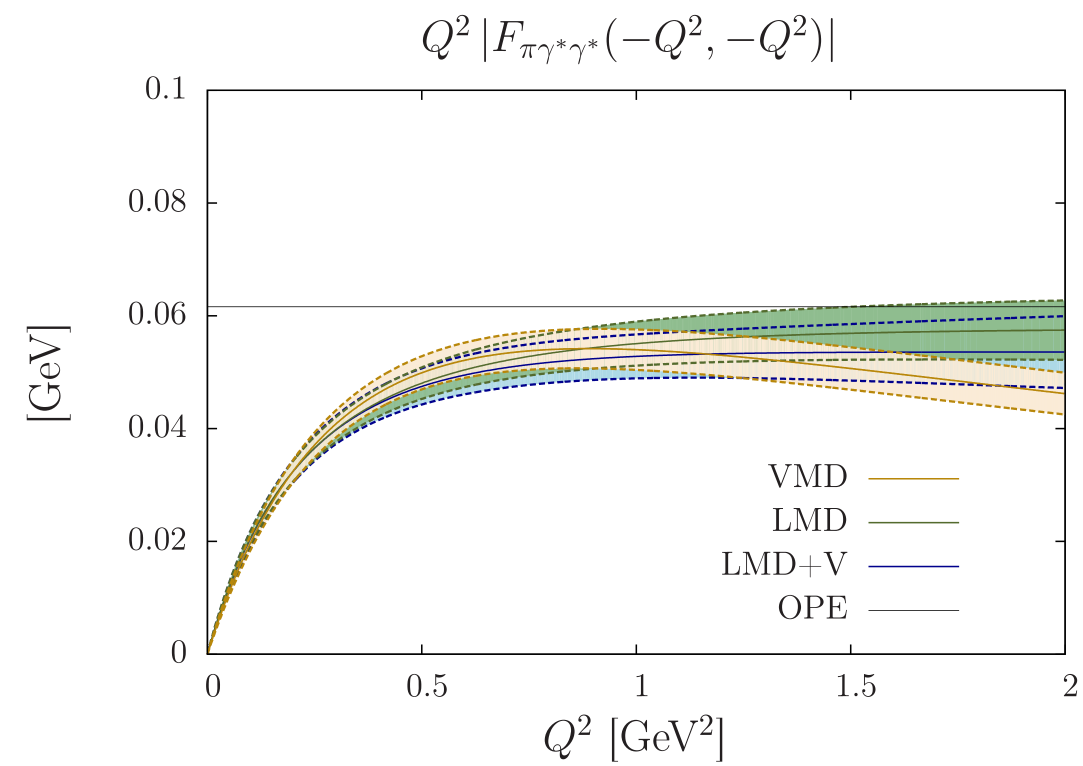
Transition form factor $\pi^0 \longrightarrow \gamma^*\gamma^*$



$$\epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta\, \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2; q_1^2, q_2^2) \equiv M_{\mu\nu}$$







* Results for π^0 contribution to hadronic light-by-light scattering:

$$(a_\mu^{\rm hlbl})_{\pi^0} = (65.0 \pm 8.3)\cdot 10^{-11} \quad \text{(LMD+V)}$$ (stat. error only)

[Gérardin, Meyer, Nyffeler, Phys Rev D94 (2016) 074507]

Summary & Outlook

Precision observables

- Provide promising hints for new physics
- Complementary to searches at high-energy colliders

Muon anomalous magnetic moment

- Beautiful interplay between theory and experiment
- New experiments will significantly increase sensitivity
- Lattice QCD: model-independent estimates for hadronic contributions
- HVP: challenge to reach sub-percent precision
- HLbL: $10–15\%$ calculation will have great impact