First Calculation of Electroweak Corrections for a Process with 6 External Particles, $e^+e^- \to$ 4 fermions

A. Denner[1], S. Dittmaier[2], M. Roth[2], L.H. Wieders[1]

[1] Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

[2] Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), 80805 München, Germany

Precision tests of the Standard Model of Particle Physics require, besides accurate measurements, precise calculations. Recently we obtained a breakthrough in the field of precision calculations for many-particle reactions. Using newly developed methods we evaluated the complete first-order electroweak corrections to charged-current 4-fermion production in electron-positron annihilation. This is, in particular, important to measure the mass of the W-boson at a linear collider with a relative accuracy below 0.01 per cent. Our calculation was the first calculation of first-order corrections to a process with six external particles.

Much progress in our understanding of the fundamental laws of nature comes from experiments at high-energy colliders. In 2007, the Large Hadron Collider will start operation, and a future electron-positron collider, the International Linear Collider (ILC), is presently under study. These colliders will allow to further test the Standard Model of particle physics, which presently describes essentially all experiments well, and to search for physics beyond. While the primary focus of these tests will be the search for and the investigation of the Higgs boson, also other parts of the Standard Model will be tested with higher accuracy or at higher energies.

Precise tests of the Standard Model require, besides accurate experiments, precise theoretical predictions. As these are obtained in perturbation theory, higher orders have to be calculated in order to match the experimental precision. In turn, this permits testing the Standard Model as a Quantum Field Theory. Since future colliders allow studying processes involving many external particles, adequate predictions for such processes are required. The calculation of higher-order corrections to processes with four external particles is routine since many years. In the last decade techniques for processes with five external particles have been developed and used to calculate perturbative corrections to processes both at hadron and lepton colliders. Processes with six external particles set the current technical frontier in this highly competitive field

.

New Methods

In the last year we succeeded to establish the first complete calculation of first-order corrections to a process with six external particles, i.e. the production of four fermions in electron-positron annihilation [1,2]. This required the developments of various new techniques and concepts. Firstly, we have introduced the complex-mass scheme for the treatment of unstable particles in higher orders of perturbation theory. In this scheme the masses of unstable particles are consistently treated as complex quantities. As a consequence perturbative calculations are straightforward, and gauge invariance is exactly conserved in this universal scheme. Secondly, in order to get numerically stable results in all parts of the complicated many-particle phase space, we have devised new methods for the calculation of appearing integrals. To this end, we used appropriate expansions or numerical integration of suitable well-behaved master integrals in problematic regions of parameter space. These methods are independent of the considered process and applicable to arbitrary processes with six external particles and can be generalized to processes with more external particles. Finally, we have developed techniques to simplify the spinor structure of the matrix elements. The constructed algorithms allow reducing the more than thousand appearing spinorial objects to only a few. These algorithms are applicable to similar processes with external massless fermions.

Physical Motivation

We have chosen the process $e^+e^- \to$ 4 fermions, because it can be studied with very high precision at the ILC. It contains W-pair production as dominant contribution and will allow measuring the mass of the W boson with an accuracy of 6 MeV from a scan of the cross section in the threshold region. This cross section and the couplings of the W boson can be measured at the per-mille level. These measurements rely on adequate theoretical predictions. The tools used to analyse this process at the Large Electron-Positron (LEP) collider are based on approximations like the double-pole approximation (DPA), which has a theoretical uncertainty of 0.5% sufficiently far above threshold, or the improved-Born approximation (IBA) with a theoretical uncertainty of 2% in the threshold region. Evidently these approximations are not accurate enough for the envisaged experimental accuracies at the ILC.

Figure 1 shows the theoretical predictions for the W-pair-production cross section in the LEP2 region with the error band of the DPA approximation and the experimental data points with errors. At the ILC this error will be reduced by an order of magnitude above threshold and by two orders of magnitude in the threshold region.

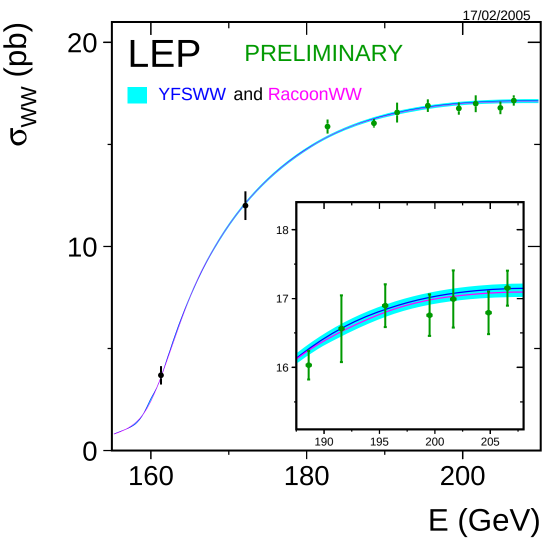

Figure 1: Total cross section for W-pair production at LEP2 [4]

Results

The relative corrections to the leading-order prediction for the cross section of the process $e^+e^- \to u\bar{d}\mu^-\nu_\mu$, which is a specific final state of W-pair production, are shown in Figure 2. The red curve corresponds to the complete corrections, the green one to the DPA, and the blue one to the IBA. The difference between the full calculation and the DPA is of the order of half a per cent in the LEP2 energy region. It gets larger with increasing energy and reaches 2% at 2 TeV. In the threshold region, where the DPA is unreliable, the full corrections differ from the IBA by almost 2%. Our calculation reduced the theoretical uncertainty to some per mille, which is a sizeable improvement and necessary for the ILC, in particular in the threshold region, where it was 2% before.

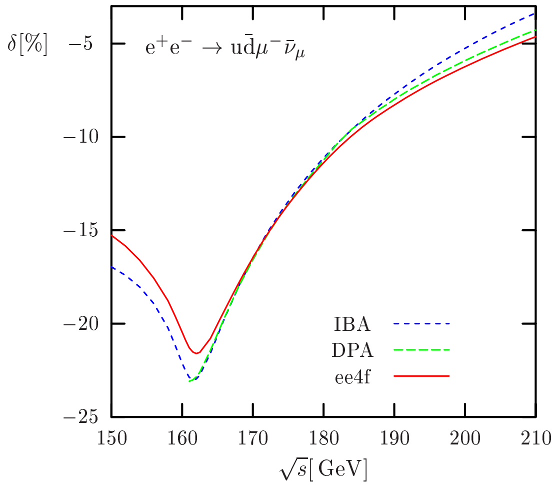

Figure 2: Relative corrections to the cross section for $e^+e^- \to u\bar{d}\,\mu^-\nu_\mu$ from the complete O(a) calculation, in DPA and in IBA

Since our calculation has been implemented in a Monte Carlo generator, we can also calculate differential distributions. It turns out that the corrections beyond the DPA distort the angular distribution of the produced W bosons and can thus fake effects of non-standard W-boson couplings, if not taken into account. Similarly, these corrections have a non-trivial effect on the invariant-mass distribution of the virtual W bosons, which can affect the determination of the W-boson mass from the decay products. In summary, our calculation is crucial for the full exploitation of the potential of the ILC for the measurements of the properties of the W boson.

References

[1] A. Denner, S. Dittmaier, LM. Roth, L.H. Wieders, Phys. Lett. B612 (2005) 223.

[2] A. Denner, S. Dittmaier, LM. Roth, L.H. Wieders, Nucl. Phys. B724 (2005) 247.

[3] J.A. Aguilar-Saavedra et al., TESLA Technical Design report Part III: Physics at an e^+e^- Linear Collider, hep-ph/0106315.

[4] The LEP Collaborations ALEPH, DELPHI, L3, OPAL, and the LEP Electroweak Working Group, LEPEWWG/2005-051, hep-ph/0511027.