Conversion of Bound Muons: Lepton Flavour and Number Violation

Tanja Geib

+ Alexander Merle: *Phys. Rev. D93 (2016) 055039* → technical details on $\mu^- - e^-$
+ Stephen King, Alexander Merle, Jose Miguel No, Luca Panizzi: *Phys. Rev. D93 (2016) 073007* → complementarity of $\mu^- - e^-$ with LHC
+ Alexander Merle, Kai Zuber: *Phys. Lett. B764 (2017) 157* → 'appetiser' $\mu^- - e^+$
+ Alexander Merle: *arXiv:1612.00452* → technical details on $\mu^- - e^+$

Max Planck Institute for Physics

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

PSI Seminar, December 16, 2016

Today's Agenda:

- What happens in a μ– e conversion?
- What are similarities and differences when considering μ^-– e^- and μ^-– e^+ conversion?
- How to tackle μ^-– e^- conversion (using the example of a realisation via doubly charged scalars)?
- Employing the complementarity between collider and low energy physics to increase the testability $\rightarrow$ Results based on the example case
- How to tackle μ^-– e^+ conversion (using the example of a realisation via doubly charged scalars)?
- Discovery potential for μ^-– e^+ conversion
- Open issues $\rightarrow$ where do we need to improve in order to get reliable predictions?
- Summary and Outlook

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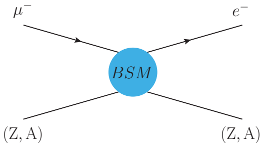
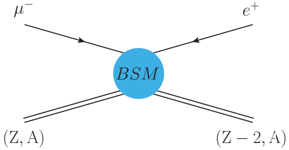
What happens in a **$\mu^- - e^\pm$ conversion** ?? $\rightarrow$ experimentally a two-step process





First Step: μ^- is captured in an 'outer' atomic shell, and subsequently de-excites to the 1s ground state

Second Step: μ^- is captured by the nucleus and reemits an $e^\pm$

$\rightarrow$ we only consider **"coherent" conversion**: initial and final state nucleus are in ground state

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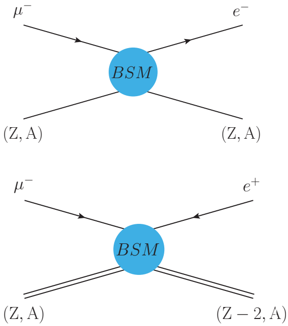

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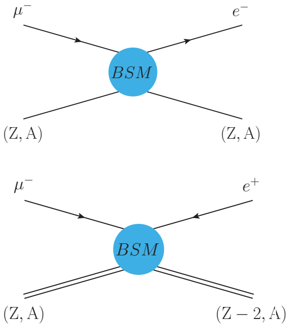

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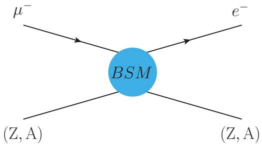

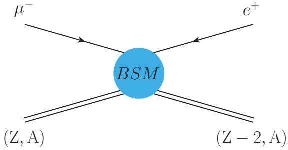

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Energy Scales of the Process

- muon **bound** in **1s state** with binding energy
  $\epsilon_B \simeq \frac{m_\mu}{m_e} \cdot 13.6 \text{ eV} \cdot Z \ll m_\mu \xrightarrow{Z \leq 100}$ **non-relativistic**
- consider **"coherent"** process → initial and final nucleus in **ground state**
  + in good approximation: both nuclei at rest

  $$\Rightarrow E_e = \underbrace{m_\mu - \epsilon_B}_{E_\mu \sim \mathcal{O}(100 \text{ MeV})} + \underbrace{E_i - E_f}_{\sim \mathcal{O}(\text{MeV})} \sim \mathcal{O}(100 \text{ MeV})$$

  $\Rightarrow e^\pm$ is **relativistic** particle under influence of Coulomb potential: $E_e \simeq E_\mu \simeq m_\mu$ and $m_e \simeq 0$
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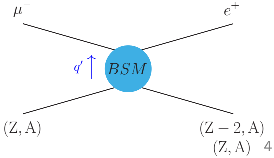
  In this set-up $\Rightarrow$ $\boxed{q'^2 \simeq -m_\mu^2}$



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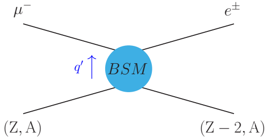

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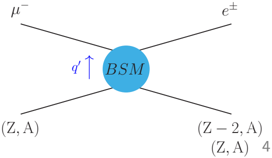

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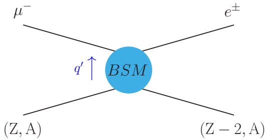

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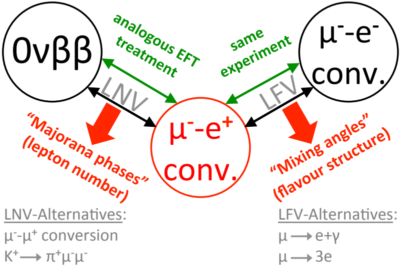

from
TG, Merle, Zuber Phys.Lett. B764 (2017) 157

$\mu^- - e^-$

- occurs at single nucleon ($\Delta Q = 0$)
- dominated by coherent process

$\mu^- - e^+$

- needs to occur at two nucleons to achieve $\Delta Q = 2 \rightarrow$ similar to $0\nu\beta\beta$
- around 40% of the process' total are g.s. $\rightarrow$ g.s.

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further investigations needed:
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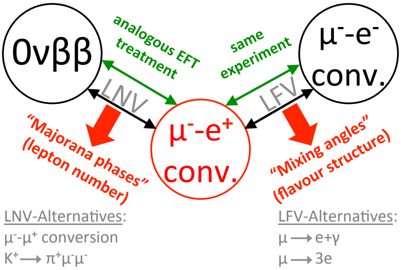

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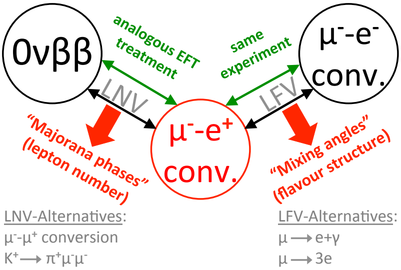

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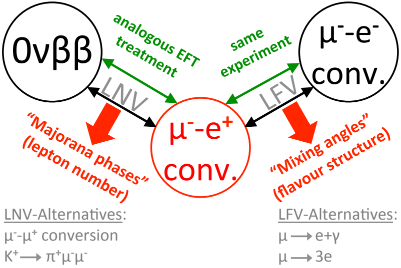

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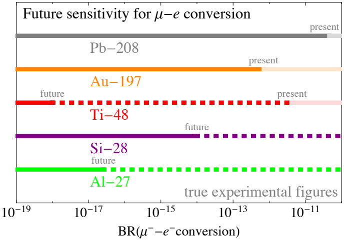
Snapshot on **current limits** and **sensitivities of upcoming experiments**:



past: SINDRUM II for ^{48}Ti (1993), ^{208}Pb (1995), ^{197}Au (2006)

future: DeeMee for ^{28}Si, COMET and Mu2e (taking data $\sim$ 2018) for ^{27}Al, PRISM/PRIME for ^{48}Ti

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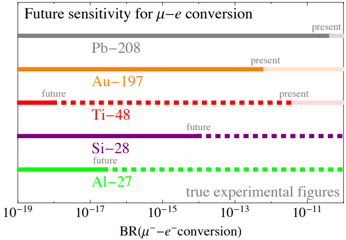

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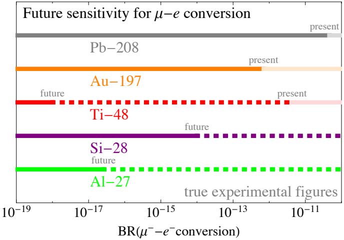

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How to tackle μ^-– e^- **conversion** (using the example of a realisation via *doubly charged scalars*)?

Effective theory of a doubly charged scalar singlet

based on King, Merle, Panizzi JHEP 1411 (2014) 124

Minimal extension of SM:

- only **one** extra particle: S^{++}
  $\rightarrow$ lightest of possible new particles (UV completion e.g. Cocktail model)
  $\rightarrow$ reduction of input parameters
- tree-level coupling to SM (to charged right-handed leptons)
  $\rightarrow$ **LNV and LFV!**
- effective Dim-7 operator (necessary to generate neutrino mass)

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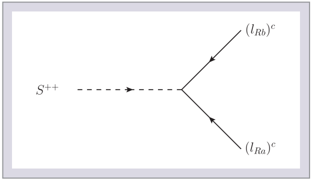

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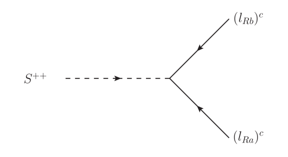
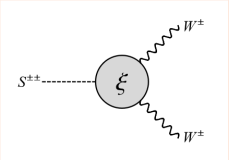
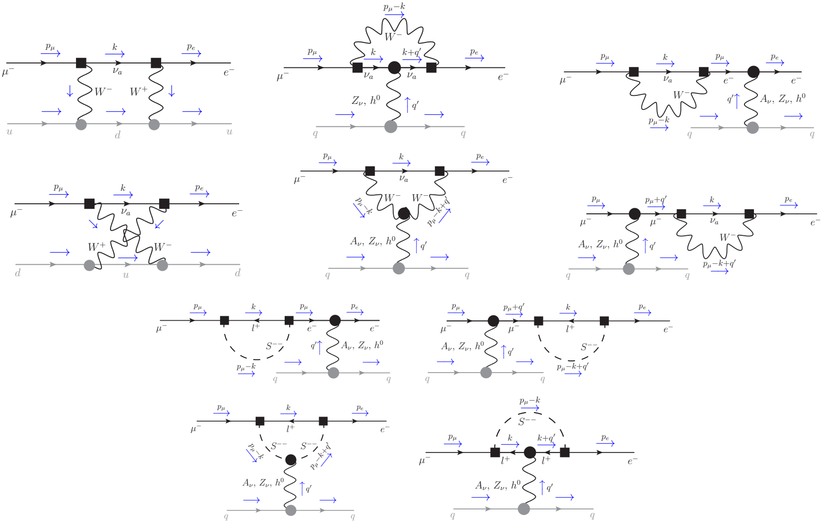




$\mu^-\!-e^-$ Conversion: Universally Valid for Models Involving Doubly Charged Singlet Scalars based on TG, Merle Phys.Rev. D93 (2016) 055039

$\mu^-\!-e^-$ conversion realised at **one-loop** level



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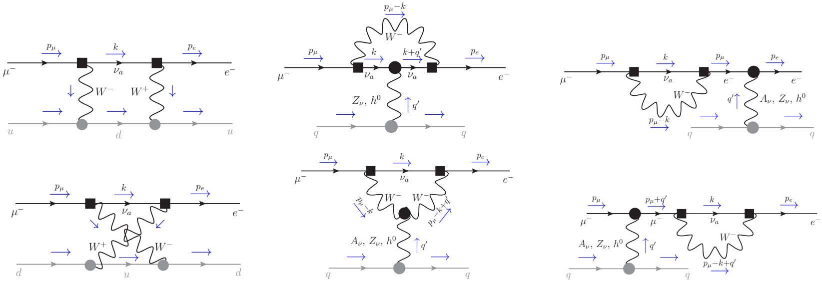
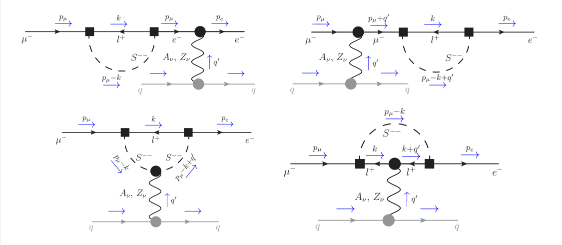
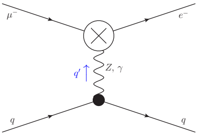




relevant diagrams

Different Contributions to $\mu^- - e^-$ Conversion

- estimate nuclear radius: $R = \overbrace{r_0}^{\sim\mathcal{O}(10^{-15}\ \mathrm{m})} A^{1/3} \sim \mathcal{O}(10^{-15}\ \mathrm{m})$
- reduced Bohr radius: $\underbrace{a_0}_{\mathcal{O}(10^{-10}\ \mathrm{m})} \frac{m_e}{m_\mu} \sim \mathcal{O}(10^{-13}\ \mathrm{m})$
- estimate interaction range: $r_\gamma \to \infty$ and $r_Z \leq 10^{-18}\ \mathrm{m}$
  $\Rightarrow$ for Z-exchange: μ^- has to be **within nucleus**! Probability?!



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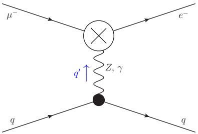

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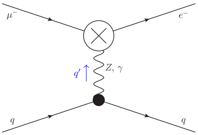

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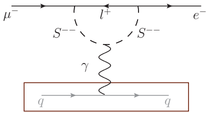

photonic contribution:
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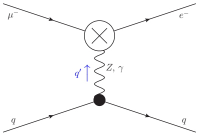

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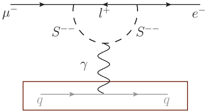

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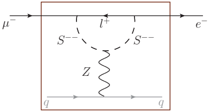

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$\Rightarrow$ contributions need to be treated **qualitatively differently**!!

Different Contributions to $\mu^- - e^-$ Conversion

- estimate nuclear radius: $R = \overbrace{r_0}^{\sim\mathcal{O}(10^{-15}\ \mathrm{m})} A^{1/3} \sim \mathcal{O}(10^{-15}\ \mathrm{m})$
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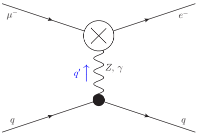

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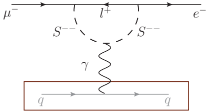

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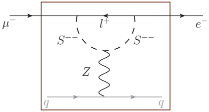

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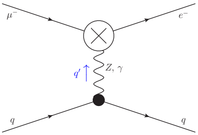

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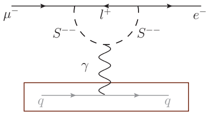

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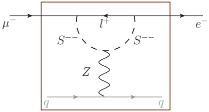

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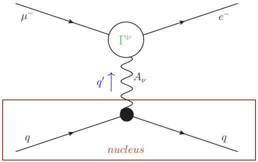

$$\mathcal{M} \propto \int \mathrm{d}^3 r \, \overline{\psi^e_{jlm}}(p_e, r) \, \Gamma^\nu \, \psi^\mu_{j_\mu l_\mu m_\mu}(p_\mu, r) \underbrace{\langle N|\overline{q}\,\gamma_\nu\, q|N\rangle}_{Ze\rho^{(P)}(r)\,\delta_{\nu 0}}$$

→ **wave functions** for μ^- and e^- obtained by solving modified Dirac equation (+ Coulomb potential)

→ Most **general** (Lorentz-) invariant **expression** for Γ^ν:

$$\Gamma^\nu = \left(\gamma^\nu - \frac{\not{q}' q'^\nu}{q'^2}\right) F_1(q'^2) + \frac{i\,\sigma^{\nu\rho}\, q'_\rho}{m_\mu} F_2(q'^2) + \left(\gamma^\nu - \frac{\not{q}' q'^\nu}{q'^2}\right)\gamma_5\, G_1(q'^2) + \frac{i\,\sigma^{\nu\rho}\, q'_\rho}{m_\mu}\gamma_5\, G_2(q'^2)$$

with $q' = p_e - p_\mu$.

In non-relativistic limit:

⇒ ψ_{jlm} and $Ze\rho^{(P)}(r)$ factorise from Γ^0 on matrix element level

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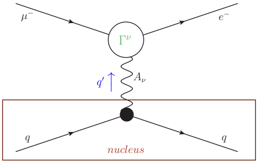

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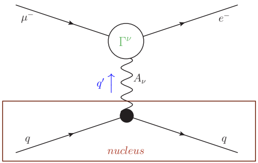

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Write **branching ratio** as product of nuclear and particle physics parts

$$\mathrm{BR}(\mu^- N \to e^- N) = \frac{8\alpha^5 m_\mu Z_{\mathrm{eff}}^4 Z F_p^2}{\Gamma_{\mathrm{capt}}} \, \Xi^2$$

see Kuno, Okada Rev. Mod. Phys. 73 (2001) 151-202

→ **factorisation** works perfectly for **photonic** contributions
→ Ξ has to be modified for **non-photonic** contributions to be a function of the nuclear characteristics (A,Z)

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$$\Xi^2 = \left| -F_1(-m_\mu^2) + F_2(-m_\mu^2) \right|^2 + \left| G_1(-m_\mu^2) + G_2(-m_\mu^2) \right|^2$$

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Photonic Contribution: Results

In good approximation (up to a **few per cent**), we use

$$F_1(q'^2) = G_1(q'^2) = -f^*_{ea}\, f_{a\mu} \left[\frac{2m_a^2 + m_\mu^2 \log\left(\frac{m_a}{M_S}\right)}{12\pi^2 M_S^2} + \frac{\sqrt{m_\mu^2+4m_a^2}(m_\mu^2 - 2m_a^2)}{12\pi^2 m_\mu M_S^2} \text{Arctanh}\left(\frac{m_\mu}{\sqrt{m_\mu^2+4m_a^2}}\right)\right]$$

$$F_2(q'^2) = -G_2(q'^2) = f^*_{ea}\, f_{a\mu} \frac{m_\mu^2}{24\pi^2 M_S^2}$$

with $q'^2 = -m_\mu^2$ for the particle physics factor:

$$\Xi^2_{\text{photonic}} = \frac{1}{288\,\pi^4\, m_\mu^2\, M_S^4} \left| \sum_{a=e,\,\mu,\,\tau} f^*_{ea}\, f_{a\mu} \left(4m_a^2\, m_\mu - m_\mu^3 + 2\left(-2m_a^2 + m_\mu^2\right)\sqrt{4m_a^2 + m_\mu^2}\ \text{Arctanh}\left[\frac{m_\mu}{\sqrt{4m_a^2+m_\mu^2}}\right] + m_\mu^3 \ln\left[\frac{m_a^2}{M_S^2}\right]\right)\right|^2$$

→ while F_2 is independent of m_a, $|F_1|$ decreases with increasing m_a
→ hierarchy: $|F_2|<|F_1|$ **but** for $M_S \sim 10$ GeV of order 10 %
→ compare to $\mu \to e\gamma$: $F_1(q'^2=0) = G_1(q'^2=0) = 0$ and $F_2(q'^2=0) = -G_2(q'^2=0) = F_2(q'^2=-m_\mu^2) \Rightarrow \mu^- - e^-$ conversion enhanced by F_1 contribution

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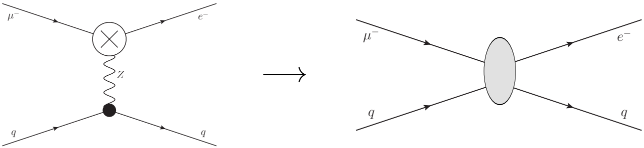
Short-range $\leftrightarrow$ takes place inside the nucleus:
EFT treatment $\Rightarrow$ **Integrating out** the Z-boson:



$\rightarrow$ four-point vertices
$\rightarrow$ consider operators up to **dimension six**
$\rightarrow$ for the coherent $\mu^- - e^-$ conversion, the only vertex realised in this model is described by

$$\mathcal{L}_{\text{short-range}} = -\frac{G_F}{\sqrt{2}} \underbrace{\frac{2(1+k_q \sin^2\theta_W)\cos\theta_W}{g} A_R(q'^2)}_{g_{RV(q)}} \overline{e_R}\,\gamma_\nu\,\mu_R\,\overline{q}\,\gamma^\nu\,q$$

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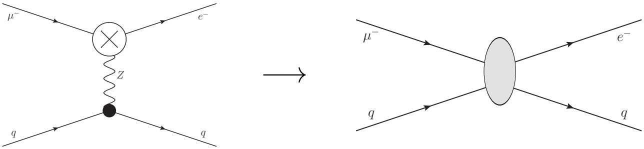

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→ instead of lines we do have bands with finite widths for Ξ

⇒ determine form factors from amputated diagrams with off-shell Z-Boson

Combining photonic and non-photonic contributions:

$$\Xi_{\text{particle}} \to \Xi_{\text{combined}}(Z, N) = \Xi_{\text{photonic}} + \Xi_{\text{non-photonic}}(Z, N)$$

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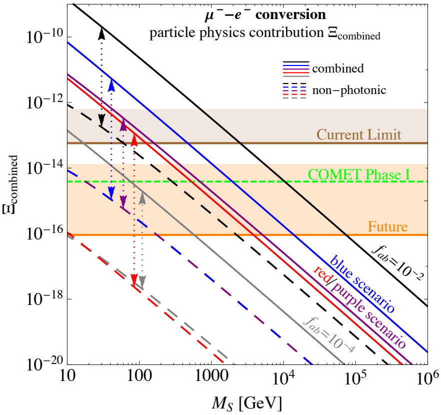
see TG, Merle Phys.Rev. D93 (2016) 055039



Benchmark Points:

f_{ab} such that LFV/LNV bounds fulfilled + suitable neutrino mass matrix reproduced

- 'red': $f_{ee} \simeq 0$ and $f_{e\tau} \simeq 0$
- 'purple': $f_{ee} \simeq 0$ and $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$
- 'blue': $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$

$\Downarrow$

choose **representative 'average' set** for each scenario to display M_S dependence

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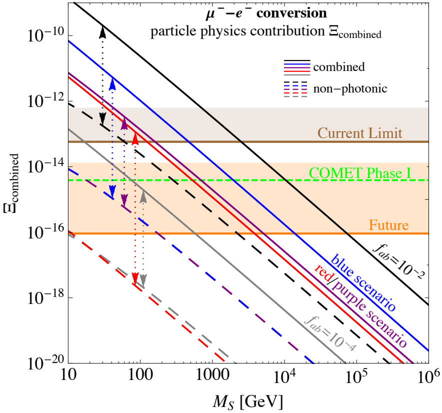


→ widths of the bands so small that appear as lines

→ non-photonic (DASHED) contributions **negligibly small**

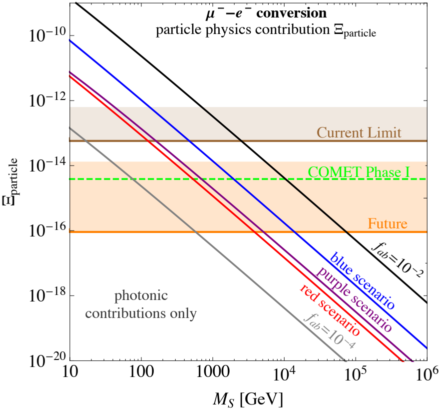
⇓

→ approximate process by its purely photonic (SOLID) contribution

→ **factorisation**: dependence on isotope only in width of limit

Results: Photonic Contribution vs $\mu \to e\gamma$

see TG, Merle Phys.Rev. D93 (2016) 055039 and King, Merle, Panizzi JHEP 1411 (2014) 124



For $\mu^+ \to e^+ \gamma$:
strongest bound for red, weakest for blue points

$$\mathcal{A} \propto \left| f_{ee} f^*_{e\mu} + f_{e\mu} f^*_{\mu\mu} + f_{e\tau} f^*_{\tau\mu} \right| \cdot C$$

$\to$ some amount of cancellation

For μ^-–e^- **conversion**:
!! other way around !!

$$\mathcal{A} \propto \left| C_e f^*_{ee} f_{e\mu} + C_\mu f^*_{e\mu} f_{\mu\mu} + C_\tau f^*_{e\tau} f_{\tau\mu} \right|$$

$\to$ flavour-dependent coefficients: prevent similar cancellations

$\to$ **shape of amplitude leads to drastical change** (not mainly off-shell contributions)

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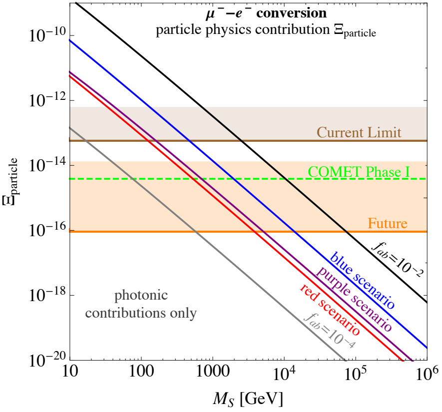

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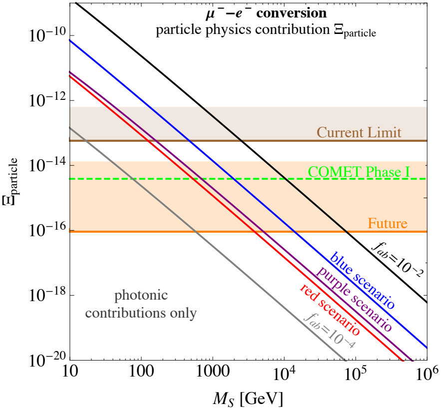
$\to$ **shape of amplitude leads to drastical change** (not mainly off-shell contributions)

Results: Photonic Contribution vs $\mu \to e\gamma$

see TG, Merle Phys.Rev. D93 (2016) 055039 and King, Merle, Panizzi JHEP 1411 (2014) 124



For $\mu^+ \to e^+ \gamma$:
strongest bound for red, weakest for blue points

$$\mathcal{A} \propto \left| f_{ee}\, f^*_{e\mu} + f_{e\mu}\, f^*_{\mu\mu} + f_{e\tau}\, f^*_{\tau\mu} \right| \cdot C$$

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!! other way around !!

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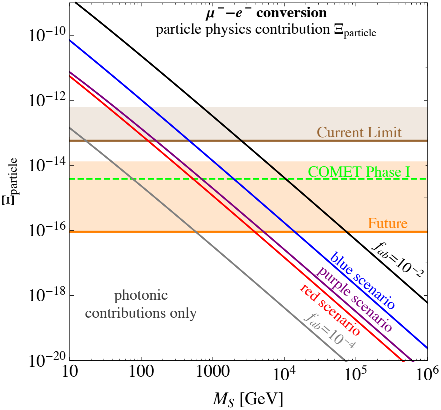

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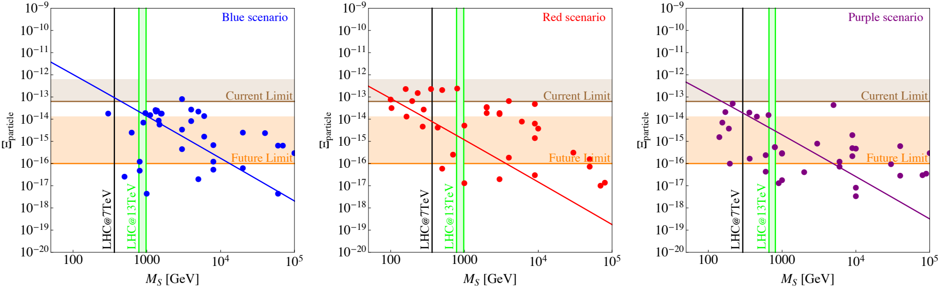
see TG, King, Merle, No, Panizzi Phys.Rev. D93 (2016) 073007



From **'average scenarios'** (depicted by lines), we can estimate the **lower limits on M_S** resulting from μ-e conversion:

	current limit [GeV]	future sensitivity [GeV]	COMET I (Al-27) [GeV]
blue curve	$M_S > 131.9 - 447.1$	$M_S > 1031.5 - 13271.3$	$M_S > 1954.1$
purple curve	$M_S > 42.5 - 152.3$	$M_S > 360.7 - 4885.2$	$M_S > 694.5$
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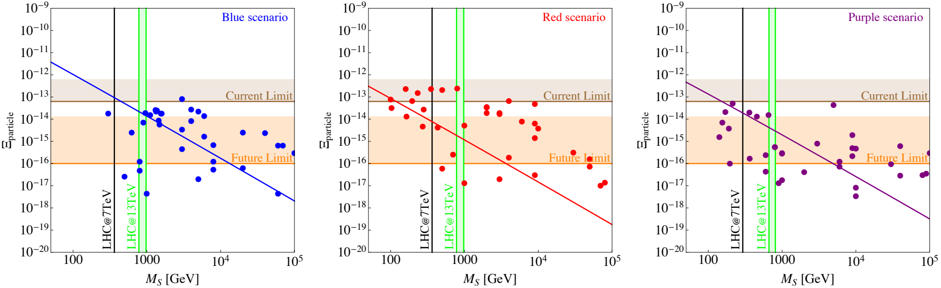

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How to tackle $\mu^{-}- e^{+}$ **conversion** (using the example of a realisation via *doubly charged scalars*)?

Motivation

In the following, we perform the computation for the decay rate for one particular short-range operator ϵ_3^{LLz}. But **why?!**

- There are a few **earlier references** available focussing on $\mu^- - e^+$ conversion from Majorana neutrinos but no uniform formalism is used:
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    → has the nuclear matrix elements (for ^{48}Ti) that we use: ϵ_3^{LLz}

    → explicit computation focussing on the nuclear physics

    ⇒ includes the formalism that we want **make accessible to the particle physics community**
- many aspects do not change if another operator was realised

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μ^-– e^+ Conversion from doubly charged scalars

- formalism to describe μ^-– e^+ conversions within **general framework**
- use **EFT** to neatly separate the nuclear physics from the respective particle physics realisation of the conversion → **factorisation**

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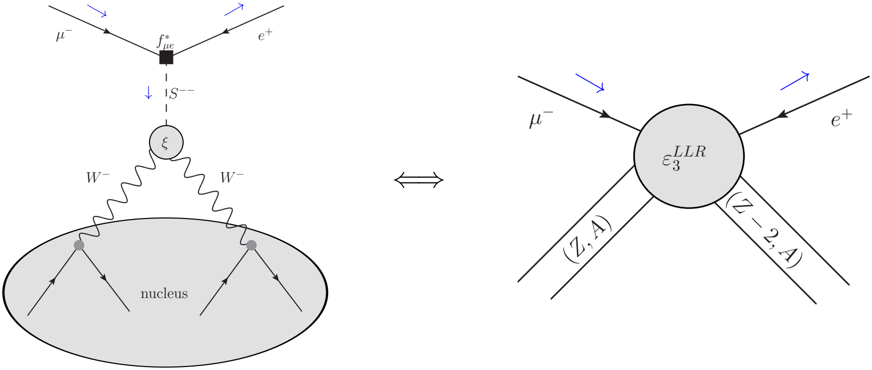

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General Formalism for $\mu^- - e^+$ Conversion from Short-Range Operators

based on Päs *et al.* Phys.Lett. B498 (2001) 35, and TG, Merle, Zuber Phys.Lett. B764 (2017) 157

Employ **EFT formalism** to generally describe $\mu^- - e^+$ conversion ⇒ dim-9 **short-range operators**:

$$\mathcal{L}^{\mu e}_{\text{short-range}} = \frac{G_F^2}{2m_p} \sum_{x,y,z=L,R} \Big[\epsilon_1^{xyz} J_x J_y j_z + \epsilon_2^{xyz} J_x^{\nu\rho} J_{y,\nu\rho} j_z + \epsilon_3^{xyz} J_x^{\nu} J_{y,\nu} j_z + \epsilon_4^{xyz} J_x^{\nu} J_{y,\nu\rho} j_z^{\rho} + \epsilon_5^{xyz} J_x^{\nu} J_y j_{z,\nu} + \epsilon_6^{xyz} J_x^{\nu} J_y^{\rho} j_{z,\nu\rho} + \epsilon_7^{xyz} J_x J_y^{\nu\rho} j_{z,\nu\rho} + \epsilon_8^{xyz} J_{x,\nu\alpha} J_y^{\rho\alpha} j_{z,\rho}^{\nu} \Big]$$

using the hadronic currents:

$$J_{R,L} = \overline{d}(1 \pm \gamma_5)u, \quad J^{\nu}_{R,L} = \overline{d}\,\gamma^{\nu}(1 \pm \gamma_5)u, \quad J^{\nu\rho}_{R,L} = \overline{d}\,\sigma^{\nu\rho}(1 \pm \gamma_5)u\,,$$

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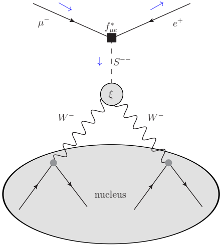
Deriving the Decay Rate for ϵ_3 based on TG, Merle arXiv:1612.00452

Start with the **amplitude** obtained from EFT diagram



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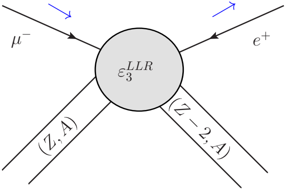

which is

$$\langle N', f | S^{(1)}_{\text{eff}} | N, i \rangle = -i \langle N', f | \int \mathrm{d}^4 x \, \widehat{T} \{ \mathcal{L}_{\text{eff}}(x) \} | N, i \rangle$$
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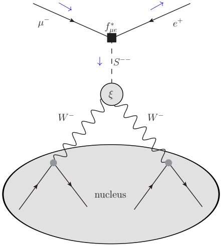

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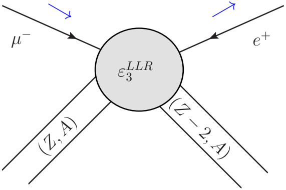

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Structure can be split into hadronic and leptonic parts:

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Leptonic part:

- muon is bound in 1s state
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$\Rightarrow$ need to modify the free spinors u and v respectively

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- two nucleon interactions $\rightarrow$ take place with finite distance $\rightarrow$ introduce *second location* $\tilde{x}$ over which we also "sum" $\int \mathrm{d}^3\tilde{x}$

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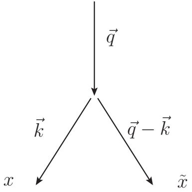

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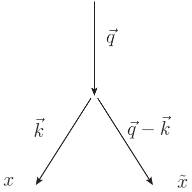

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Next:

- perform x^0 integration
  $\rightarrow$ **conservation of external energies** $2\pi\delta(E_i + E_\mu - E_f - E_e)$
- write non-relativistic currents in term of **effective transition operators**:

$$\tilde{F}(\vec{k}^{\,2}, \Lambda_i)\, J_{L\nu}(\vec{x}\,) = \sum_m \tau_m^- \left(g_V\, \tilde{F}(\vec{k}^{\,2}, \Lambda_V)\, g_{\nu 0} + g_A\, \tilde{F}(\vec{k}^{\,2}, \Lambda_A)\, g_{\nu j}\, \sigma_m^j \right) \delta^{(3)}(\vec{x} - \vec{r}_m\,)$$

with nuclear isospin raising operator τ_m^- and the dominant spin structures given by the Fermi operator and the Gamow-Teller operator

$\Rightarrow$ allows for **factorisation** of nuclear physics from respective particle physics model:

$$\mathcal{M} = \frac{G_F^2 \epsilon_3^{LLR} g_A^2 m_e}{2R} \sqrt{F(Z-2, E_e)}\, \delta(E_f - E_i + E_e - E_\mu)\, \overline{v_e}(k_e)\, \mathrm{P_R}\, u_\mu(k_\mu)\, \mathcal{M}^{(\mu^-, e^+)\,\phi}$$

with $\mathcal{M}^{(\mu^-, e^+)\,\phi}$ being the *nuclear matrix element*.

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$$\tilde{F}(\vec{k}^{\,2}, \Lambda_i)\, J_{L\nu}(\vec{x}\,) = \sum_m \tau_m^- \left(g_V\, \tilde{F}(\vec{k}^{\,2}, \Lambda_V)\, g_{\nu 0} + g_A\, \tilde{F}(\vec{k}^{\,2}, \Lambda_A)\, g_{\nu j}\, \sigma_m^j \right) \delta^{(3)}(\vec{x} - \vec{r}_m\,)$$

with nuclear isospin raising operator τ_m^- and the dominant spin structures given by the Fermi operator and the Gamow-Teller operator

$\Rightarrow$ allows for **factorisation** of nuclear physics from respective particle physics model:

$$\mathcal{M} = \frac{G_F^2 \epsilon_3^{LLR} g_A^2 m_e}{2R} \sqrt{F(Z-2, E_e)}\, \delta(E_f - E_i + E_e - E_\mu)\, \overline{v_e}(k_e)\, \mathrm{P_R}\, u_\mu(k_\mu)\, \mathcal{M}^{(\mu^-, e^+)\,\phi}$$

with $\mathcal{M}^{(\mu^-, e^+)\,\phi}$ being the *nuclear matrix element*.

Deriving the Decay Rate for ϵ_3 based on TG, Merle arXiv:1612.00452

Next:

- perform x^0 integration
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From amplitude to decay rate using **Fermi's Golden rule**:

$$\Gamma = 2\pi \frac{1/T}{(2\pi)^3} \int \mathrm{d}^3 k_e \left|\mathcal{M}\right|^2$$

So, we need to

- spin sum/average $\to 1/4$
- rewrite *nuclear matrix element* using that the muon wave function varies only slowly within nucleus: $\left|\mathcal{M}^{(\mu^-,e^+)\,\phi}\right|^2 = \langle\phi_\mu\rangle^2 \left|\mathcal{M}^{(\mu^-,e^+)}\right|^2$
- square delta-function: "$\delta(E_f - E_i + E_e - E_\mu)^2$" $= \frac{T}{2\pi}\,\delta(E_f - E_i + E_e - E_\mu)$

and obtain the **decay rate**:

$$\Gamma = \frac{g_A^4\, G_F^4\, m_e^2\, m_\mu^2\, |\epsilon_3^{LLR}|^2}{32\pi^2 R^2} \left|F(Z-2, E_e)\right| \langle\phi_\mu\rangle^2 \left|\mathcal{M}^{(\mu^-,e^+)}\right|^2$$

$\to$ can be generalised to ϵ_3^{xyz} for $x = y$
$\to$ for $x \neq y$ there is a relative sign switched in the nuclear matrix element

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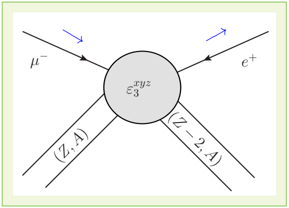
Further Realisations of ϵ_3

Cheng-Geng-Ng model
Cheng, Geng, Ng Phys.Rev. D75 (2007) 053004

EFT with doubly charged scalar King, Merle, Panizzi JHEP 1411 (2014) 124



Heavy Majorana neutrinos Domin, Kovalenko, Faessler, Simkovic Phys.Rev. C70 (2004) 065501

Left-Right symmetric models Pritimita, Dash, Patra JHEP 1610 (2016) 147

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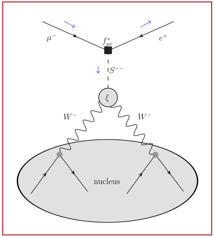

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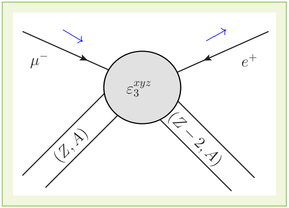

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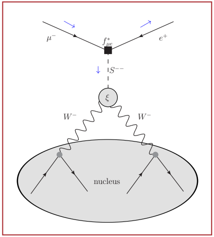

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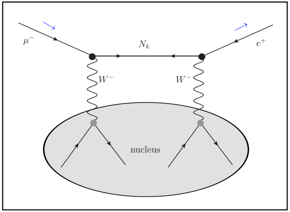

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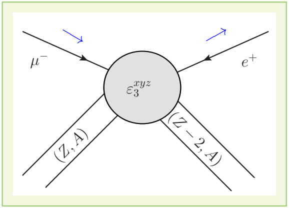

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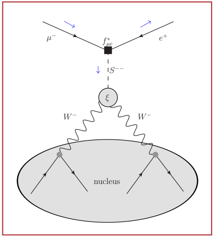

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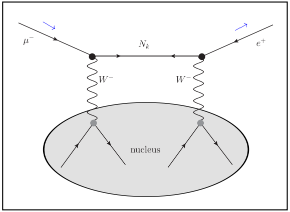

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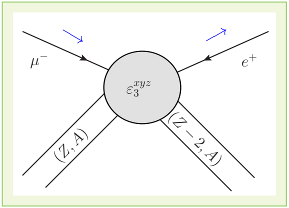

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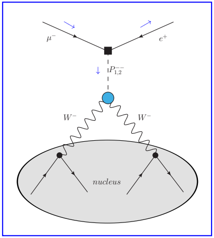

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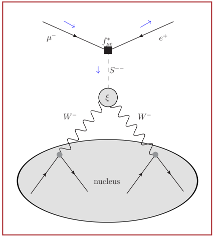

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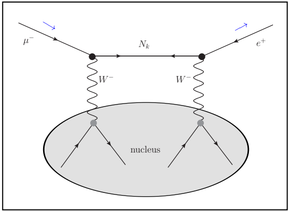

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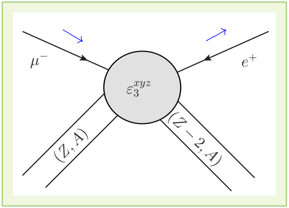

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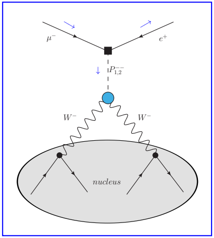

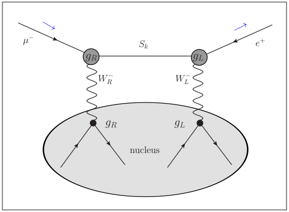

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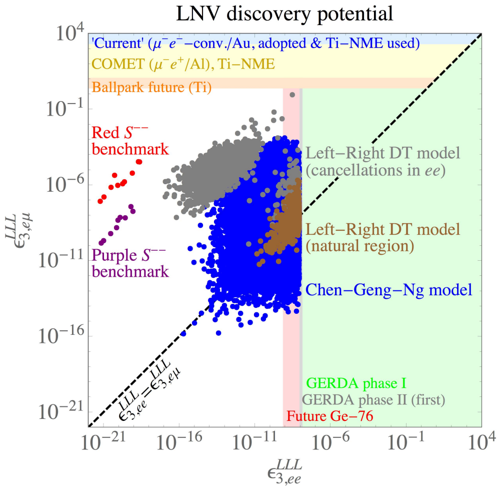
Reach of Future Experiments for ϵ_3

based on TG, Merle, Zuber Phys.Lett. B764 (2017) 157



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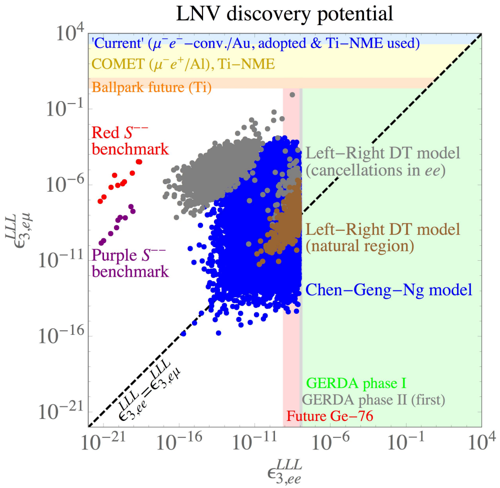

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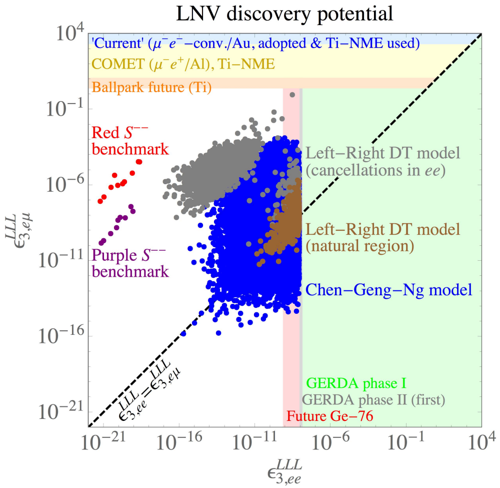

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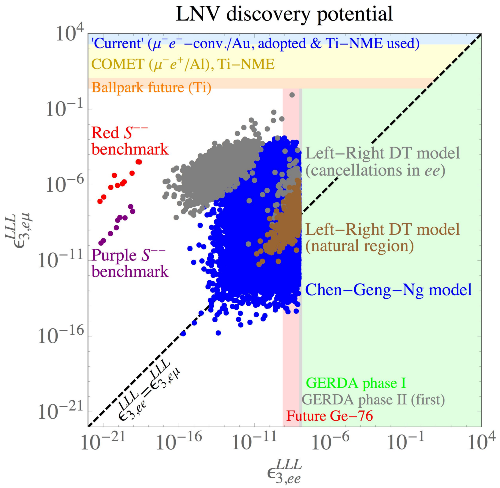

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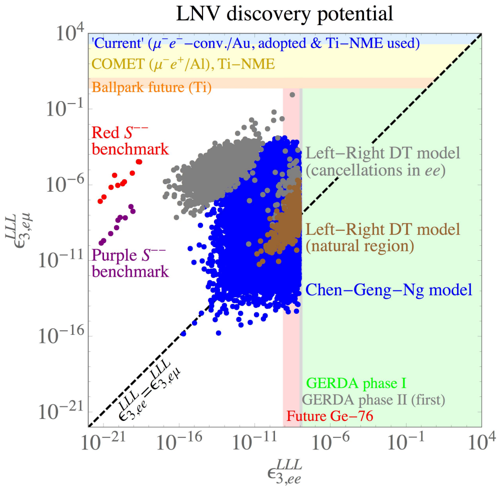

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Open Issues of $\mu^- - e^+$ based on TG, Merle, Zuber Phys.Lett. B764 (2017) 157

However: several **key pieces of information are missing**!!
We are in dire need of **improvements** from different areas:

- **Experiment:** more detailed sensitivity studies for $\mu^- - e^+$ conversion
- **Nuclear Matrix Elements:**
  - detailed study on percentage of process that is "coherent"
  - hardly any **nuclear matrix elements** (NMEs) are available
    $\rightarrow$ need for NMEs for further element, e. g. ^{27}Al, and for other operators like $\epsilon_{1,2}$

  $\Rightarrow$ there are promising models but we cannot judge them properly
- **Particle Physics:** for many models there are no (detailed) studies on LNV in the $e\mu$ sector and no information on which effective operators are realised

$\Rightarrow$ Only if all three communities pull **together**, advances will be achieved!!

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- $\mu^- - e^-$ **conversion**:
  - **FIRST work** that treats $\mu^- - e^-$ conversion in such **detail**, i. e. beyond previous EFT treatment/approximations $\rightarrow$ analytic expression for Ξ_{particle}
  - **complementarity**: rich phenomenology of loop models $\rightarrow$ high- and low-energy processes $\rightarrow$ $\mu^- - e^-$ conversion important part of study
- $\mu^- - e^+$ **conversion**:
  - complete computation of the rate for the lepton flavour and number violating conversion process, mediated by the **effective operator** ϵ_3
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  - **complementarity**: rich phenomenology of loop models $\rightarrow$ high- and low-energy processes $\rightarrow$ $\mu^-- e^-$ conversion important part of study
- $\mu^-- e^+$ **conversion**:
  - complete computation of the rate for the lepton flavour and number violating conversion process, mediated by the **effective operator** ϵ_3
  - pointed out **open issues** and further models/operators
  - LNV possibly more prominent in $e\mu$ **sector** $\rightarrow$ experiments could make a countable physics impact
  - open issues need to be addressed in order to proceed
- **COMET**: expecting to take first data in **2018**

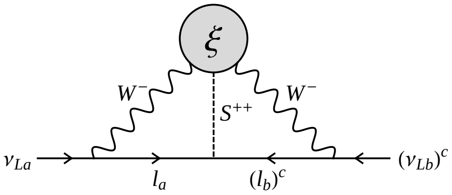
Thank you for your attention!!

Any questions?

Backup Slides

Generating the Neutrino Mass

The mass is generated at **two-loop level** via the diagram



which leads to the **neutrino mass**

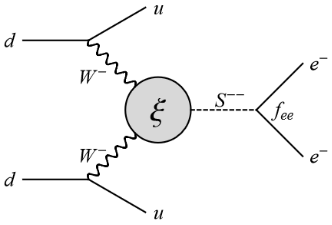
$$\mathcal{M}^{\text{2-loop}}_{\nu,ab} = \frac{2\,\xi\, m_a\, m_b\, M_S^2\, g_{ab}(1+\delta_{ab})}{\Lambda^3}\, \mathcal{I}[M_W,\, M_S,\, \mu]$$

$\longrightarrow$ Majorana mass term
$\longrightarrow$ further LNV processes

Testing the Model

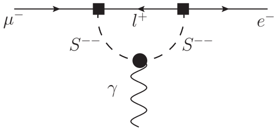
based on King, Merle, Panizzi arXiv:1406.4137

Selection of interesting processes: **low energy physics**



- neutrinoless double beta decay:

$$\frac{\xi\, f_{ee}}{M_S^2\, \Lambda^3} < \frac{4.0\cdot 10^{-3}}{\mathrm{TeV}^5}$$



- $\mu^- \to e^- \gamma$:

$$\left| f_{ee}^*\, f_{e\mu} + f_{e\mu}^*\, f_{\mu\mu} + f_{e\tau}^*\, f_{\mu\tau} \right| < 3.2 \cdot 10^{-4}\, M_S^2 [\mathrm{TeV}]$$

Testing the Model

based on King, Merle, Panizzi arXiv:1406.4137

benchmark points:

f_{ab} such that bounds fulfilled + suitable light neutrino mass matrix reproduced

- 'red': $f_{ee} \simeq 0$ and $f_{e\tau} \simeq 0$
- 'purple': $f_{ee} \simeq 0$ and $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$
- 'blue': $f_{e\mu} \simeq \frac{f^*_{\mu\tau}}{f^*_{\mu\mu}} f_{e\tau}$

$\downarrow$

complementary check with **high energy experiments**:
compute cross sections for e.g.

- $S^{\pm\pm} \to W^{\pm\pm}$
- $S^{\pm\pm} \to l_a^{\pm\pm} \, l_b^{\pm\pm}$
- ...

$\to$ some of the benchmark points already excluded by LHC data (7 TeV run)

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based on King, Merle, Panizzi arXiv:1406.4137

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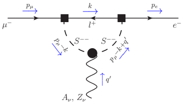
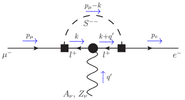
Photonic Contribution: Cross Check via UV Divergences

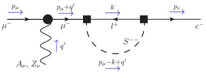
In form of $i\mathcal{M} = e\, f_{ea}^*\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)$:



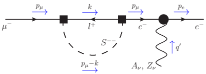
$$-4Q_S \int \frac{d^dk}{(2\pi)^d} \frac{P_L \not k (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{d^dk}{(2\pi)^d} \frac{P_L (\not k + \not q' + m_a)\, \gamma^\nu\, (\not k + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2\varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{d^dk}{(2\pi)^d} \frac{P_L\, \not k\, (\not p_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not p_e (\not p_e + m_\mu) \gamma^\nu$$



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$$\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2\varepsilon}[(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-}) P_L \gamma^\nu] = 0 \quad \surd$$

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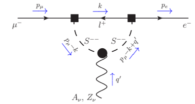

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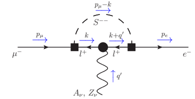

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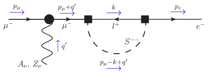
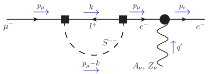


$$-4Q_{\mu^-} \int \frac{d^dk}{(2\pi)^d} \frac{P_L \not k\,(\not p_e+m_\mu)\,\gamma^\nu}{[p_e^2-m_\mu^2][(p_e-k)^2-M_S^2][k^2-m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2\varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not p_e(\not p_e+m_\mu)\gamma^\nu$$



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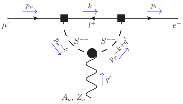

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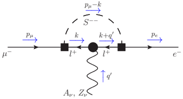

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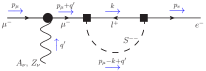

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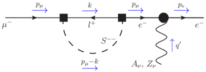

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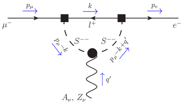

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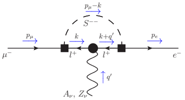

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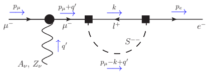

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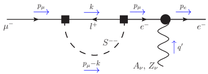

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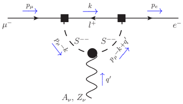

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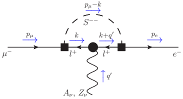

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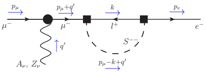

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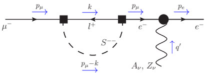

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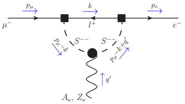

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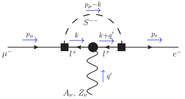

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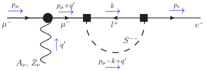
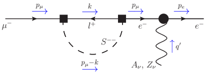


$$-4Q_{\mu^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L\, \not k\, (\not p_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not p_e (\not p_e + m_\mu) \gamma^\nu$$



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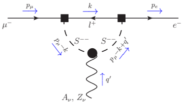
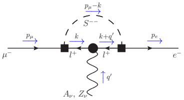
Photonic Contribution: Cross Check via UV Divergences

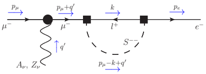
In form of $\boxed{i\mathcal{M} = e\, f^*_{ea}\, f_{a\mu}\, A_\nu(q')\, \overline{u}_e(p_e)\, \mathcal{I}^\nu\, u_\mu(p_\mu)}$:



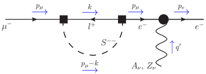
$$-4Q_S \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L \not{k} (2p_\mu - 2k + q')^\nu}{[k^2 - m_a^2][(p_\mu - k + q')^2 - M_S^2][(p_\mu - k)^2 - M_S^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} Q_S P_L \gamma^\nu$$



$$-4Q_{l^+} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L (\not{k} + \not{q}' + m_a)\, \gamma^\nu\, (\not{k} + m_a) P_R}{[k^2 - m_a^2][(p_\mu - k)^2 - M_S^2][(k + q')^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-i}{(4\pi)^2 \varepsilon} Q_{l^+} P_L \gamma^\rho \gamma^\nu \gamma_\rho P_R$$



$$-4Q_{\mu^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{P_L\, \not{k}\, (\not{p}_e + m_\mu)\, \gamma^\nu}{[p_e^2 - m_\mu^2][(p_e - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{2i}{(4\pi)^2 \varepsilon} \frac{Q_{\mu^-}}{m_\mu^2} P_L \not{p}_e (\not{p}_e + m_\mu) \gamma^\nu$$



$$4Q_{e^-} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\gamma^\nu\, \not{p}_\mu\, P_L\, \not{k}}{[p_\mu^2][(p_\mu - k)^2 - M_S^2][k^2 - m_a^2]} \xrightarrow{\text{div}} \frac{-2i}{(4\pi)^2 \varepsilon} \frac{Q_{e^-}}{m_\mu^2} \gamma^\nu \not{p}_\mu P_L \not{p}_\mu$$

$$\boxed{\Rightarrow \sum \mathcal{I}^\nu = \frac{i}{(4\pi)^2 \varepsilon} [(2Q_S + 2Q_{l^+} - Q_{e^-} - Q_{\mu^-}) P_L \gamma^\nu] = 0} \quad \checkmark$$

Photonic Contribution: Results I

Determine **form factors** with help of Mathematica package *Package*-X (Patel, arXiv:1503.01469):

$$\mathbf{F}_1(-m_\mu^2) = \mathbf{G}_1(-m_\mu^2) =$$

$$= -\frac{1}{128\,\pi^2 m_\mu^4} \sum_{a=e,\,\mu,\,\tau} f_{ea}^* f_{a\mu} \Bigg[2\, m_\mu^2 \Big(-5m_a^2 + 6m_\mu^2 + 5M_S^2 \Big) - 2\, S_a\, m_\mu^2 \Big(m_a^2 + 3m_\mu^2 - M_S^2 \Big) \ln\left[\frac{2m_a^2}{2m_a^2 + m_\mu^2(1+S_a)}\right] + 4\, S_S\, m_\mu^2 \Big(m_a^2 + m_\mu^2 - M_S^2 \Big) \ln\left[\frac{2M_S^2}{2M_S^2 + m_\mu^2(1+S_S)}\right] + \Big(3m_a^2 \Big(2m_a^2 - m_\mu^2 - 4M_S^2 \Big) + 5m_\mu^4 - 7m_\mu^2 M_S^2 + 6M_S^4 \Big) \ln\left[\frac{m_a^2}{M_S^2}\right] + 2\, T_a \Big(-6m_a^2 + m_\mu^2 + 6M_S^2 \Big) \ln\left[\frac{2m_a M_S}{m_a^2 - m_\mu^2 + M_S^2 - T_a}\right]$$

$$+ 2\, m_\mu^2 \Bigg[\Big(m_a^4 + 8m_a^2 m_\mu^2 + M_S^4 - 2M_S^2 \Big(m_a^2 + 2m_\mu^2 \Big) \Big)\, C_0\Big[0,\, -m_\mu^2,\, m_\mu^2;\, m_a,\, M_S,\, m_a\Big] + 2\Big(m_a^4 - 2M_S^2 \Big(m_a^2 - 2m_\mu^2 \Big) + M_S^4 \Big)\, C_0\Big[0,\, -m_\mu^2,\, m_\mu^2;\, M_S,\, m_a,\, M_S\Big] \Bigg] \Bigg]$$

$$\xrightarrow{M_S \gg m_a} -f_{ea}^* f_{a\mu} \left[\frac{2m_a^2 + m_\mu^2 \log\left(\frac{m_a}{M_S}\right)}{12\pi^2 M_S^2} + \frac{\sqrt{m_\mu^2 + 4m_a^2}\,(m_\mu^2 - 2m_a^2)}{12\pi^2 m_\mu M_S^2} \,\mathrm{Arctanh}\left(\frac{m_\mu}{\sqrt{m_\mu^2 + 4m_a^2}}\right) \right] + \mathcal{O}(M_S^{-4})$$

Note: $\mathcal{O}(M_S^{-4})$ gives corrections of up to a **few per cent**

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$$\ln\left[\frac{2m_a^2}{2m_a^2 + m_\mu^2(1+S_a)}\right] + 4\, S_S\, m_\mu^2 \Big(m_a^2 + m_\mu^2 - M_S^2\Big) \ln\left[\frac{2M_S^2}{2M_S^2 + m_\mu^2(1+S_S)}\right] + \Big(3m_a^2\Big(2m_a^2 - m_\mu^2$$
$$-4M_S^2\Big) + 5m_\mu^4 - 7m_\mu^2 M_S^2 + 6M_S^4\Big) \ln\left[\frac{m_a^2}{M_S^2}\right] + 2\, T_a\Big(-6m_a^2 + m_\mu^2 + 6M_S^2\Big) \ln\left[\frac{2m_a\, M_S}{m_a^2 - m_\mu^2 + M_S^2 - T_a}\right]$$
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$$+2\Big(m_a^4 - 2M_S^2\Big(m_a^2 - 2m_\mu^2\Big) + M_S^4\Big)\, C_0\Big[0,\, -m_\mu^2,\, m_\mu^2;\, M_S,\, m_a,\, M_S\Big]\bigg]\Bigg]$$

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$$\ln\left[\frac{2m_a\,M_S}{m_a^2-m_\mu^2+M_S^2-T_a}\right] + 2\,m_\mu^2 \Big[\Big(-3m_a^4 - 3M_S^4 + 2M_S^2 \big(3m_a^2 + 2m_\mu^2 \big) \Big)\, C_0\left[0,\, -m_\mu^2,\, m_\mu^2;\, m_a,\, M_S,\, m_a\right]$$

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$$+\Big(m_a^2\Big(-2m_a^2-7m_\mu^2+4M_S^2\Big)+m_\mu^4+5m_\mu^2\,M_S^2-2M_S^4\Big)\ln\left[\frac{m_a^2}{M_S^2}\right]+2\,T_a\Big(2m_a^2-3m_\mu^2-2M_S^2\Big)$$
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'Average Scenario' Couplings

	red	purple	blue
f_{ee}	10^{-16}	10^{-15}	10^{-1}
$f_{e\mu}$	10^{-2}	10^{-3}	10^{-4}
$f_{e\tau}$	10^{-19}	10^{-2}	10^{-2}
$f_{\mu\mu}$	10^{-4}	10^{-3}	10^{-3}
$f_{\mu\tau}$	10^{-5}	10^{-4}	10^{-4}
$f_{ee}\, f_{e\mu}$	10^{-18}	10^{-18}	$\mathbf{10^{-5}}$
$f_{e\mu}\, f_{\mu\mu}$	$\mathbf{10^{-6}}$	$\mathbf{10^{-6}}$	10^{-7}
$f_{e\tau}\, f_{\mu\tau}$	10^{-24}	$\mathbf{10^{-6}}$	10^{-6}

Table: First part: 'average scenario' couplings for the benchmark points as extracted from Tab. 7 in *King, Merle, Panizzi: arXiv:1406.4137*. Second part: combination of couplings that enter the μ–e conversion amplitude. The bold values indicate the dominant photonic contribution.

Non-Photonic Bands

- The amplitude that enters the non-photonic Ξ takes the form

$$\mathcal{A} \propto \left| f_{ee}^* \, f_{e\mu} \, D(m_e) + f_{e\mu}^* \, f_{\mu\mu} \, D(m_\mu) + f_{e\tau}^* \, f_{\tau\mu} \, D(m_\tau) \right| .$$

- The function $D(m_a)$ strongly varies with m_a.
  $\rightarrow$ **dominant term** stems from the tau propagating within the loop, i.e. $\boldsymbol{D(m_\tau)}$
  $\rightarrow$ exeeds the muon and electron contribution by three to four orders of magnitude
- blue/purple scenario: neither $f_{ee}^* f_{e\mu}$ nor $f_{e\mu}^* f_{\mu\mu}$ bypasses this difference
  $+$ **identic** $\boldsymbol{f_{e\tau}^* f_{\tau\mu}}$ in both scenarios
  $\rightarrow$ indistinguishable curves
- red/grey scenario:
  dominant contributions: $f_{e\mu}^* \, f_{\mu\mu} \, D(m_\mu) \sim f_{e\tau}^* \, f_{\tau\mu} \, D(m_\tau)$
  $\rightarrow$ same order of magnitude, i.e. **comparable values** of non-photonic contribution