Anomalous dimensions from the S-matrix

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30th September 2020

Based on J. Elias-Miro, J. Ingoldby, MR; 2005.06983
XXth Century particle physics from a XXIst Century perspective:

- DM
- GUT
- Connection with String Theory
- Explain why EW symm. Broken
- CPV, Baryogenesis
- ...
- Why not?

Starting from the discovery of muon decay, path towards Higgs & beyond is straightforward:
Particle Physics is back to the origin, is again the exploration of the unknown.
EFT operators encode information about the heavy dynamics, and tells us in which way the SM is deformed.

\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda} \mathcal{O}_i \]

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While waiting for the next collider, we might get some hints from precision experiments.

- Flavour
- mu to e transitions (several orders of mag. improvement)
- EDMs
- ...

$L$?
While waiting for the next collider, we might get some hints from precision experiments.

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Current bounds already testing dynamics at several TeV even if only affect the dipole at two loops [Pomarol, Panico, MR]

Precision experiments might explore dynamics at two loops... but how to get this precision?

This talk is about a new way to compute anomalous dimensions, potentially reaching higher loops.

But before, I will exemplify the potential usefulness of this approach for the case of the electron EDM.
Evolution of electron EDM constraints

Current: ACME II \( |d_e| < 1.1 \cdot 10^{-29} \text{ e cm} \)
Translation of ACME constraints to particle physics:

\[ \frac{d_e}{e} \sim \frac{1}{(16\pi^2)^2} \frac{m_e}{\Lambda^2} \quad \rightarrow \quad \Lambda > 3 \text{ TeV} \]

Relevant constraints even at two loops.
We want to characterize all effects that enter with

\textbf{Two loops} \hspace{1cm} \textbf{Chirality flip} \hspace{1cm} \textbf{log enhanced}

This is the key to help organize the contributions

G. Panico, A. Pomarol, MR [1810.09413]
ACME-II implications for BSM:

**Fix** $\Lambda = 10$ TeV.

### tree-level
- $C_{eW} = 5.5 \times 10^{-5} y_e g$
- $C_{eB} = 5.5 \times 10^{-5} y_e g'$

### one-loop
- $C_{luqe} = 1.0 \times 10^{-3} y_e y_t$
- $C_{W\tilde{W}} = 4.7 \times 10^{-3} g^2$
- $C_{B\tilde{B}} = 5.2 \times 10^{-3} g' \cdot 2$
- $C_{W\tilde{B}} = 2.4 \times 10^{-3} g g'$
- $C_{\tilde{W}} = 6.4 \times 10^{-2} g^3$

### two-loops
- $C_{lequ} = 3.8 \times 10^{-2} y_e y_t$
- $C_{\tau W} = 260 y_\tau g$
- $C_{\tau B} = 380 y_\tau g'$
- $C_{tW} = 6.9 \times 10^{-3} y_t g$
- $C_{tB} = 1.2 \times 10^{-2} y_t g'$
- $C_{bW} = 64 y_b g$
- $C_{bB} = 47 y_b g'$
- $C_{\text{le}\tilde{d}q} = 10 y_e y_t (y_t/y_b)$
- $C_{\text{le}\tilde{e}\tilde{l}'} = 0.63 y_e y_t (y_t/y_\tau)$

### two-loops finite
- $C_{ye} = 14 y_e \lambda_h$
- $C_{yt} = 14 y_t \lambda_h$
- $C_{yb} = 2.9 \times 10^3 y_b \lambda_h$
- $C_{y_\tau} = 3.1 \times 10^4 y_\tau \lambda_h$

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G. Panico, A. Pomarol, MR [1810.09413]
- After some time of promises of improvements with nothing happening, it seems that there will be further progress in a short time scale.
- If there is a positive signal, we'll have confirmation very quickly.
- There are some proposals for a total breakthrough.
So two-loop RGE relevant for the electron EDM.

Might be other experiments are also sensitive to some two-loop RGEs

Only if there was a way to compute anomalous dimensions easily...
SMEFT anomalous dimensions from the S-matrix

Joan Elias Miró, James Ingoldby, MR [2005.06983]
Form Factors of specific operators give the response of a given state once we insert that operator.

\[ F_{O_i} = \langle p_1, p_2, p_3, p_4 | O_i | 0 \rangle \overset{e.g.}{=} \delta_{ab} (s + t) \]

\[ p_i^2 = 0 \]

\[ \sum_i p_i \neq 0 \]
They receive, of course, loop corrections.

\[ F_{\mathcal{O}_i} = \left\langle p_1, p_2, p_3, p_4 | \mathcal{O}_i | 0 \right\rangle \overset{\text{e.g.}}{=} \delta_{ab} (s + t) \left( 1 + c_j \frac{g^2}{16\pi^2} \log \frac{s}{\Lambda} \right) \]

\[ p_i^2 = 0 \]

\[ \sum_i p_i \neq 0 \]
In perturbation theory (and perhaps beyond), a Form Factor is related across both sides of the cut by the **reality condition**

\[ F(s_{ij} + i\epsilon) = F^*(s_{ij} - i\epsilon) \]

This is generated by a complex rotation of the momenta,

\[ F = e^{-i\pi \sum_i p_i \frac{\partial}{\partial p_i}} F^* = e^{-i\pi D} F^* \]

On the other hand, **unitarity** implies

\[ F = \sum_{\text{out}} \langle \alpha | \mathcal{O} | 0 \rangle = \sum_{\text{in}} \langle \alpha | \beta \rangle \sum_{\text{in}} \langle \beta | \mathcal{O} | 0 \rangle = S F^* \]

So,

\[ e^{-i\pi D} F^* = S F^* \]

(This is a generalization of the Watson equation)

The dilatation operator is proportional to the phase of the S–matrix

see [Caron-Huot, Wilhelm ‘16] for alternative derivation
\[ e^{-i\pi D} F = SF \]

- Dilatation operator related to anomalous dimensions by RG equation:
  \[ DF \sim \mu \frac{\partial}{\partial \mu} F \sim (\gamma_{UV} - \gamma_{IR} + \beta(g^2) \frac{\partial}{\partial g})F \]

- Convolution of FF with S-matrix:
  \[ S = 1 + i\mathcal{M} \]

At LO, dependence on beta ignored,

\[ (\gamma_{UV} - \gamma_{IR}) \langle \alpha | \mathcal{O} | 0 \rangle = -\frac{1}{\pi} \langle \alpha | \mathcal{M} \otimes \mathcal{O} | 0 \rangle \]

For this talk, we focus on elements with no IR divergences

\[ \gamma_{i \leftarrow j} = -\frac{1}{\pi} \frac{\langle \alpha | \mathcal{M} \otimes \mathcal{O}_j | 0 \rangle}{\langle \alpha | \mathcal{O}_i | 0 \rangle} \]
Example 1: Self renormalization of \( \mathcal{O}_{FF} = |H|^2 F_{\mu\nu}^2 \)

\[
\langle \alpha | \mathcal{M} \otimes \mathcal{O}_{FF} | 0 \rangle = \]

\[
\langle 1_i 2^*_j | \mathcal{M} | 3_k 4^*_l \rangle = 2\lambda (\delta_{ij} \delta_{kl} + \delta_{il} \delta_{kj})
\]

\[
\langle 1_{\phi_1} 2_{\phi_2} 3^{-} 4^{-} | H^\dagger H F_{\mu\nu} F_{\mu\nu} | 0 \rangle = 2\delta_{ij} \langle 34 \rangle^2
\]

\[
= \int [dp_{1'}][dp_{2'}] 4\lambda \langle 34 \rangle^2 (\delta_{ij} \delta_{i'j'} + \delta_{ii'} \delta_{jj'}) \delta_{i'j'}
\]

\[
= (2\delta_{ij} \langle 34 \rangle^2) 4\lambda (n_s + 1) \int [dp_{1'}][dp_{2'}]
\]

\[
= (2\delta_{ij} \langle 34 \rangle^2) 4\lambda (n_s + 1) \frac{1}{16\pi}
\]

So, \( \gamma_{FF \leftrightarrow FF} = \frac{\lambda}{16\pi^2} 4(n_s + 1) \)

next slide I'll tell you. Not important now.
Phase space integral:

The previous integral was trivial, but in general it is not. It will be useful to write it as

\[ p_\mu \sigma_{ab}^\mu = p_{ab} = -|p|_a \langle p |_b, \quad |p\rangle^\Lambda = \sqrt{2E} \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} \]

Spinor notation for massless momenta:

See [Elvang-Huang] for a review

The important point is that angle and square brackets carry opposite little group weight, or helicity.

Example:

\[ \bar{f} \gamma_\mu f \phi \rightarrow \bar{u}_\pm(p_1) (\not{p}_3 - \not{p}_4) v_\mp(p_2) \]

\[ \begin{array}{c}
\begin{array}{c}
+ - \\
- +
\end{array}
\end{array} \rightarrow \begin{array}{c}
\begin{array}{c}
[13] \langle 32 \rangle + [14] \langle 42 \rangle \\
\langle 13 \rangle [32] + \langle 14 \rangle [42]
\end{array}
\end{array} \]

Phase space integral:

The previous integral was trivial, but in general it is not. It will be useful to write it as

\[ \int [dp][dq] = \frac{1}{16\pi} \int_0^\pi 2 \cos \theta \sin \theta d\theta \int_0^{2\pi} \frac{d\phi}{2\pi} \]

The angles parametrize the rotation to base spinors,

\[ \begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta e^{i\phi} \\ s_\theta e^{-i\phi} & c_\theta \end{pmatrix} \begin{pmatrix} |p\rangle \\ |q\rangle \end{pmatrix} \]
Example 2:

4-fermions

- There are two fundamentally different types of 4-fermions, with and without net helicity

\((\bar{\ell}\gamma_{\mu}\ell)(\bar{\ell}\gamma_{\mu}\ell) = \langle 1|\gamma_{\mu}|2\rangle\langle 3|\gamma_{\mu}|4\rangle = 2 \cdot \langle 13\rangle[24]\)

\((\bar{\ell}u)(\bar{q}e) = \langle 12\rangle\langle 34\rangle\)

- We'll compute anomalous dimensions of the second type due to a \(U(1)\):

\[ \mathcal{A}(f^- f^+ f^- f^+) = g^2\langle 13\rangle[24]\left(\frac{1}{s} + \frac{1}{u}\right) \]

- Only two flavour structures will be independent due to Schouten identity:

\[ 0 = \langle \ell u\rangle\langle q e\rangle + \langle \ell q\rangle\langle e u\rangle + \langle \ell e\rangle\langle u q\rangle \]

\[
\gamma = \sum_{f_1 f_2} \langle f_1 f_2|\mathcal{M}|x y\rangle\langle f_3 f_4 x y|\mathcal{O}_{l e q u}\rangle|0\rangle
\]

\[
= \int d\Omega g^2 \left( Y_{\ell} Y_u \frac{\langle \ell u\rangle[x y]\langle x e\rangle[q y]}{s_{l x}} + Y_e Y_q \frac{\langle e q\rangle[x y]\langle l q\rangle[\ell u]}{s_{e x}} + \cdots \right)
\]

\[
= \int d\Omega g^2 \left( Y_{\ell} Y_u + Y_{\ell} Y_q + Y_e Y_u + Y_e Y_q \right) \langle \ell u\rangle\langle q e\rangle + \langle \ell e\rangle\langle q u\rangle \cdots
\]

\[
= \frac{g^2}{16\pi^2} (Y_{\ell} + Y_e) (Y_q + Y_u) \langle \ell u\rangle\langle q e\rangle
\]
Non-renormalization theorems

This language trivializes the non-renormalization theorems of

\[ \text{[Alonso, Jenkins, Manohar]} \]
\[ \text{[Elias Miro, Espinosa, Pomarol]} \]
\[ \text{[Cheung, Shen]} \]

See also \[ \text{[Bern, Parra-Martinez, Sawyer]} \]

The spinor language naturally splits the dim-6 operators into 5 categories:

\[
\left\langle \cdot \right\rangle^3, \quad \left\langle \cdot \right\rangle^2, \quad \left\langle \cdot \right\rangle, \quad 1 \quad \text{and} \quad \left\langle \cdot \right\rangle[\cdot]
\]

<table>
<thead>
<tr>
<th>Operator</th>
<th>Operator Expression</th>
<th>MFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_{3F} )</td>
<td>( \frac{f^{ABC}}{23!} F_{A \nu}^\mu F_{B \rho}^\nu \tilde{F}_{C \mu}^\rho )</td>
<td>( F_3(1_{0}^{-2} 2^{-}\gamma_{3}^{3} 3_{C}^{\gamma}) )</td>
</tr>
<tr>
<td>( O_{FF} )</td>
<td>( \frac{1}{2} H^\dagger H F_{A \mu}^\nu F_{A \mu}^\nu )</td>
<td>( F_{FF}(1_{1}^{-2} 2^{-3} 3_{4}^{4} 3_{B}^{4}) )</td>
</tr>
<tr>
<td>( O_{qF} )</td>
<td>( \tilde{Q} \sigma^{\mu \nu} T^{A} q H F_{\mu \nu}^A )</td>
<td>( F_{qF}(1_{1}^{-2} 2^{-3} 3_{4}^{4} 3_{A}^{4}) )</td>
</tr>
<tr>
<td>( O_{4F_1} )</td>
<td>( (\bar{Q} i u) \epsilon_{ij}(\bar{Q} j d) )</td>
<td>( F_{4F_1}(1_{1}^{-2} 2^{-3} 3_{4}^{4} 3_{A}^{4}) )</td>
</tr>
<tr>
<td>( O_y )</td>
<td>(</td>
<td>H</td>
</tr>
<tr>
<td>( O_6 )</td>
<td>(</td>
<td>H</td>
</tr>
<tr>
<td>( O_{4F_2} )</td>
<td>( (\bar{Q} T^{A} \gamma^\mu Q)(\bar{Q} T^{A} \gamma_\mu Q) )</td>
<td>( F_{4F_2}(1_{1}^{-2} 3_{4}^{4} 3_{5}^{5} 6_{f}^{6}) )</td>
</tr>
<tr>
<td>( O_{QH} )</td>
<td>( (\bar{Q} T^{A} \gamma^\mu Q)(i H^\dagger T^{A} D_\mu H) )</td>
<td>( F_{QH}(1_{1}^{-2} 3_{4}^{4} 3_{5}^{5} 6_{f}^{6}) )</td>
</tr>
<tr>
<td>( O_\perp )</td>
<td>( (H^\dagger D_\mu H)(D^\mu H)^\dagger H )</td>
<td>( F_\perp(1_{1}^{-2} 3_{4}^{4} 3_{5}^{5} 6_{f}^{6}) )</td>
</tr>
<tr>
<td>( O_\parallel )</td>
<td>(</td>
<td>H</td>
</tr>
</tbody>
</table>

Similar to \[ \text{[Henning, Melia]} \]
Non-renormalization theorems

One loop structure:

A two-particle cut removes two legs of the operator and the S-matrix insertion adds, at least, two legs. So operators can only renormalize operators with the same number of legs or more.

Between the two 4-particle class (and from $F^3$ to $<>[]$) there is another effect. Transitions between them require 4-particle amplitudes to violate helicity, but there is only one of them in the SM.

The only non-vanishing transition is between 4-fermion operators.

[Azatov, Contino, Machado, Riva]
Non-renormalization theorems

Two loop structure:

At two loops, RGEs can shorten operator legs by one, or avoid helicity selection rules. The former type are particularly easy to compute with this method!
Example 3: A 2–loop example, Yukawa to dipole

Two types of diagrams, involving 3–particle cut and 5–point amplitudes:

This will be zero so forget it

The entire difficulty of this calculation is to write down the 5–point amplitude in a simple way so that the integral is easily doable. Let’s focus on the pure gauge part:

\[
\mathcal{A}(f^- f^+ \phi \phi \gamma^-) = g^3 \left( Q_f Q_\phi^2 \frac{[23][24]}{[12][35][45]} - Q_f^2 Q_\phi \frac{[23][24]}{[15][25][34]} \right)
\]
Example 3: A 2–loop example, Yukawa to dipole

\[ A(f^−f^+ϕϕγ^−) = g^3 \left( Q_f Q_ϕ^2 \frac{[23][24]}{[12][35][45]} - Q_f^2 Q_ϕ \frac{[23][24]}{[15][25][34]} \right) \]

Since integral symmetric under 3, 4 exchange, and this term is odd

\[ \text{integral} = s_{14} \int dμ \frac{[xy][xz]}{[1x][y4][z4]} \langle x2 \rangle = \langle 14 \rangle \langle 42 \rangle \cdot N \]

with

\[ N = \int_0^{\pi/2} 2s_1 c_θ_1 dθ_1 \int_0^{\pi/2} 4s_2^3 c_θ_2 dθ_2 \int_0^{\pi/2} 2s_3 c_θ_3 dθ_3 \frac{c_θ_3^2}{s_2^2} = 1 \]

Adding the flavour structure, one gets the result in the literature,

\[ \frac{d}{d \ln μ} \left( \frac{C_{eB}}{C_{eW}} \right) = \frac{g^3}{(16π^2)^2} \frac{3}{4} \left( t_{θ_w} Y_H + 4t_{θ_w}^3 Y_H^2 (Y_L + Y_e) \right) \frac{1}{2 + \frac{2}{3} t_{θ_w}^2 Y_H (Y_L + Y_e)} C_{y_e} \]

from [Pomarol, Panico, MR]
Example 3:

My brain is not a standard candle, but to get an idea...

The Verdict

One month+ of struggle, of sign-chasing, of looking for factors of two and comparing with collaborators...

VS

One day to get the amplitude in a nice form. 30min of writing the rotation and do the integrals in Mathematica.
We computed all the length-shortening transitions.

Since they are the leading contribution, they are a single log and in this method they are especially simple: a tree amplitude and a tree FF with a 3-particle cut.
Last but not least, an important aspect of the method is that it is GREEN

\[\psi^2 \phi F \leftarrow F^3\]

\[\phi^2 F^2 \leftarrow \psi^2 \phi^3\]

\[F^3 \leftarrow \psi^2 \phi F\]

Same amplitude appears in several computations, in this case

\[\mathcal{A}(f^- f^- \phi g^+ g^+ )\]
Electron EDM searches are not the only low energy probes to get a boost in the next years,

As you all know much better than me, muon to electron conversion searches (+photon, +ee, conversion in hadrons) are expected to improve by orders of magnitude.

\[ \text{Br}(\mu \to e\gamma) < 6 \cdot 10^{-14} \implies \Lambda \geq 10\text{TeV}, \text{ for } \mathcal{L} \supset \frac{\sqrt{y_e y_\mu}}{\Lambda^2} \left( \frac{g^4}{(16\pi^2)^2 \log \frac{m_\omega^2}{\Lambda^2}} \right) \bar{\mu} \sigma_{\mu\nu} e H F_{\mu\nu} \]
The full map of two-loop effects for mu to e transition experiments seem to look like this:

Would be great to learn if there are new effects compared with previous literature, e.g. A. Crivellin et al [1702.03020]
Conclusions:

- Unitarity and analyticity give us new perspectives on the computation of anomalous dimensions.

- The method is particularly useful for computing a set of two-loop transitions.

- Two loop precision is required to correctly interpret the electron EDM constraints, and might be useful for other future experiments.

Thank you!