

Neutrino Oscillation & Other Quantum Oscillations

Fumihiko Suekane

RCNS/Tohoku Univ., Japan

Blaise Pascal Chair @APC Lab., France

Seminar @ PSI, 1/3/2018

Flow of this Seminar

Part-I

- * Neutrino Oscillation
- * Oscillation measurements
 - Status
 - Future

Part-II

- * Collection of Quantum Oscillations
 - Cabbibo Angle: θ_C ,
 - Weinberg Angle: θ_W
 - precession of magnetic and electric dipole meoment
 - Chirality Oscillation (why μ_R^- can decay)
 - What is Parity and Isospin?

Part-I: Neutrino Oscillation

Fermion (spin=1/2) Flavor

Name	Charge	Flavor		
		1 st generation	2 nd generation	3 rd generation
Lepton	-1	e	μ	τ
	0	ν_e	ν_μ	ν_τ
Quark	+2/3	u	c	t
	-1/3	d	s	b

← neutrino

Gauge boson (spin=1)

charge	EM	W	S
0	γ	Z^0	G
± 1		W^\pm	

Higgs boson (spin=0)

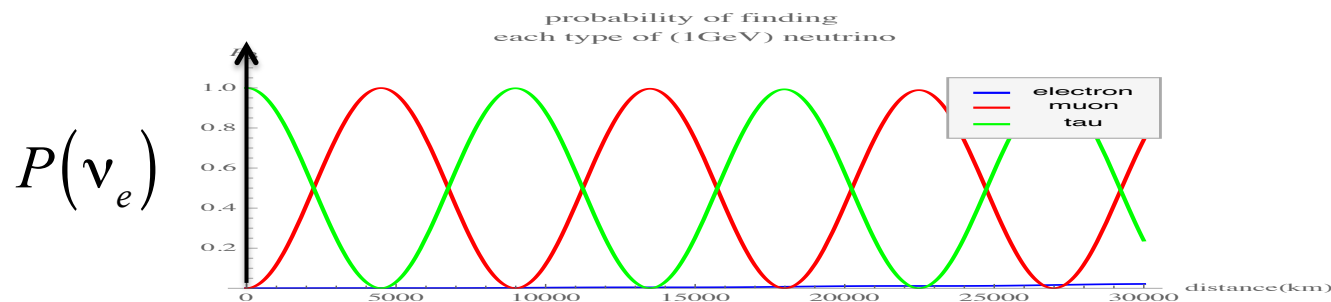
charge	
0	H^0

What is Neutrino Oscillation?

Electron stays as electron while it travels in space.



However, neutrinos change their flavors periodically.

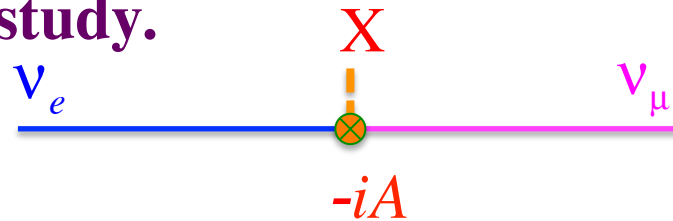


This phenomenon is called neutrino oscillation

What causes the neutrino to oscillate?

We do not know yet.

In order for N.O. to happen, something(**X**) has to change the neutrino flavor. To know what is **X** is the important purpose of N.O. study.



"A" indicates the strength of the transition (amplitude).

In this case

State equation

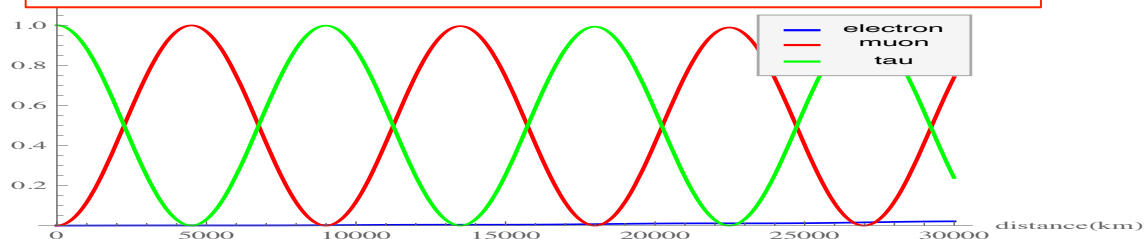
$$\frac{d}{dt} \nu_e = -iA \nu_\mu, \quad \frac{d}{dt} \nu_\mu = -iA \nu_e$$

Initial condition

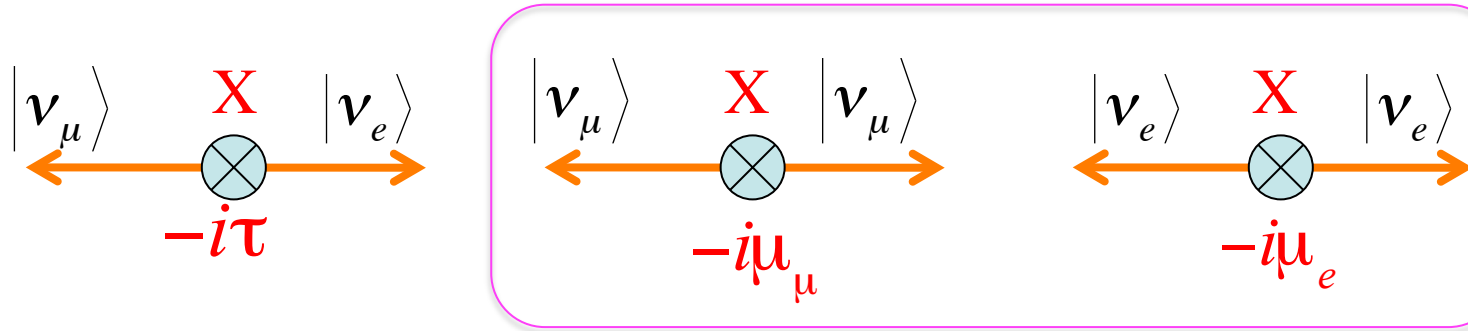
$$\nu_e [t = 0] = 1, \quad \nu_\mu [t = 0] = 0$$

➔ Oscillation

$$|\nu_e [t]|^2 = \cos^2 At, \quad |\nu_\mu [t]|^2 = \sin^2 At$$



General transition amplitudes & mass eigenstates

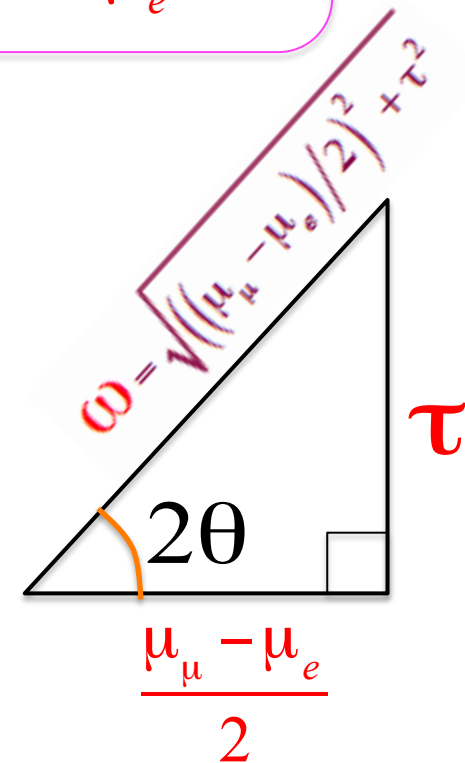


If there are **self-transitions**,
The mass eigenstate becomes

$$\begin{cases} \nu_1 [t] = (\cos \theta |\nu_e\rangle - \sin \theta |\nu_\mu\rangle) \exp[-im_1 t] \\ \nu_2 [t] = (\sin \theta |\nu_e\rangle + \cos \theta |\nu_\mu\rangle) \exp[-im_2 t] \end{cases}$$

The neutrino masses are

$$m_1 = \frac{\mu_\mu + \mu_e}{2} - \omega, \quad m_2 = \frac{\mu_\mu + \mu_e}{2} + \omega$$

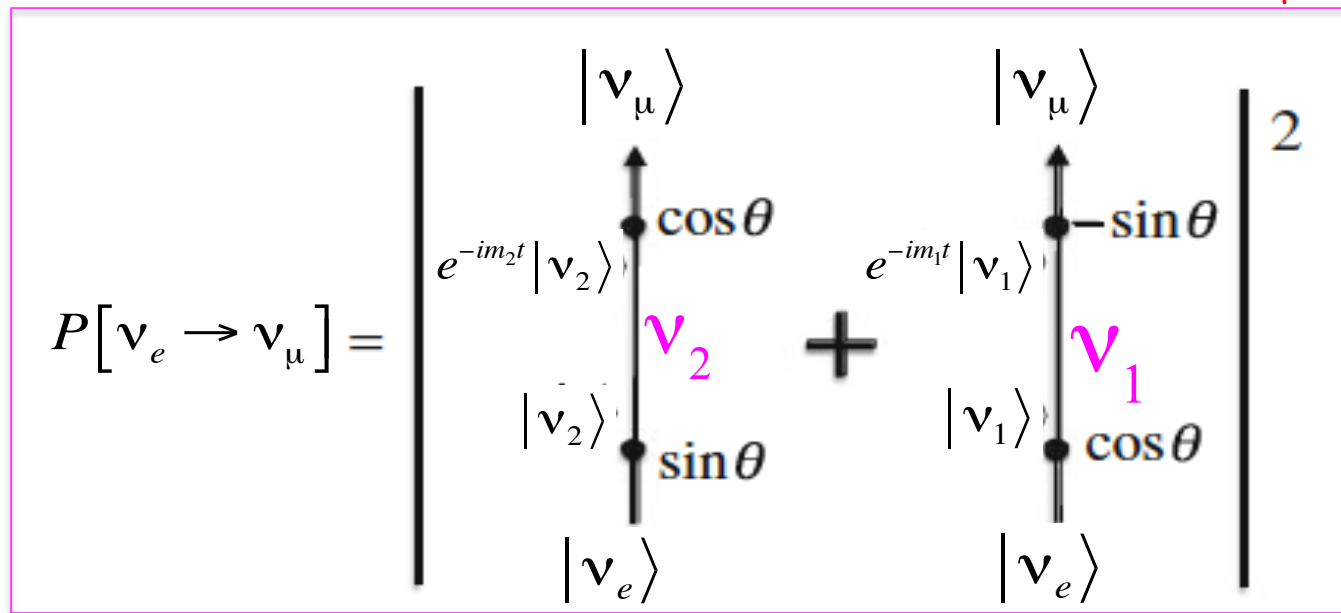


Mixing triangle₆

ν oscillation (non relativistic case)

A Q.M. principle: Probability for something to happen is the absolute square of the sum of amplitudes of all possible diagrams.

There are 2 amplitudes for $\nu_e \rightarrow \nu_\mu$



$$= \left| \sin \theta \cos \theta e^{-im_2t} - \sin \theta \cos \theta e^{-im_1t} \right|^2$$

$$= \sin^2 2\theta \sin^2 \frac{m_2 - m_1}{2} t$$

Oscillation in time

Relativistic Neutrino Oscillation

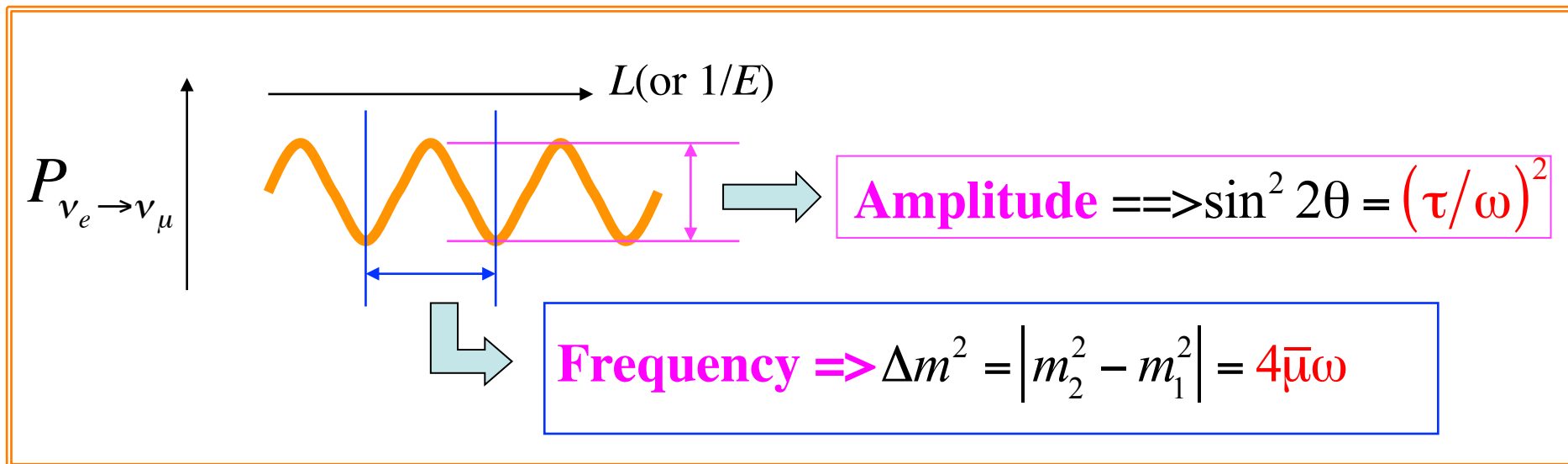
In experimental condition, neutrino is traveling relativistic

Lorentz Boost ($\gamma = E/m$)

$$mt \rightarrow m \frac{t}{\gamma} = \frac{m^2}{E} t = \frac{m^2 L}{E}$$

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} L$$

What we can measure,



Why we measure ν oscillations?

There are many oscillations (irrespective to it is observable or not).

- * $K^0 \leftrightarrow \overline{K^0}$ oscillation. \rightarrow CP violation
- * $|u\bar{u}\rangle \leftrightarrow |d\bar{d}\rangle$ oscillation in π^0, ρ , etc. \rightarrow Hadron mass pattern
- * Cabibbo angle, quark mass $\leftarrow d' \leftrightarrow s'$ oscillation
- * Weinberg angle, W, Z^0 mass $\leftarrow B \leftrightarrow W_3$ oscillation

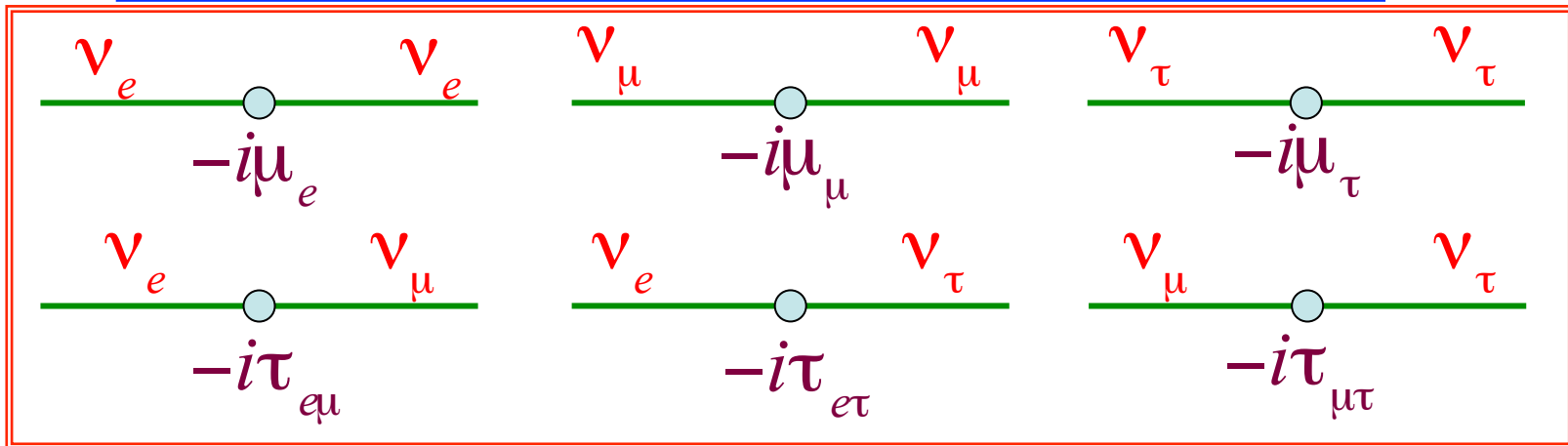
\leftarrow We have learned a lot from these "Oscillations"

We can expect to learn more from ν oscillations;

$$\nu_\alpha \leftrightarrow \nu_\beta$$

What is X??

3 Flavor Neutrino Oscillation



→ The mixing matrix becomes 3x3 & there are 3 mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad m_1, m_2, m_3 = \dots$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

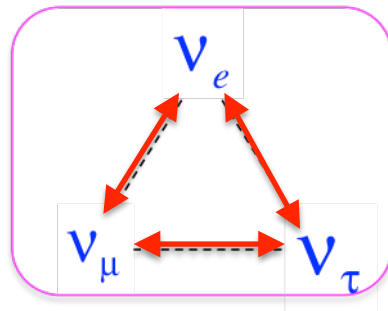
Neutrino Oscillation Experiments

Parameters
to measure

Mixing angles: $\theta_{12}, \theta_{23}, \theta_{13}$

Square mass differences: $\Delta m_{12}^2, \Delta m_{23}^2$ ($, \Delta m_{13}^2$)

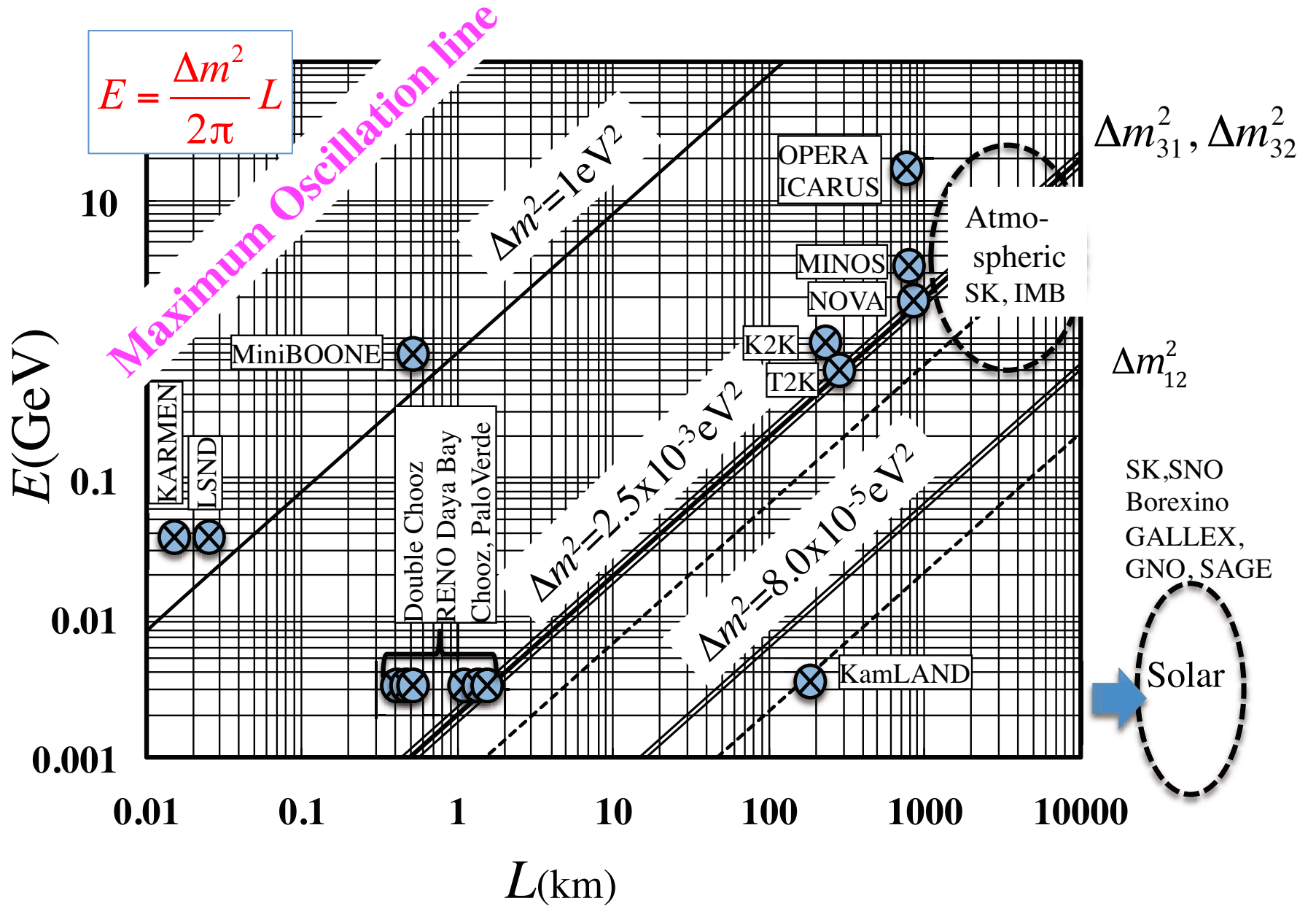
CP violation phase: δ_{CP}



It is a long history with labors, efforts, brilliant ideas,
severe competitions and sometimes errors.

But allow me to make it short

E-L relation of N.O. experiments

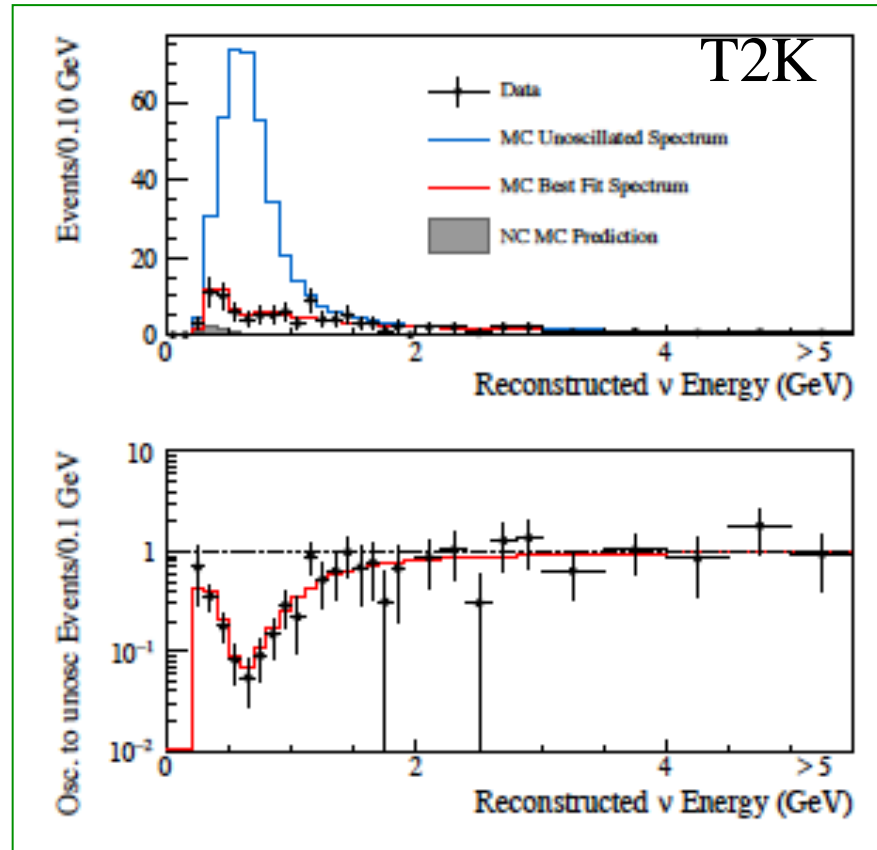
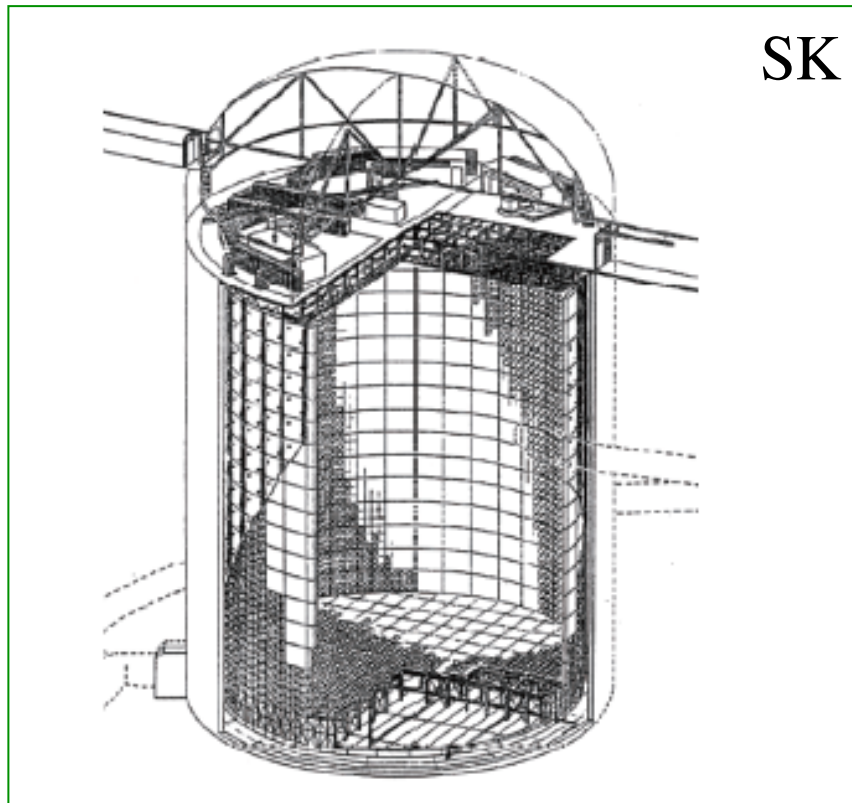


$$\theta_{23}, \Delta m_{32}^2$$

Atmospheric (SK. etc.), T2K, MINOS, NOVA ...

$$P(\nu_{\mu} \rightarrow \nu_{\mu}) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{32}^2 L}{4E}$$

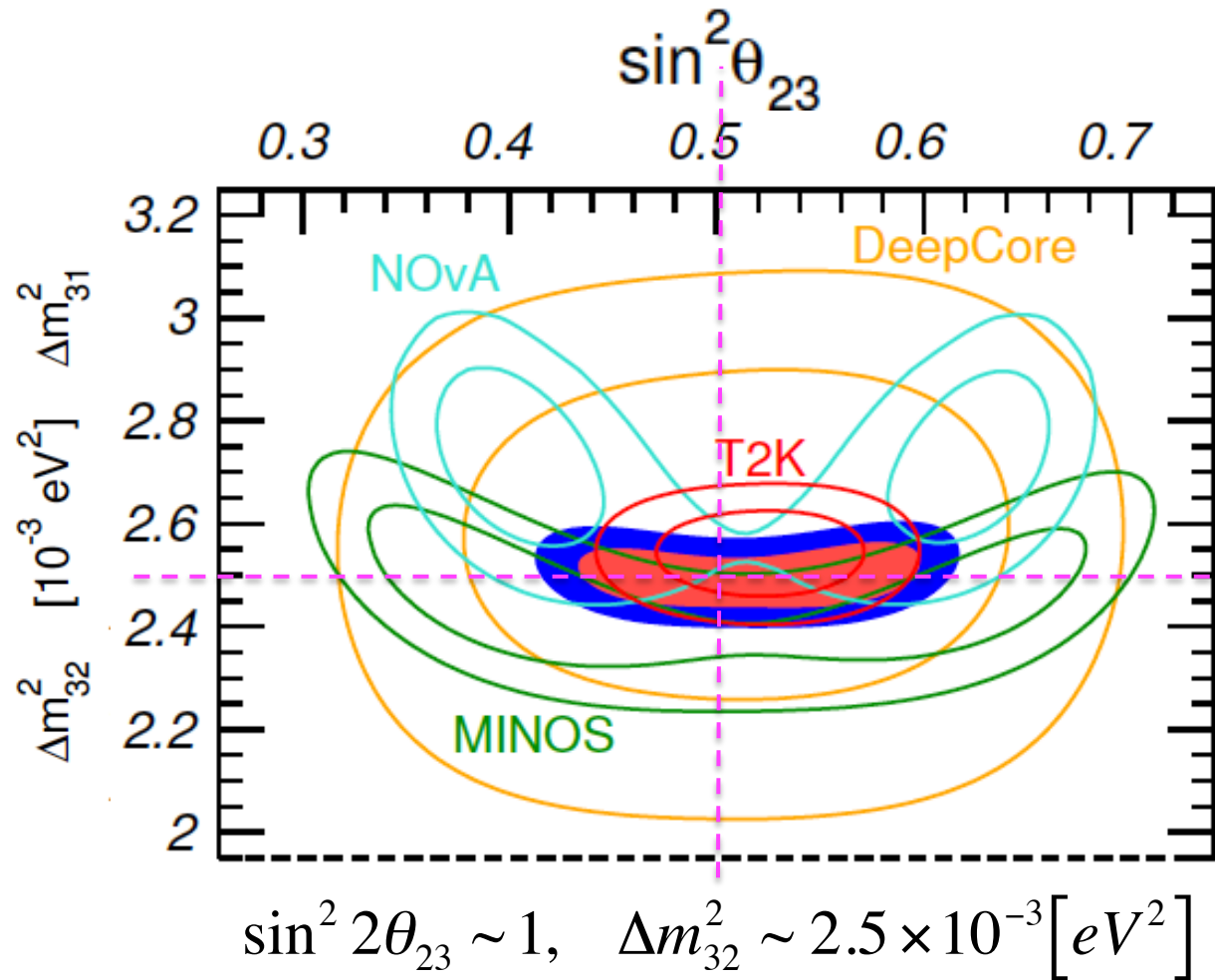
arXiv:1502.01550v1



→ Discovery of N.O. : Nobel prize in 2015 (T.Kajita)

$$\theta_{23}, \Delta m_{32}^2$$

$$P(\nu_\mu \rightarrow \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{32}^2}{4E} L$$



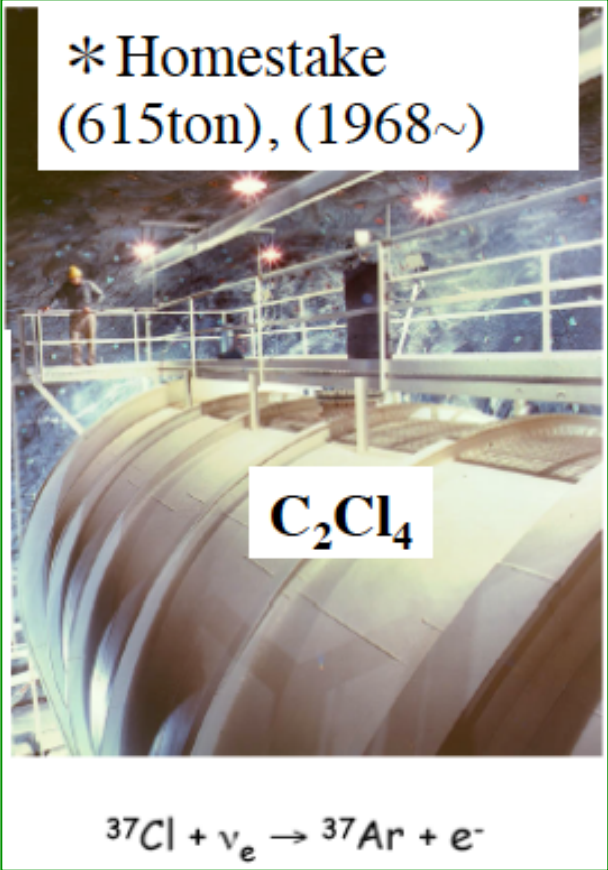
Normal Hierarchy
($m_3 > m_2$) case

NuFIT 3.1(2017),
www.nu-fit.org,
JHEP01(2017)087,

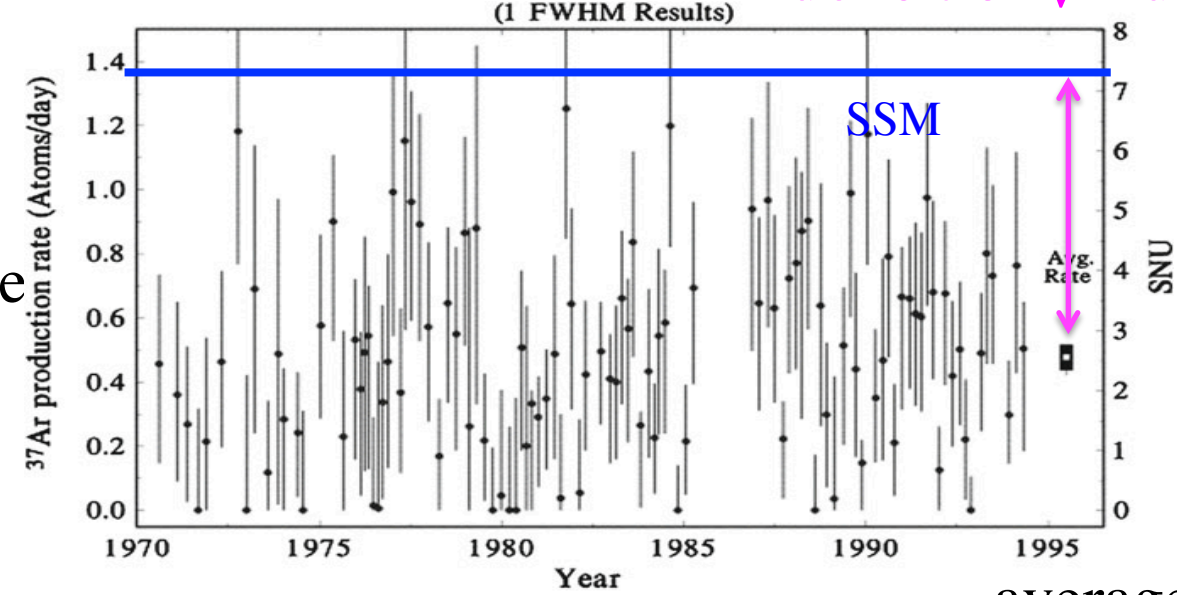
$$\theta_{12}, \Delta m_{12}^2$$

Solar Neutrino Experiments
(Homestake, SuperK, SAGE, GALLEX, BNO,
Borexino, SNO, etc.)

$$P(\nu_e \rightarrow \nu_e; @ \Delta m_{21}^2) \sim 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{12}^2 L}{4E}$$



deficit of ν flux

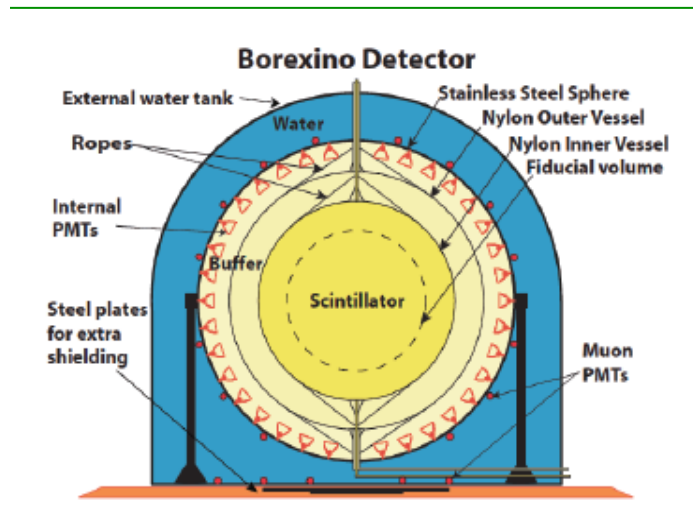


average

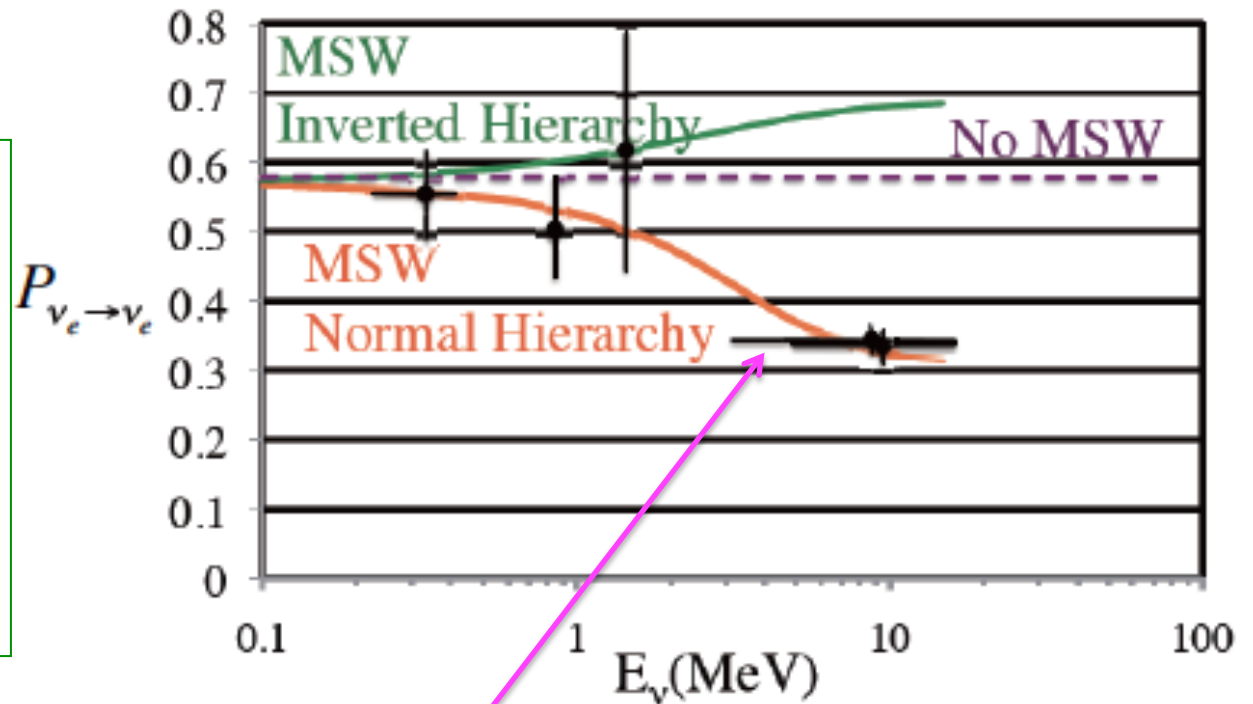
→ Nobel prize in 2002 (R.Davis)

Δm_{21}^2 mass hierarchy ($m_2 > m_1$ or $m_2 < m_1$)

Genuine N.O. can not resolve it but
Matter Effect depends on M.H. and it can be used.



etc.

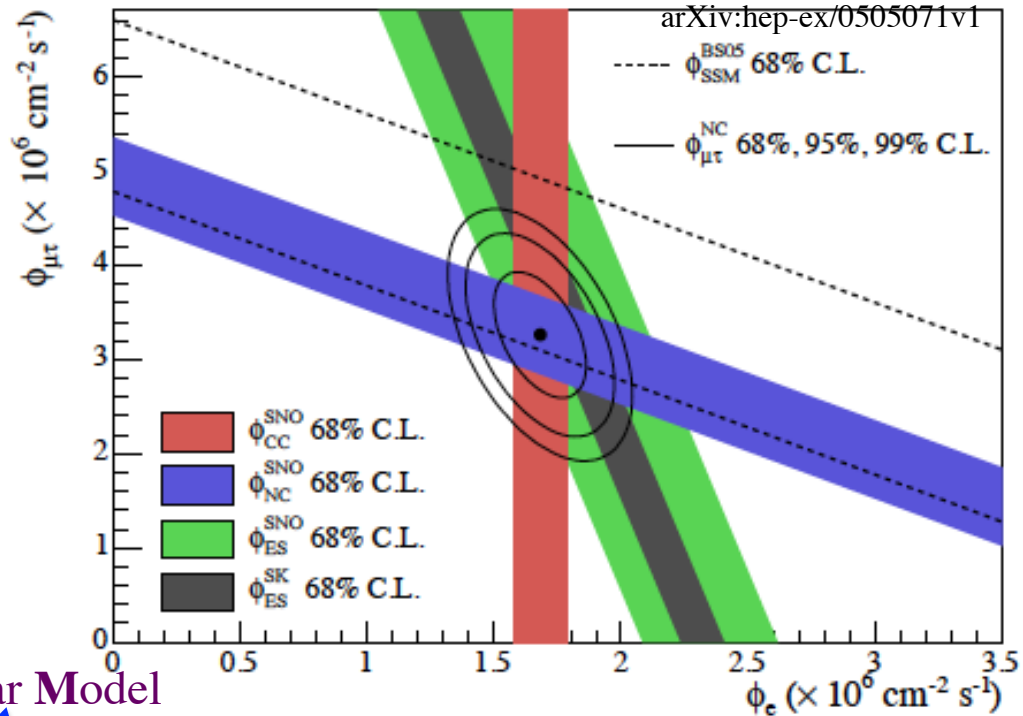
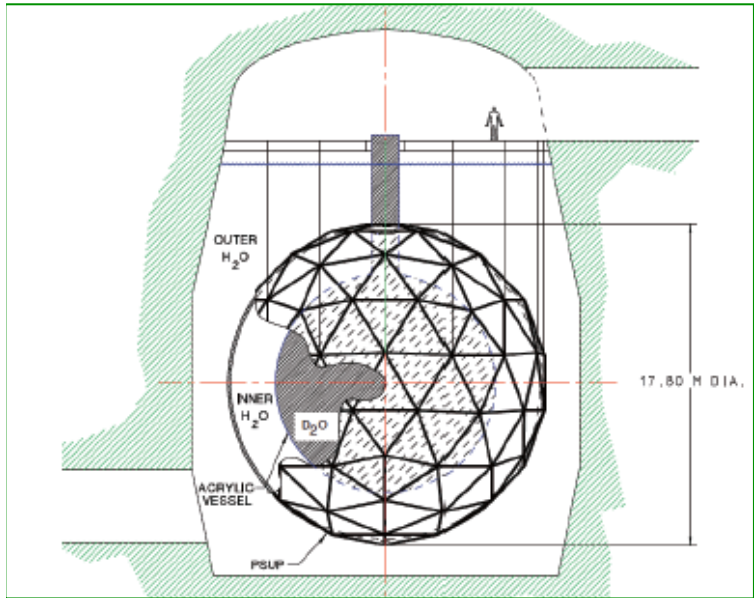


$m_2 > m_1$ determined

Flavor Transmutation: SNO experiment

$$\nu_x + D \rightarrow \nu_x + p + n$$

NC interaction: possible to count all flavors



Standard Solar Model

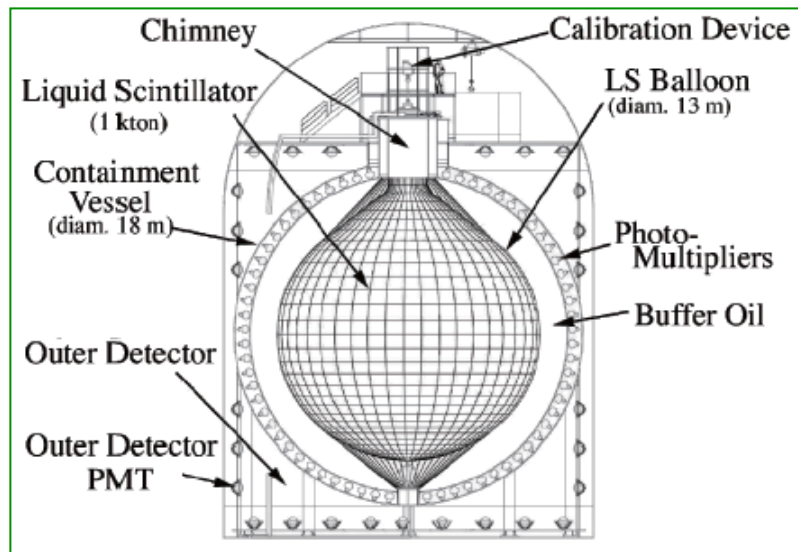
Although $\Phi(\nu_e) < \Phi(SSM)$, $\Phi(\nu_e) + \Phi(\nu_\mu) + \Phi(\nu_\tau) = \Phi(SSM)$

➔ Total # of ν does not change. ν_e changed to other neutrinos.

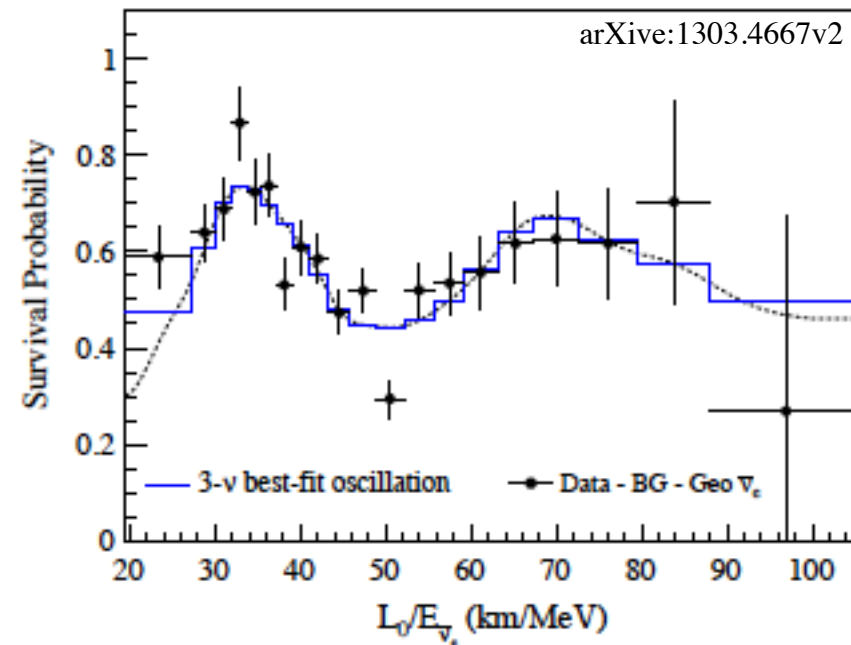
$$\theta_{12}, \Delta m_{12}^2$$

KamLAND Reactor Neutrino Oscillation

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; @ \Delta m_{21}^2) \sim 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{12}^2 L}{4E}$$



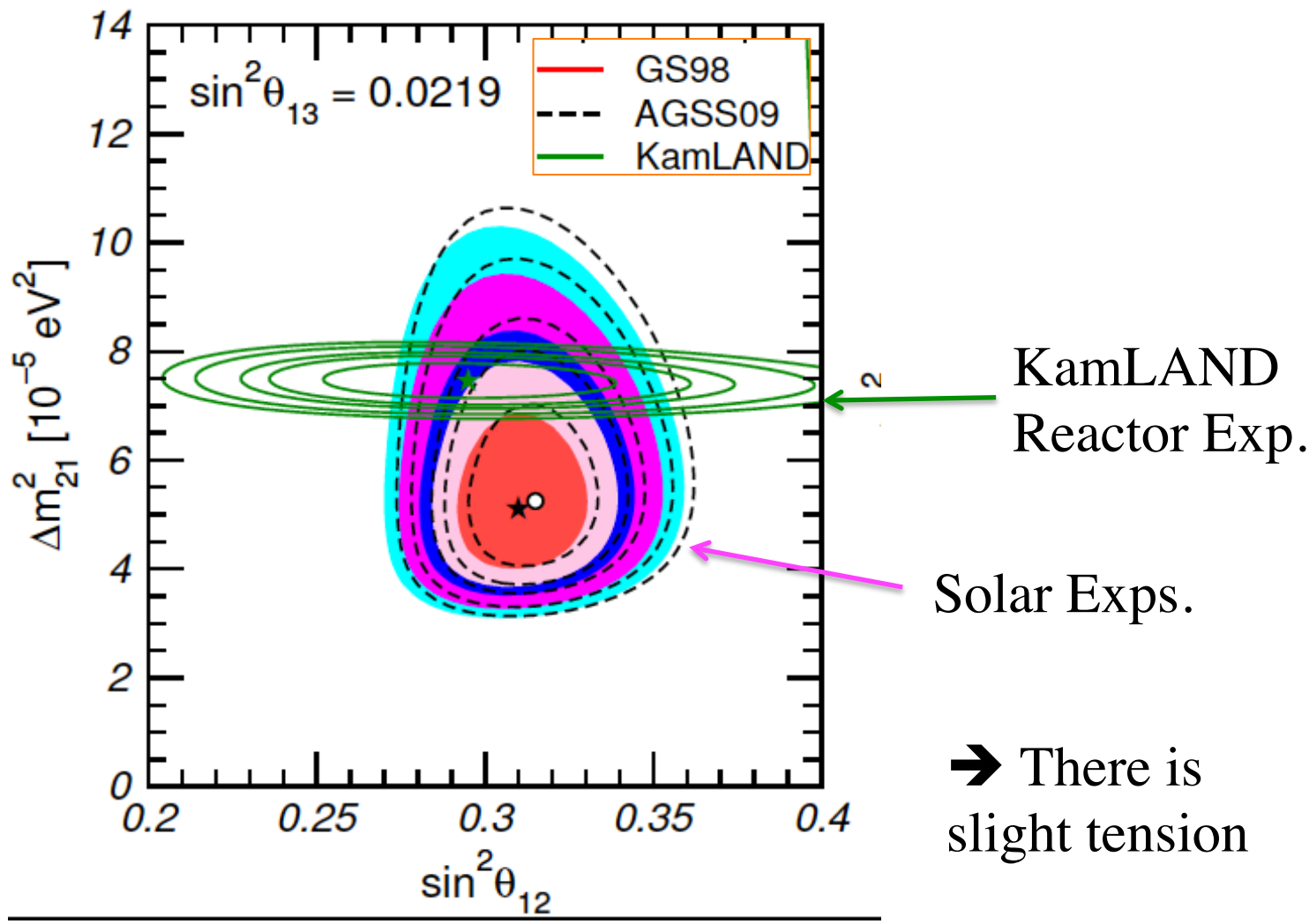
$L \sim 180 \text{ km}$



$$\text{KamLAND: } \tan^2 \theta_{12} = 0.436_{-0.025}^{+0.029}, \quad \left| \Delta m_{21}^2 \right| = 7.53_{-0.18}^{+0.18} \times 10^{-5} eV^2$$

$$\theta_{12}, \Delta m_{12}^2$$

NuFIT 3.1(2017), www.nu-fit.org,
JHEP01(2017)087,

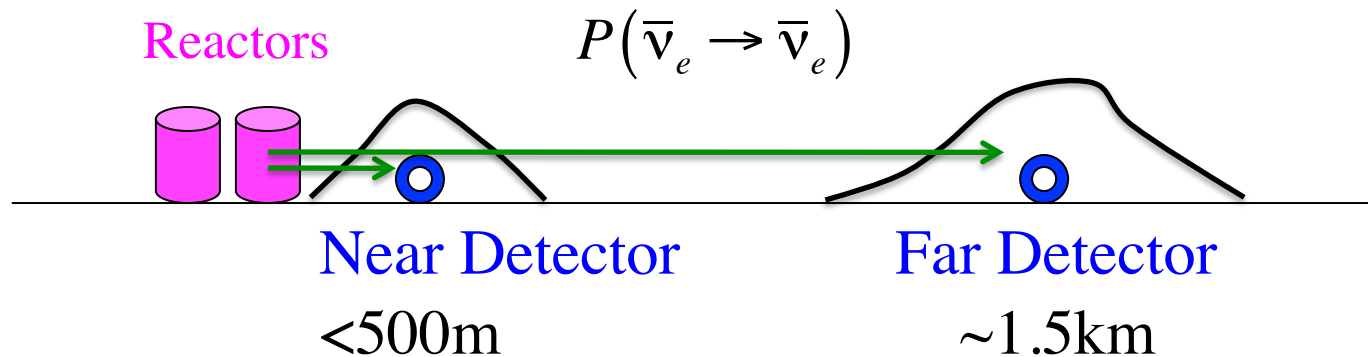


$$\theta_{13}, \Delta m_{31}^2$$

Reactor- θ_{13} Experiment

→ Reactor measurement of θ_{13}

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L \sim 1.5\text{km}) \sim 1 - \sin^2 2\theta_{13}$$

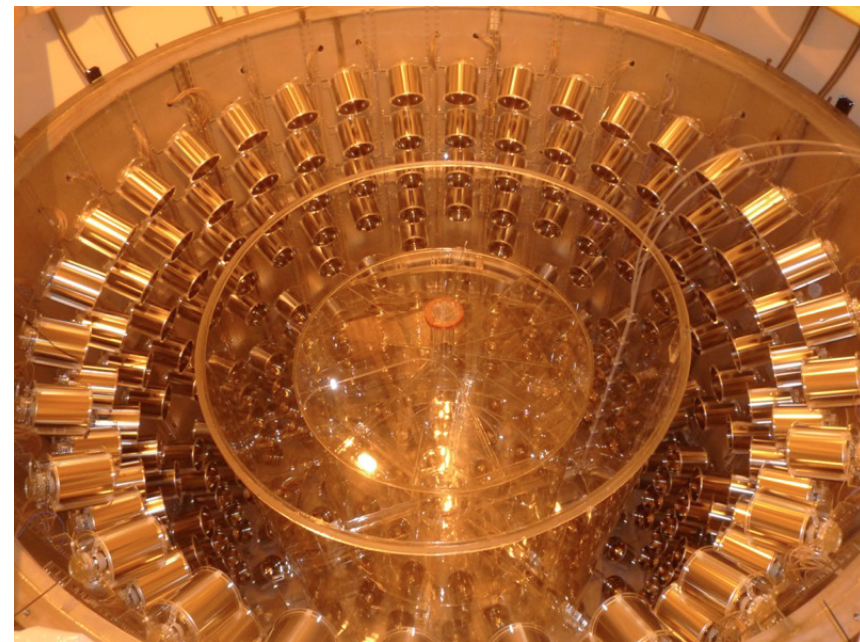
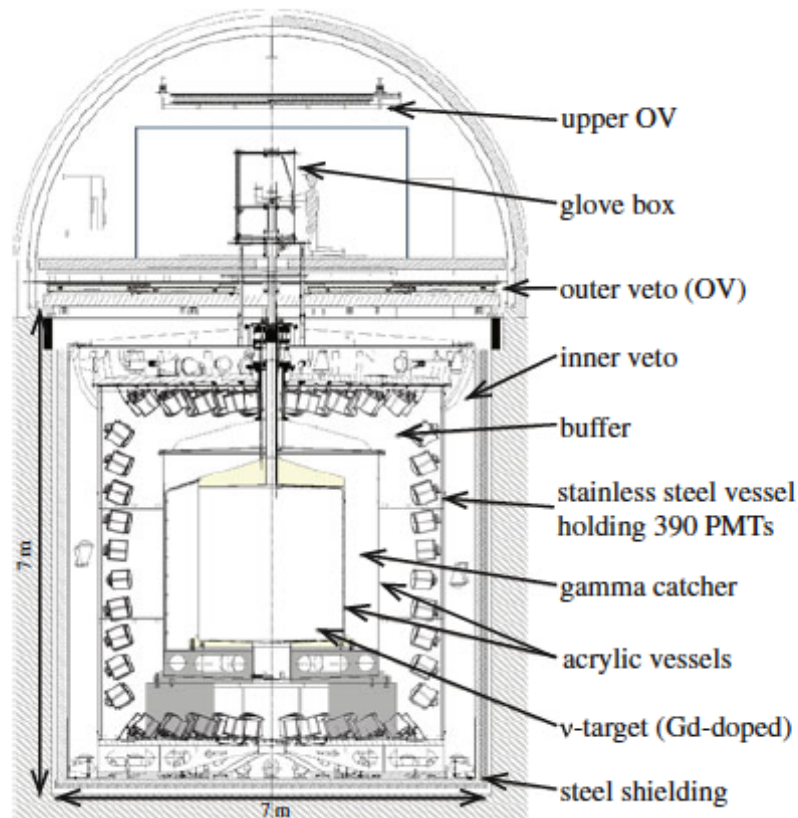


Two detector concept: Cancel uncertainty of neutrino flux and detection efficiency by comparing near & far detector

$$\theta_{13}, \Delta m_{31}^2 :$$

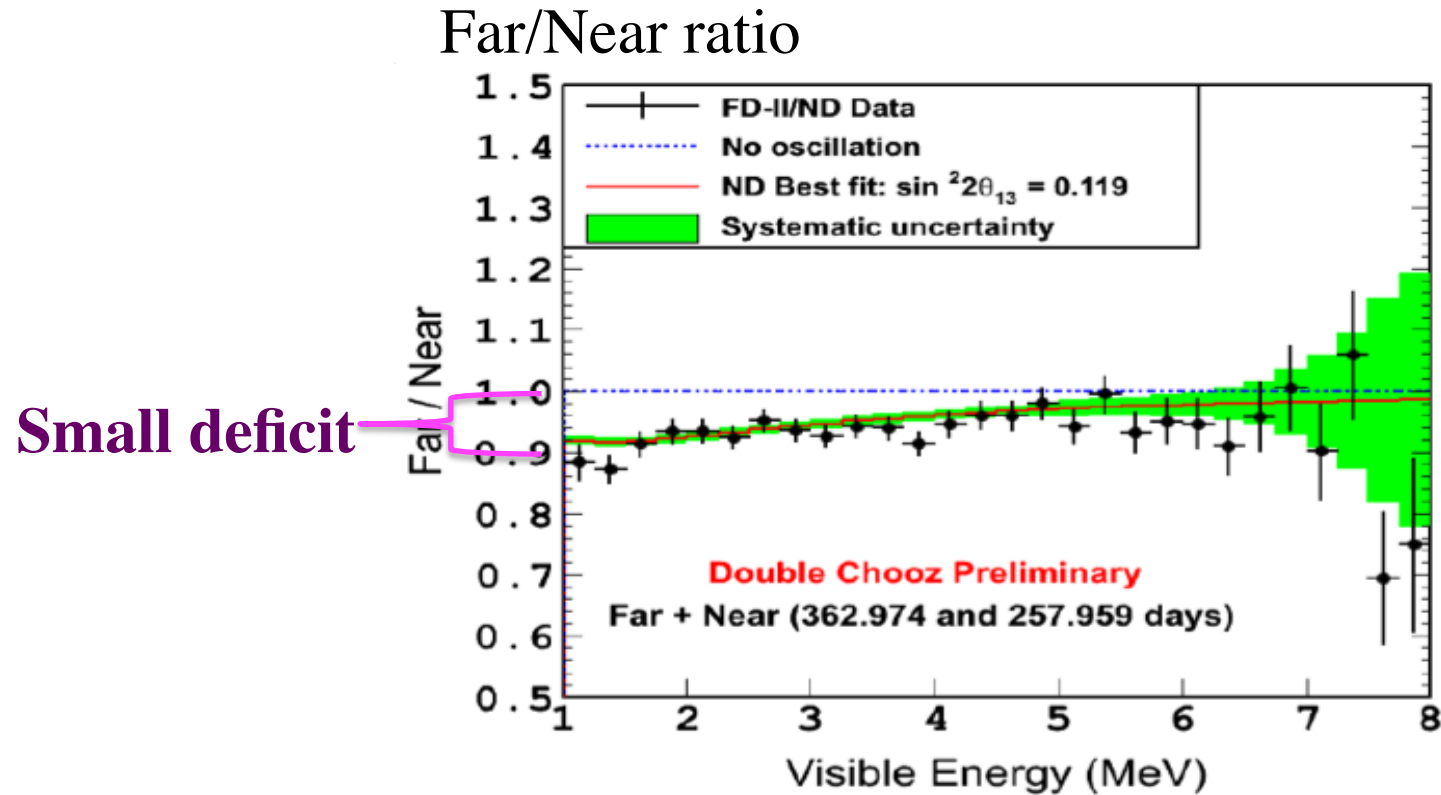
Our experiment: Double Chooz

Most of the ideas of the reactor θ_{13} experiment/detector were proposed first by the DC group members.



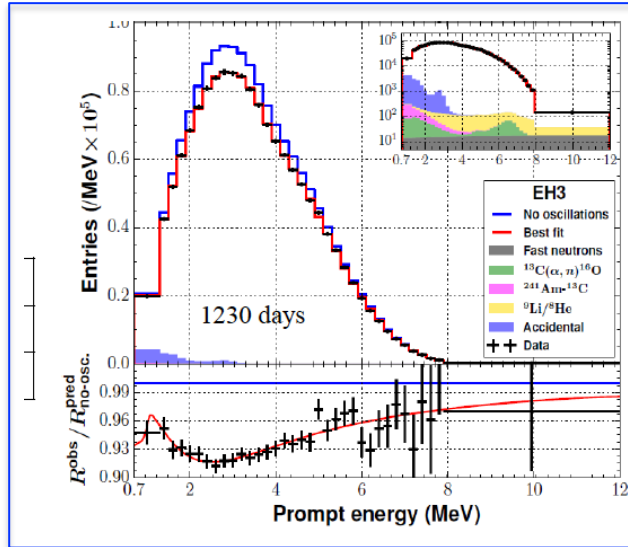
Double Chooz Oscillation fit result

Far detector/Near detector concept to cancel most of the systematics.



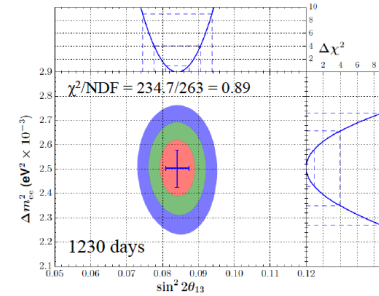
$\sin^2 2\theta_{13} = 0.119 \pm 0.016$ with $\chi^2/\text{ndf} = 236.2/114$
(preliminary)

Daya Bay Result



Logan Lebanowski @ 2016.11 NNN16

Oscillation analysis result



$$\sin^2 2\theta_{13} = 0.0841 \pm 0.0027(\text{stat.}) \pm 0.0019(\text{syst.})$$

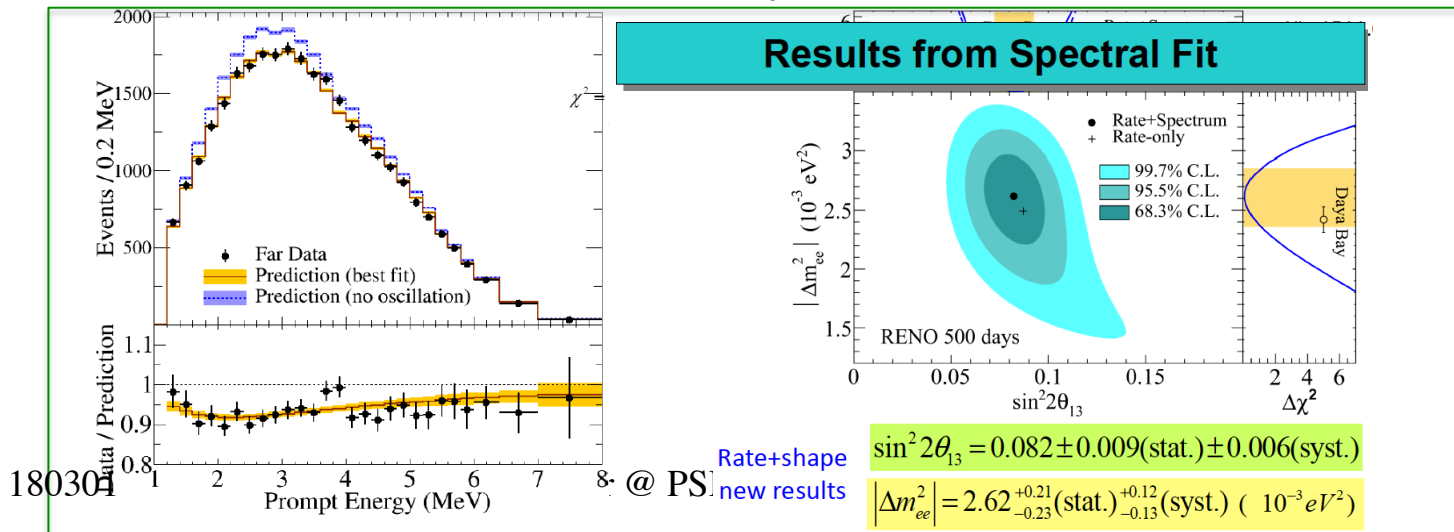
$$|\Delta m_{ee}^2| = [2.50 \pm 0.06(\text{stat.}) \pm 0.06(\text{syst.})] \times 10^{-3} \text{ eV}^2$$

Multiple analyses yield consistent results.

[arXiv:1610.04802]

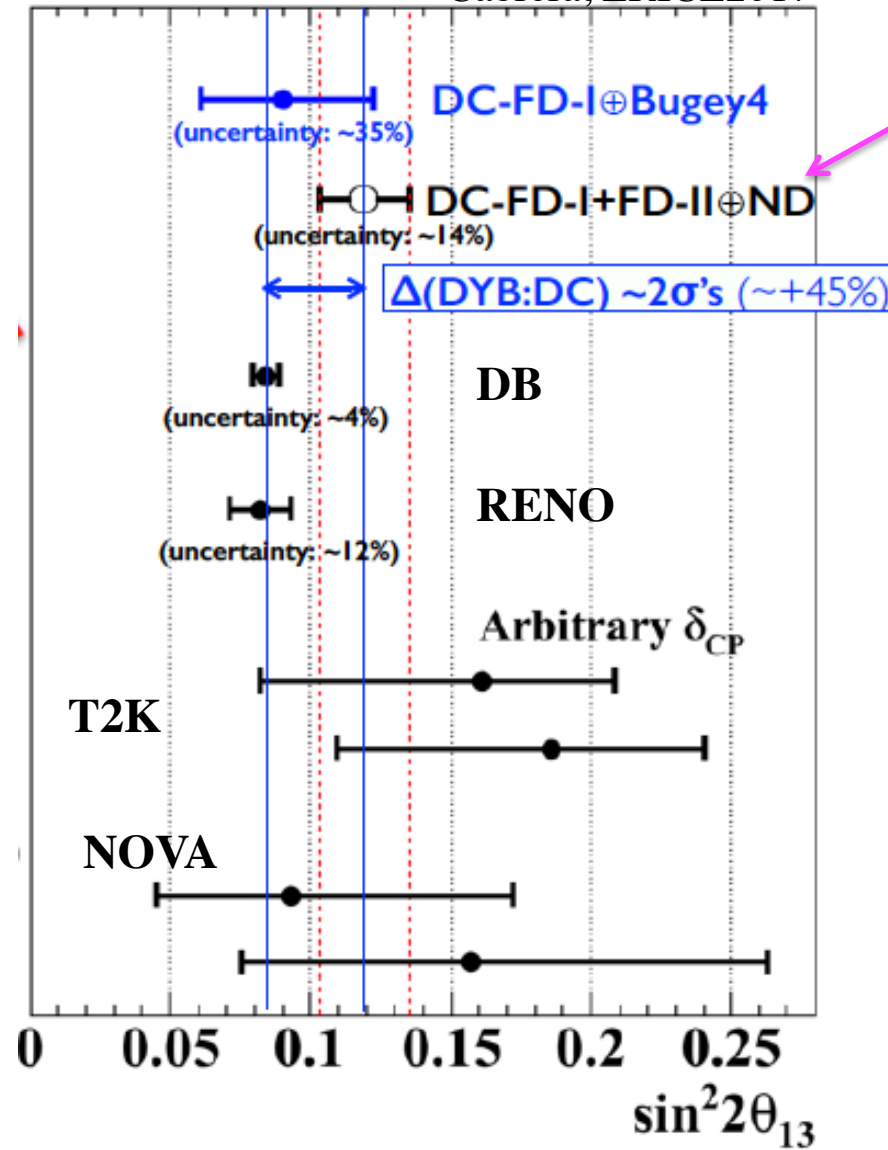
RENO result

Hyunkwan Seo @ 2016.11 NNN16



There may be a tension between DC and DB, RENO

Cabrera, ERICE2017

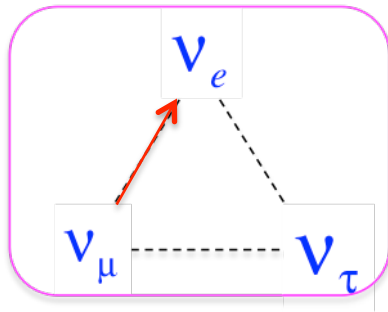


DC-IV-PRELIMINARY @ CERN

CP violation δ

T2K and NOVA measure

$$\left(\nu_{\mu} \rightarrow \nu_e\right), \left(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e\right)$$

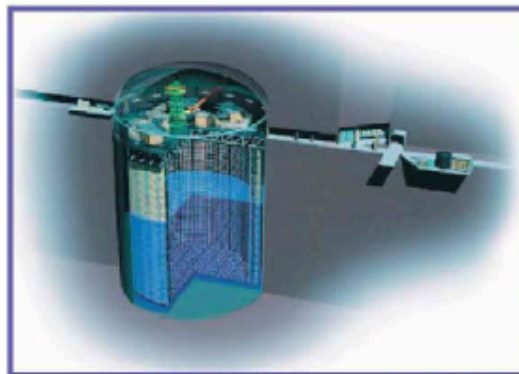
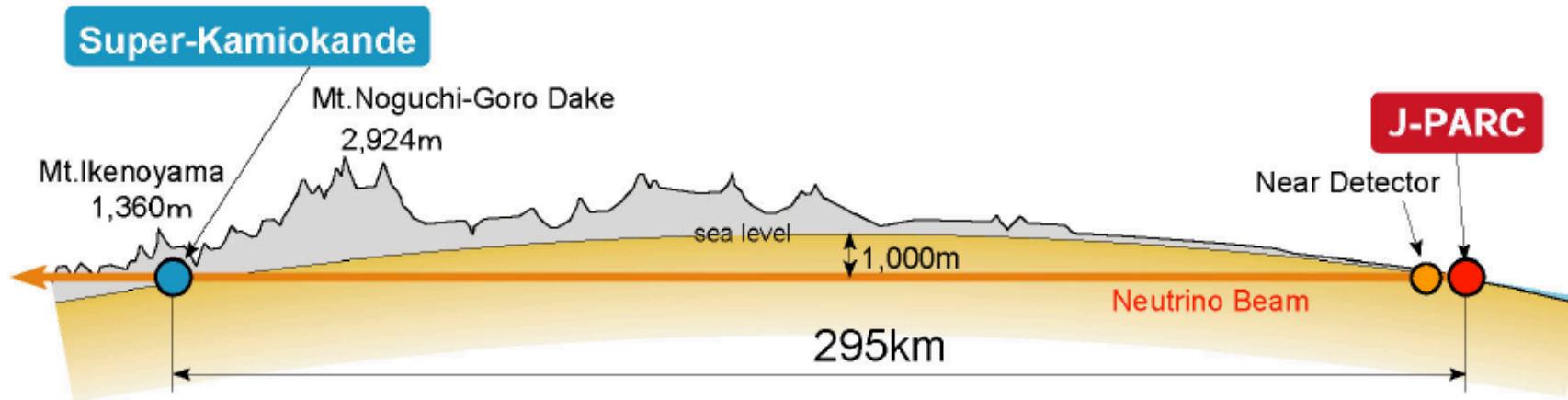


$$\nu_{\mu} \rightarrow \nu_e \quad P_A \sim 0.5 \sin^2 2\theta_{13} - 0.043 \sin 2\theta_{13} \sin \delta$$

$$\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e \quad \bar{P}_A \sim 0.5 \sin^2 2\theta_{13} + 0.043 \sin 2\theta_{13} \sin \delta$$

$$A_{CP} = \frac{P_A - \bar{P}_A}{P_A + \bar{P}_A} \sim 0.3 \sin \delta$$

T2K Experiment



Super-Kamiokande
(ICRR, Univ. Tokyo)

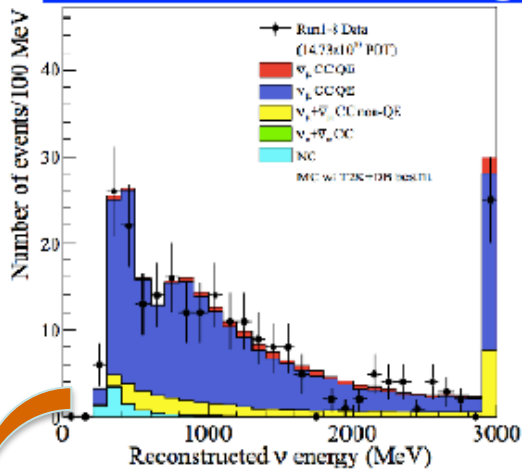


J-PARC Main Ring
(KEK-JAEA, Tokai)

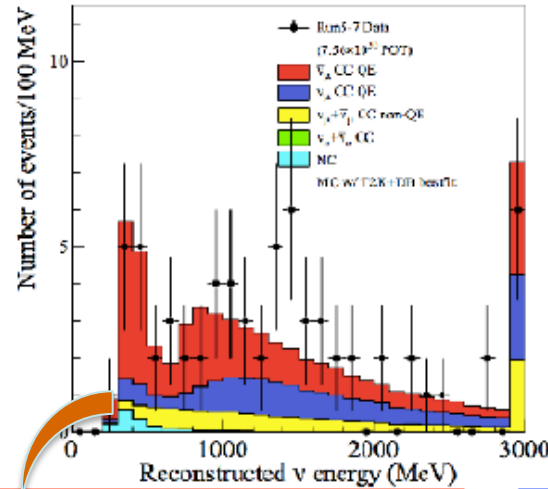


Observation at Super-K

Neutrino 1 μ -like ring

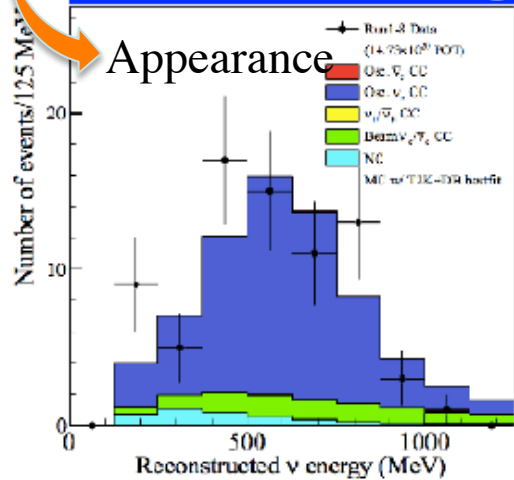


Antineutrino 1 μ -like ring

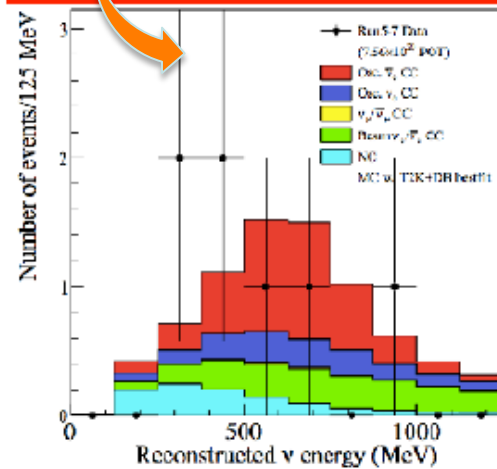


T2K Preliminary

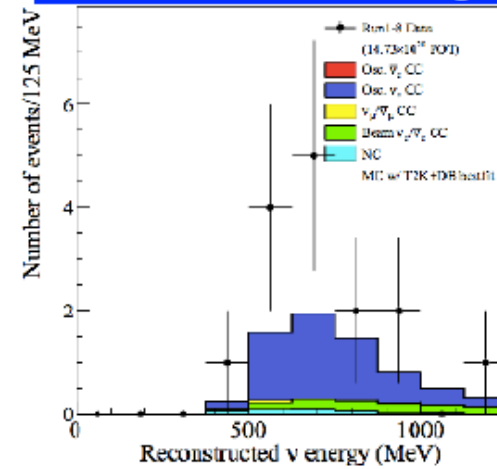
Neutrino 1 e-like ring



Antineutrino 1 e-like ring



Neutrino 1 e-like ring + π



θ_{13} and δ_{CP}

- Fit without the reactor constraint: closed contours in δ_{CP} at 90% CL
- The T2K value for $\sin^2 \theta_{13}$ is consistent with the PDG 2016

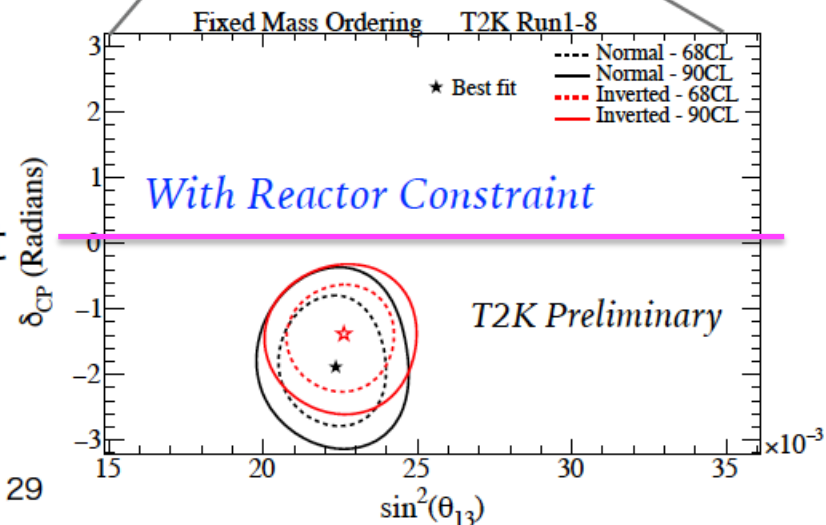
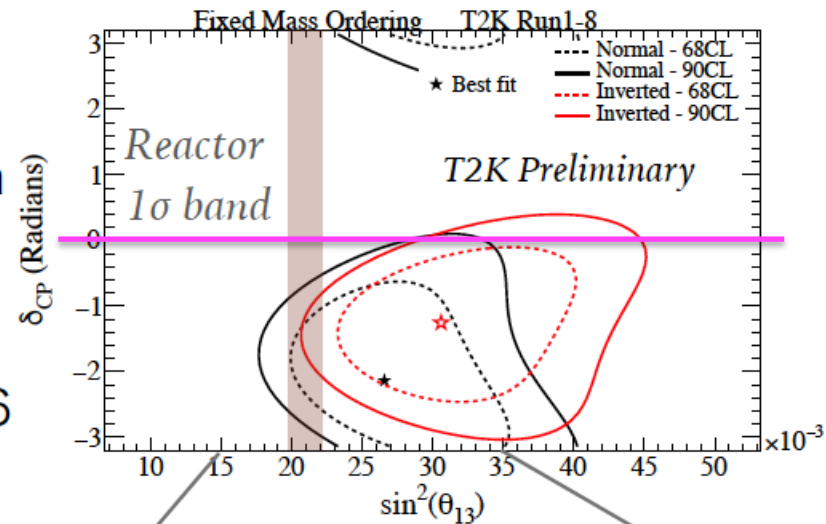
T2K Best Fit:

$$\sin^2 \theta_{13} = 0.0277^{+0.0054}_{-0.0047} \text{ (NH)}$$

PDG 2016:

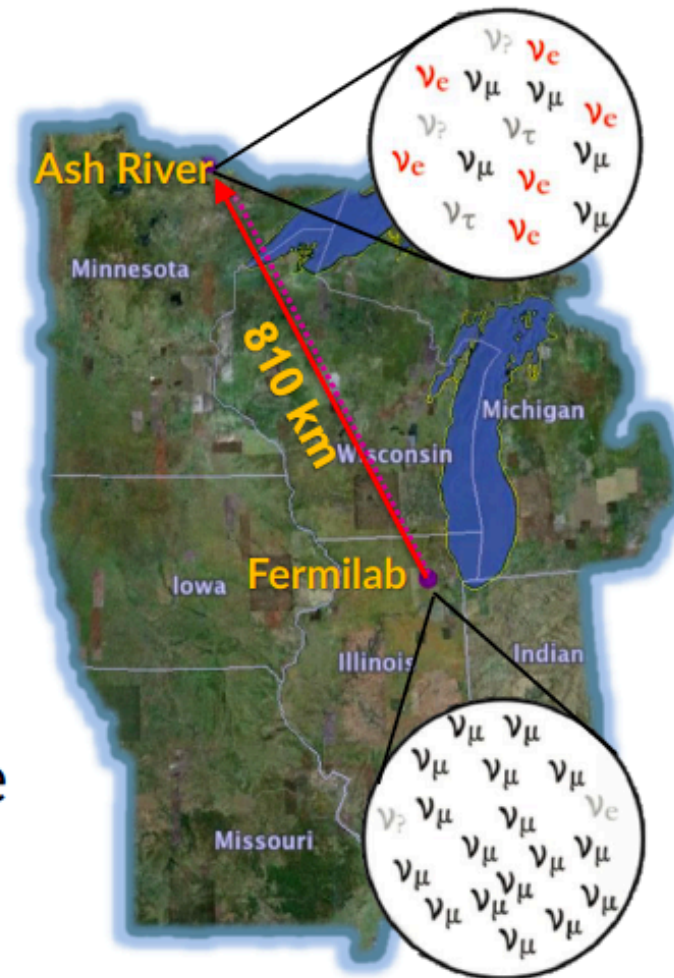
$$\sin^2 \theta_{13} = 0.0210 \pm 0.0011$$

- Adding the reactor constraint improves the constraint on δ_{CP} average:



NuMI Off-axis ν_e Appearance Experiment

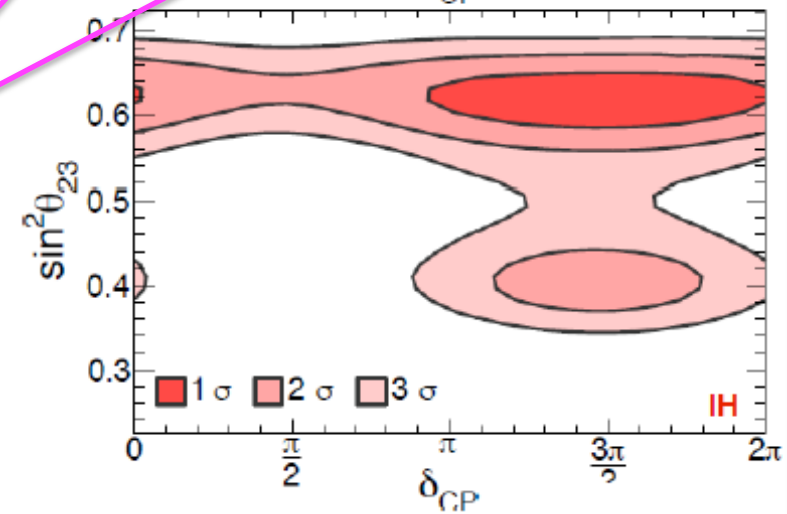
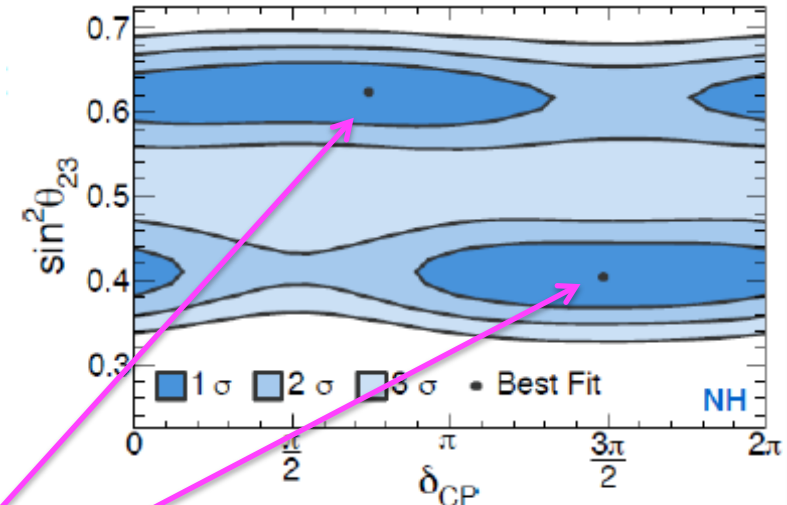
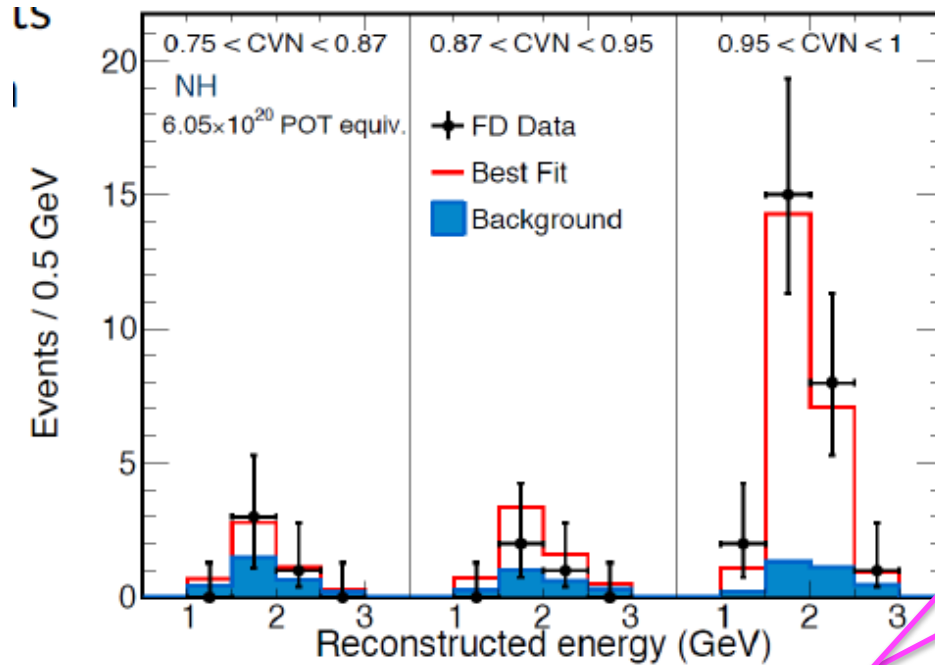
- Long-baseline, two-detector ν oscillation experiment
- Looks for ν_e in ν_μ NuMI beam
- 14 mrad off-axis
- 2 liquid scintillator detectors
- FD (14 kton), ND (0.3 kton)
- Cooled APD readout (live)
- Appearance & disappearance
- Exotics, non-beam...



ν_e appearance results

NOvA

- Observe 33 events on a background of 8.2 ± 0.8



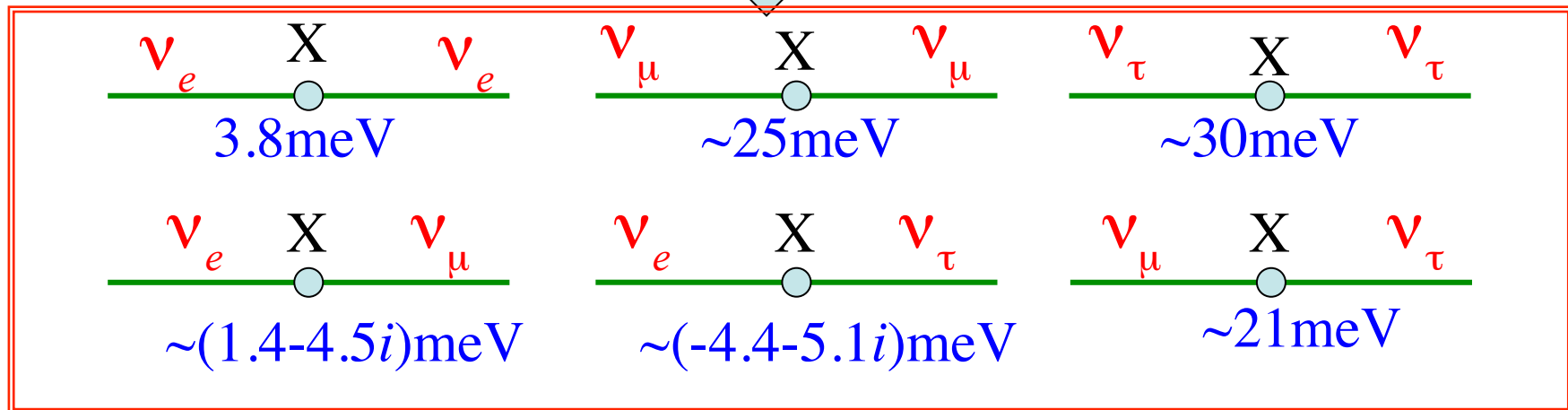
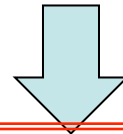
- 2 degenerate best fit points:
 - NH, $\delta_{CP} = 1.48\pi$
 $\sin^2\theta_{23} = 0.404$
 - NH, $\delta_{CP} = 0.74\pi$
 $\sin^2\theta_{23} = 0.623$

Our Current Knowledge of Neutrino Transition Amplitude

For Example: If NH and $\delta_{CP} = -\pi/2$

$$U_{NH} \sim \begin{pmatrix} 0.82 & 0.55 & -0.09 + 0.13i \\ -0.36 + 0.07i & 0.65 + 0.05i & 0.67 \\ 0.43 + 0.08i & -0.53 + 0.05i & 0.73 \end{pmatrix}$$

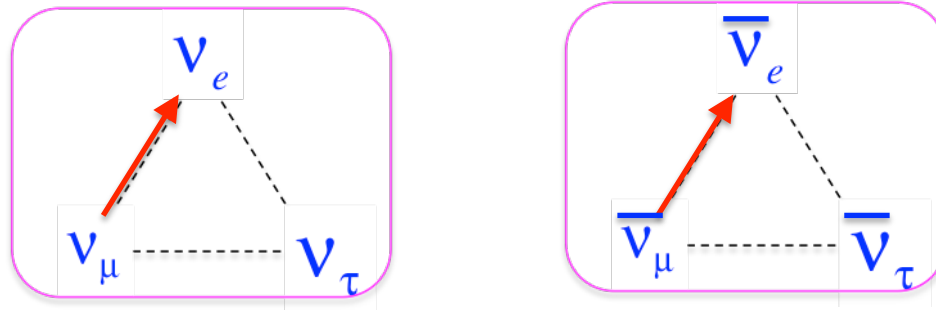
Assumption: $m_1 \sim 0, \rightarrow m_2 = 8.7\text{meV}, m_3 = 50\text{meV}$



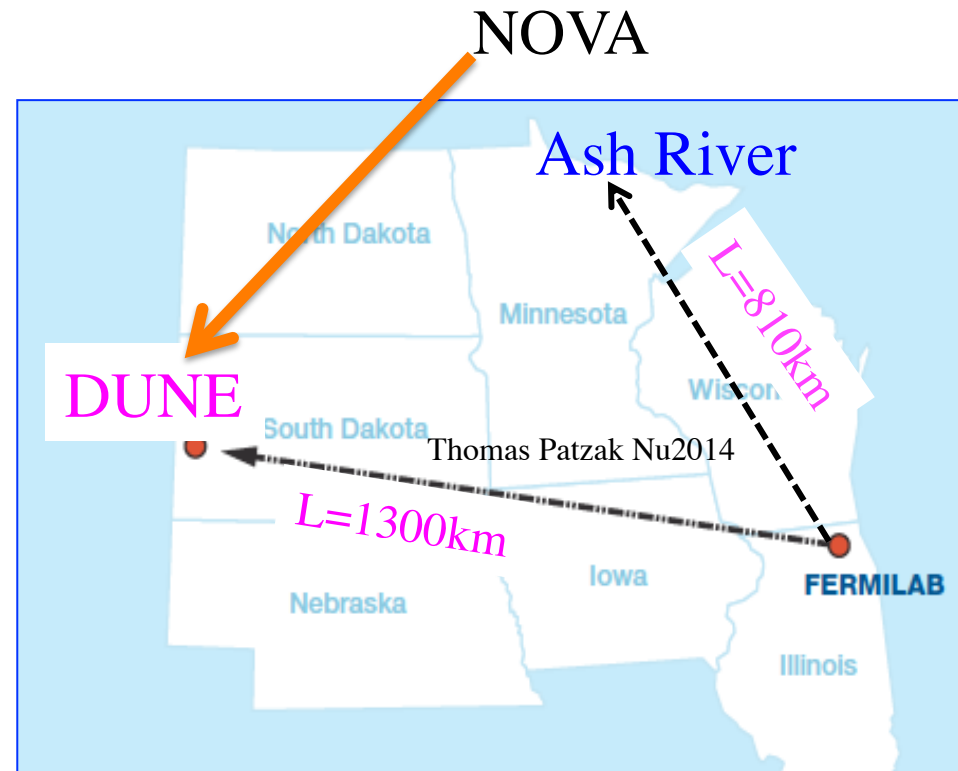
Future

More precise CP Asymmetry
Mass Hierarchy determination

:



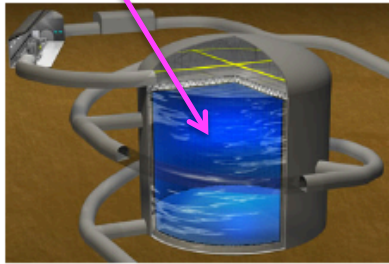
CP asymmetry by Future Long baseline experiments



Hyper Kamiokande:

516kt W.C.

T2HK



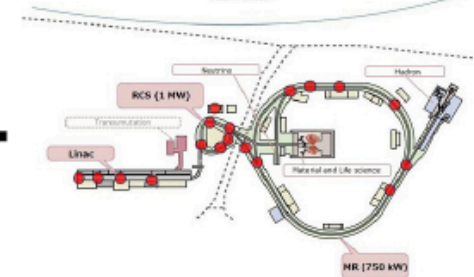
Hyper-K detector (Kamioka)



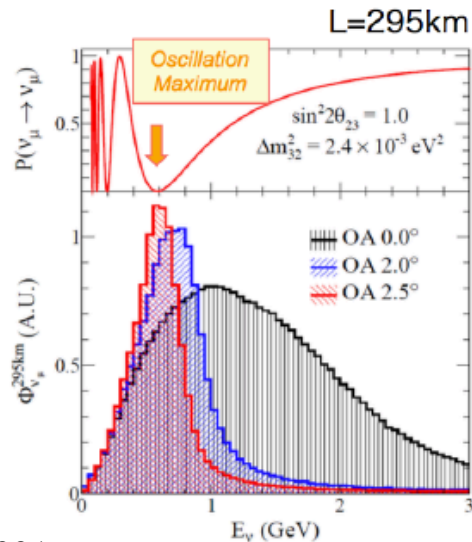
295 km

T2K →

- *Larger target
- *Larger acc. power



J-PARC neutrino Beam (Tokai)

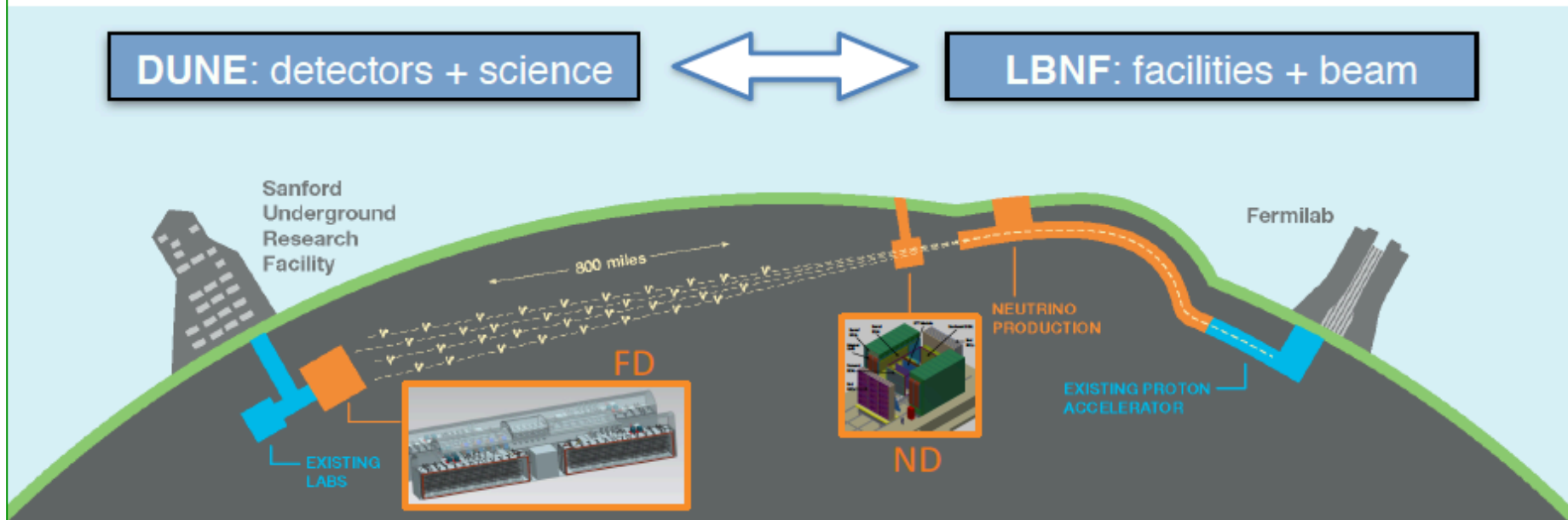


Building off the successful T2K program, Hyper-K also plans to measure neutrinos from the J-PARC neutrino beam

Data taking expected in 2026

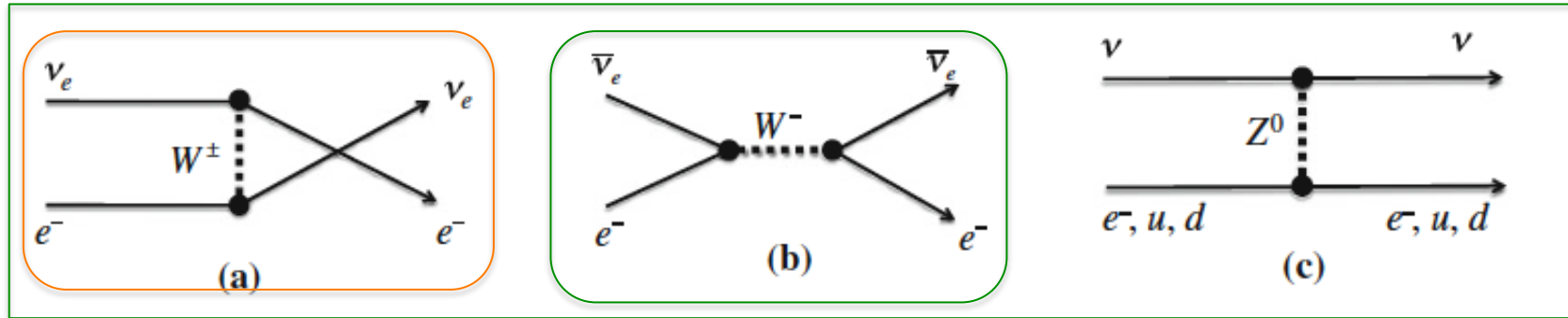
DUNE at LBNF

Deep Underground Neutrino Experiment at the Long Baseline Neutrino Facility



- High intensity, wide-band, neutrino beam from Fermilab
- Highly capable neutrino near detector at Fermilab
- 40-kt fiducial mass far detector at SURF based on LAr-TPCs

CP asymmetry with the matter effect



Difference of ν_e and $\bar{\nu}_e$ interactions with Earth matter

$$A_{CP} (@ \Phi_{13}) \sim -0.3 \sin \delta_{CP} \pm 2(L/L_0)$$

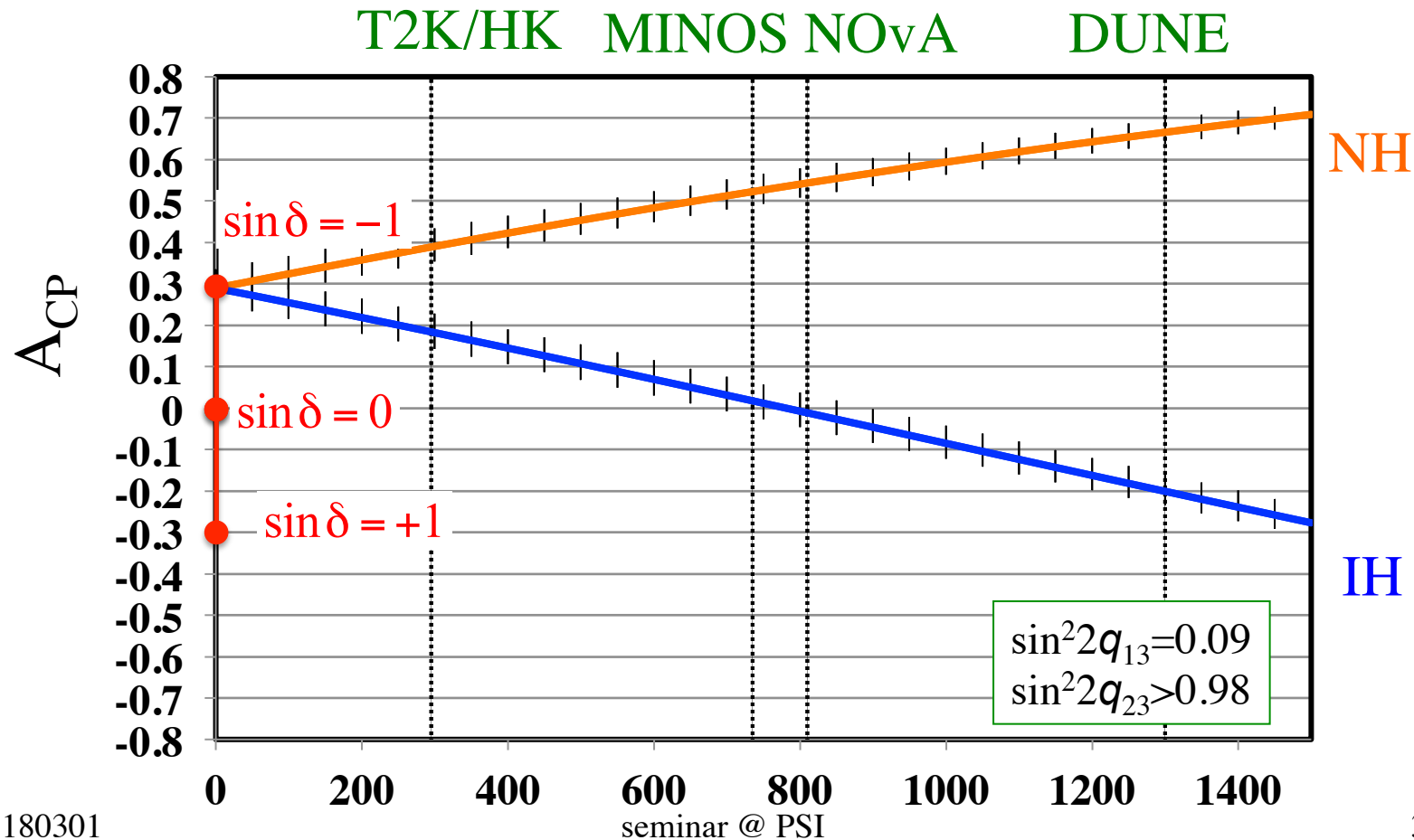
fake CP asymmetry

	$L[\text{km}]$	$A_{FK}=2(L/L_0)$
T2K/HK	295	± 0.11
NOVA	810	± 0.30
DUNE	1,300	± 0.48

The sign depends on mass hierarchy

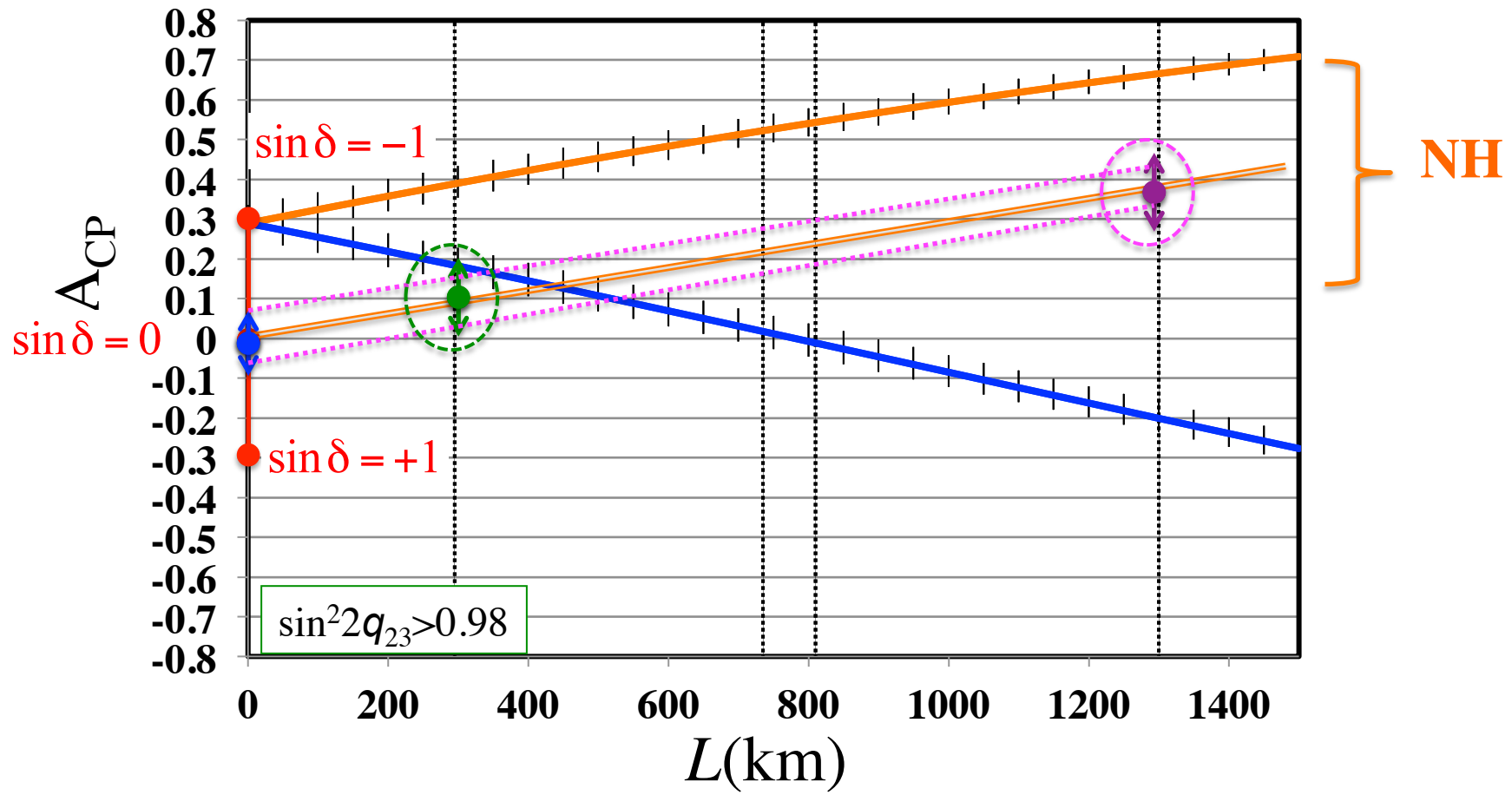
Baseline Dependence of Matter effect

$$A_{CP} \sim -0.29 \sin \delta \pm 2 \left(\frac{L}{L_0} \right)$$



HK+DUNE

T2K/HK MINOS NO_vA DUNE



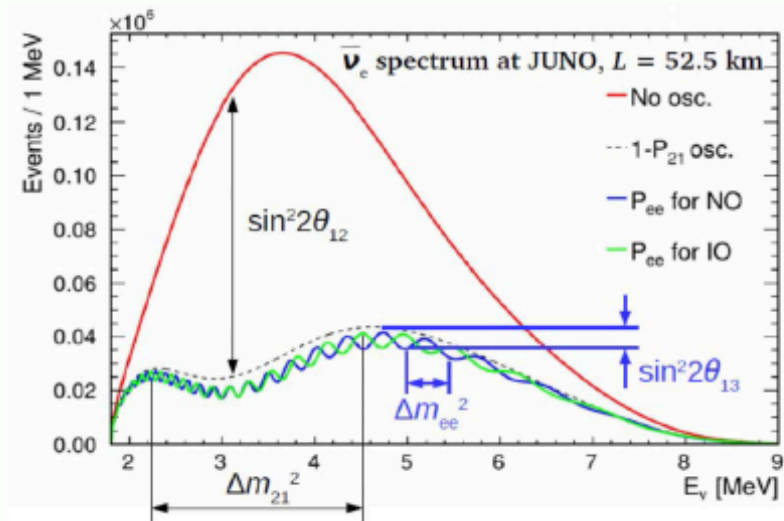
* good to have the both experiments.

Δm_{23}^2 mass hierarchy:

JUNO

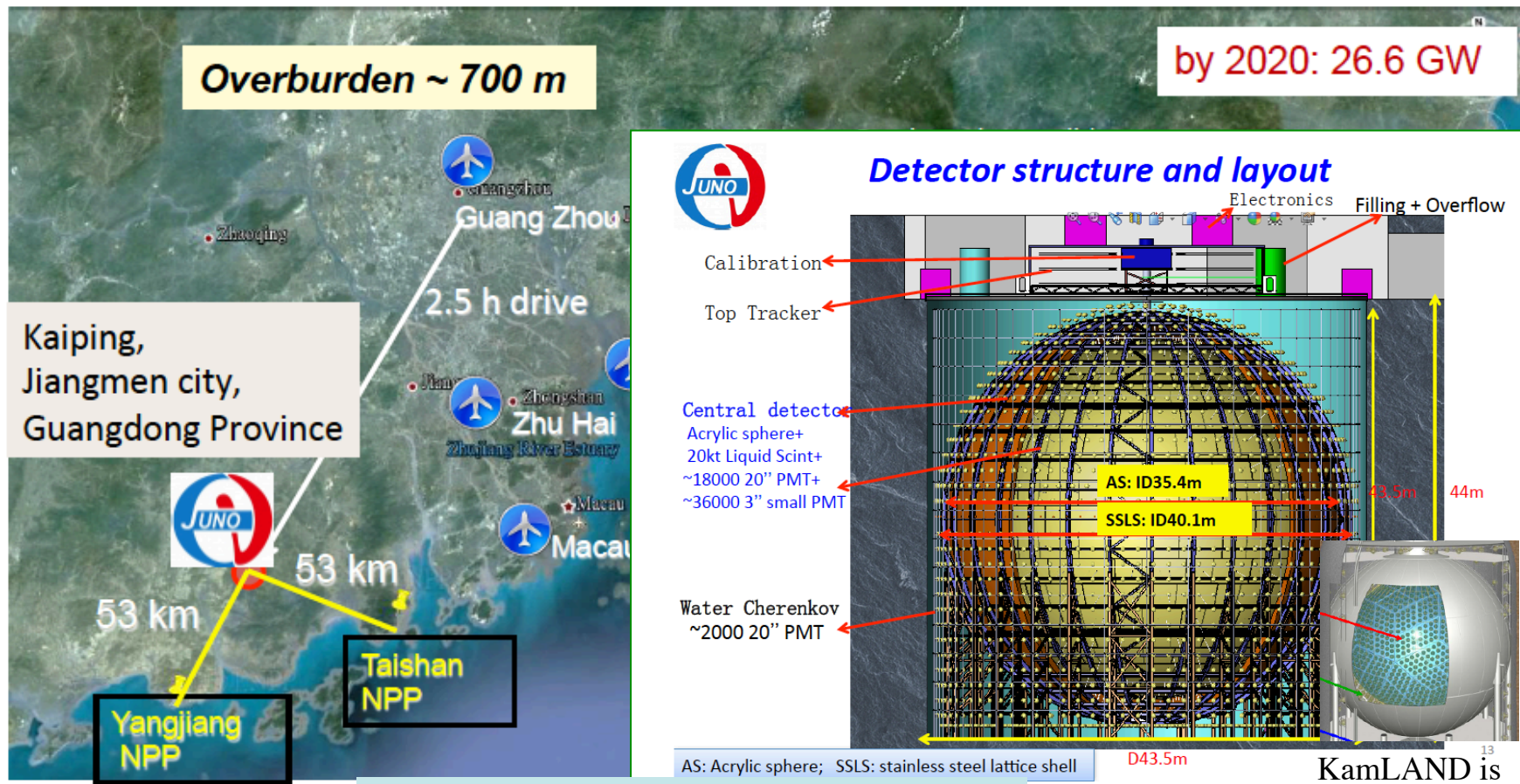
- The **J**iangmen **U**nderground **N**eutrino **O**bservatory (JUNO) is a multipurpose experiment under construction in China:
 - Rich physics program: neutrino mass hierarchy, sub-% measurement of oscillation parameters, astrophysical neutrinos, geo-neutrinos, atmospheric neutrinos, search for exotic physics... etc.
- Main keys to accomplishing the physics goals:

- Optimal baseline
- High statistics
- Superb energy resolution (3% @ 1 MeV)
- Excellent control of energy response systematics
- Background reduction



Location of JUNO

NPP	Daya Bay	Huizhou	Lufeng	Yangjiang	Taishan
Status	Operational	Planned	Planned	Under construction	Under construction
Power	17.4 GW	17.4 GW	17.4 GW	17.4 GW	18.4 GW



Filling and data taking 2020

Part-I Summary:

- * Thanks to the huge experimental efforts, $\theta_{12}, \theta_{23}, \theta_{13}$
 $\Delta m_{12}^2, |\Delta \tilde{m}_{32}^2|, |\Delta \tilde{m}_{31}^2|$ have been measured.
- * Decisive measurements of $\delta, M.H.$ are planned
- * There are several tensions.
 - ➔ Redundant experiments to check each other are important.
 - ➔ New physics might be behind them.

Part-II

Other Quantum Oscillations: = A collection of various oscillations & mixings =

The origin of the neutrino oscillation is transitions between different flavor neutrinos, such as,

$$\nu_e \Leftrightarrow \nu_\mu$$

In fact, many kinds of transitions take place in various physics phenomena; many of them bear important physics effects. Such important physics can be understood as the same way as neutrino oscillation mechanism.

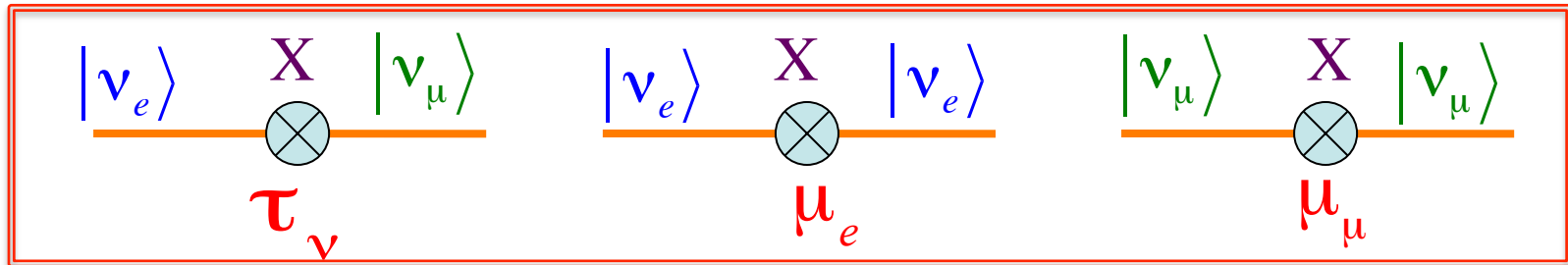
In some cases, abstract concepts, such as Parity, can be understood by a concrete idea of oscillation and mixing.

It should be useful to teach various physics using such unified and concrete point of view.

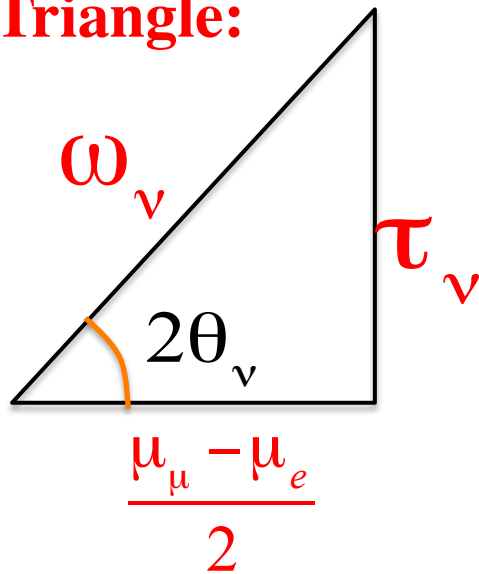
Cabbibo angle θ_c

Neutrino Oscillation case

Something (**X**) changes ν_μ to ν_e and gives self-transition



Mixing Triangle:



Mass eigenstate:

$$\begin{cases} \nu_1 = (\cos\theta_\nu |\nu_e\rangle - \sin\theta_\nu |\nu_\mu\rangle) \exp[-im_1 t] \\ \nu_2 = (\sin\theta_\nu |\nu_e\rangle + \cos\theta_\nu |\nu_\mu\rangle) \exp[-im_2 t] \end{cases}$$

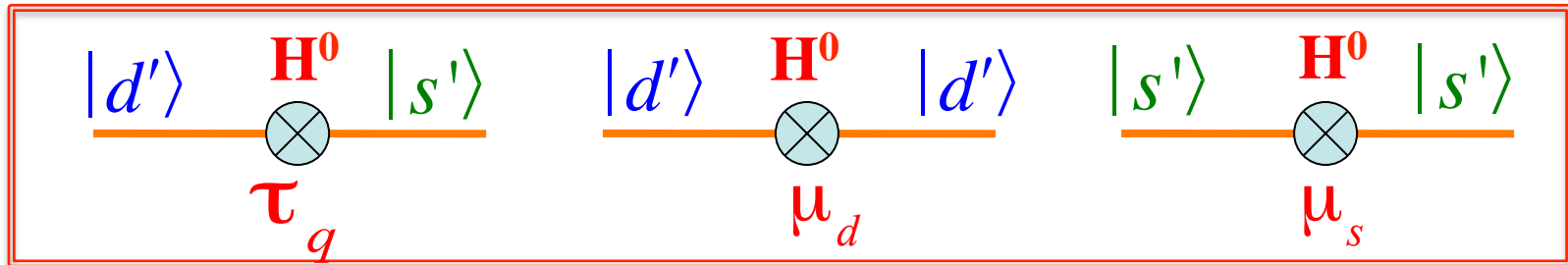
Masses:

$$m_{1,2} = \bar{\mu}_\nu \mp \omega_\nu$$

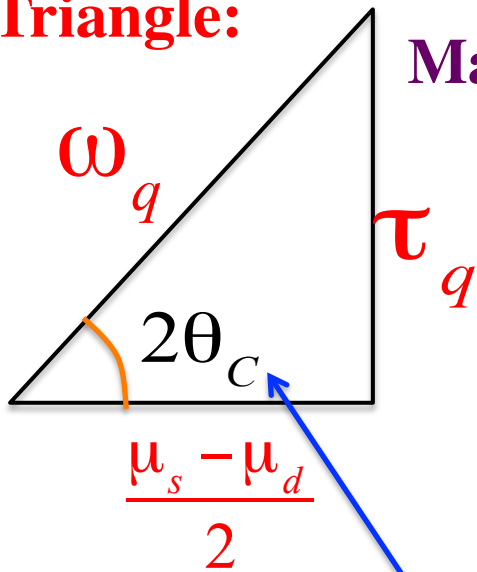
$$\bar{\mu}_\nu = \frac{\mu_e + \mu_\mu}{2}, \quad \omega_\nu = \frac{1}{2} \sqrt{(\mu_\mu - \mu_e)^2 + 4\tau_\nu^2}$$

Quark case $(\nu_e, \nu_\mu) \rightarrow (d', s')$

Higgs potential (H^0) changes d' to s' and gives self-transition



Mixing Triangle:



Mass eigenstate:

$$\begin{cases} d = (\cos \theta_c |d'\rangle - \sin \theta_c |s'\rangle) \exp[-im_d t] \\ s = (\sin \theta_c |d'\rangle + \cos \theta_c |s'\rangle) \exp[-im_s t] \end{cases}$$

Masses:

$$m_{d,s} = \bar{\mu}_q \mp \omega_q$$

$$\bar{\mu}_q = \frac{\mu_d + \mu_s}{2}, \quad \omega_q = \frac{1}{2} \sqrt{(\mu_s - \mu_d)^2 + 4\tau_q^2}$$

Cabbibo angle is quark version of ν mixing angle

Important Difference

neutrino and quark oscillations are two extreme cases of the uncertainty principle.

$$P[t] = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} t$$

Oscillation length for 1GeV particle

$$\lambda_\nu \sim 10^6 m$$

$$\lambda_q \sim 10^{-14} m$$

ν : Measurement of the flavor evolution pattern possible but it is impossible to distinguish ν_1 and ν_2 by measuring masses

q : It is possible to distinguish d and s by their masses but the oscillation is too quick to observe the evolution of d' and s'

Weinberg Angle θ_W

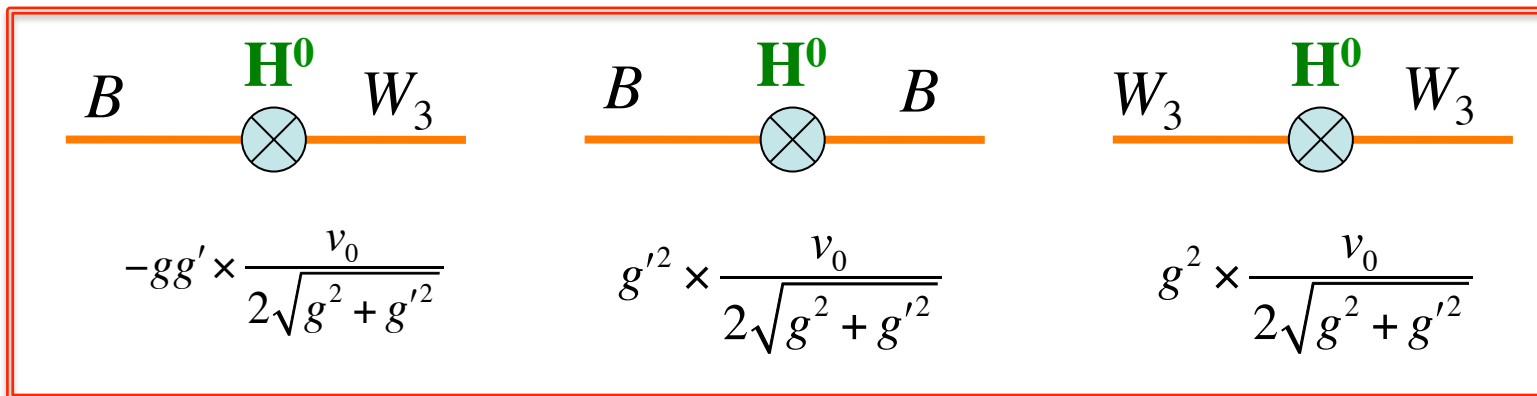
The Lagrangian for the interaction of the gauge boson and Higgs fields is written as,

$$\mathcal{L}_{\Phi G} = \frac{1}{4} \left| (g' B^\mu + g(\vec{W}^\mu \cdot \vec{\sigma})) \Phi \right|^2 \xrightarrow{\text{SSB}} \text{Neutral component}$$

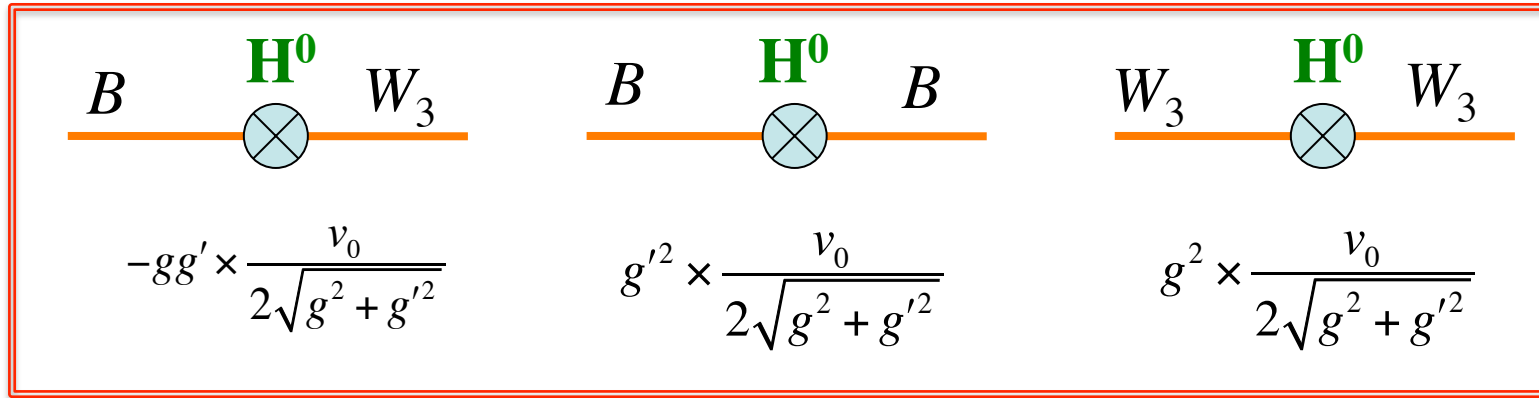
$$\frac{(v_0 + h)^2}{8} (g^2(|W^+|^2 + |W^-|^2) + \underline{(g^2 W_3^2 + g'^2 B^2 - gg'(W_3 B + B W_3))})$$

State equation,

$$i \frac{d}{dt} \begin{pmatrix} B \\ W_3 \end{pmatrix} = \frac{v_0}{2\sqrt{g^2 + g'^2}} \begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}$$

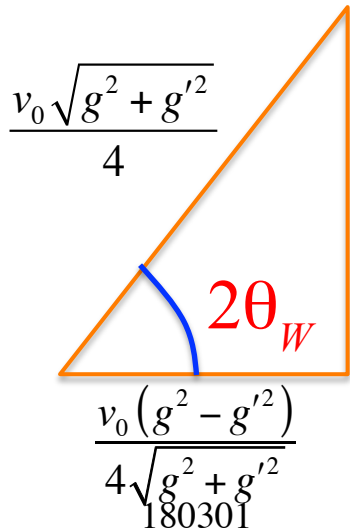


Oscillation View of Gauge Bosons



Mass eigenstate

$$\begin{cases} \psi_1 = (B \cos \theta_W - W_3 \sin \theta_W) \exp[-i \times 0 \times t] \Rightarrow A \\ \psi_2 = (B \sin \theta_W + W_3 \cos \theta_W) \exp[-i M_Z t] \Rightarrow Z^0 \end{cases}$$



$$\frac{v_0 g g'}{2\sqrt{g^2 + g'^2}}$$

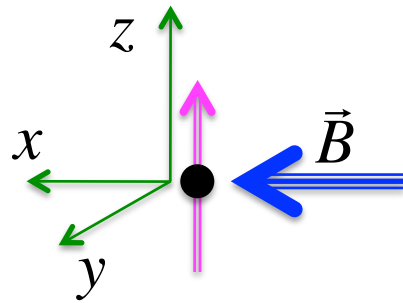
$$M_Z = \frac{v_0}{2} \sqrt{g^2 + g'^2}, \quad M_A = 0$$

$$\tan 2\theta_W = \frac{2gg'}{g^2 - g'^2} \quad \text{or} \quad \tan \theta_W = \frac{g'}{g}$$

In A and Z^0 , B and W_3 are oscillating very quickly.

$$P[B \Leftrightarrow W_3] = \sin^2 2\theta_W \sin^2 \left[\frac{1}{2} M_Z t \right]$$

Spin precession under magnetic field

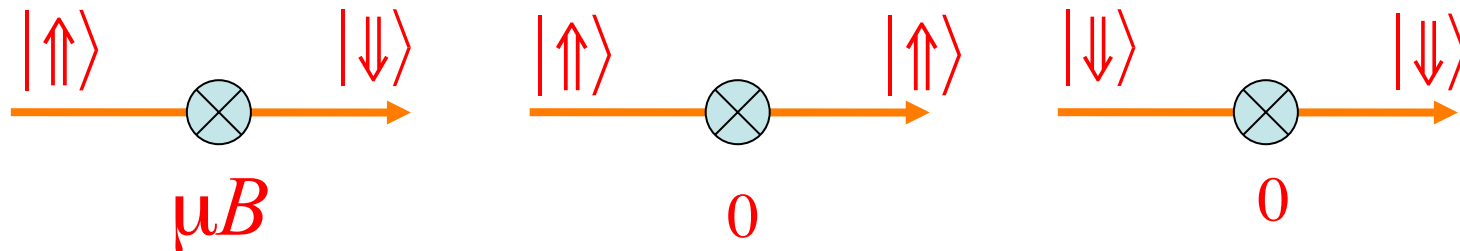


spin-1/2 wave function

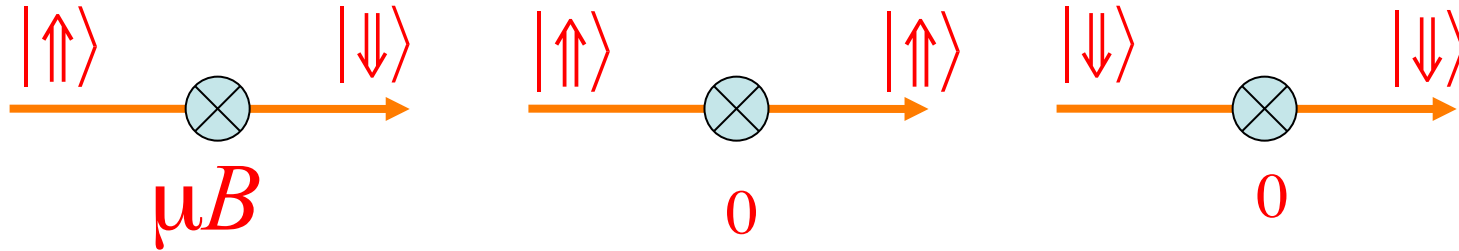
$$\Psi[t] = \alpha[t]|\uparrow\rangle + \beta[t]|\downarrow\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

If magnetic field $\vec{B} = B(1, 0, 0)$ is applied

Pauli Equation $i \frac{d}{dt} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \mu B \sigma_x \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \mu B \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$



➔ The magnetic field flips the spin direction



General wave function:

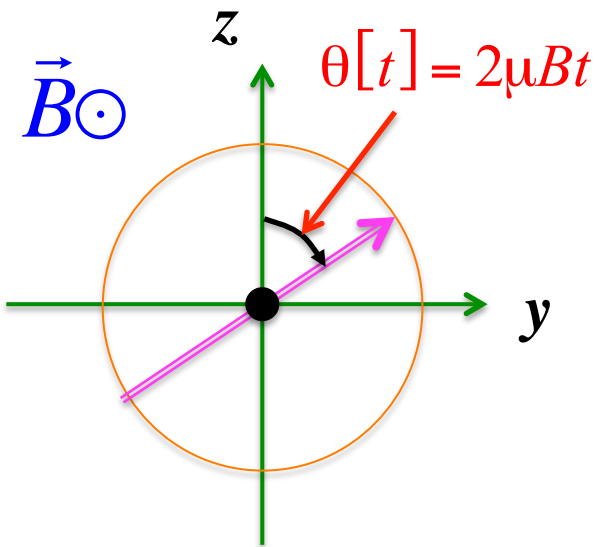
mass eigenstates

$$\Psi[t] = C_+ \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} e^{-i\mu B t} + C_- \frac{|\uparrow\rangle - |\downarrow\rangle}{\sqrt{2}} e^{+i\mu B t}$$

If the spin direction is $|\uparrow\rangle$ at $t=0$,

$$\Psi[t] = \cos[\mu B t] |\uparrow\rangle - i \sin[\mu B t] |\downarrow\rangle$$

Spin precession happens in the y - z plane with angular velocity $\omega = 2\mu B$



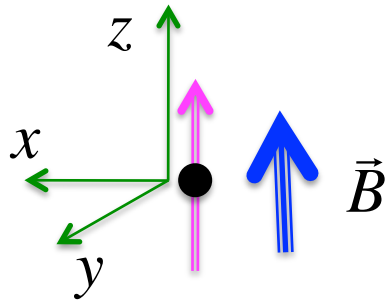
(I saw the edm experiment this morning and added this subject)

Precession of electric dipole moment in electric field.

**Magnetic dipole moment in magnetic field can be treated by
the Pauli equation.**

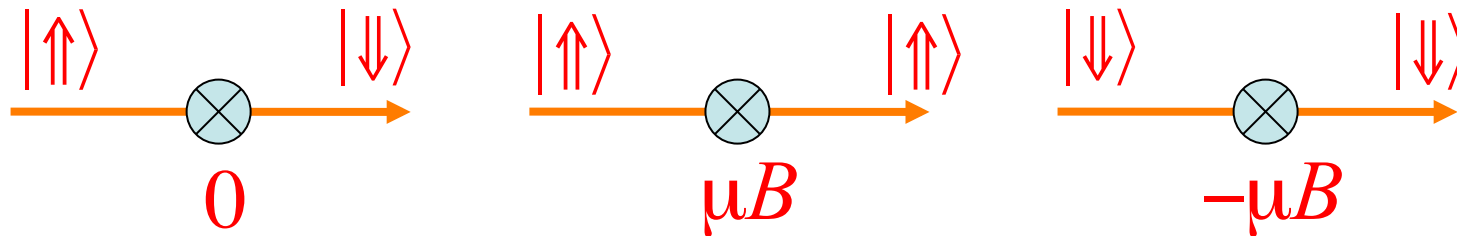
How we can treat the electric dipole moment in electric field?

Excise by magnetic moment

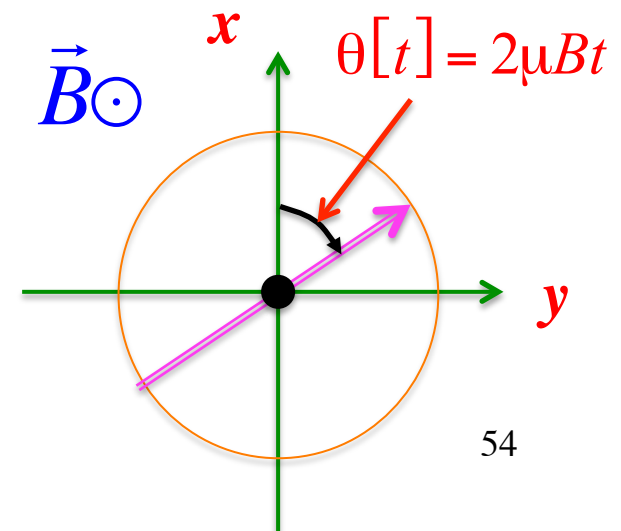


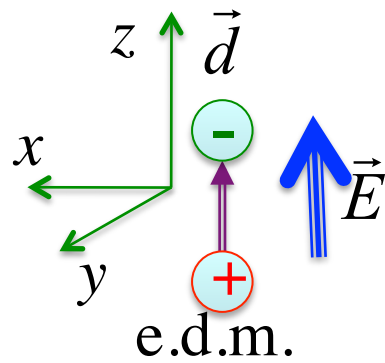
If magnetic field $\vec{B} = B(0, 0, 1)$ is applied

Pauli Equation
$$i \frac{d}{dt} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \mu B \sigma_z \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \mu B \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

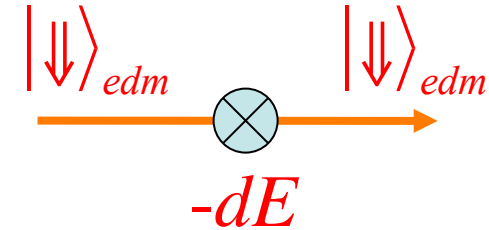
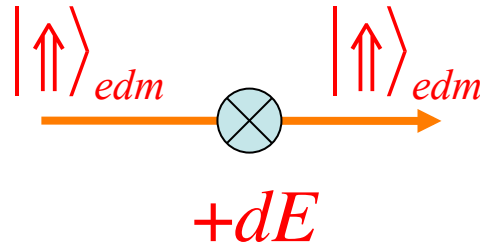
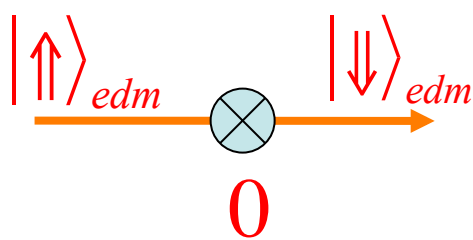
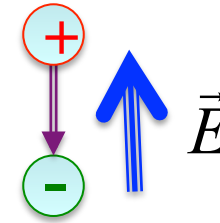
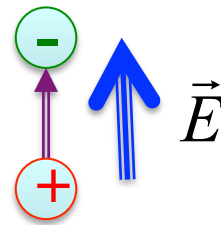


This is just rotation of coordinates from the $\vec{B} = B(1, 0, 0)$ case and we can conclude the spin precesses in the x - y plane

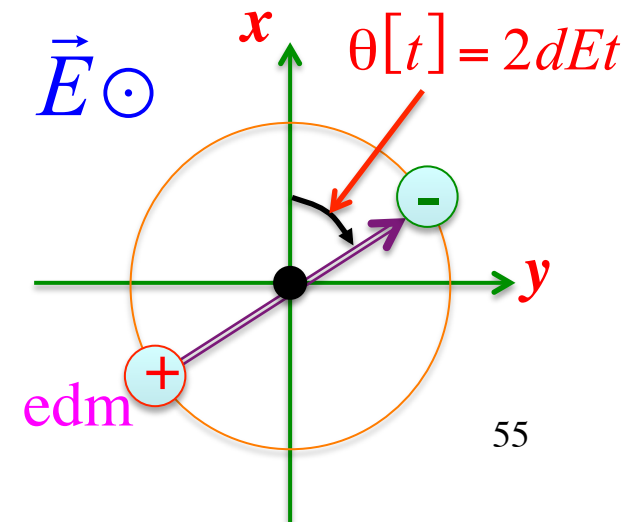




If electric field $\vec{E} = E(0, 0, 1)$ is applied



This transition pattern is the same as before, and we can assume the edm in electric field also precesses.

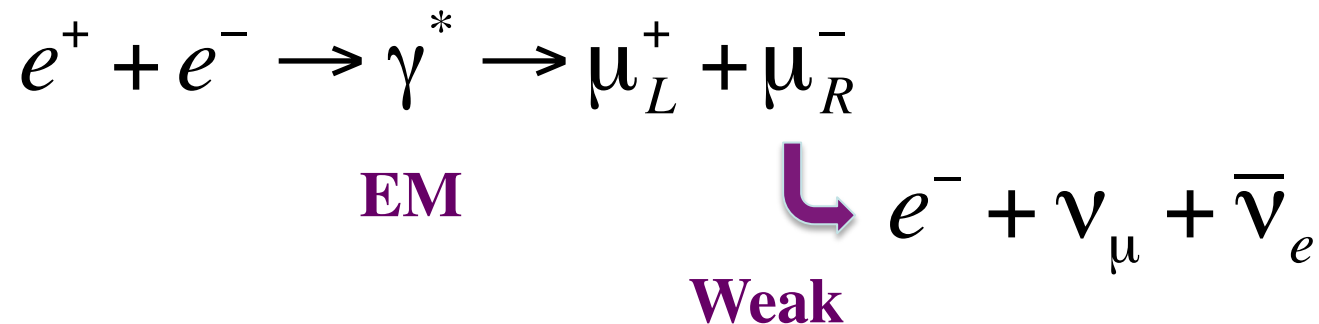


Chirality is Oscillating

A problem:

μ_R^- can be produced by EM interactions.

$\sim 2\mu\text{s}$ after, it decays weakly.



But why this μ_R^- can decay weakly?

Chirality Oscillation & Muon Decay

Definition of Chirality State:

$$\left\{ \begin{array}{l} \psi_R = \frac{1+\gamma^5}{2} \begin{pmatrix} u \\ \nu \end{pmatrix} = \frac{u+\nu}{\sqrt{2}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \equiv \frac{u+\nu}{\sqrt{2}} |R\rangle, \\ \psi_L = \frac{1-\gamma^5}{2} \begin{pmatrix} u \\ \nu \end{pmatrix} = \frac{u-\nu}{\sqrt{2}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{u-\nu}{\sqrt{2}} |L\rangle \end{array} \right.$$

Muon satisfies the Dirac equation

$$\frac{d}{dt}\psi_\mu = -im_\mu\gamma_0\psi_\mu$$

Wave function can be expressed in chirality basis

$$\frac{d}{dt}\psi_\mu = \dot{C}_R|\mu_R\rangle + \dot{C}_L|\mu_L\rangle$$

The right-hand side of the Dirac equation is expressed as

$$\gamma_0\psi_\mu = \begin{pmatrix} u \\ -v \end{pmatrix} = C_L|\mu_R\rangle + C_R|\mu_L\rangle$$

Therefore, the Dirac equation can be expressed as

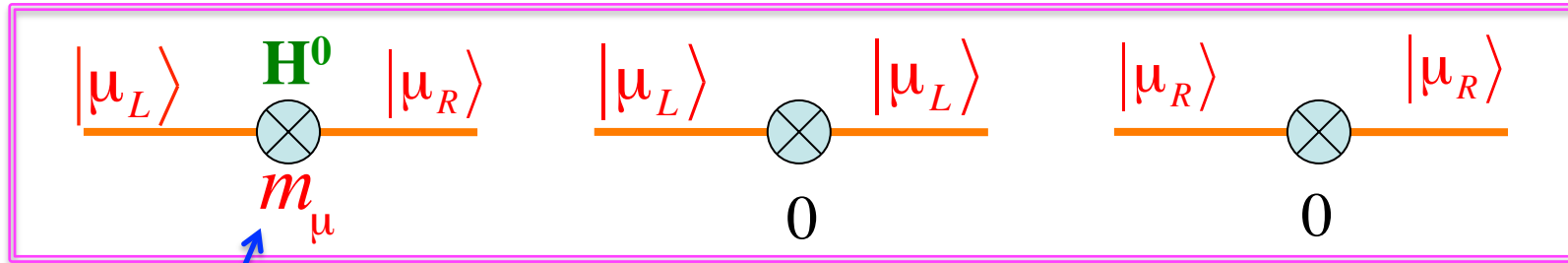
$$\Rightarrow i(\dot{C}_L|\mu_L\rangle + \dot{C}_R|\mu_R\rangle) = m_\mu(C_R|\mu_L\rangle + C_L|\mu_R\rangle)$$

or,

$$i\frac{d}{dt}\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix}\begin{pmatrix} C_L \\ C_R \end{pmatrix}$$

← The Dirac equation is actually chirality swapping equation

$$i \frac{d}{dt} \begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0 & m_\mu \\ m_\mu & 0 \end{pmatrix} \begin{pmatrix} C_L \\ C_R \end{pmatrix}$$



Dirac mass

mass eigenstates:

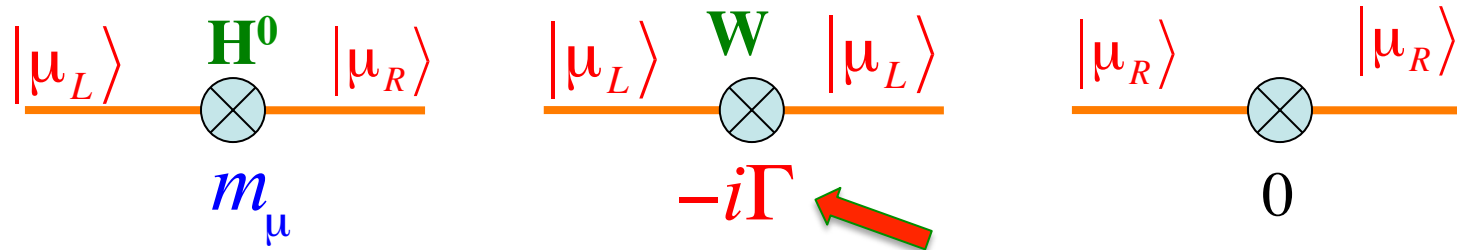
$$\begin{cases} \mu[E > 0] = \frac{1}{\sqrt{2}} (|\mu_L\rangle + |\mu_R\rangle) \exp[-im_\mu t] \\ \mu[E < 0] = \frac{1}{\sqrt{2}} (|\mu_L\rangle - |\mu_R\rangle) \exp[+im_\mu t] \end{cases}$$

$\mu_R \Leftrightarrow \mu_L$ oscillation is taking place

$$P[\mu_R \Leftrightarrow \mu_L] = \sin^2 m_\mu t$$

Muon decays weakly while it is in μ_L state

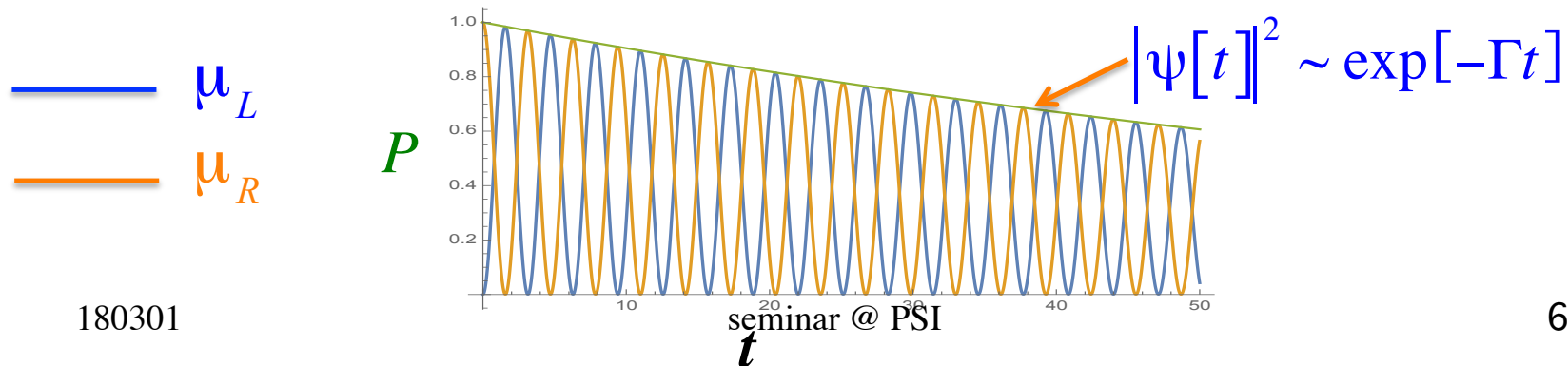
The weak decay effect can be included by putting imaginary amplitude to the μ_L self transition



State equation:
$$i \begin{pmatrix} \dot{C}_L \\ \dot{C}_R \end{pmatrix} = \begin{pmatrix} -i\Gamma & m_\mu \\ m_\mu & 0 \end{pmatrix} \begin{pmatrix} C_L \\ C_R \end{pmatrix} \quad \Gamma = \frac{1}{\tau_\mu} = 3 \times 10^{-10} \text{ eV} \ll m_\mu$$

The solution for the condition $\psi[0] = |\mu_R\rangle$ is,

$$\psi[t] \sim \left(\cos[m_\mu t] |\mu_R\rangle - i \sin[m_\mu t] |\mu_L\rangle \right) \exp\left[-\frac{\Gamma}{2} t\right]$$

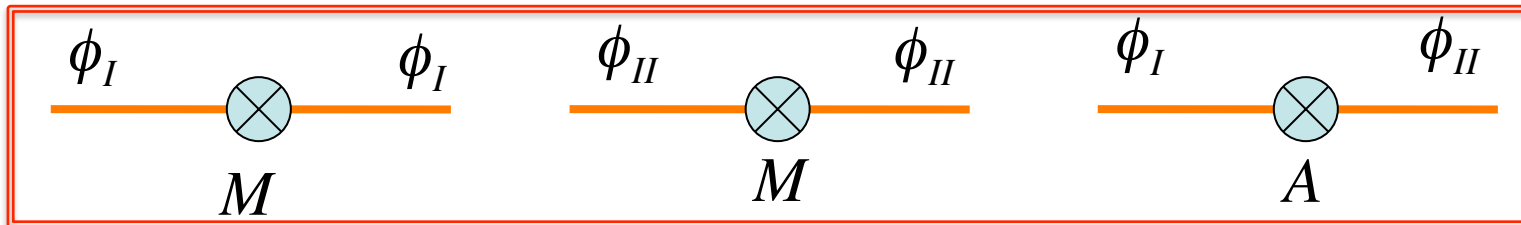
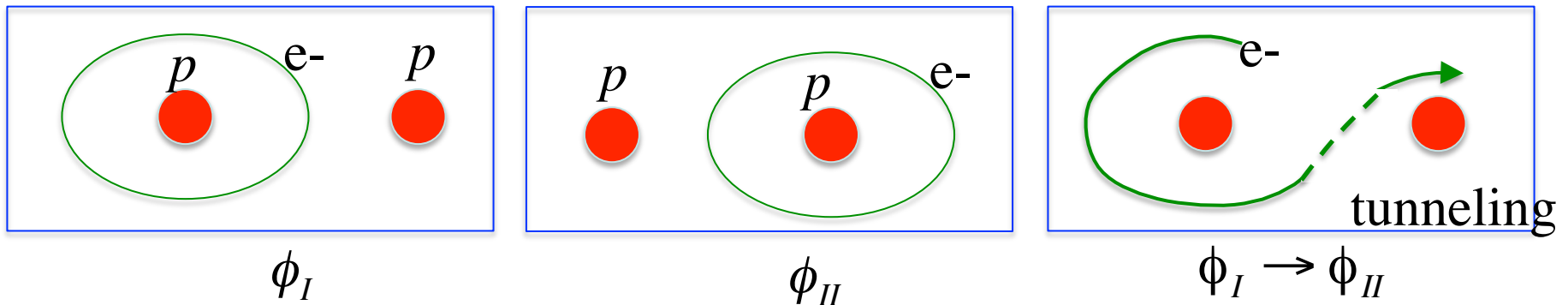


The oscillation and mixing view is useful to understand abstract properties concretely

- * Parity**
- * C-Parity**
- * Isospin**

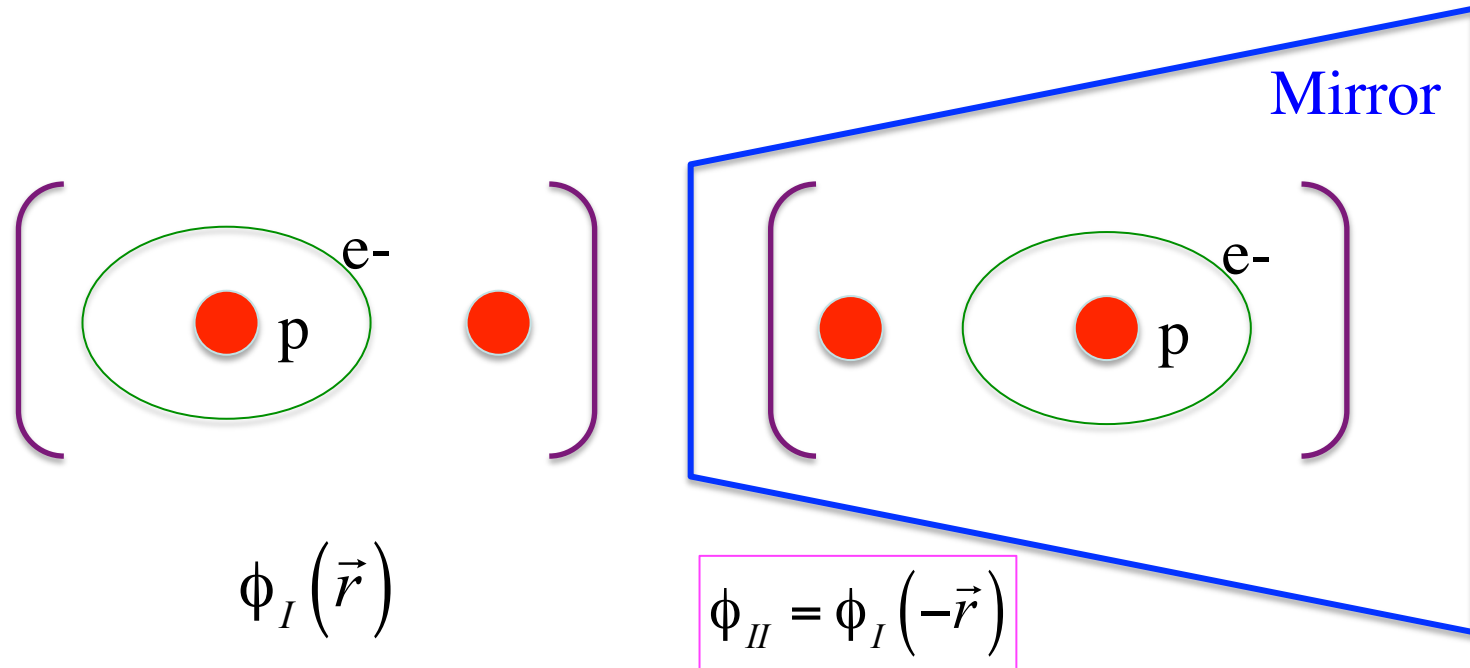
What is parity? (Feynman's explanation)

An Hydrogen ion H_2^+ has two basis states: ϕ_I and ϕ_{II}



→ The energy eigenstates

$$\begin{cases} \Phi_+ = \frac{1}{\sqrt{2}} (\phi_I + \phi_{II}) e^{-i(M+A)t} \\ \Phi_- = \frac{1}{\sqrt{2}} (\phi_I - \phi_{II}) e^{-i(M-A)t} \end{cases}$$



Actually, ϕ_{II} is a mirror image of ϕ_I

Therefore, the energy eigenstates are

$$\begin{cases} \Phi_+(\vec{r}) = \frac{1}{\sqrt{2}}(\phi_I(\vec{r}) + \phi_I(-\vec{r}))\exp[-i(M + A)t] \\ \Phi_-(\vec{r}) = \frac{1}{\sqrt{2}}(\phi_I(\vec{r}) - \phi_I(-\vec{r}))\exp[-i(M - A)t] \end{cases}$$

$$\Phi_{\pm} = \left(\left[\begin{array}{c} \text{e-} \\ \text{p} \\ \phi_I \end{array} \right] \pm \left[\begin{array}{c} \text{e-} \\ \text{p} \\ \phi_{II} \end{array} \right] \right) / \sqrt{2}$$

If parity of the mass eigenstates is reversed

$$\left\{ \begin{array}{l} \Phi_+(-\vec{r}) = \frac{1}{\sqrt{2}} (\phi_I(-\vec{r}) + \phi_I(\vec{r})) \exp[-i(M+A)t] = +\Phi_+(\vec{r}) \\ \Phi_-(-\vec{r}) = \frac{1}{\sqrt{2}} (\phi_I(-\vec{r}) - \phi_I(\vec{r})) \exp[-i(M-A)t] = -\Phi_-(\vec{r}) \end{array} \right.$$

Parity = + structure

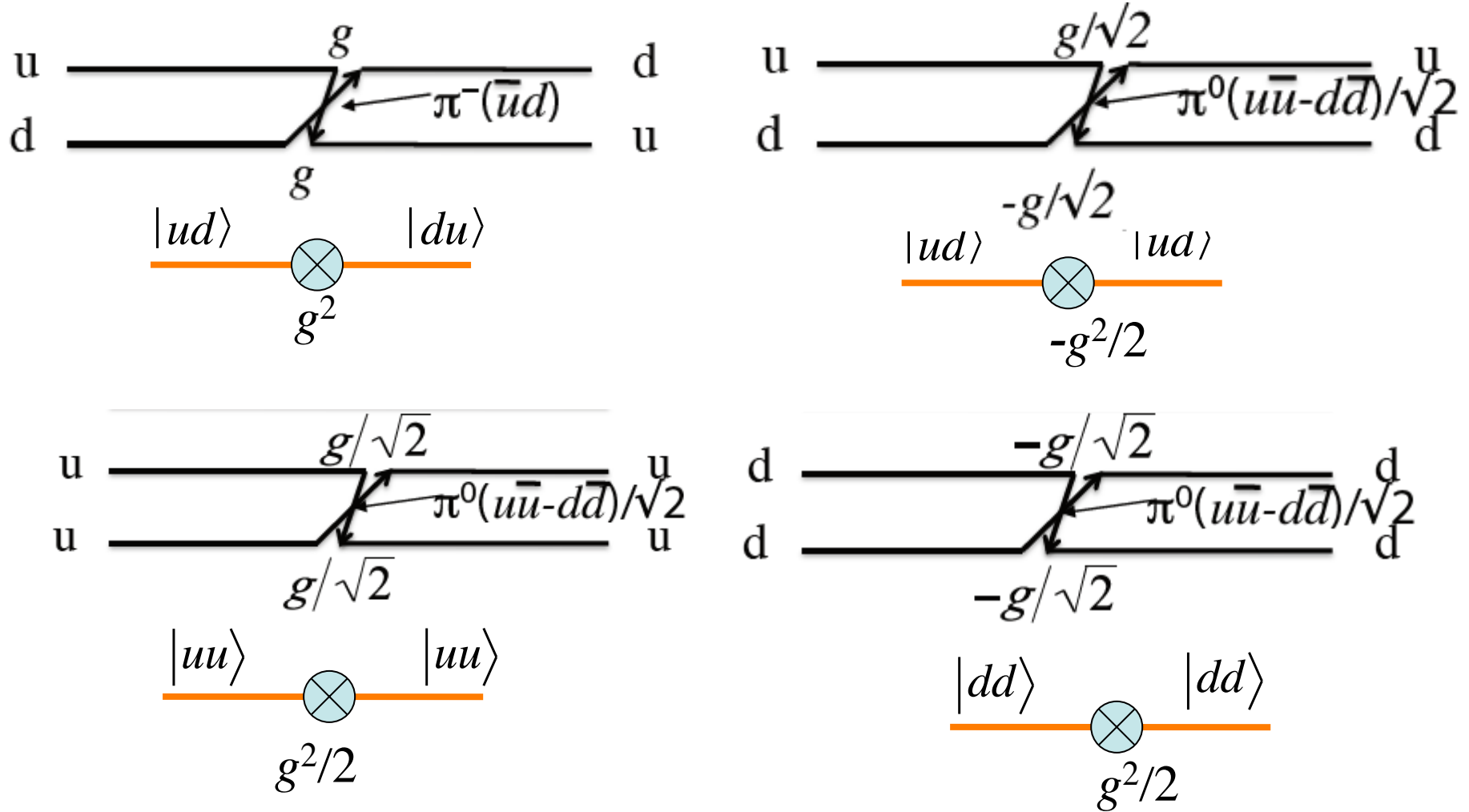
Parity = - structure

=> Energy eigenstates have fixed parities.

Isospin

Isospin

For u, d system, pion exchange changes the basis system



State equation

$$\frac{d}{dt} C_{uu} = -i \frac{g^2}{2} C_{uu}, \quad \frac{d}{dt} C_{dd} = -i \frac{g^2}{2} C_{dd}$$

$$\frac{d}{dt} \begin{pmatrix} C_{ud} \\ C_{du} \end{pmatrix} = -i \begin{pmatrix} -g^2/2 & g^2 \\ g^2 & -g^2/2 \end{pmatrix} \begin{pmatrix} C_{ud} \\ C_{du} \end{pmatrix}$$

← This is the same form as spin dipole moment interaction (cf. 21cm line)

Mass eigenstates

$$\left\{ \begin{array}{l} \psi_U = |uu\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_D = |dd\rangle \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_+ = \frac{|ud\rangle + |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 + \frac{1}{2}g^2\right)t\right] \\ \psi_- = \frac{|ud\rangle - |du\rangle}{\sqrt{2}} \exp\left[-i\left(M_0 - \frac{3}{2}g^2\right)t\right] \end{array} \right.$$

Analogy to the Spin combination

3 same mass state:
I=1 state

$$\left\{ \begin{array}{l} |\Sigma^+\rangle = |uu\rangle |s\rangle \\ |\Sigma^-\rangle = |dd\rangle |s\rangle \\ |\Sigma^0\rangle = \frac{|ud\rangle + |du\rangle}{\sqrt{2}} |s\rangle \end{array} \right.$$

singlet state:
I=0

$$|\Lambda\rangle = \frac{|ud\rangle - |du\rangle}{\sqrt{2}} |s\rangle$$

There are a lot more interesting oscillations and mixings . . .

Name	Origin	Transition	Energy eigenstate
Neutrino Oscillation	X	$ v_e\rangle \Leftrightarrow v_\mu\rangle \Leftrightarrow v_\tau\rangle$	v_1, v_2, v_3
Cabbibo Angle	Higgs	$ d'\rangle \Leftrightarrow s'\rangle$	d, s
Chirality Osc.	Higgs	$ L\rangle \Leftrightarrow R\rangle$	$ R\rangle \pm L\rangle$
Majorana Neutrino	X	$ v_L\rangle \Leftrightarrow \bar{v}_R\rangle$	$ v_L\rangle \pm \bar{v}_R\rangle$
Seesaw Mechanism	X	$ v_R\rangle \Leftrightarrow \bar{v}_L\rangle, v_L\rangle \Leftrightarrow v_R\rangle$	$v = v_L\rangle - \bar{v}_R\rangle, N = v_R\rangle + \bar{v}_L\rangle$
Weinberg angle	Higgs	$W_3 \Leftrightarrow B$	γ, Z^0
Hydrogen 21 cm line	$\vec{\mu}_p \cdot \vec{\mu}_e$	$ p(\uparrow)e(\downarrow)\rangle \Leftrightarrow p(\downarrow)e(\uparrow)\rangle$	$ \uparrow\downarrow\rangle \pm \downarrow\uparrow\rangle$
$\pi^+ - \rho^+$ mass difference	Strong	$ \uparrow\downarrow\rangle_S \Leftrightarrow \downarrow\uparrow\rangle_S$	π^+, ρ^+
CPV	Weak	$K_{CP+} \Leftrightarrow K_{CP-}$	K_1, K_2
Hydrogen Ion (H_2^+)	tunneling	$ (pe^-)p\rangle \Leftrightarrow p(e^-p)\rangle$	$ (pe^-)p\rangle \pm p(e^-p)\rangle$
Positronium	EM	$ e^+e^-\rangle \Leftrightarrow e^-e^+\rangle$	o-Ps, p-Ps
Isospin	S	$ ud\rangle \Leftrightarrow du\rangle, u\bar{u}\rangle \Leftrightarrow d\bar{d}\rangle$	$(\Lambda, \Sigma), (\rho, \omega)$
Baryon Color	Strong	$ RGB\rangle \Leftrightarrow GRB\rangle$	$ RGB\rangle - RBG\rangle + BRG\rangle - \dots$
ρ^0, ω, ϕ structure	Strong	$ u\bar{u}\rangle \Leftrightarrow d\bar{d}\rangle \Leftrightarrow s\bar{s}\rangle$	ρ^0, ω, ϕ
Spin precession in \vec{B}	$\vec{\mu}\vec{B}$	$ \uparrow\rangle \Leftrightarrow \downarrow\rangle$	$ \uparrow\rangle_\theta$
Deuteron	S	$ pn\rangle \Leftrightarrow np\rangle$	$(pn\rangle - np\rangle) \uparrow\uparrow\rangle$
sp^3 hybrid orbit	EM	$\Psi_{2S} \Leftrightarrow \Psi_{2P_i}$	$\Psi_{2S} \pm \Psi_{2P_x} \pm \Psi_{2P_y} \pm \Psi_{2P_z}$
\vdots	\vdots	\vdots	\vdots

Part-II Summary:

- * The same mechanism as neutrino oscillation is working in various other places and is playing important physics roles.**
- * Many important physics can be understood by analogy of neutrino oscillation mechanism (or vice versa).**
- * Abstract properties, such as parity, etc. can be attributed to the structure of the mixings.**
- * It should be educative to teach such ideas.**