General relativity experiment with frozen spin (EDM) rings

András László
laszlo.andras@wigner.mta.hu
Wigner Research Centre for Physics, Budapest

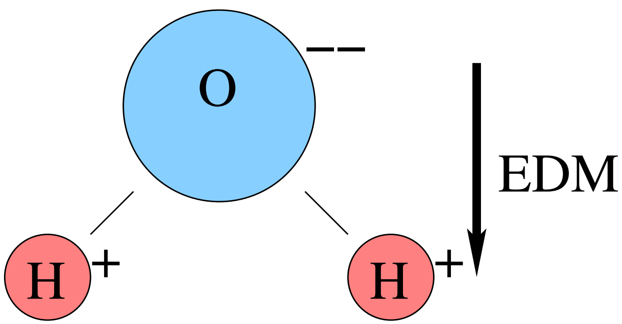
(based on: A.László,Z.Zimborás:*Class.Quant.Grav.***35**(2018)175003)

PSI, Zürich

20 September 2018

Introduction

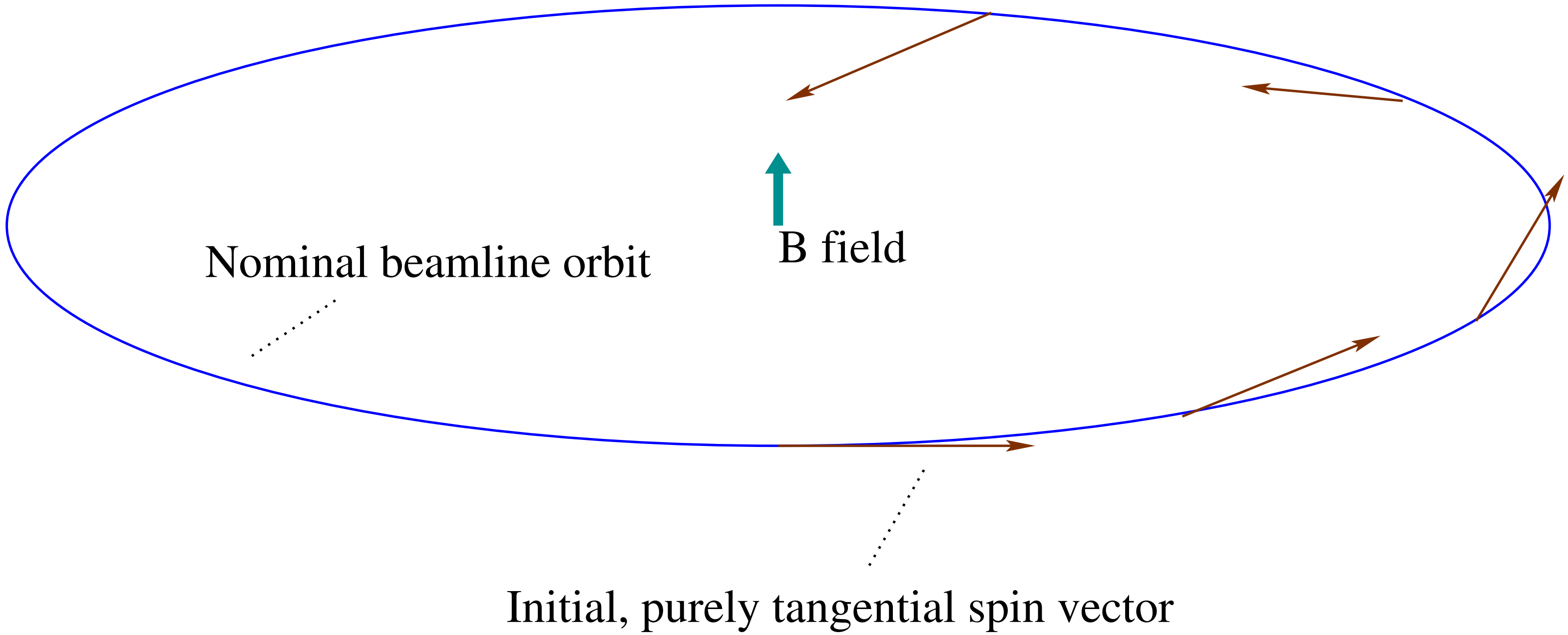
- Magnetic dipole moment is an important quantity: sensitive to radiative corrections.
  For elementary particle obeying classical Dirac equation: $\mathrm{g} := \frac{2\,m\,\mu}{q\,s} = 2$.
  QFT: $\mathrm{g} \approx 2$ but $\mathrm{g} \neq 2$ due to radiative corrections, sensitive to model details.
  For composite particles: g grossly deviates from 2, due to internal angular momenta.
- So, $\mathrm{g}{-}2$ is a sensitive probe for SM / BSM physics ($\Rightarrow$ $\mathrm{g}{-}2$ experiment).
  E.g. for muons: $\frac{(\mathrm{g}-2)}{2} \approx 0.0011659209\ldots$ in $3\,\sigma$ tension with SM.
- Electric Dipole Moment (EDM) is also sensitive to internal structure.
  Crude analogy: water molecule. (Or at the level of elementary Lagrangian: CP violation.)



- SM gives negligible EDM, so can be SM / BSM discriminator ($\Rightarrow$ CPEDM collaboration).
  Planned sensitivity: $10^{-29}\,\mathrm{e\,cm}$, can see new physics up to $\approx 3000\,\mathrm{TeV}$ mass.

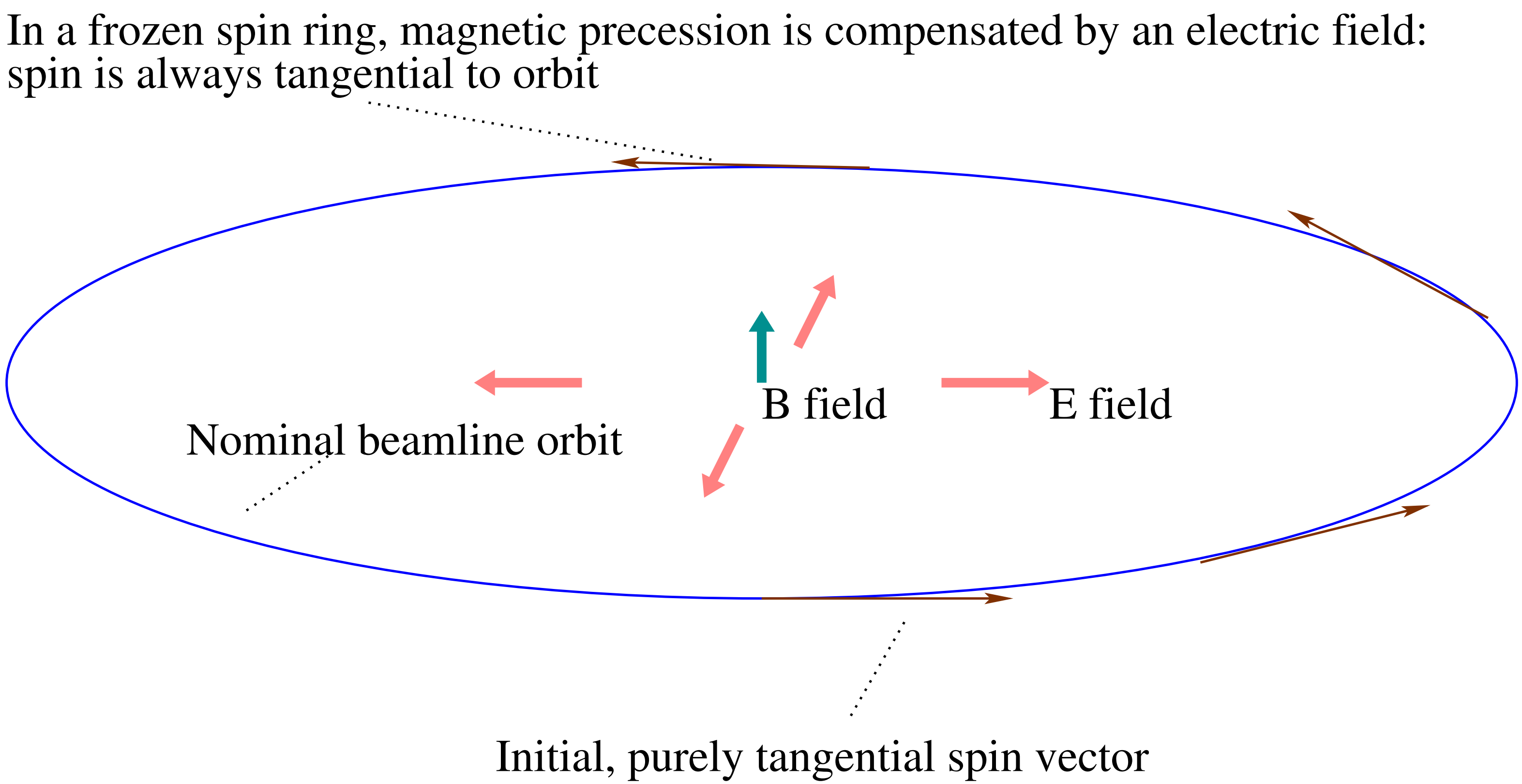
How the magnetic moment anomaly $a := \frac{g-2}{2}$ is measured?

Principle of g–2 measuring storage rings:
in vertical magnetic field, spin precesses in orbital plane at a rate $a\gamma\omega$

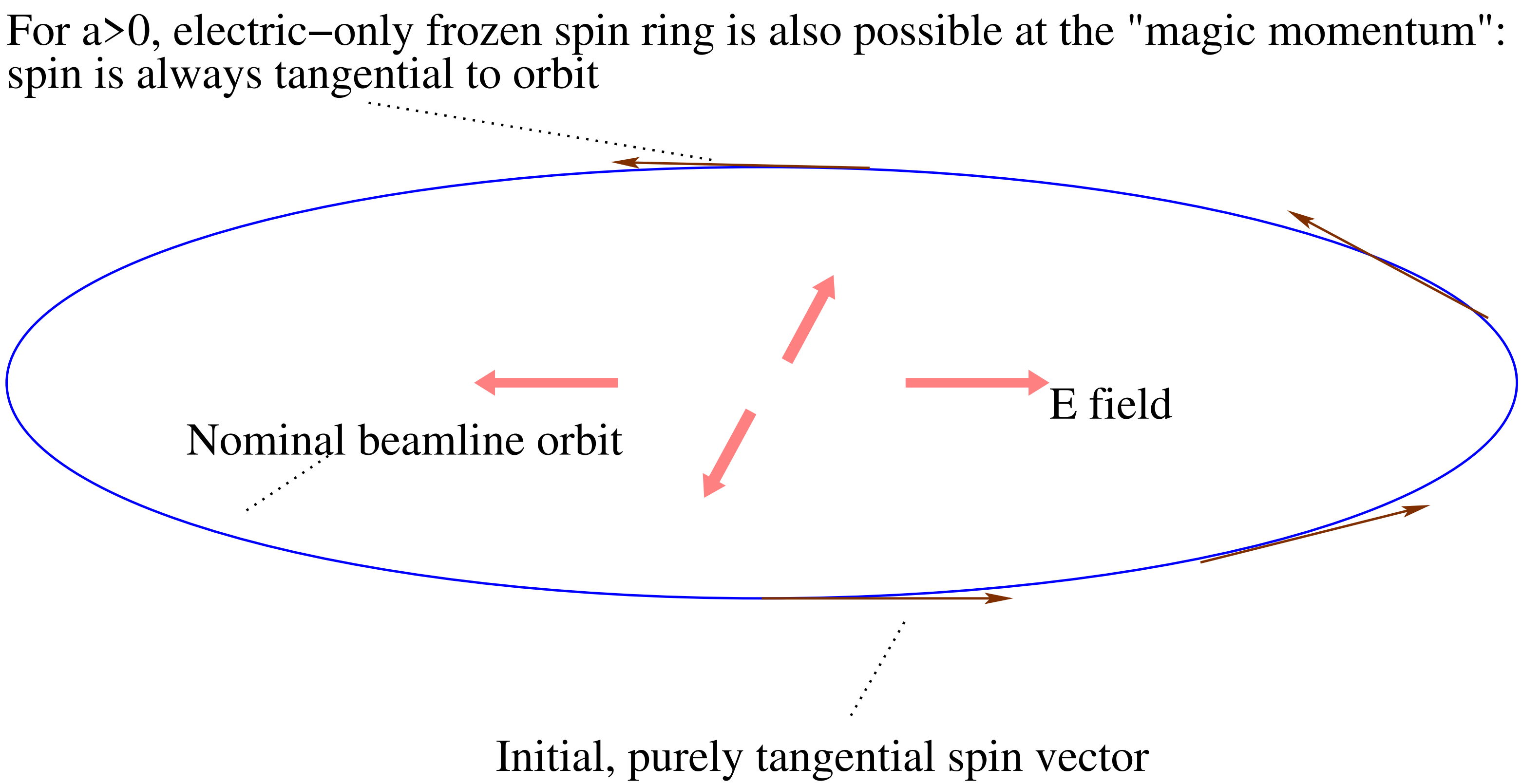


If the particle circular velocity is ω, the spin vector is ahead of momentum vector by $a\,\gamma\,\omega$.
$(\gamma\,\omega = B\,\frac{q}{m})$

How EDM is measured?



Magnetic precession is compensated by an electric field ("frozen spin"): spin$||$momentum.
If EDM existed, it would slowly elevate spin out of the orbital plane.



For $a > 0$, electric-only frozen spin ring can be made at “magic momentum” $\beta\gamma = \frac{1}{\sqrt{a}}$.
If EDM existed, it would slowly elevate spin out of the orbital plane.

Beam evolution equations in lab frame
(Newton + Thomas-Barmann-Michel-Telegdi equations):

$$\frac{\mathrm{d}\vec{\beta}}{\mathrm{d}t_{\mathrm{lab}}} = \frac{q}{m\,\gamma}\left(\vec{E} - (\vec{\beta}\cdot\vec{E})\,\vec{\beta} + \vec{\beta}\times\vec{B}\right),$$

$$\frac{\mathrm{d}\vec{S}_{\mathrm{lab,corot.}}}{\mathrm{d}t_{\mathrm{lab}}} = -\frac{q}{m}\left(\underbrace{a\,\vec{B}}_{\text{magnetic term}} + \underbrace{\overbrace{\left(\frac{1}{(\beta\gamma)^2} - a\right)}^{\text{zero at “magic momentum”}}\vec{\beta}\times\vec{E}}_{\text{electric term}} + \underbrace{\frac{1}{2}\,\eta\,\left(\vec{E}+\vec{\beta}\times\vec{B}\right)}_{\text{EDM term}}\right) \times \vec{S}_{\mathrm{lab,corot.}}$$

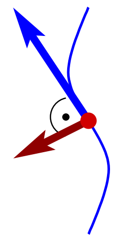
(g $:= \frac{2\,m\,\mu}{q\,s}$, $a := \frac{\mathrm{g}-2}{2}$ is magnetic moment anomaly, $\eta := \frac{2\,m\,d}{q\,s}$ is the “g” of EDM.)

Magic momentum: $\beta\gamma = \frac{1}{\sqrt{a}}$ (only possible for $a > 0$).

Frozen spin condition: $\frac{\mathrm{d}\vec{S}_{\mathrm{lab,corot.}}}{\mathrm{d}t_{\mathrm{lab}}} = 0$ (assuming $\eta = 0$).
(Special case: with $\vec{B} = 0$, then magic momentum ⇔ frozen spin condition.)

Relativistic motion of particle with spin in electromagnetic field:

u^a denotes the four velocity of the particle at points of trajectory.



w^a denotes the spin direction four vector at points of trajectory.

(For quantum mechanical reasons, always: $u_a \, w^a = 0$.)

Then the equation of motion is Newton + Thomas-Bargmann-Michel-Telegdi equation.

$$u^a \nabla_a u^b = -\frac{q}{m} F^{bc} u_c \qquad (\leftarrow \text{ Newton equation with Lorentz force}),$$

$$\begin{aligned} D_u^F w^b &= -\frac{\mu}{s} \left(F^{bc} - u^b u_d \; F^{dc} - \; F^{bd} \, u_d u^c \right) w_c \qquad (\leftarrow \text{ TBMT equation}) \\ &\quad + \frac{d}{s} \left({}^\star F^{bc} - u^b u_d \, {}^\star F^{dc} - {}^\star F^{bd} \, u_d u^c \right) w_c . \end{aligned}$$

$D_u^F w^b := u^a \nabla_a w^b + w^a \, u^b u^c \nabla_c u_a - w^a \, u_a u^c \nabla_c u^b$ is the Fermi-Walker derivative.
(Conserves the constraint $u_a \, w^a = 0$. Free gyroscope equation would be $D_u^F w^b = 0$.)

Some historical terminology on the free gyroscope equation $D_u^F w^b = 0$:

- For free, i.e. geodesic motion ($u^a \nabla_a u^b = 0$) in gravitational field:
  - In the field of nonrotating object (Schwarzschild): *de Sitter* precession.
  - In the field of rotating object (Kerr): *Lense-Thirring* precession. (See also: *Gravity Probe B* satellite experiment.)
- For forced orbit ($u^a \nabla_a u^b =$ some force):
  - It is called the *Thomas* precession. (Already gives effect in special relativity, i.e. in absence of gravity.)

Our case:

$$\underbrace{D_u^F w^b}_{\text{causes **Thomas** precession}} = \underbrace{-\frac{\mu}{s}\left(F^{bc} - u^b u_d \; F^{dc} - \; F^{bd} \, u_d u^c \right) w_c}_{\text{causes **Larmor** precession in addition}}$$

So we have: precession of kinematic origin (Thomas) + of electromagnetic origin (Larmor).

GR corrections?!

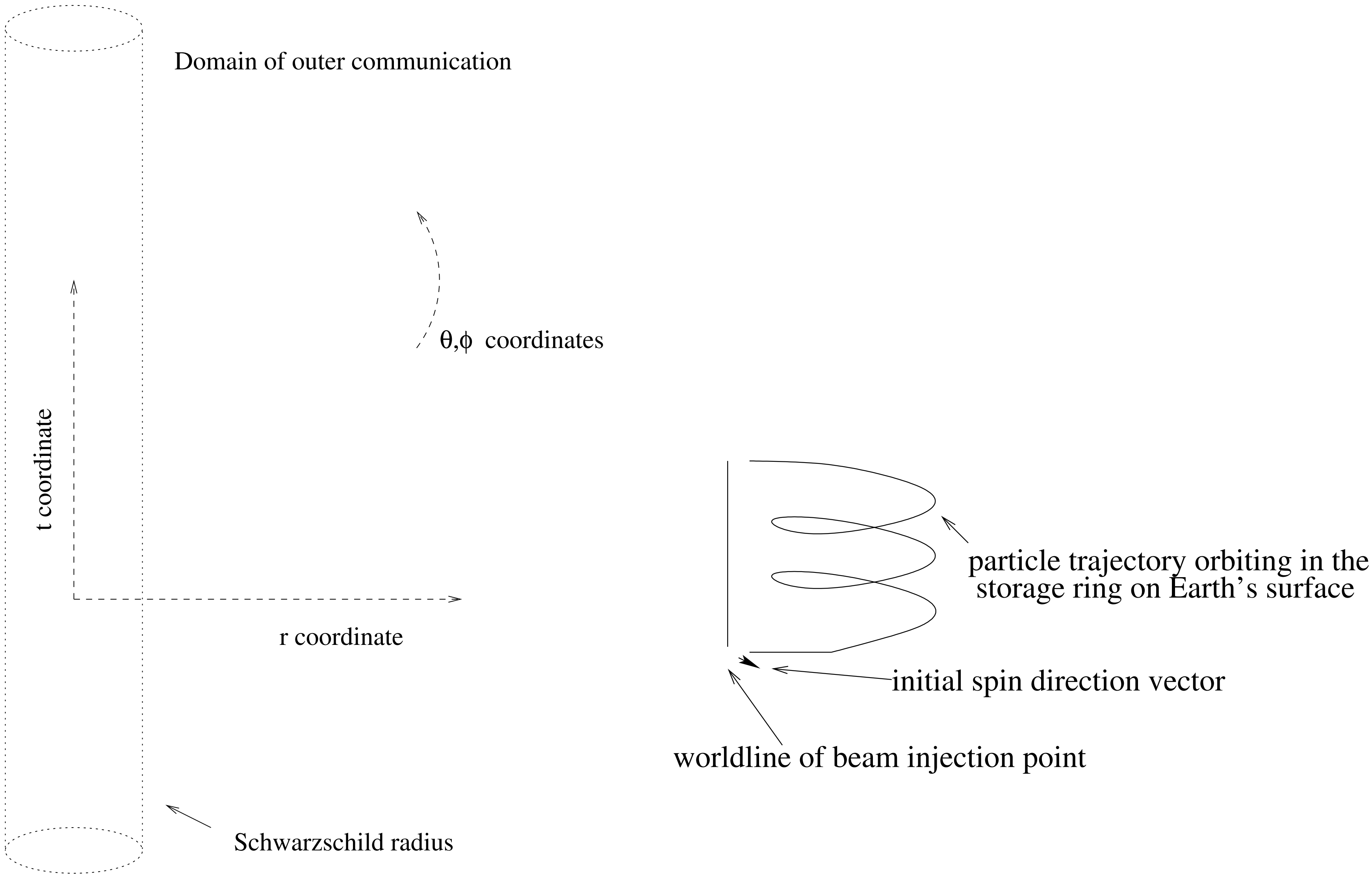
For $\mathbf{g-2}$:

- In February, a series of preprints appeared on arXiv, claiming that GR gives an unaccounted systematic error to $g-2$ experiment, resolving the $3\,\sigma$ tension against SM.
  T. Morishima, T. Futamase, H. M. Shimizu:
  `arXiv:1801.10244`, `arXiv:1801.10245`, `arXiv:1801.10246`.
- Other authors responded: GR correction is much smaller than exp.sensitivity 10^{-7}.
  M. Visser: `arXiv:1802.00651`, P. Guzowski: `arXiv:1802.01120`.
- Further authors: the effect is exactly zero.
  H. Nikolic: `arXiv:1802.04025`.
- Which is true? From first principles it is difficult to judge.

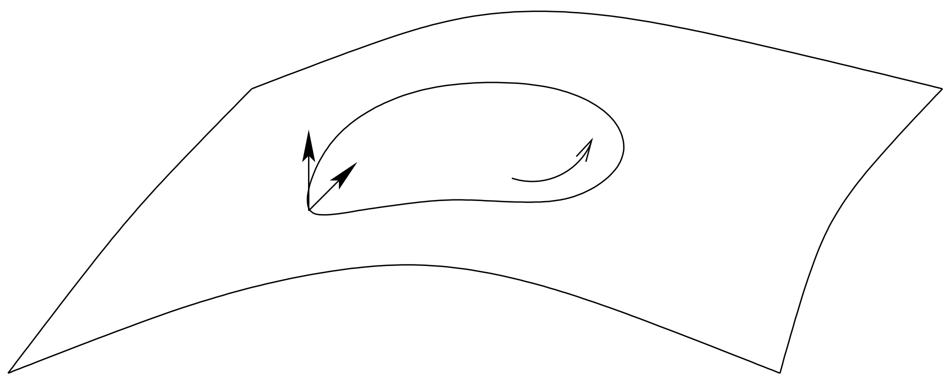
For EDM:

- Earlier papers already warned about possibility for a GR systematics on precession!
  PRD71(2005)064016, PRD76(2007)061101, NPB911(2016)206, PRD94(2016)044019...
  Perturbative calculations with approximations. Qualitatively OK.
- Explicitely first calculated for electric-only frozen spin ring in PLA376(2012)2822.
  Perturbative. Quantitatively OK!! But not done for mixed ring.
- One thing for sure from first principles: only Earth can contribute (equivalence principle).

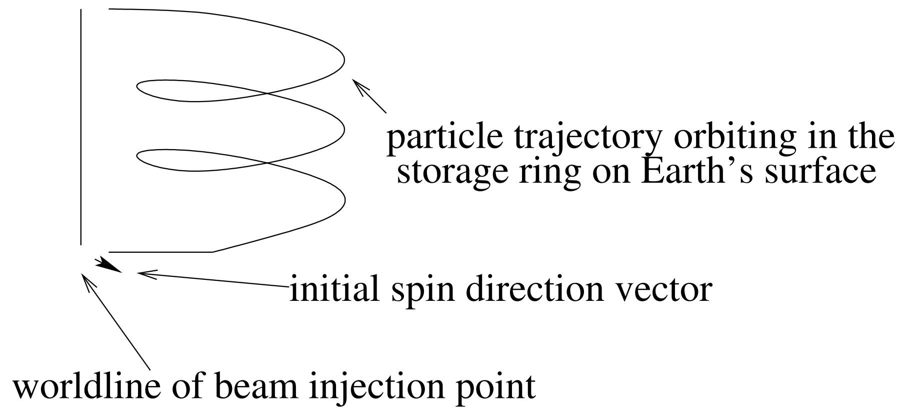
The kinematic configuration in GR setting



Since vectors are transported along closed curves, GR effect is very likely nonzero.



difference is expected due to curvature



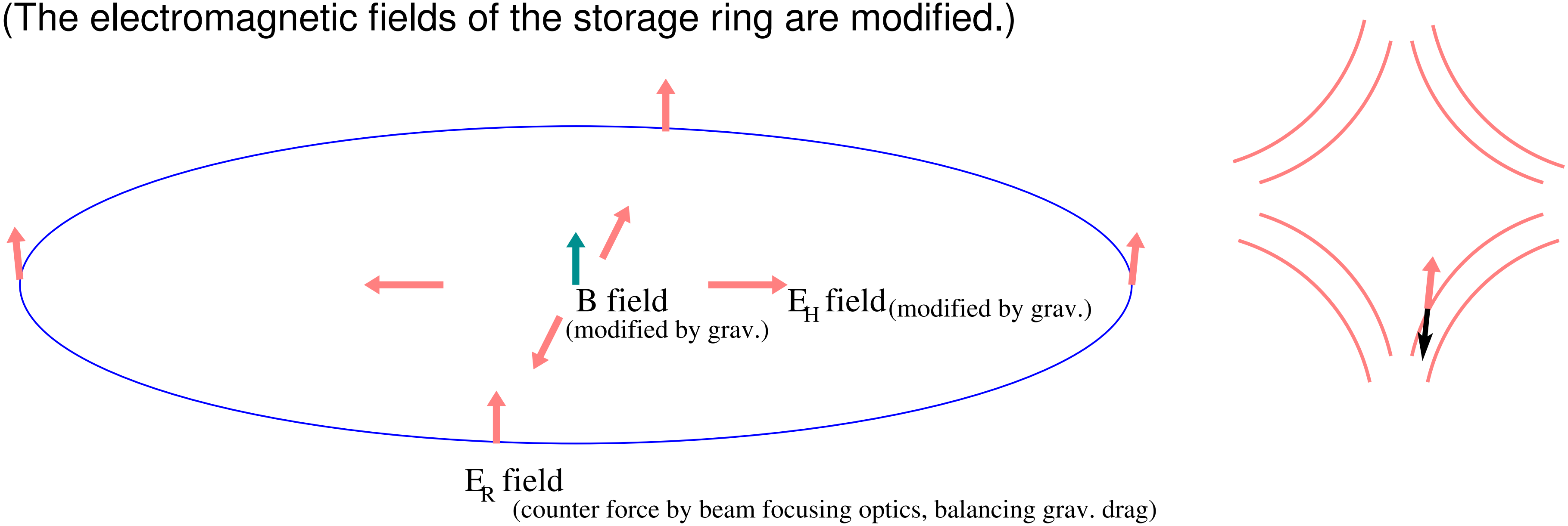
Because curvature just measures that.

But the effect can be small.

Question is: can this be substantially large?

What does the GR modify?

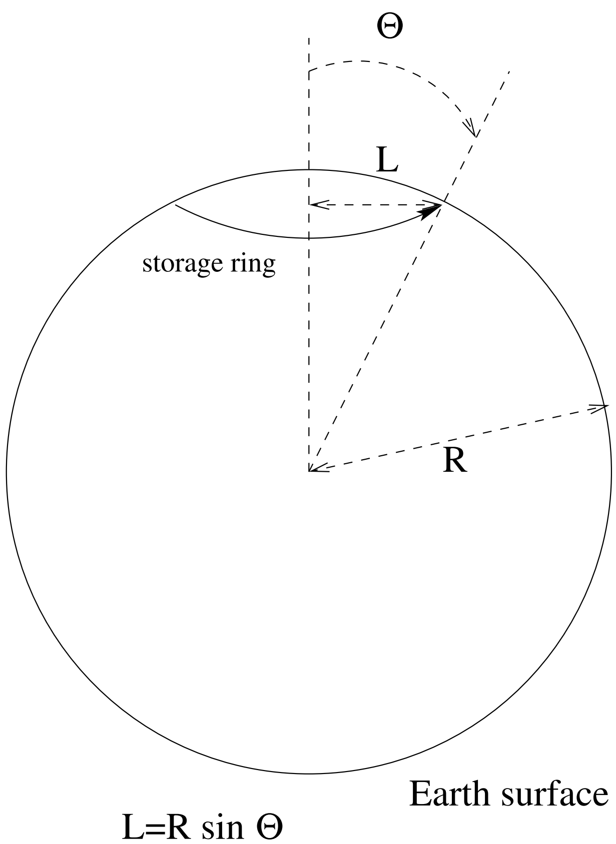
- The spacetime metric, and thus the parallel transport ∇_a, Fermi-Walker derivative D^F. (Newton and TBMT equations of motion are modified.)
- The Maxwell equations $\nabla_a F^{ab} = 0,\ \nabla_a {}^\star F^{ab} = 0$. (The electromagnetic fields of the storage ring are modified.)



Notion of “vertical homogeneous magnetic field”, “horizontal cylindrical electric field”, and “Earth-radial electric field” makes sense and calculable over Schwarzschild.

These electomagnetic fields are necessary for modeling fields in idealized storage ring.

Coordinate conventions:

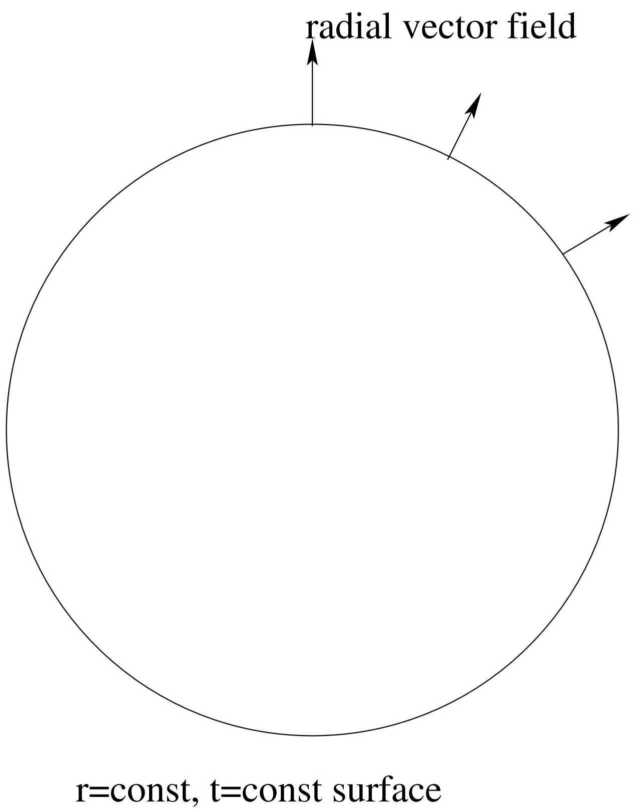


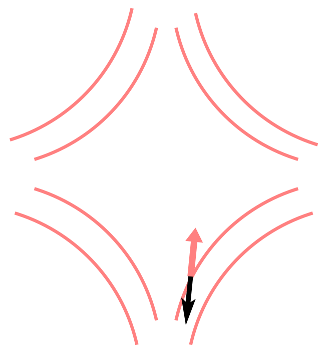
Schwarzschild metric:

$$g_{\mathsf{ab}}(t,r,\vartheta,\varphi) = \begin{pmatrix} 1-\frac{r_S}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{r_S}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\,\sin^2\vartheta \end{pmatrix}$$

Schwarzschild metric is time translation (t) and rotationally (ϑ,φ) invariant.
Earth surface at: $r = R = const$.
The storage ring: $r = R = const$, $\vartheta = \Theta = const$.
Time: in terms of proper time along curves (along laboratory and particle trajectory).

Earth-radial electrostatic field effectively exerted by beam focusing optics:

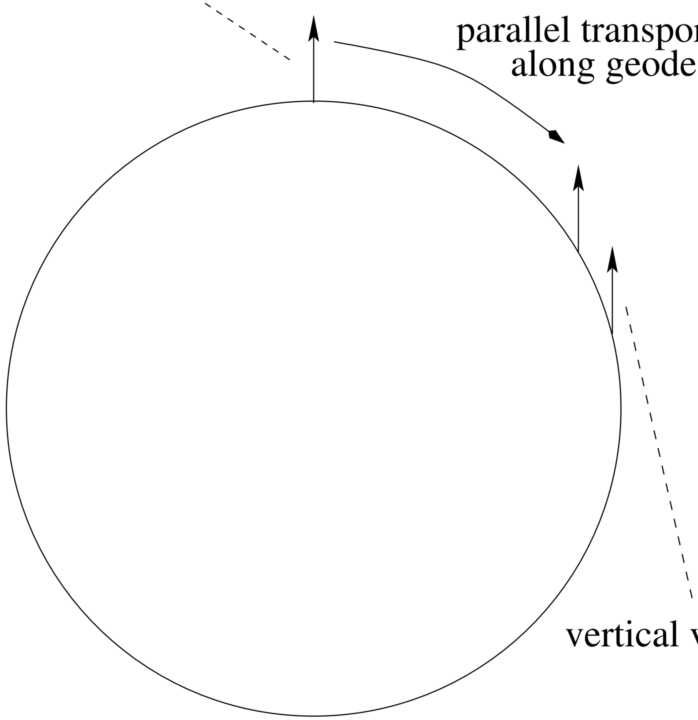




$$E_R{}^{a}(t,r,\vartheta,\varphi) = E_R\,\frac{R^2}{r^2}\begin{pmatrix} 0 \\ \sqrt{1-\frac{r_S}{r}} \\ 0 \\ 0 \end{pmatrix}$$

Holds the beam against falling. (Field of charged spherical shell around graviting center.)

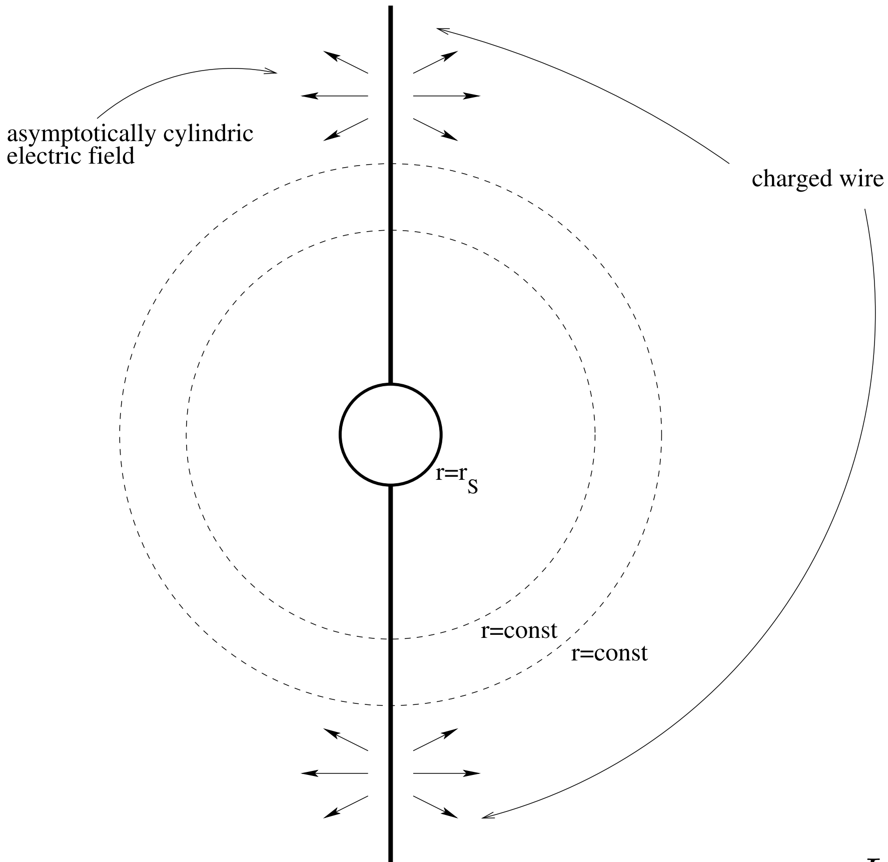
Vertical magnetic field (field inside infinite solenoid):



$$B^{\mathsf{a}}(t,r,\vartheta,\varphi) = B\sqrt{\frac{1-\frac{r_S}{r}}{1-\frac{r_S}{R}\left(\frac{L}{R}\right)^2}}\begin{pmatrix}0\\ \cos\vartheta\\ -\frac{1}{r}\ \sin\vartheta\\ 0\end{pmatrix}$$

Bending field.

Horizontal electrostatic field (field of infinite uniformly charged suspended wire):



$$E_H{}^a(t,r,\vartheta,\varphi) = E_H\,\frac{L}{r\,\sin\vartheta}\,\sqrt{1-\frac{r_S}{r}}\,\mathcal{N}_{r_S}\left(\begin{array}{c} 0 \\ \sin\vartheta\,\left(1+\frac{r_S}{r}\,\ln(\frac{1}{2}\sin\vartheta)\right) \\ \frac{1}{r}\,\cos\vartheta \\ 0 \end{array}\right),$$

$$\text{with } \mathcal{N}_{r_S} := \left(\left(\frac{L}{R}\right)^2\left(1+\frac{r_S}{R}\ln\left(\frac{L}{2R}\right)\right)^2 + \left(1-\left(\frac{L}{R}\right)^2\right)\left(1-\frac{r_S}{R}\right)\right)^{-\frac{1}{2}}$$

Bending field.

Results

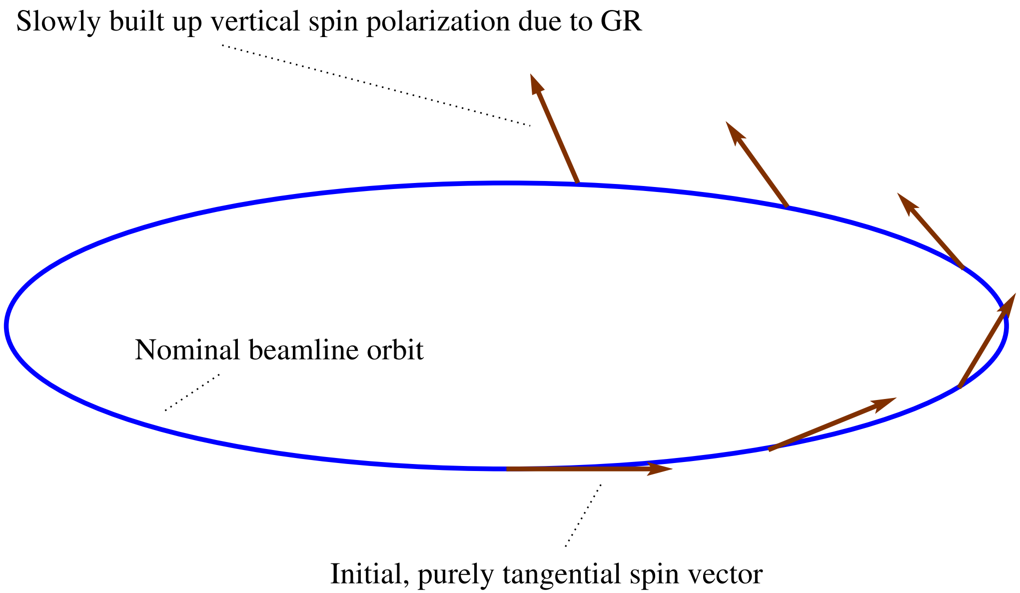
A. László, Z. Zimborás: *Class.Quant.Grav.* **35**(2018)175003.

(R: Earth radius, r_S: Schwarzschild radius, L: storage ring radius, g: grav.accel.)

- **g − 2:** Systematic errors by GR is $\sim \frac{r_S}{R}\frac{L^2}{R^2} \approx 10^{-21}$, pretty much negligible. Qualitative reason: precession due to magnetic moment anomaly is relatively large effect, plus GR mainly modifies precession in the other direction.

- **EDM:** In a “frozen spin” storage ring GR torques the spin vector out of the orbital plane, at a rate $-a\,\beta\gamma\,\underbrace{g/c}_{\approx 33\,\mathrm{nrad/sec}}$. This fake EDM signal is $\approx 10\times$ foreseen sensitivity.

(What about optimizing for specific GR experiment?)



Some philosophy…

To what extent it is gravitational modification of kinematics vs Larmor precession?

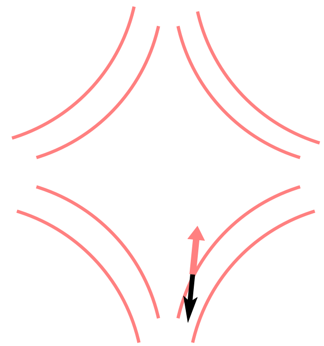
$$-\overbrace{\frac{g-2}{2}}^{a}\ \beta\gamma\ \ g/c$$ (← vertical polarization buildup rate by GR)

$$\underbrace{D^F_u\, w^b}_{\text{from kinematics (Thomas)}} = \underbrace{-\,g\,\frac{q}{2\,m}\left(F^{bc} - u^b u_d\ F^{dc} - F^{bd}\,u_d u^c\right) w_c}_{\text{of electrodynamic origin (Larmor)}}$$ (← TBMT equations)

$$\left(\quad F_{ab} = \underbrace{F^{B}_{ab}}_{\text{magnetic bending}} + \underbrace{F^{E_H}_{ab}}_{\text{electric bending}} + \underbrace{F^{E_R}_{ab}}_{\text{just compensating ``weight'' of beam}}\quad\right)$$

Part of the contribution is coming merely from “weight”:

$$-\frac{\mu}{s}\,\left(F_{bc}^{E_R} - u_b u^d\, F_{dc}^{E_R} - F_{bd}^{E_R}\, u^d u_c\right) \qquad (\leftarrow \text{ Larmor precession by } E_R)$$



E_R merely compensates the gravitational drag of Earth:

Little to do with GR, kind of “classical” effect.

What is the “real GR” contribution, other than E_R ?

Answer:

$$\text{“real GR”} \;:\; E_R \qquad = \qquad -1 \;:\; (1+a)$$

for all $\beta\gamma$.

Optimalization

We need to optimize for large $|a\,\beta\gamma|$. Grows unboundedly with γ, but becomes expensive.

- Given $a := \frac{\mathrm{g}-2}{2}$ and $a\,\beta\gamma$, the Newton equation for circular motion and the "frozen spin" condition uniquely determines the B and E_H.
  (2 equations, for 2 variables.)

$$\begin{aligned} E_H\,L &= -\,\mathrm{sign}(a)\,\frac{m\,c^2}{q}\,\frac{(a\,\beta\gamma)^2\sqrt{a^2+(a\,\beta\gamma)^2}}{a^2\,(1+a)}, \\ B\,L &= \frac{m\,c}{q}\,\frac{(a\,\beta\gamma)(a-(a\,\beta\gamma)^2)}{a^2\,(1+a)} \end{aligned}$$

- Observe:
  The necessary $|E_H|$ grows monotonically as $\sim |a\,\beta\gamma|^3$, for large $|a\,\beta\gamma|$.
  The necessary $|E_H|$ decreases as $\sim |a|^{-2}$, for large $|a|$.

  Experimental limitation is in $|E_H|$: above $8\,\mathrm{MV/m}$, essentially impossible.
- Experimental idea:
  Use large $|a|$ particle (nucleus), so that too large $|E_H|$ can be avoided.

Experimental / financial constraints:

ring radius L maximum $\sim 10\,\mathrm{m}$,

magnetic field $|B|$ maximum $\sim 1\,\mathrm{Tesla}$,

electric field $|E_H|$ maximum $\sim 8\,\mathrm{MV/m}$.

Let us aim for a GR signal strength $|a\,\beta\gamma| = 0.4 \quad (13.1\,\mathrm{nrad/sec})$.

Assume a surely realistic electric field $|E_H| = 4.10\,\mathrm{MV/m}$.

Possible settings:

particle	$a\ (\approx)$	L [m]	$\lvert B\rvert$ [Tesla]	p [MeV/c]	$\mathcal{E}_{\mathrm{kin}}$ [MeV]
triton	7.92	**1.55**	0.0335	141.9	3.58
helion3	−4.18	**4.13**	0.0353	268.5	12.8
proton	1.79	**7.50**	0.0304	209.7	23.1

Not realistic settings:

particle	$a\ (\approx)$	L [m]	$\lvert B\rvert$ [Tesla]	p [GeV/c]	$\mathcal{E}_{\mathrm{kin}}$ [GeV]
deuteron	−0.142	1796	0.0243	5.283	3.731
electron	0.00116	5942	0.0136	0.1765	0.1760
muon	0.00116	1228520	0.0136	36.497	36.391

Observation: triton, helion3, or proton beam can be OK.
For triton beams, it can be “tabletop” experiment!

Open questions:

- spin polarized ion source (seems to be feasible),
- acceleration without depolarization (seems to be feasible),
- precision storage ring with large spin coherence time (seems to be feasible),
- polarimetry (probably doable, is carbon polarimetry OK at these low energies?)

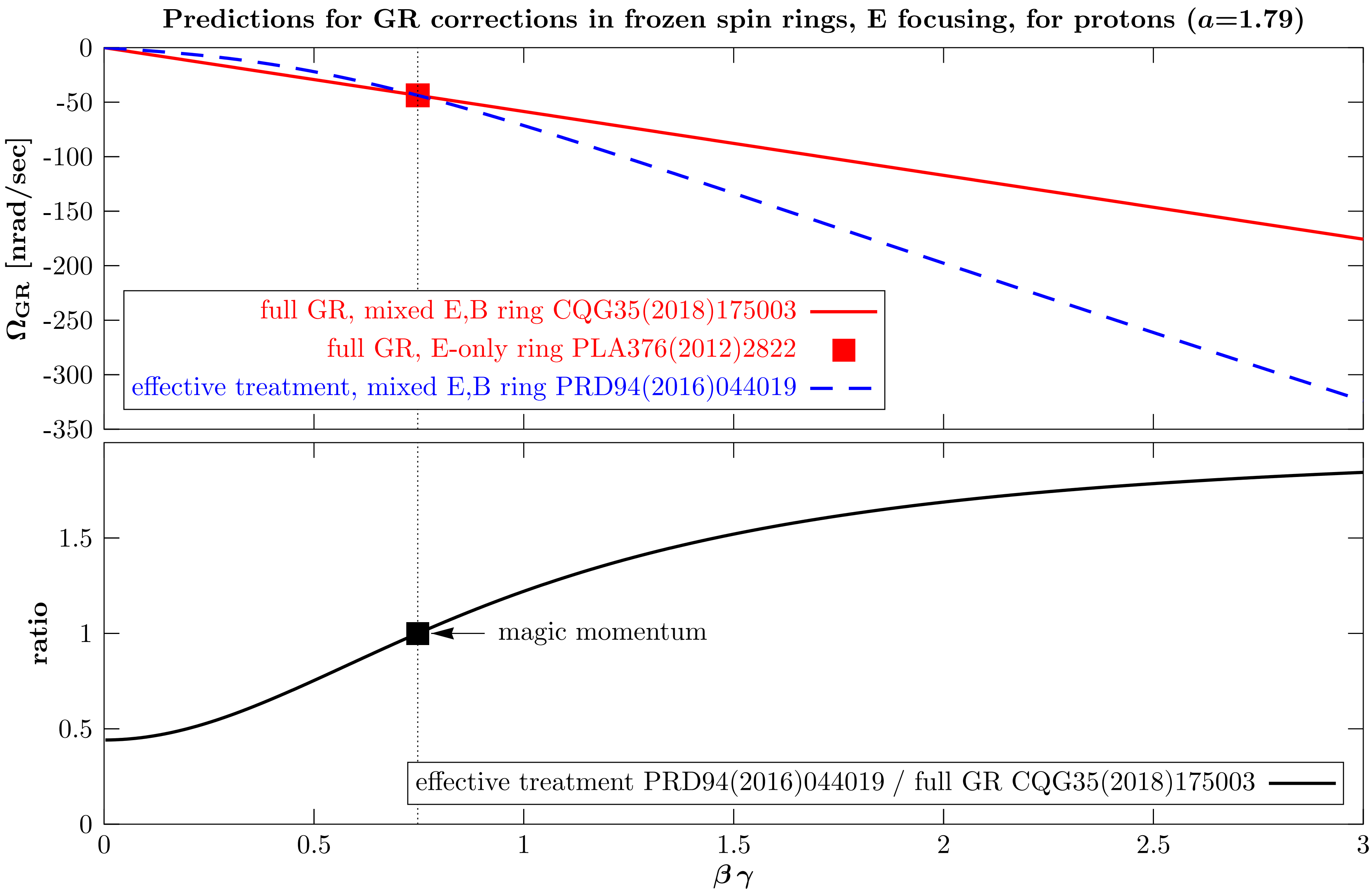
Comparison of predictions for frozen spin rings...

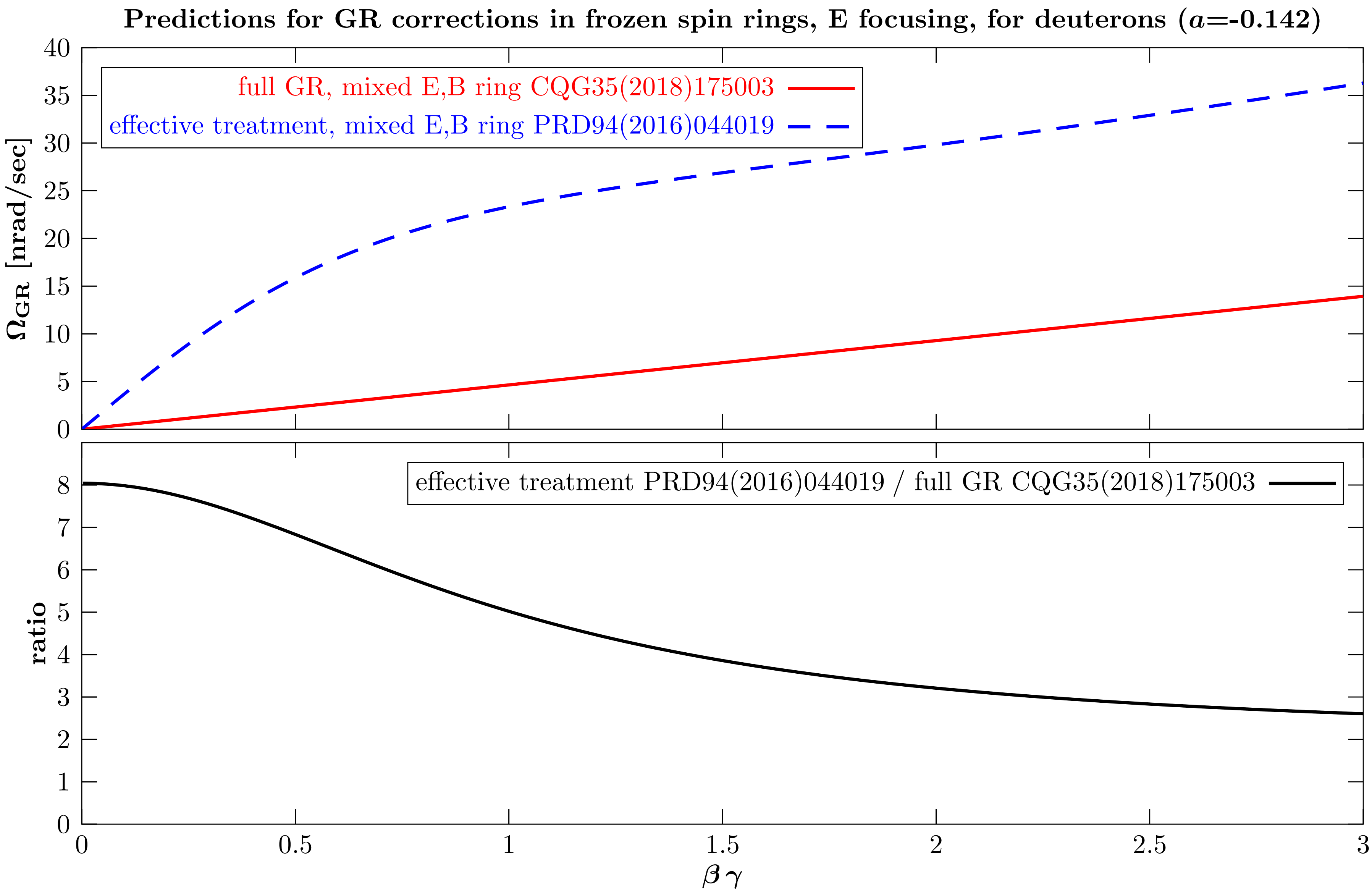
full GR treatment:

- Y.Orlov,E.Flanagan,Y.Semertzidis:*Phys.Lett.***A376**(2012)2822.
  Via linearized GR, for electrostatic-only ring, with electrostatic vertical focusing.
  Main objective of experimental program (lowest systematics for EDM).
  Only possible for $a > 0$ and at magic momentum $\beta\gamma = \frac{1}{\sqrt{a}}$. Prediction: $\Omega_{\mathrm{GR}} = -\sqrt{a}\,\frac{g}{c}$.
- A.László,Z.Zimborás:*Class.Quant.Grav.***35**(2018)175003.
  Via full GR, for mixed magnetic - electric ring, with electrostatic vertical focusing.
  Experimentally more difficult, but allows any $\beta\gamma$. Prediction: $\Omega_{\mathrm{GR}} = -a\,\beta\gamma\,\frac{g}{c}$.
  Meant as a *baseline prediction*. Any approximative treatment must reproduce it!
  (E.g. weak field approximation, which should be enough, can be cross-checked with it.)

effective treatment of gravitational modifications:

- *PRD*71(2005)064016, *PRD*76(2007)061101, *NPB*911(2016)206, culminating in
  Y.N.Obukhov,A.J.Silenko,O.V.Terayaev:*Phys.Rev.***D94**(2016)044019.
  Effective treatment, for mixed magnetic - electric ring, with electric or magnetic focusing.
  Prediction for electric vertical focusing (for situation as above): $\Omega_{\mathrm{GR}} = \frac{1-a\,(2\gamma^2-1)}{\gamma}\,\beta\,\frac{g}{c}$.
  Differs from full GR prediction. (Agrees at magic momentum.)
  Possibly we want good spin dynamics for all $\beta\gamma$?


Predictions for GR corrections in frozen spin rings, E focusing, for protons (a=1.79)
Ω_{GR} [nrad/sec]
0
-50
-100
-150
-200
-250
-300
-350
full GR, mixed E,B ring CQG35(2018)175003
full GR, E-only ring PLA376(2012)2822
effective treatment, mixed E,B ring PRD94(2016)044019
ratio
1.5
1
0.5
0
magic momentum
effective treatment PRD94(2016)044019 / full GR CQG35(2018)175003
0
0.5
1
1.5
2
2.5
3
$\beta\gamma$


Predictions for GR corrections in frozen spin rings, E focusing, for deuterons (a=-0.142)
full GR, mixed E,B ring CQG35(2018)175003
effective treatment, mixed E,B ring PRD94(2016)044019
effective treatment PRD94(2016)044019 / full GR CQG35(2018)175003
Ω_GR [nrad/sec]
ratio
βγ
0
0.5
1
1.5
2
2.5
3
0
5
10
15
20
25
30
35
40
1
2
3
4
6
7
8

Further sanity checks: gravitational counter force.

In full GR treatment:

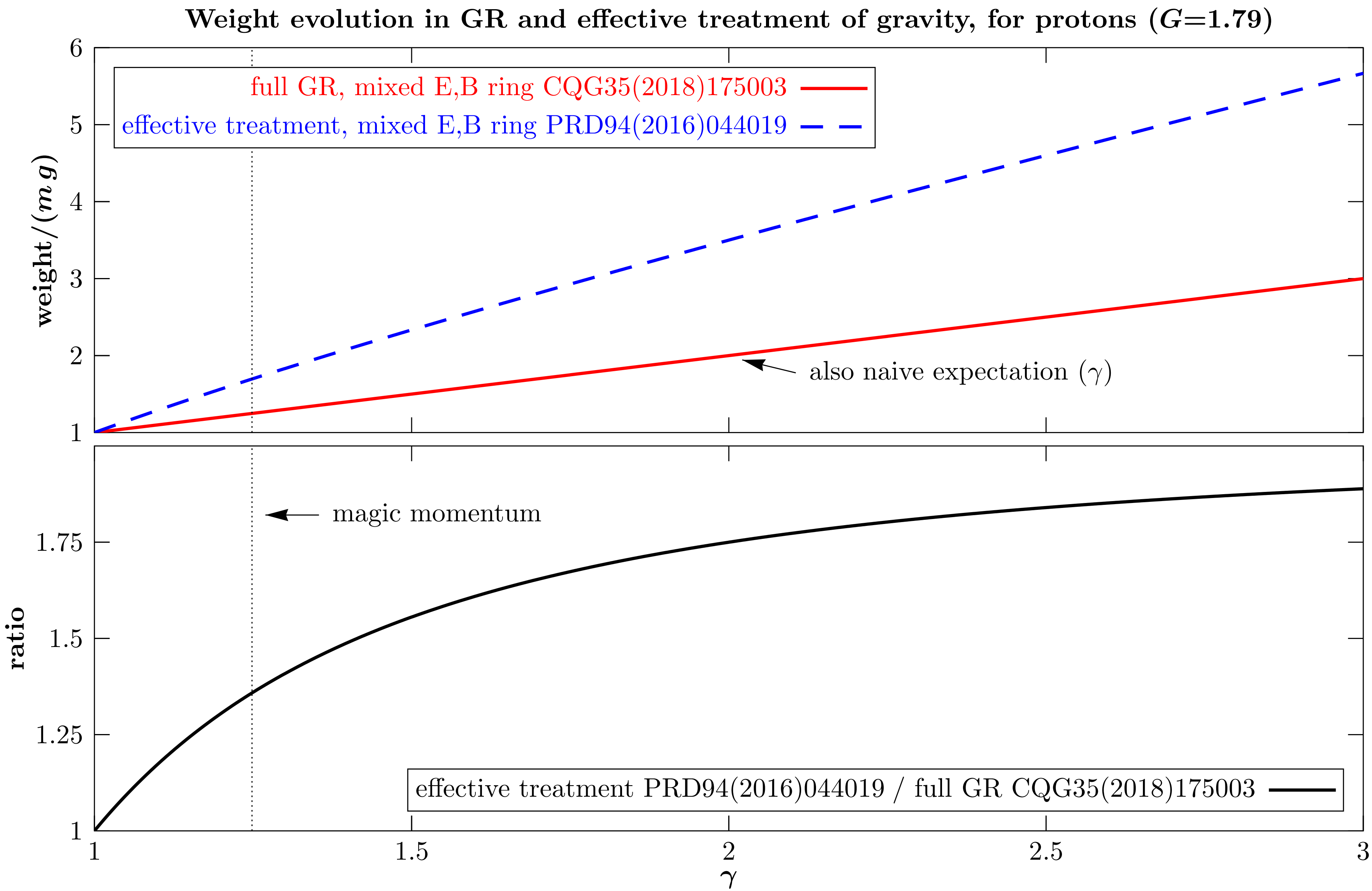
$$q\,E_R \quad = \quad m\gamma\,g$$

is the electrostatic force to keep the balance, CQG35(2018)175003.
(== naive expectation)

In effective treatment of gravitational modifications:

$$q\,E_R \quad = \quad m\frac{2\gamma^2 - 1}{\gamma}\,g$$

is the electrostatic force to keep the balance, PRD94(2016)044019.
(substantially heavier beam than expected, doesn't it violate equivalence principle?)


Weight evolution in GR and effective treatment of gravity, for protons (G=1.79)
full GR, mixed E,B ring CQG35(2018)175003
effective treatment, mixed E,B ring PRD94(2016)044019
also naive expectation (γ)
magic momentum
effective treatment PRD94(2016)044019 / full GR CQG35(2018)175003
weight/($m\,g$)
ratio
γ
6
5
4
3
2
1
1.75
1.5
1.25
1
1
1.5
2
2.5
3

Summary

- GR gives substantial contribution to "frozen spin" EDM experiments, of magnitude $-a\,\beta\gamma\,\frac{g}{c}$.

  A.László,Z.Zimborás:*Class.Quant.Grav.***35**(2018)175003

  (Y.Orlov,E.Flanagan,Y.Semertzidis:*Phys.Lett.***A376**(2012)2822 for purely electrostatic)
- Dedicated GR experiment via maximizing this contribution? The signal grows unboundedly with γ.
- Technical limiting factor is the necessary electric field. This can be decreased via using large $|a|$ particle.
- With modest energy triton, helion3, or proton beams, seems to be OK (?).
- For electric-only frozen spin ring: with triton, slightly less electric bending power is enough in comparison to proton, but GR effect would be $2\times$ bigger.

Backup