Differential predictions for Top-quark production at NNLO

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In collaboration with
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arXiv: 1906.06535, arXiv:2005.00557 and paper in preparation

PSI Particle Theory Seminar, November 20, 2020

Outline

- Introduction
- The q_T subtraction formalism
  - the MATRIX framework
- Extension to heavy-quarks
  - computation of missing soft contributions
- Results
  - validation
  - comparison with CMS predictions in the lepton+jets channel
- Predictions in the $\overline{\text{MS}}$ scheme
  - running mass effects
- Summary

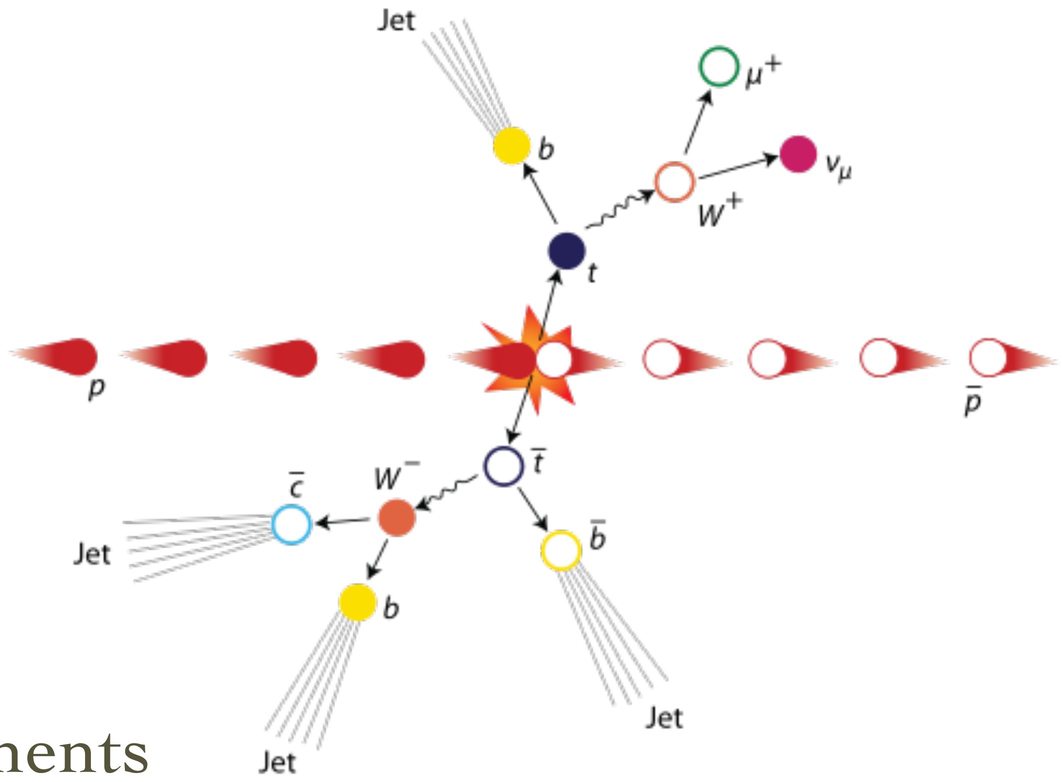
Introduction

- Top-quark production is a crucial process at high-energy colliders

- Possible window on new physics

- Top mass fundamental parameter

- Ubiquitous background to Higgs measurements and new physics searches

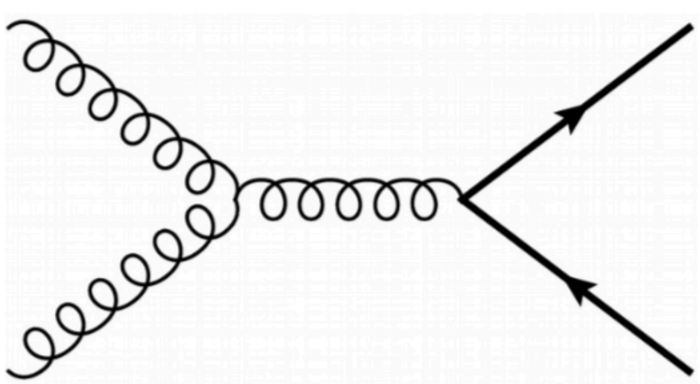


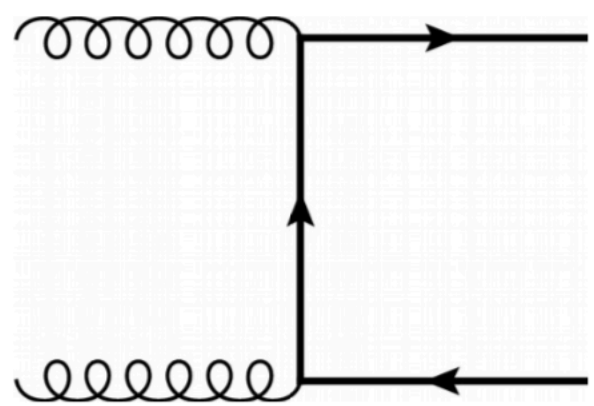
- Standard candle at the LHC

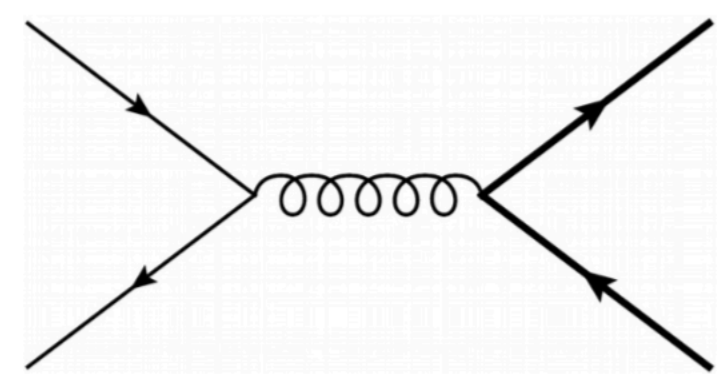
Introduction

Main source of top-quark event at hadron colliders is $t\bar{t}$ production

About 15 $t\bar{t}$ events per second at the LHC !







gg contribution dominant
at the LHC (85% at LO)

Cross section known at NNLO in QCD + resummations

(no attempt to compile a list of references……)

Why a new NNLO calculation ?

- Very complex calculation, only one group able to complete it till recently

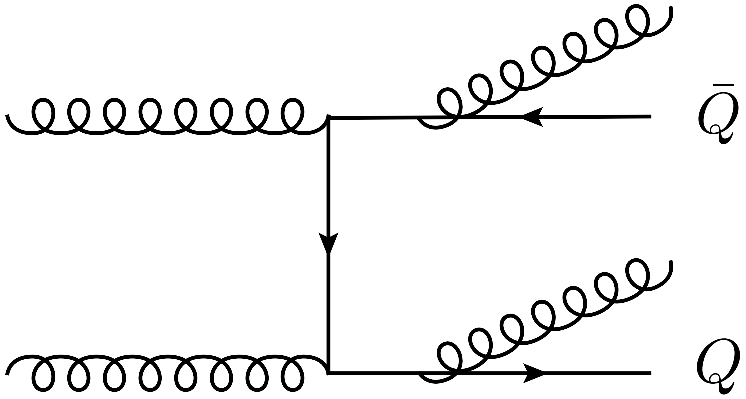
  Bärnreuther, Czakon, Mitov (2012)
  Czakon, Mitov (2012)
  Czakon, Fiedler, Mitov (2013)
  Czakon, Fiedler, Heymes, Mitov (2015,2016)

- Experience shows that NNLO calculations are difficult and that an independent check is always useful

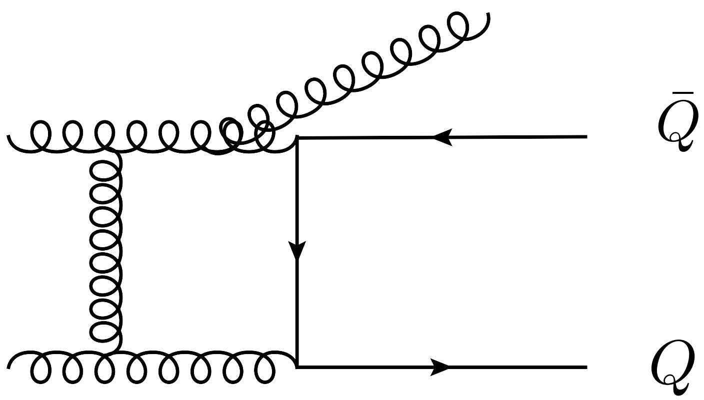
  - Drell-Yan — Hamberg, Matsuura, Van Neerven (1991); Harlander, Kilgore (2000)
  - $e^+ e^- \rightarrow 3$ jets — Gehrmann-De Ridder, Gehrmann, Glover, Heinrich (2008) ; Weinzierl (2008)
  - Diphoton hadroproduction — Catani, Cieri, Ferrera, de Florian, MG (2012); Campbell, Ellis, Williams (2016)
  - Higgs production in VBF — Cacciari et al. (2015); Cruz-Martinez, Gehrmann, Glover, Huss (2018)
  - Higgs+jet(s) — Boughezal et al (2015); Caola, Melnikov et al (2015); Chen, Gehrmann, Glover, Jaquier (2015)

- No public parton level event generator was available

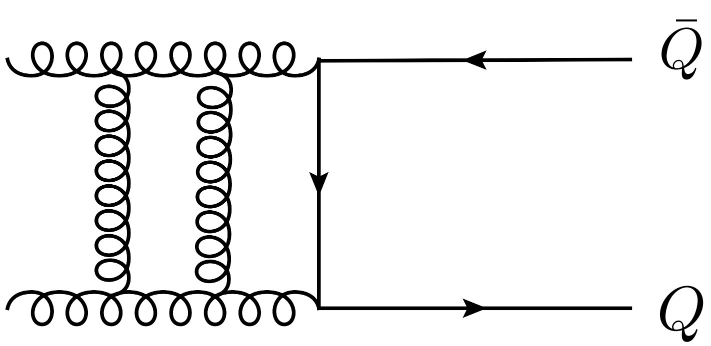
NNLO: building blocks



Tree-level amplitudes with two additional partons



One-loop amplitudes with one additional parton (to be evaluated in unresolved regions where instabilities may arise)



Two-loop amplitudes → currently the major bottleneck (new class of functions, charting new territory…)

+ one-loop squared amplitudes

All the three contributions separately divergent !

Crucial to keep the calculation fully differential: corrections for fiducial and inclusive rates may be significantly different

NNLO: building blocks

- Tree-level amplitudes with two additional partons and one-loop amplitudes with one additional parton are the same entering the computation of $Q\bar{Q}$+jet

  Dittmaier, Uwer, Weinzierl (2007,2008)

  → Nowadays they can be obtained with automatic generators like Openloops, Recola….

- The one loop squared contribution is known

  Korner, Merebashvili, Rogal (2008)
  Anastasiou, Aybat (2008)
  Kniehl, Merebashvili, Korner, M. Rogal (2008)

- Two-loop amplitude only available numerically

  Czakon (2008)
  Barnreuther, Czakon, and Fiedler (2013)

  Recent progress in the computation of non-planar master integrals suggests that the analytic calculation can be completed soon

  Bonciani et al (2019)
  Gehrmann et al (2019)

All the contributions in principle available but separately divergent !

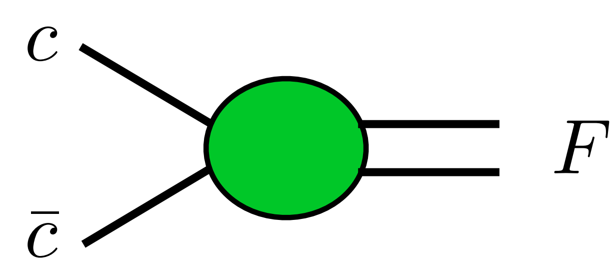
→ **Subtraction scheme needed !**

The q_T subtraction method

Catani, MG (2007)

Let us consider the production of a colourless high-mass systems F in hadron collisions (F may consist of lepton pairs, vector bosons, Higgs bosons......)

At LO it starts with $c\bar{c} \to F$



Strategy: start from NLO calculation of F+jet(s) and observe that as soon as the transverse momentum of the F $q_T \neq 0$ one can write:

$$d\sigma^{F}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{(N)LO}$$

Define a counterterm to deal with singular behaviour at $q_T \to 0$

But.....
the singular behaviour of $d\sigma^{F+\text{jets}}_{(N)LO}$ is well known from the resummation program of large logarithmic contributions at small transverse momenta

Parisi, Petronzio (1979)
Collins, Soper, Sterman (1985)
Catani, de Florian, MG (2000)

→ choose $d\sigma^{CT} \sim d\sigma^{(LO)} \otimes \Sigma^F(q_T/Q)$

where $\Sigma^F(q_T/Q) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$

Perturbative coefficients known up to N3LO

de Florian, MG (2000); Becher, Neubert (2011)
Li, Zhu (2017); Vladimirov (2016)

Then the calculation can be extended to include the $q_T = 0$ contribution:

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right]$$

where I have subtracted the truncation of the counterterm at (N)LO and added a contribution at $q_T = 0$ to restore the correct normalization

The function $\mathcal{H}^F$ can be computed in QCD perturbation theory

$$\mathcal{H}^F = 1 + \left(\frac{\alpha_S}{\pi}\right)\mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \ldots\ldots$$

The hard-collinear coefficients

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

$$\mathcal{H}^F = \left[H^F C_1 C_2 \right]_{c\bar{c};a_1 a_2}$$

Universal collinear functions:
fully known up to NNLO
and recently extended to N3LO

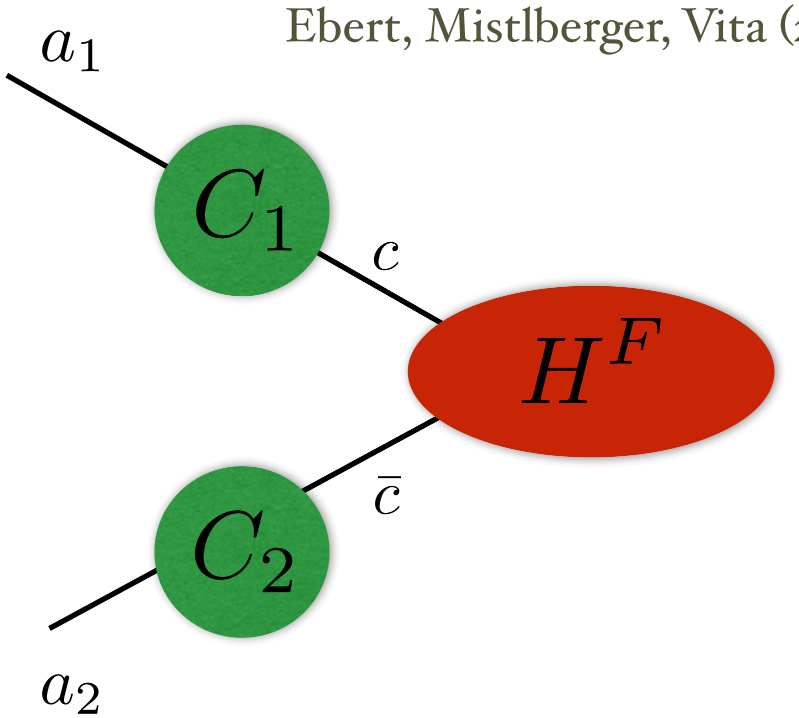
Catani, MG (2011)
Catani, Cieri, de Florian, Ferrera, MG (2012)
Luo, Yang, Zhu,Zhu (2019)
Ebert, Mistlberger, Vita (2020)

$$H^F \sim \langle \tilde{\mathcal{M}} | \tilde{\mathcal{M}} \rangle$$

All order virtual amplitude

$$|\tilde{\mathcal{M}}\rangle = (1 - \tilde{I}) |\mathcal{M}\rangle$$

Suitable subtraction operator (fully known up to NNLO)

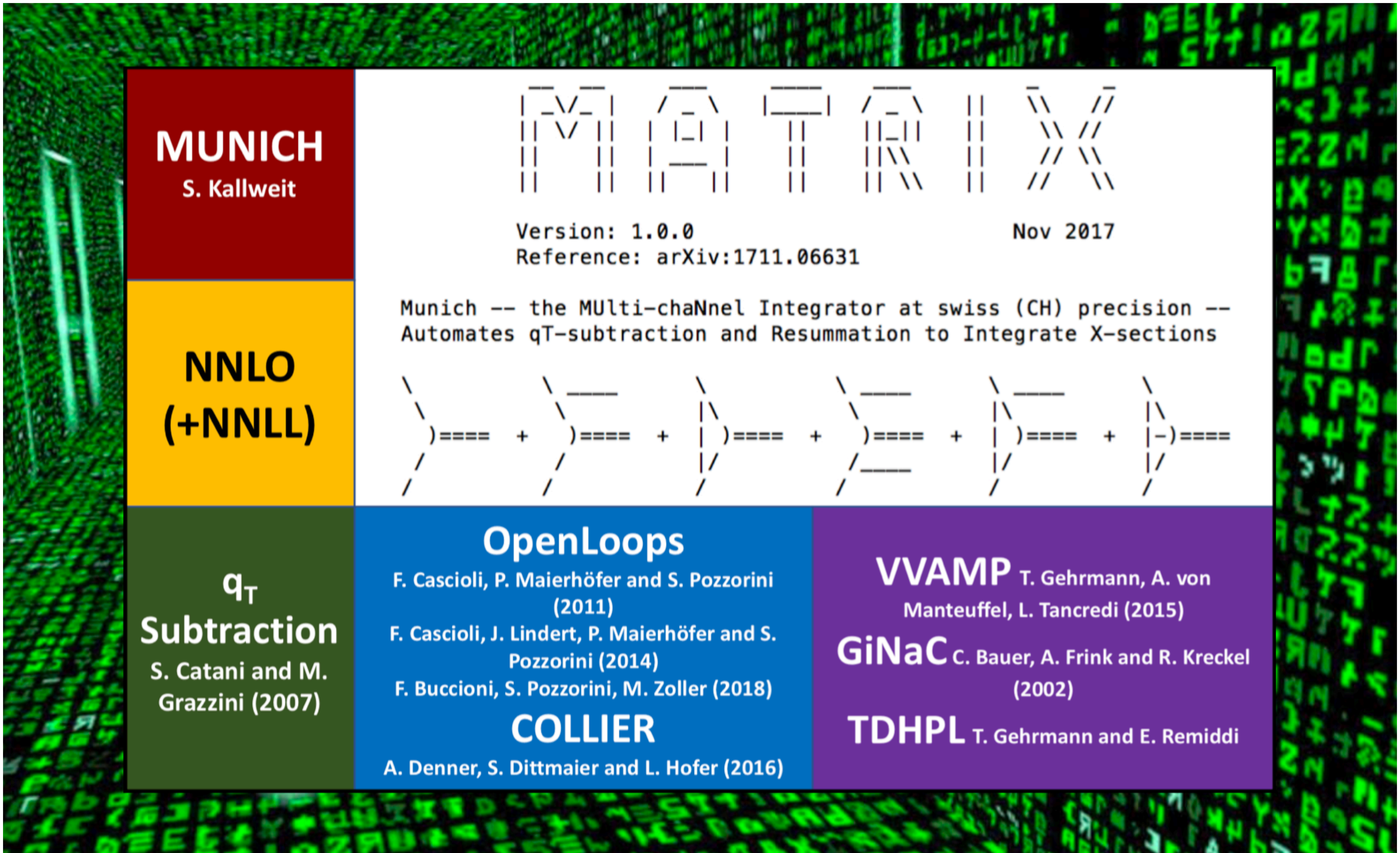


The method can be implemented in general terms for any colourless final state provided the two loop amplitude is available

S. Catani, L.Cieri, D. de Florian, G.Ferrera, MG (2013)

The MATRIX project

Kallweit, Wiesemann, MG (2017)
+ Buonocore, Devoto, Fabre, Mazzitelli, Rathlev, Sargsyan, Yook



Status

- $pp \to Z/\gamma^* (\to l^+l^-)$ ✅
- $pp \to W(\to l\nu)$ ✅
- $pp \to H$ ✅
- $pp \to \gamma\gamma$ ✅
- $pp \to W\gamma \to l\nu\gamma$ ✅
- $pp \to Z\gamma \to l^+l^-\gamma$ ✅
- $pp \to ZZ(\to 4l)$ ✅
- $pp \to WW \to (l\nu l'\nu')$ ✅
- $pp \to ZZ/WW \to ll\nu\nu$ ✅
- $pp \to WZ \to l\nu ll$ ✅
- $pp \to HH$ (✅)

First public release out in November 2017

Kallweit, Wiesemann, MG (2017)

NLO for gluon fusion for ZZ and WW

Kallweit, Wiesemann, Yook, MG (2018, 2020)

Combination with EW corrections

Kallweit, Lindert, Pozzorini, Wiesemann, MG (2019)

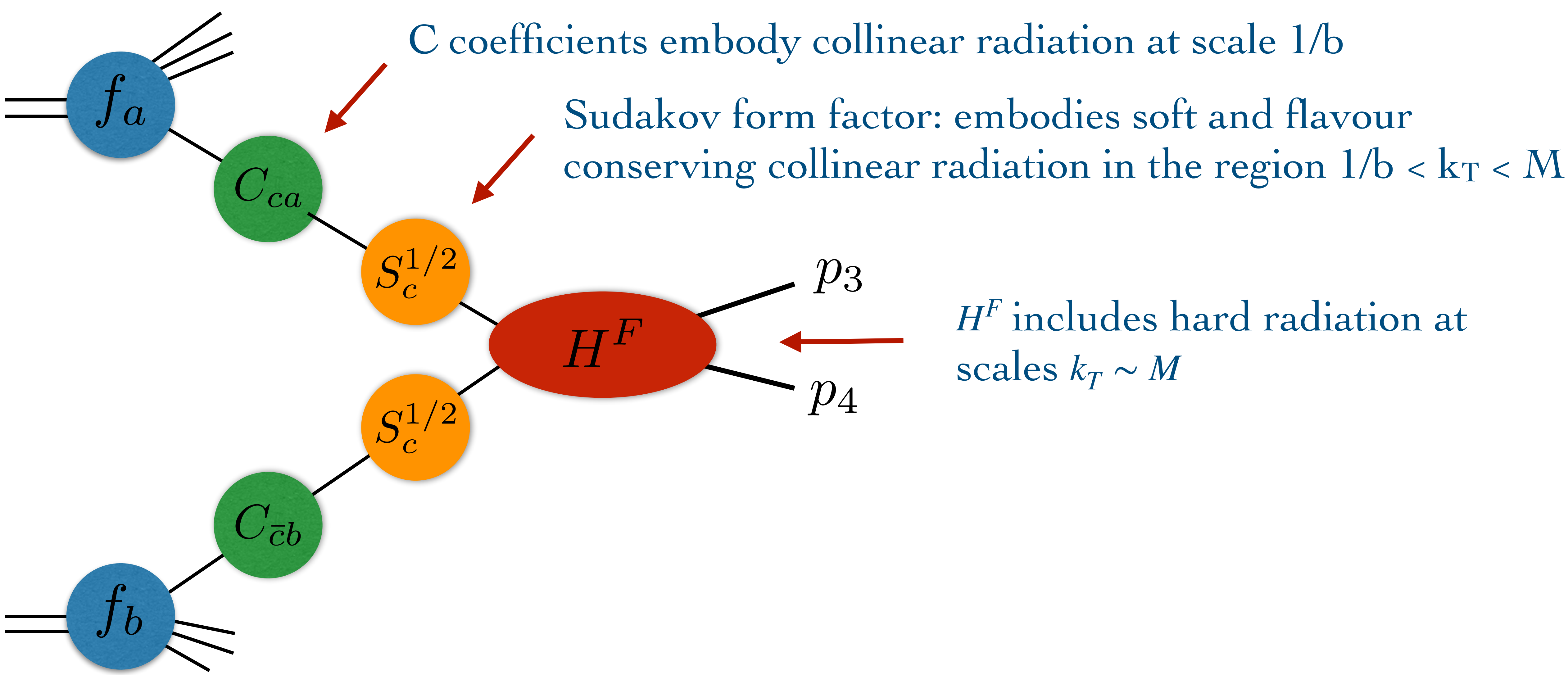
↓

Included in v2 beta version (currently under test)

Extension to heavy-quark production

S.Catani, A.Torre, MG (2014)

In the case of colourless final states the small-q_T singularities are entirely due to initial state soft and collinear radiation



Extension to heavy-quark production

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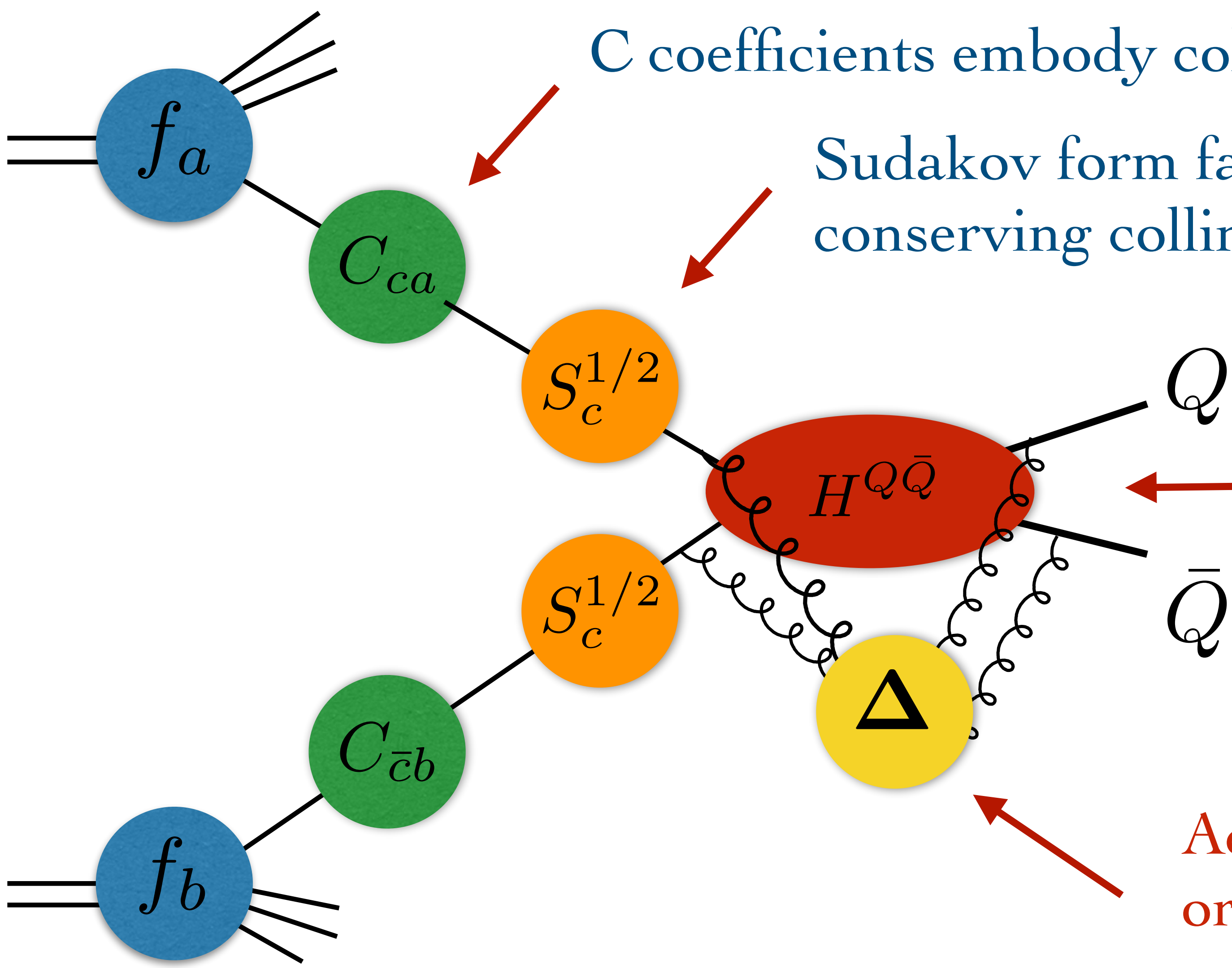
In the case of heavy-quark production additional soft singularities appear that need to be taken into account



C coefficients embody collinear radiation at scale 1/b

Sudakov form factor: embodies soft and flavour conserving collinear radiation in the region $1/b < k_T < M$

H^F includes hard radiation at scales $k_T \sim M$

Additional radiative factor of purely soft origin (starts to contribute at NLL)

We obtain an analogous structure for the subtraction formula ($q\bar{q}$ and gg channels contribute at the same order) with some differences

$$d\sigma^{Q\bar{Q}}_{(N)NLO} = \mathcal{H}^{Q\bar{Q}}_{(N)NLO} \otimes d\sigma^{Q\bar{Q}}_{LO} + \left[d\sigma^{Q\bar{Q}+\mathrm{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

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- Modified subtraction counterterm fully known (enough to compute NNLO corrections in all the off-diagonal channels)

Bonciani, Catani, Torre, Sargsyan, MG (2015)

Additional perturbative ingredient: soft anomalous dimension Γ_t entering the soft radiative factor: known at NNLO

Mitov, Sterman, Sung (2009)
Neubert et al (2009)

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- Structure of hard collinear function is analogous but now

$$\mathcal{H}^{Q\bar{Q}} \sim \langle \tilde{\mathcal{M}} | \boldsymbol{\Delta} | \tilde{\mathcal{M}} \rangle$$

Additional radiative factor of purely soft origin

equivalently $\tilde{I}_c \to \tilde{I}_{c\bar{c}\to Q\bar{Q}}$

Subtraction operator for colourless final states

Modified subtraction operator

The missing contributions can be computed by integrating a suitably subtracted soft current

The calculation at NLO

Catani, Torre, MG (2014)

Standard soft current contains the correct soft behaviour but also additional initial state collinear singularities

$$-\mathbf{J}(k)^2 = \sum_{i,j=1}^{4} \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} \mathbf{T}_i \cdot \mathbf{T}_j$$

These singular contributions are already accounted for in the calculation of colour-singlets

➡ We define a suitably subtracted soft current

$$-\mathbf{J}(k)^2|_{\rm sub} = \sum_{J=3,4} \left[\frac{p_J^2}{(p_J \cdot k)^2} \mathbf{T}_J^2 + \sum_{i=1,2} \left(\frac{p_i \cdot p_J}{p_J \cdot k} - \frac{p_1 \cdot p_2}{(p_1 + p_2) \cdot k} \right) \frac{2\,\mathbf{T}_i \cdot \mathbf{T}_J}{p_i \cdot k} \right] + \frac{2 p_3 \cdot p_4}{(p_3 \cdot k)(p_4 \cdot k)} \mathbf{T}_3 \cdot \mathbf{T}_4$$

final state (heavy-quark) emitters

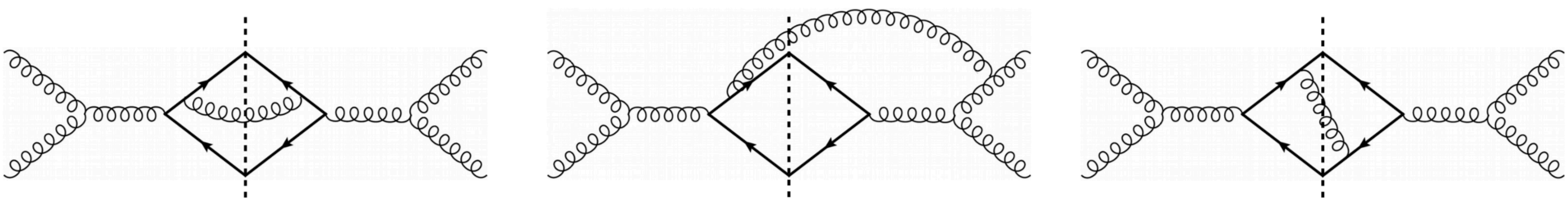
Initial state (massless) emitters

The calculation at NLO

Catani, Torre, MG (2014)

We need to compute the integral of the subtracted soft current over the phase space of the unresolved gluon

$$\int d^d k \, \delta_+(k^2)\, e^{i\mathbf{b}\cdot\mathbf{k}_T}\, \mathbf{J}^2(k)|_{\rm sub}$$



$$\widetilde{\mathbf{I}}^{(1)}_{c\bar{c}\to Q\bar{Q}}\left(\epsilon, \frac{M^2}{\mu_R^2}\right) = -\frac{1}{2}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon}\left\{\left(\frac{1}{\epsilon^2} + i\pi\frac{1}{\epsilon} - \frac{\pi^2}{12}\right)(\mathbf{T}_1^2 + \mathbf{T}_2^2) + \frac{2}{\epsilon}\gamma_c - \frac{4}{\epsilon}\,\mathbf{\Gamma}_t^{(1)}(y_{34}) + \mathbf{F}_t^{(1)}(y_{34})\right\}$$

Singular structure from initial state radiation

Additional soft contribution obtained from integration of the subtracted soft current

NLO results

$$\mathbf{F}_t^{(1)}(y_{34}) = (\mathbf{T}_3^2 + \mathbf{T}_4^2)\ \ln\left(\frac{m_T^2}{m^2}\right) + \ (\mathbf{T}_3 + \mathbf{T}_4)^2\ \mathrm{Li}_2\left(-\frac{\mathbf{p}_\mathbf{T}^2}{m^2}\right) + \mathbf{T}_3 \cdot \mathbf{T}_4\ \frac{1}{v}\, L_{34}$$

$$\begin{aligned}\mathbf{\Gamma}_t^{(1)}(y_{34}) = -\frac{1}{4}\Bigg\{&(\mathbf{T}_3^2 + \mathbf{T}_4^2)\ (1 - i\pi) + \sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i \cdot \mathbf{T}_j\ \ln\frac{(2p_i \cdot p_j)^2}{M^2 m^2}\\ &+ 2\ \mathbf{T}_3 \cdot \mathbf{T}_4\left[\frac{1}{2v}\ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v} + 1\right)\right]\Bigg\}\ .\end{aligned}$$

$$v = \sqrt{1 - \frac{m^4}{(p_3 \cdot p_4)^2}}$$

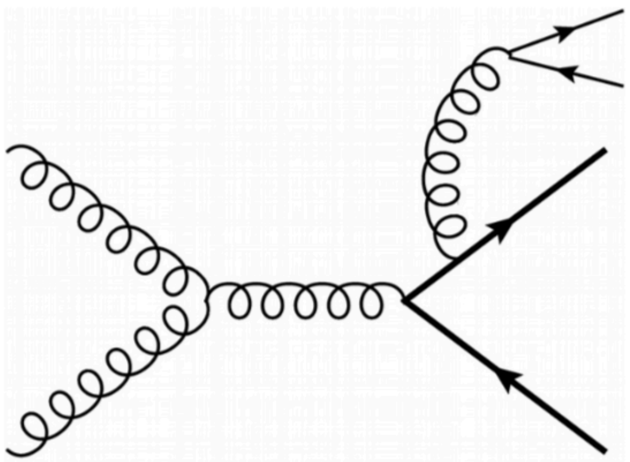
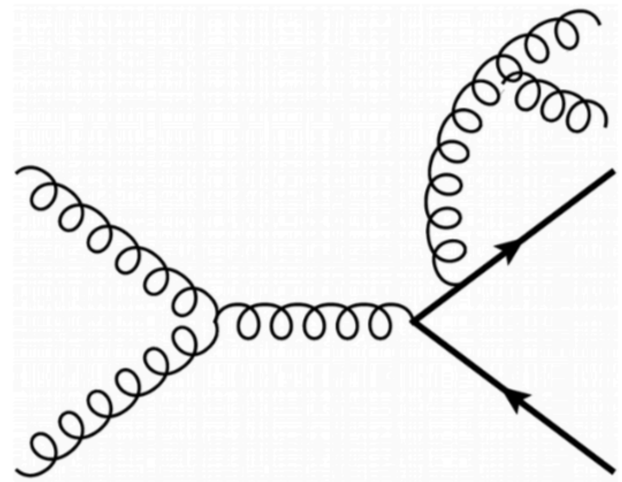
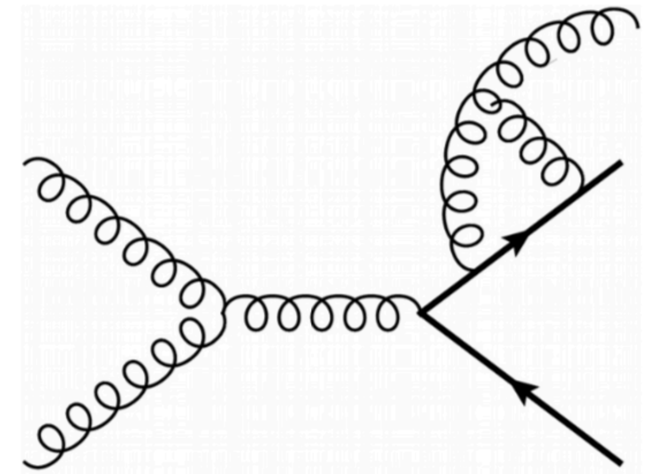
Relative velocity

$$\begin{aligned}L_{34} = &\ln\left(\frac{1+v}{1-v}\right)\ \ln\left(\frac{m_T^2}{m^2}\right) - 2\,\mathrm{Li}_2\left(\frac{2v}{1+v}\right) - \frac{1}{4}\ln^2\left(\frac{1+v}{1-v}\right)\\ &+ 2\left[\mathrm{Li}_2\left(1 - \sqrt{\frac{1-v}{1+v}}\, e^{y_{34}}\right) + \mathrm{Li}_2\left(1 - \sqrt{\frac{1-v}{1+v}}\, e^{-y_{34}}\right) + \frac{1}{2}\, y_{34}^2\right]\end{aligned}$$

$$y_{34} = y_3 - y_4$$

The calculation at NNLO

Catani, Devoto, Mazzitelli, MG , to appear

Three classes of contributions: singular structure fully known

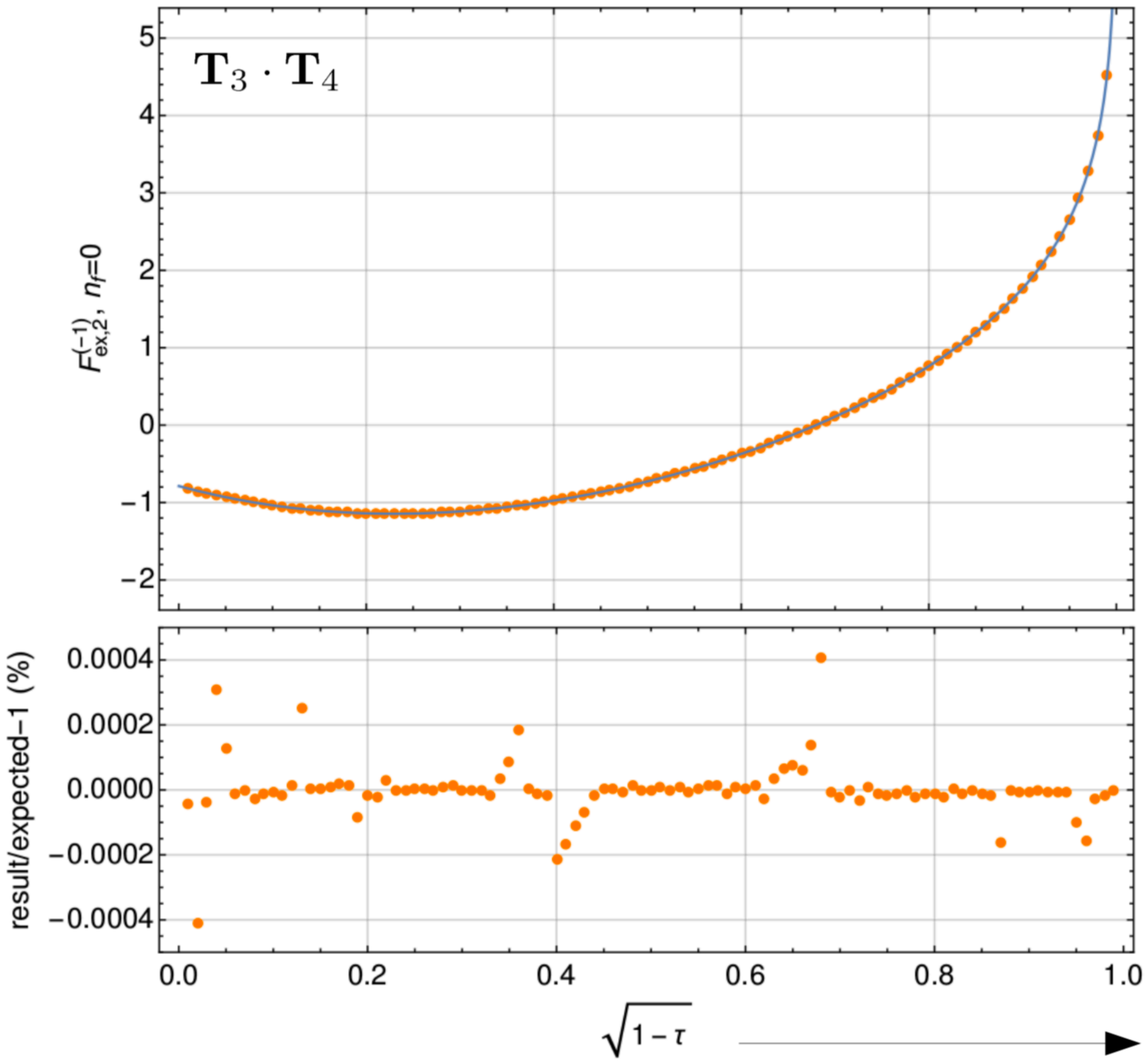
- Emission of a soft quark-antiquark pair — Catani, MG (2000)
- Emission of two soft gluons — Catani, MG (2000); Czakon (2011)
- Soft-gluon emission at one loop — Catani, MG (2000); Bierenbaum, Czakon, Mitov (2011); Czakon, Mitov (2018)

Construct suitably subtracted soft current for each of these contribution

Intermediate results contain $1/\varepsilon^3$ poles ➡ add up to $1/\varepsilon^2$ in the end

Pole cancellation

We managed to obtain analytic cancellation of all the poles except for the $1/\varepsilon$ pole in the $\mathbf{T}_3 \cdot \mathbf{T}_4$ contribution



Poles can be predicted to cancel the remaining singularities of 2-loop amplitude

$$\beta = \sqrt{1-\tau}$$

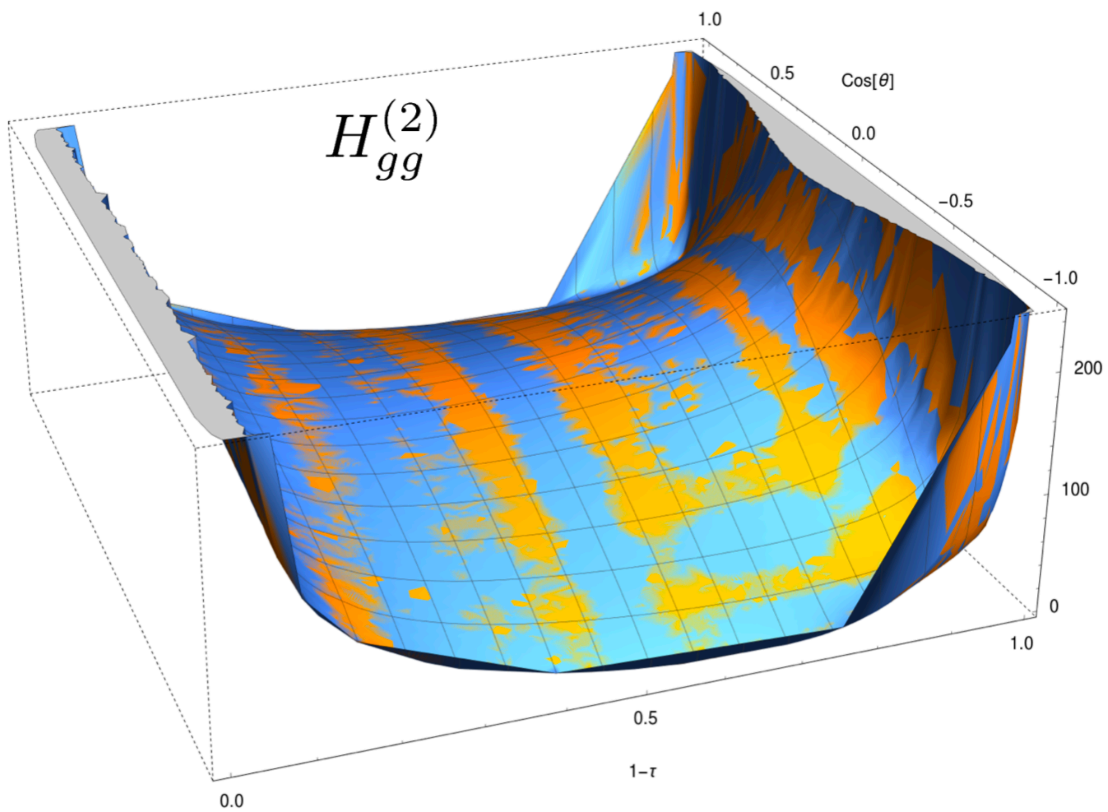
Pole independent on cosθ

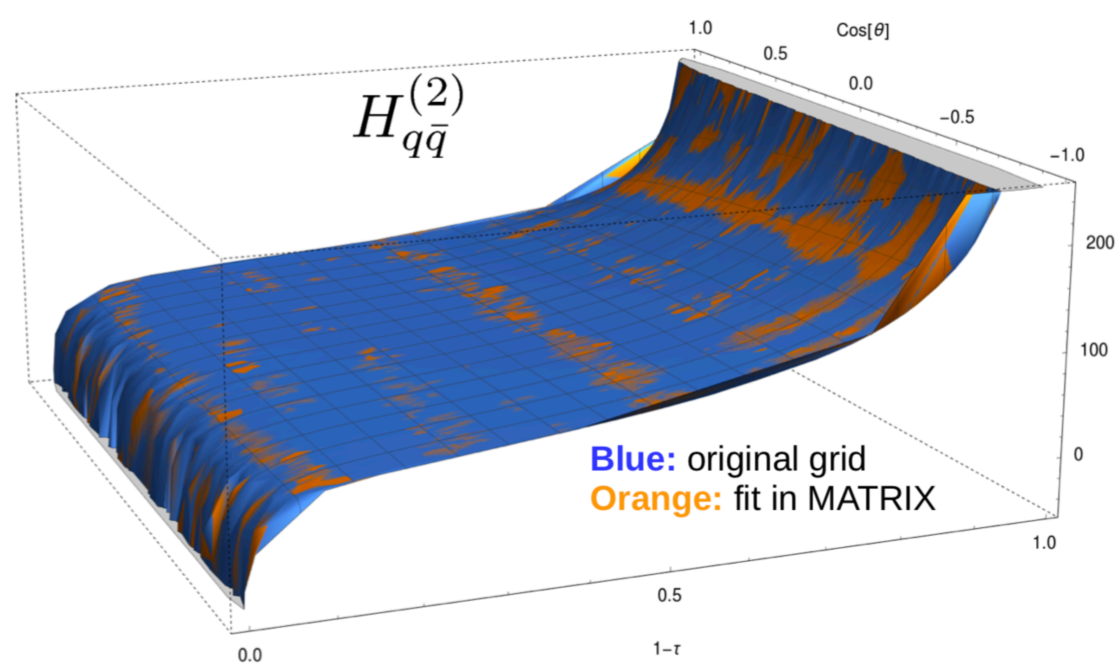
Result for $H^{(2)}$

We combine analytical results with a numerical evaluation of the remaining terms

Final result for $H^{(2)}$ coefficients including the two-loop amplitudes

Czakon (2008)
Barnreuther, Czakon, and Fiedler (2013)





We then construct a grid which is directly implemented in MATRIX

We have carried out several studies (removing a fraction of the points) that show that the procedure is extremely robust

➡ Crucial to have most of the result in analytic form

Implementation

As for the other NNLO calculations in MATRIX all spin and colour correlated tree-level and one loop amplitudes are obtained with **Openloops**

→ Excellent numerical stability in IR singular regions

Four parton tree-level colour correlations are computed analytically

Real-virtual contributions cross checked with **Recola**

The calculation is now fully implemented into the MATRIX framework

Automatic evaluation of scale uncertainties

Cross sections at 0.1 % accuracy computable with O(1000) CPU days

Inclusive results

Use NNPDF3.1 NNLO PDFs and M_t=173.3 GeV

σ_{NNLO} [pb]	MATRIX	TOP++
8 TeV	$238.5(2)^{+3.9\%}_{-6.3\%}$	$238.6^{+4.0\%}_{-6.3\%}$
13 TeV	$794.0(8)^{+3.5\%}_{-5.7\%}$	$794.0^{+3.5\%}_{-5.7\%}$
100 TeV	$35215(74)^{+2.8\%}_{-4.7\%}$	$35216^{+2.9\%}_{-4.8\%}$

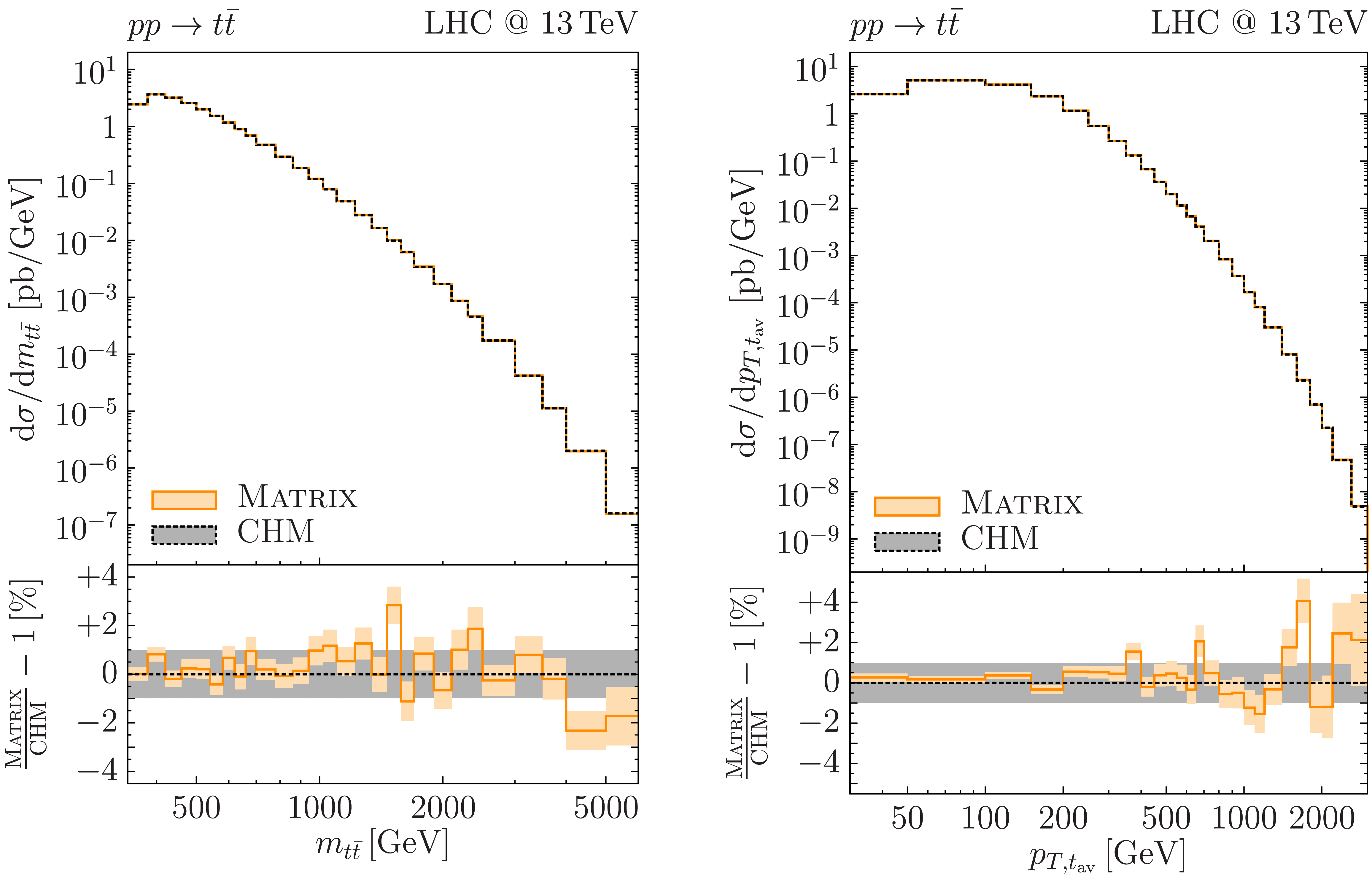
statistical+systematic uncertainties

Excellent agreement with Top++ !

We find that the quantitative impact of the two-loop amplitude is extremely small (0.1% of the full NNLO cross section at 13 TeV)

→ (Almost) completely independent computation !

Going differential: validation



Excellent agreement even in extreme kinematical regions

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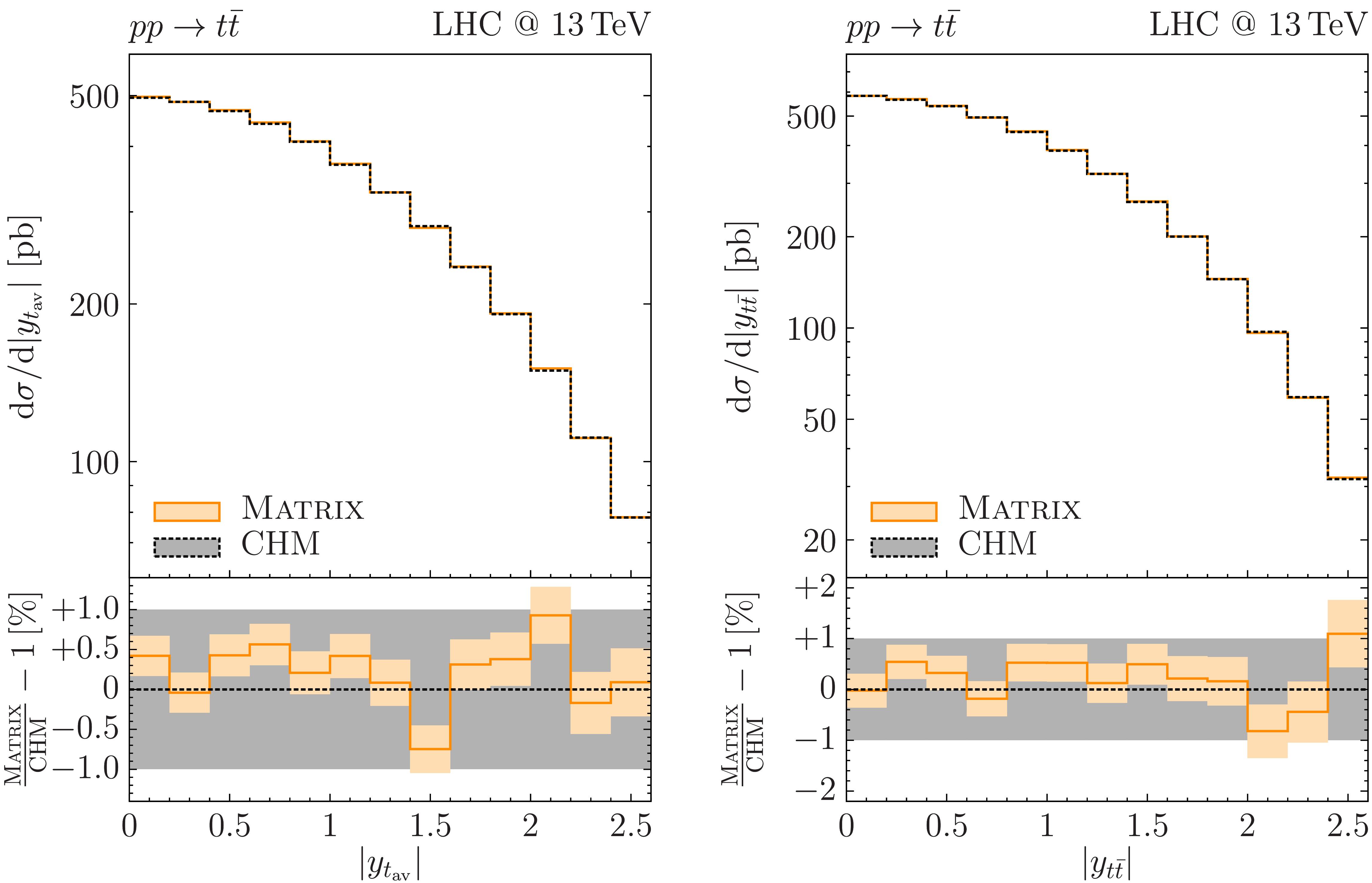


Excellent agreement even in extreme kinematical regions

Going differential: results

LO, NLO and NNLO predictions obtained using NNPDF3.1 PDFs with $\alpha_S(m_Z)$=0.118 at the corresponding order

CMS data of CMS-TOP-17-002 in the lepton+jets channel

Extrapolation to parton level in the inclusive phase space

→ Our calculation is carried out without cuts

To compare with data we multiply our absolute predictions by 0.438 (semileptonic BR of the $t\bar{t}$ pair) times 2/3 (only electrons and muons)

The choice of scales

Perturbative results depend on the choice of the renormalisation and factorisation scales μ_R and μ_F

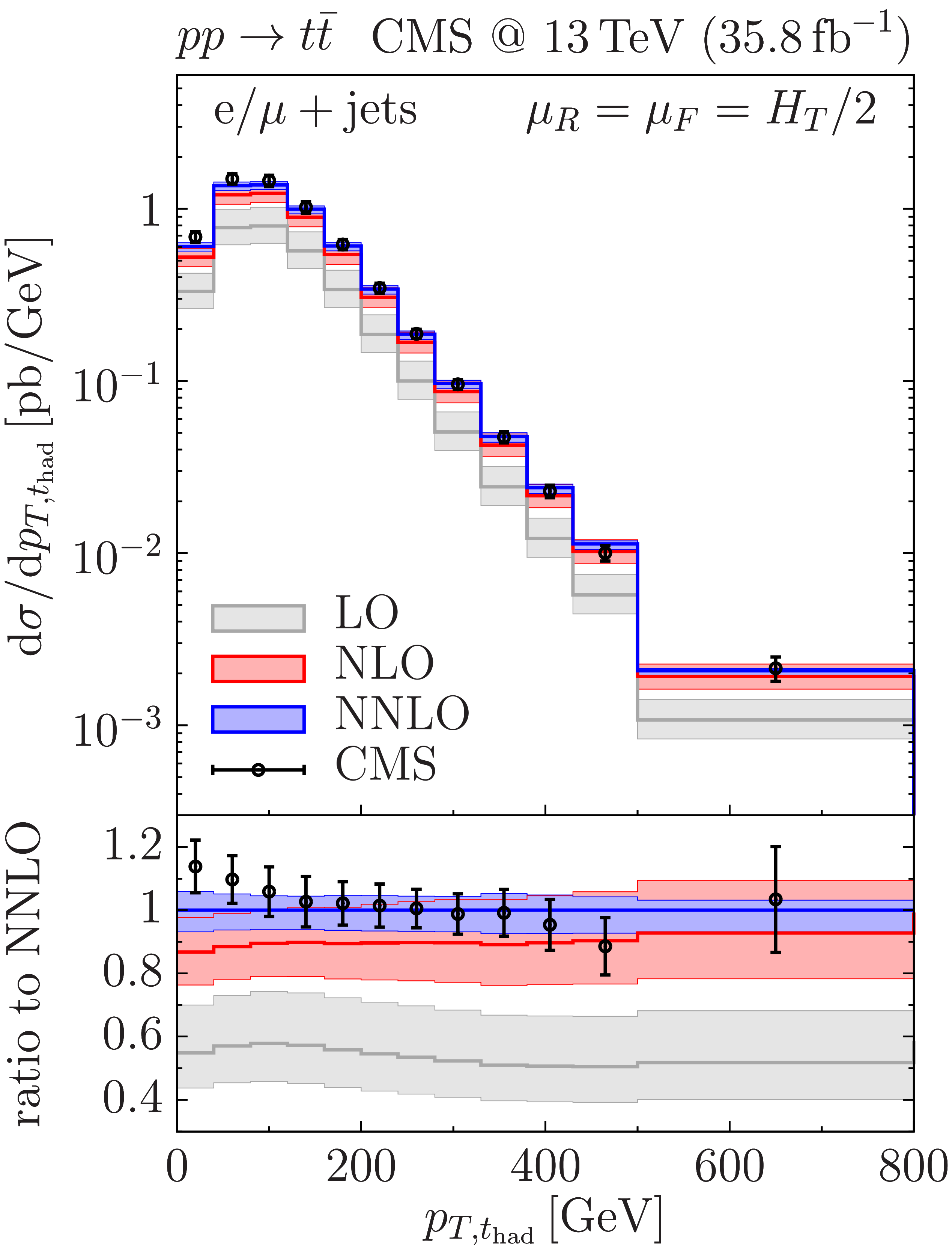
These scales should be chosen of the order of the characteristic hard scale

- Total cross section: the hard scale is the top mass m_t
- The same can be said for the rapidity distributions
- Invariant mass distribution: m_{tt}
- Tranverse momentum distributions: m_T

A dynamical central scale like $\mu_0 = H_T/2 = (m_{T,t} + m_{T,\bar{t}})/2$ turns out to be a good approximation of all these characteristic scales

Scale uncertainties: $\mu_0/2 < \mu_F, \mu_R < 2\mu_0$ $\quad$ $0.5 < \mu_F/\mu_R < 2$

Single-differential distributions

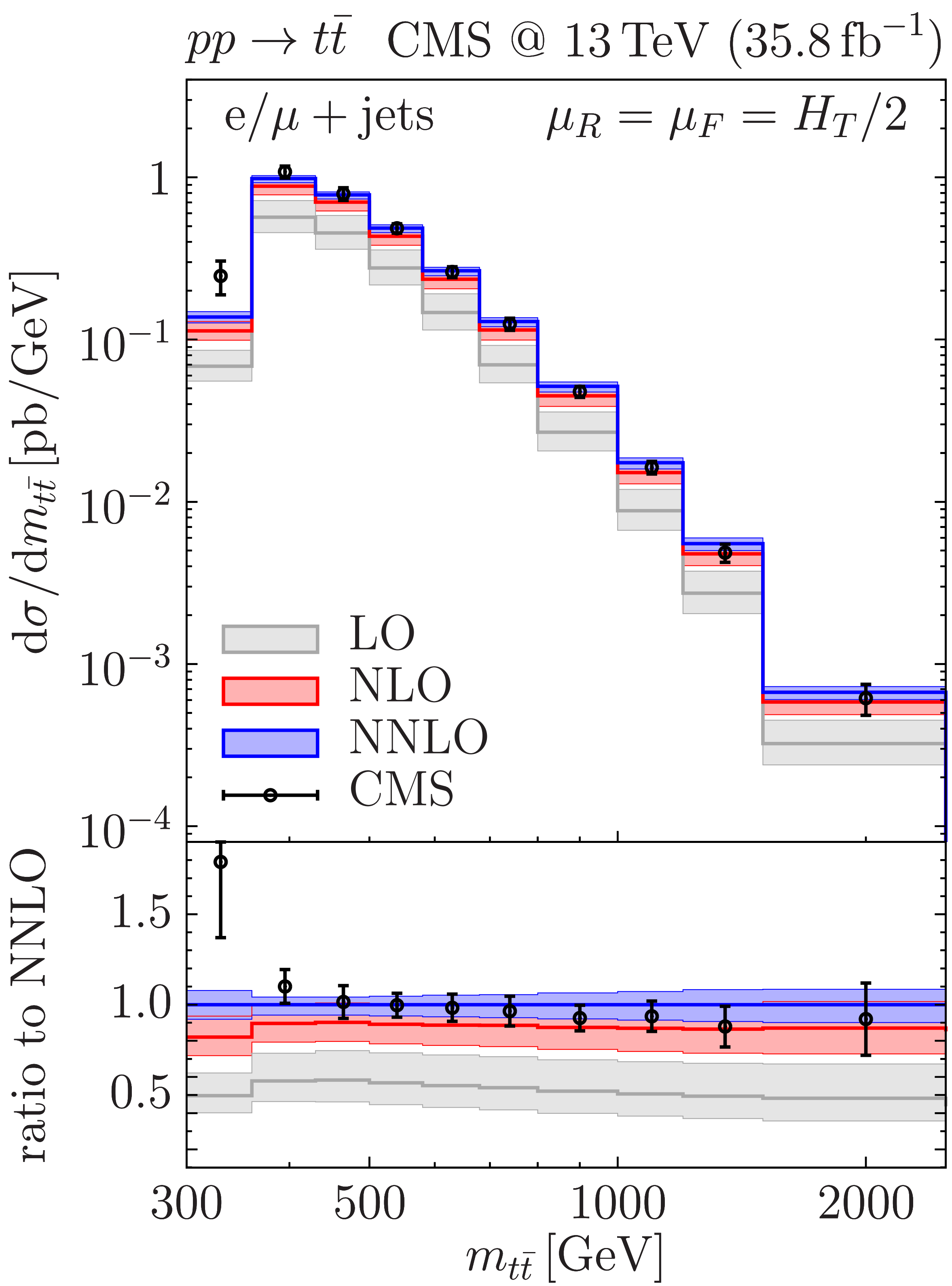


As noted in various previous analyses the measured p_T distribution is slightly softer than the NNLO prediction

Perturbative prediction relatively stable when going from NLO to NNLO

Data and theory are consistent within uncertainties

Single-differential distributions



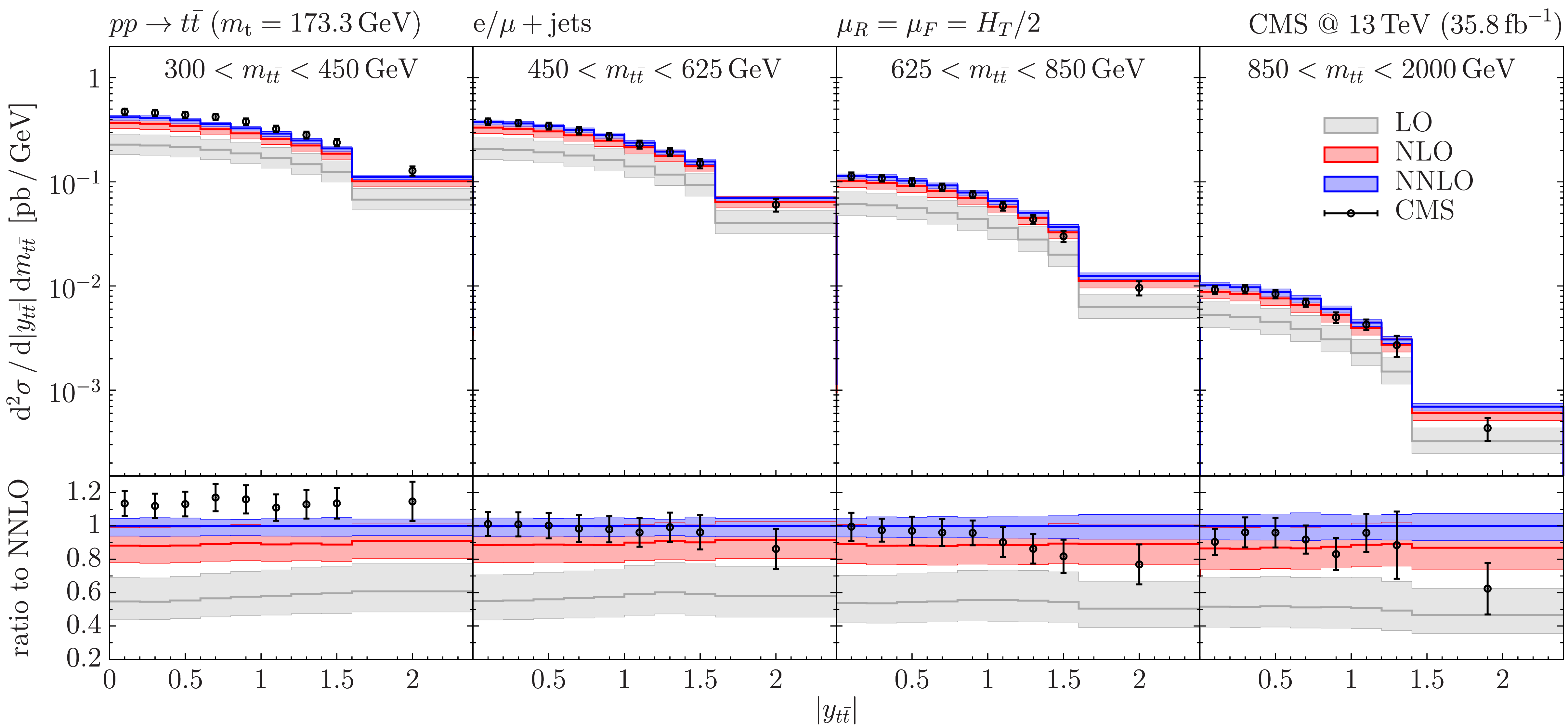
Good description of the data except in the first bin

Issues in extrapolation ? Smaller m_t ?

A smaller m_t (just by about 2 GeV) leads to a higher theoretical prediction in this bin and to small changes at higher m_{tt}

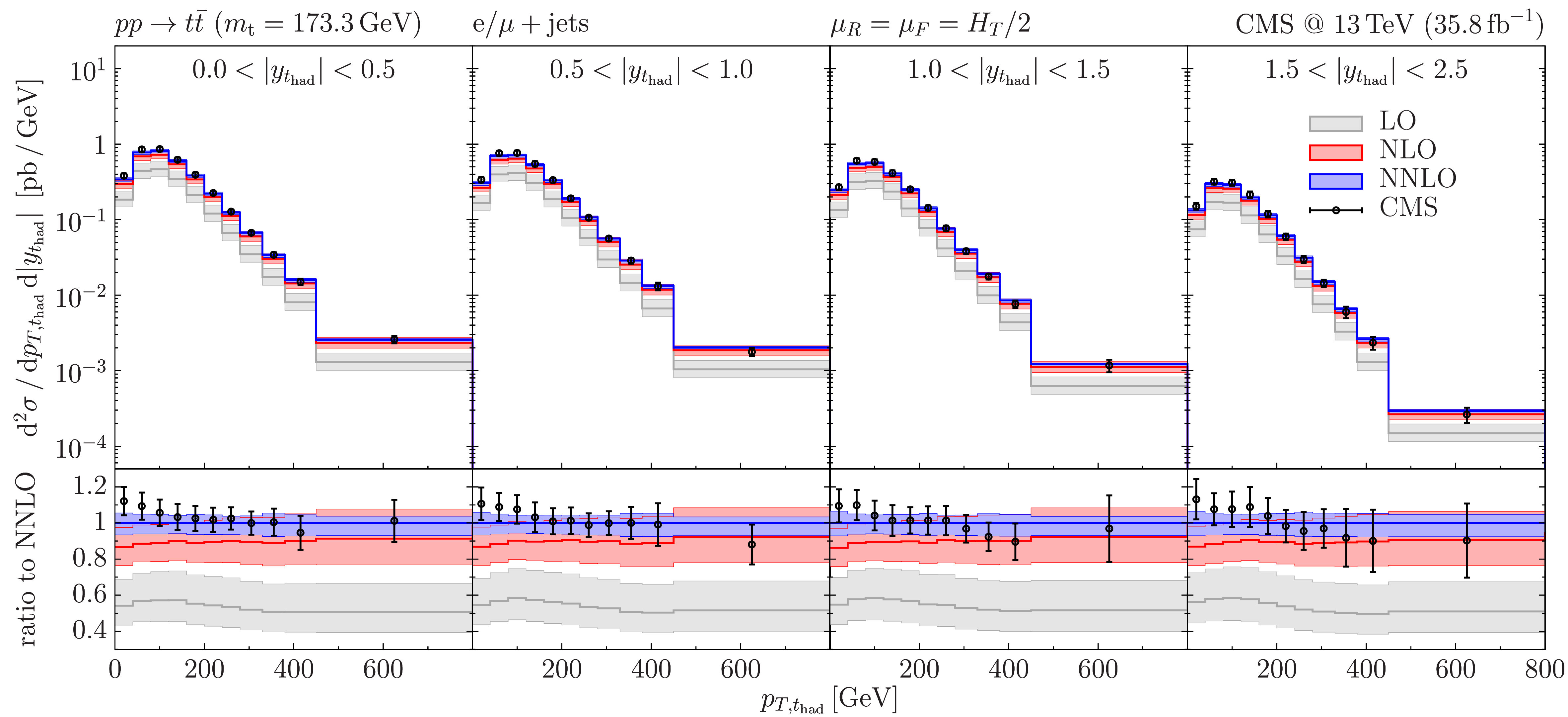
CMS-TOP-18-004: leptonic channel: a fit with the same PDFs leads to m_t=170.81 ± 0.68 GeV

Double-differential distributions



The first m_{tt} interval now extends up to 450 GeV → better agreement with the data

Double-differential distributions



As for the single-differential distribution the p_T distribution is softer than the NNLO prediction in all the rapidity intervals

Results in the $\overline{\text{MS}}$ scheme for the top mass

The top mass in the $\overline{\text{MS}}$ scheme

The top mass is a fundamental parameter of the SM to be properly defined by renormalization of related UV divergences

The results shown up to now are obtained in the pole scheme: the renormalisation procedure fixes the pole of the quark propagator, at any order in perturbation theory, to the same value M_t

In the $\overline{\text{MS}}$ sheme the renormalised mass $m_t(\mu_m)$ is defined by subtracting UV divergences in dimensional regularization, and, therefore, the pole of the quark propagator receives corrections at any order in perturbation theory

Different renormalisation schemes are perturbatively related

$$M_t = m_t(\mu_m)\, d(m_t(\mu_m), \mu_m) = m_t(\mu_m) \left(1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_S(\mu_m)}{\pi}\right)^k d^{(k)}(\mu_m)\right)$$

coefficients $d^{(k)}$ known for $k \leq 4$

Chetyrkin, Steinhauser (1999); Melnikov, Ritbergen (1999)
Marquard et al (2016)

The $\overline{\rm MS}$ mass depends on arbitrary renormalization scale μ_m (similarly to the QCD coupling $\alpha_S(\mu_R)$) and such scale dependence is perturbatively computable

$$\frac{d \ln m_t(\mu_m)}{d \ln \mu_m^2} = - \sum_{k=0}^{\infty} \gamma_k \left(\frac{\alpha_S(\mu_m)}{\pi} \right)^{k+1}$$

Note: scale dependence of $\overline{\rm MS}$ mass much slower than α_S

$$\frac{d \ln m_t(\mu)}{d \ln \mu} \sim \frac{1}{2} \frac{d \ln \alpha_S(\mu)}{d \ln \mu} \qquad \text{at LO}$$

The $\overline{\rm MS}$ mass can be specified by fixing a reference scale + RG evolution

Customary scale $\bar{m}_t$ defined such that $m_t(\bar{m}_t) = \bar{m}_t$ (no special meaning !)

Typical values: $M_t = 173$ GeV $\longleftrightarrow$ $\bar{m}_t = 164$ GeV (~ 10 GeV difference)

[Note: at scale $\mu_m = \bar{m}_t/2$ ➡ $m_t(\mu_m) = M_t + \mathcal{O}(1 \text{ GeV})$, simply because at this scale $d^{(1)} \sim 0$]

Two main consequences of scale dependence of $\overline{\rm MS}$ mass

- perturbative QCD predictions unavoidably depend on μ_m (in addition to renormalization scale μ_R from $\alpha_S(\mu_R)$ and factorization scale μ_F from PDFs)
- μ_m can possibly be set to a scale very different from $M_t \sim \bar{m}_t$ to embody ("resum") higher-order corrections ➡ running mass effects

From pole to $\overline{\text{MS}}$ predictions

Start from on-shell cross section $\sigma(M_t, X)$ (total or differential) with pole mass M_t

e.g. up to NNLO $$\sigma_{\text{NNLO}}(\alpha_S(\mu_R), \mu_R, \mu_F; M_t; X) = \sum_{i=0}^{2} \left(\frac{\alpha_S(\mu_R)}{\pi}\right)^{i+2} \sigma^{(i)}(M_t; \mu_R, \mu_F; X)$$

Perform all-order replacement $M_t \to m_t(\mu_m)$ and define $\overline{\text{MS}}$ scheme cross section through the all-order equality

$$\bar{\sigma}(\alpha_S(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sigma(\alpha_S(\mu_R), \mu_R, \mu_F; M_t = m_t(\mu_m)\, d(m_t(\mu_m), \mu_m); X)$$

↑ $\overline{\text{MS}}$ scheme

↑ Pole scheme

Expand in $\alpha_S(\mu_R)$ (e.g. up to NNLO) at fixed $m_t(\mu_R)$:

$$\bar{\sigma}_{\text{NNLO}}(\alpha_S(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sum_{i=0}^{2} \left(\frac{\alpha_S(\mu_R)}{\pi}\right)^{i+2} \bar{\sigma}^{(i)}(m_t(\mu_m); \mu_m, \mu_R, \mu_F; X)$$

within this formulation, pole scheme and $\overline{\text{MS}}$ scheme results are formally equivalent to all orders in α_S but different if expanded at fixed orders

Explicit expressions up to NNLO

Langenfeld, Moch, Uwer (2009)
Dowling, Moch (2014)

$$\bar{\sigma}^{(0)}(m_t(\mu_m);\mu_F;X) = \Big[\sigma^{(0)}(m;\mu_F;X)\Big]_{m=m_t(\mu_m)}$$

At NLO: 1st derivative of the LO

$$\bar{\sigma}^{(1)}(m_t(\mu_m);\mu_m,\mu_R,\mu_F;X) = \Big[\sigma^{(1)}(m;\mu_R,\mu_F;X) + d^{(1)}(\mu_m)\, m\, \partial_m \sigma^{(0)}(m;\mu_F;X)\Big]_{m=m_t(\mu_m)}$$

At NNLO: 1st derivative of the NLO and 2nd derivative of the LO

$$\bar{\sigma}^{(2)}(m_t(\mu_m);\mu_m,\mu_R,\mu_F;X) = \Big[\sigma^{(2)}(m;\mu_R,\mu_F;X) + m\Big(d^{(1)}(\mu_m)\,\partial_m\sigma^{(1)}(m;\mu_R,\mu_F;X) + \frac{1}{2}\left(d^{(1)}(\mu_m)\right)^2 m\,\partial_m^2\sigma^{(0)}(m;\mu_F;X) + d^{(2)}(\mu_m)\,\partial_m\sigma^{(0)}(m;\mu_F;X) + \beta_0\, d^{(1)}(\mu_m)\ln\left(\frac{\mu_R^2}{\mu_m^2}\right)\partial_m\sigma^{(0)}(m;\mu_F;X)\Big)\Big]_{m=m_t(\mu_m)}$$

The results depend on renormalization coefficients $d^{(k)}$, perturbative terms $\sigma^{(k)}$ of on-shell cross section and their mass derivatives $\partial_m^n \sigma^{(k)}$

Note that: the mass derivatives can be very sizeable and spoil the perturbative convergence of the $\overline{\text{MS}}$ cross section $\bar{\sigma}$ (see e.g. the invariant mass of $t\bar{t}$ pair close to its threshold region)

General expectations

at low orders, σ and $\bar{\sigma}$ can give consistent (within scale uncertainties) results (differences can be larger for observables close to kinematical thresholds for $t\bar{t}$ on-shell production)

at higher orders, σ and $\bar{\sigma}$ can be quantitatively very similar

→ equivalent perturbative description

Setup

Our results depend on 3 auxiliary scales $\mu_i = \{\mu_R, \mu_F, \mu_m\}$ independently varied by a factor of two around central μ_0 :

$\mu_i = \xi_i \mu_0$, $\xi_i = \{1/2, 1, 2\}$ with constraints $\mu_i/\mu_j \leq 2$

→ 15-point scale variation in $\overline{\text{MS}}$ scheme
(customary 7-point in pole scheme with 2 auxiliary scales)

We compare pole scheme and $\overline{\text{MS}}$ scheme by setting

- pole scheme: $M_t = 173.3$ GeV and use $\mu_0 = M_t$

- $\overline{\text{MS}}$ scheme: $\overline{m}_t = 163.7$ GeV (mass evolution at NNLO) and use $\mu_0 = \overline{m}_t$
  (varying μ_m with $0.5 < \mu_m/\mu_0 < 2 \longrightarrow 155 \text{ GeV} < m_t(\mu_m) < 173 \text{ GeV}$)

We use NNPDF31 and $\sqrt{s} = 13$ TeV

Results: total cross section

scheme	pole	$\overline{\text{MS}}$				
variation	7-point	15-point	$\mu_m = \mu_0$	$\mu_{R/F} = \mu_0$	$\mu_{R/F} = \mu_m$
LO (pb)	$478.9\,^{+29.6\%}_{-21.4\%}$	$625.7\,^{+29.4\%}_{-21.9\%}$	$^{+29.4\%}_{-21.3\%}$	$^{+24.7\%}_{-21.9\%}$	$^{+1.5\%}_{-1.5\%}$
NLO (pb)	$726.9\,^{+11.7\%}_{-11.9\%}$	$826.4\,^{+7.6\%}_{-9.7\%}$	$^{+7.6\%}_{-9.6\%}$	$^{+5.6\%}_{-9.7\%}$	$^{+1.2\%}_{-1.2\%}$
NNLO (pb)	$794.0\,^{+3.5\%}_{-5.7\%}$	$833.8\,^{+0.5\%}_{-3.1\%}$	$^{+0.4\%}_{-2.9\%}$	$^{+0.3\%}_{-3.1\%}$	$^{+0.0\%}_{-0.3\%}$

- order-by-order consistency of the results and very similar at NNLO
- $\overline{\text{MS}}$ typically higher at central scale and with smaller uncertainties at (N)NLO
  [μ_R and μ_m dependences have similar size but opposite sign (cancellations)]
- $\overline{\text{MS}}$ results have faster apparent convergence

$$\frac{\text{NLO}}{\text{LO}} = 1.52\ (\text{pole}),\ 1.32\ (\overline{\text{MS}}) \qquad \frac{\text{NNLO}}{\text{NLO}} = 1.09\ (\text{pole}),\ 1.01\ (\overline{\text{MS}})$$

first noticed by Langenfeld, Moch, Uwer (2009)

- Technical explanation: at LO the $\overline{\text{MS}}$ cross section is obtained by evaluating the pole cross section with $\bar{m}_t = 163.7$ GeV and is thus much larger than the pole cross section; at NLO there is a further negative effect from $\partial_m \sigma^{(0)}$

Results: total cross section

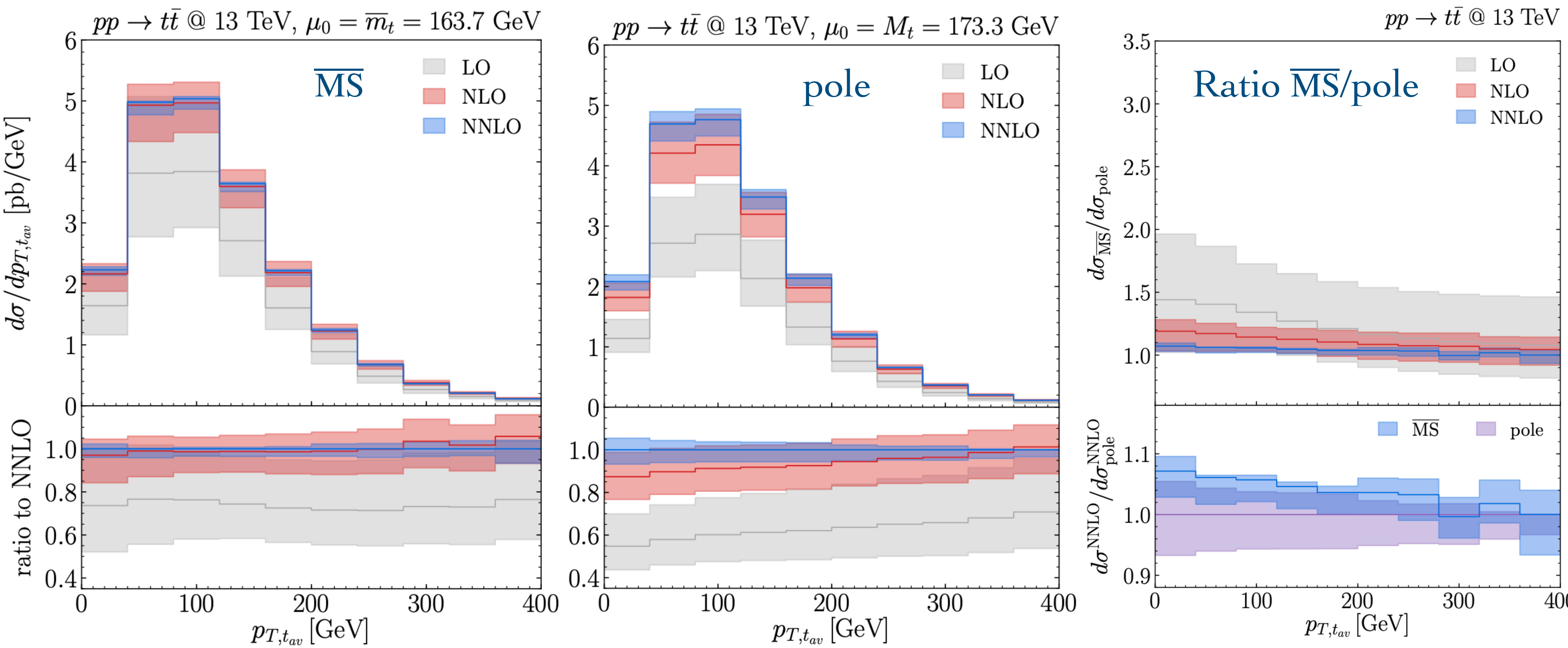
scheme	pole	$\overline{\text{MS}}$	$\overline{\text{MS}}$	pole
central scale choice	$\mu_{R/F} = M_t$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t/2$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t$	$\mu_{R/F} = M_t/2$
LO (pb)	478.9	488.9	625.7	619.8
NLO (pb)	726.9	746.4	826.4	811.4
NNLO (pb)	794.0	808.0	833.8	822.4

Such apparent convergence strongly depends on the choice of the central scale μ_0

- Slower: $\overline{\text{MS}}$ scheme ($\mu_{0,m} = \overline{m}_t/2$) and pole scheme ($\mu_0 = M_t$) behave similarly

- Faster: $\overline{\text{MS}}$ scheme ($\mu_{0,m} = \overline{m}_t$) and pole scheme ($\mu_0 = M_t/2$) behave similarly

Results: differential distributions



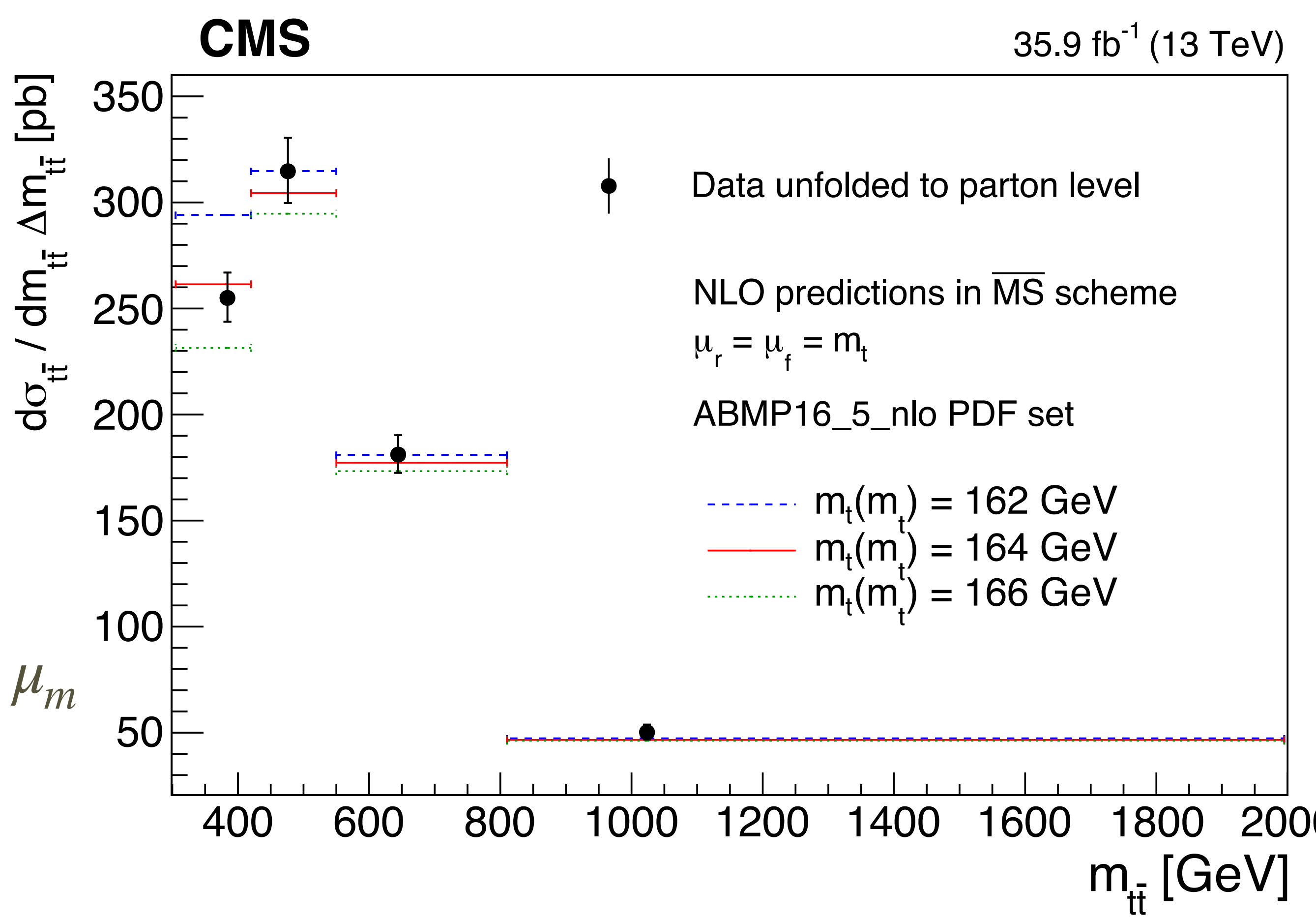
comparison pole scheme ($\mu_0 = M_t$) vs. $\overline{\text{MS}}$ scheme ($\mu_0 = \overline{m}_t$)

- overall features similar to those for total cross sections: at NNLO shape differences are quite small and within scale uncertainties

→ the results in the two schemes behave similarly at (sufficiently) high order

The invariant-mass distribution

CMS precisely measured the $m_{t\bar{t}}$ distribution and compare their data with NLO calculation with fixed $\overline{\text{MS}}$ mass $\bar{m}_t$
(i.e. $\mu_m = \bar{m}_t$ in all bins)
and fit value of $\bar{m}_t$ to data in each bin



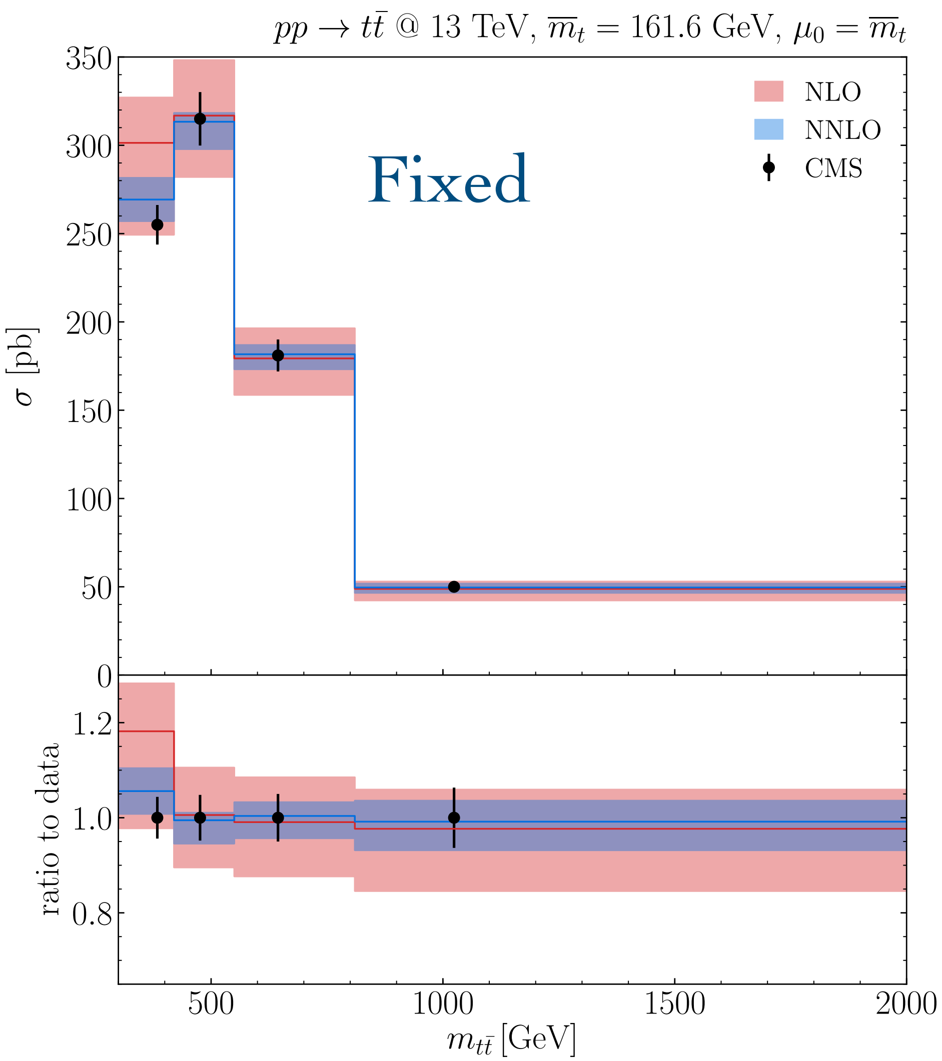
Studying running-mass effects requires using a running (bin-dependent) value of μ_m

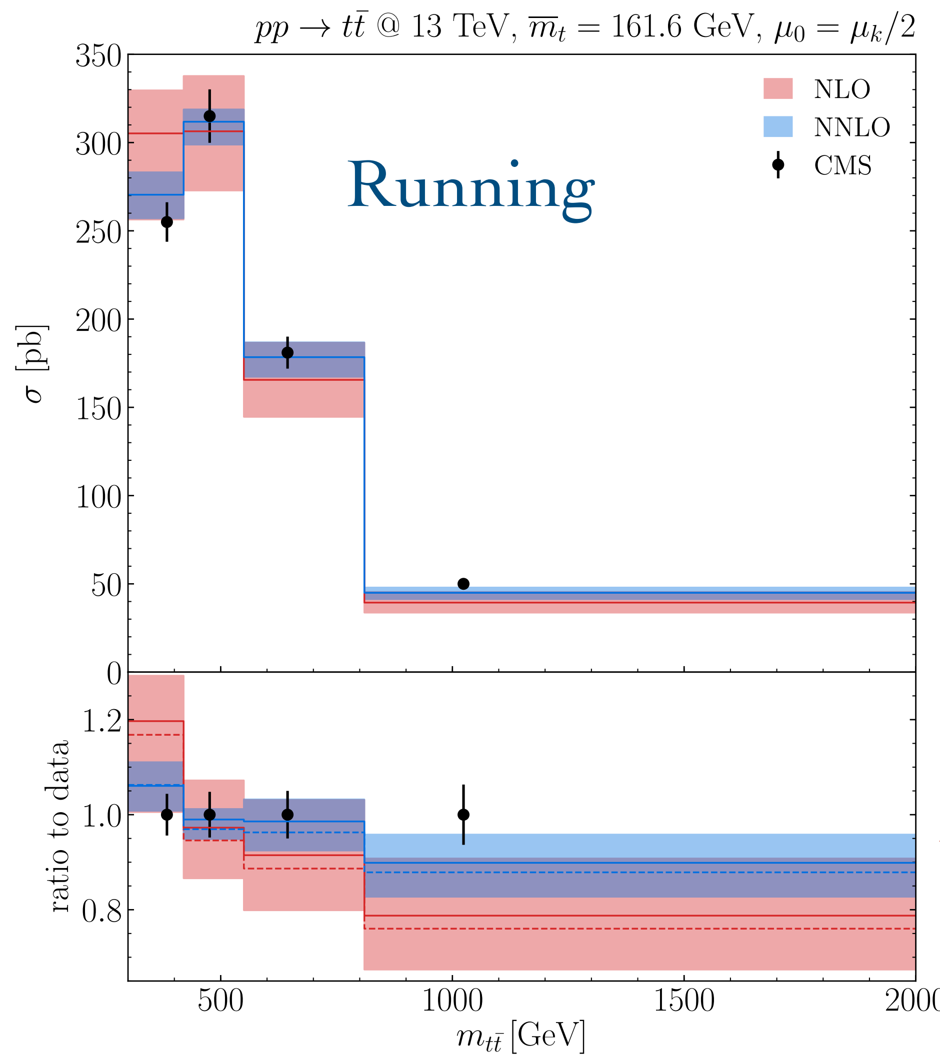
Two different options for central scale μ_0:

- FIXED mass : set $\mu_0 = \bar{m}_t$ (for μ_m, μ_R, μ_F)
  [NNLO extension of CMS NLO calculation]
- RUNNING mass : set $\mu_0 \simeq m_{t\bar{t}}/2$ (for μ_m, μ_R, μ_F)
  (i.e. $m_t(m_{t\bar{t}}/2)$ is bin-dependent and it varies by about 10 GeV :
  from $m_t \sim 160$ GeV in 1-st. bin $\rightarrow$ to $m_t \sim 150$ GeV in 4-th. bin)

Setup: ABMP16 PDFs (as done by CMS) and $\bar{m}_t = 161.6$ GeV as extracted at NNLO by CMS from the same data with the same PDFs

Running mass effects





$\mu_0 \simeq m_{t\bar{t}}/2$

dashed lines: results with $\mu_R = \mu_F \simeq m_{t\bar{t}}/2$ but keeping $\mu_m = \bar{m}_t$

- practically ("by definition") no theory differences at low $m_{t\bar{t}}$
- differences at high $m_{t\bar{t}}$ are small and mainly driven by running of α_S and PDFs

→ **NNLO corrections lead to reduced theoretical uncertainties and to an improved agreement with data but no significant sensitivity to running mass effects**

Note: very high invariant masses $m_{t\bar{t}} \gg M_t$ a resummation of soft and collinear effects would be needed

Ahrens et al (2010; Ferroglia et al (2012); Czakon et al (2018)

Summary

- We have presented a new computation of heavy-quark production at NNLO
- The calculation is carried out with the q_T subtraction formalism and it is the first complete application of the method for a colourful final states at NNLO
- The missing ingredient to apply q_T subtraction to this process are of purely soft origin and were computed with a semi analytical method
- First NNLO results for the inclusive cross section and multi differential distributions
- The inclusive results nicely agree with those obtained with Top++
- Excellent agreement with Czakon-Mitov-Heymes also at differential level

  → Absolutely non-trivial check given that the computations are carried out with two completely independent methods

- Nice description of parton level CMS data in the inclusive phase space

Summary

- We have extended our computation to consider the $\overline{\text{MS}}$ scheme for the top mass: this is obtained from a formal reorganisation of the perturbative expansion
- Perturbative predictions in such scheme depend on three scales: we have used a 15-point scale variations to assess perturbative uncertainties
- The $\overline{\text{MS}}$ results show an apparent faster convergence with respect to the results in the pole scheme: this is however strongly depends on the central scale choice
- Shape differences between the pole and $\overline{\text{MS}}$ scheme results are reduced by the inclusion of the high-order contributions, and they are quite small at NNLO
- First study of running mass effects ($m_t(\mu_m)$ with $\mu_m \sim m_{t\bar{t}}/2$) for the invariant-mass distribution of $t\bar{t}$ pair in region up to $m_{t\bar{t}} \sim 1$ TeV

→ No significant sensitivity to running mass effects