

Can μ SR access the phase of the longitudinal
spin density wave - a parameter inaccessible by
neutron diffraction?

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Elastic scattering of polarized neutrons

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2 \hbar^4} \text{Sp}\{\rho_\sigma V^\dagger V\} = W_1 + W_2 + W_3 + W_4$$

$$V = V_{\text{nucl}} + V_{\text{mag}}$$

Pure nuclear scattering

Cd

$$W_1 \propto \delta(\vec{q} - \vec{\tau}) \left| \sum_j A_j \exp(i \vec{q} \cdot \vec{\rho}_j) \right|^2$$

Magnetic non-polarized scattering

$$W_2 \propto \left| \sum_v P_v(q) e^{i \vec{q} \cdot \vec{R}_v} \vec{M}_v \right|^2 \propto \left| \sum_{jn} \vec{Q}_K P_j(q) e^{i(\vec{q} + \vec{K}) \cdot \vec{r}_n} e^{i(\vec{q} + \vec{K}) \cdot \vec{\rho}_j} e^{i\varphi} \right|^2$$

$$\vec{M}_v = \vec{S}_v - (\vec{e} \cdot \vec{S}_v) \vec{e} \quad \propto \delta(\vec{q} - (\vec{\tau} + \vec{K})) \sum_j Q_K^2 P_j(q) e^{i(\vec{q} + \vec{K}) \cdot \vec{\rho}_j}$$

$$\vec{S}_v = \sum_{\vec{K}} \vec{Q}_K e^{i \vec{K} \cdot \vec{R}_v + i\varphi}$$

Nucl/magnetic interference

$$W_3 \propto \sum_{v,l} A_l P_v(q) e^{-i \vec{q} \cdot \vec{R}_l} e^{i \vec{q} \cdot \vec{R}_v} (\vec{M}_v \cdot \vec{p}_0) \propto \sum_{v,l} A_l P_v(q) (\vec{Q}_K \cdot \vec{p}_0) e^{-i \vec{q} \cdot \vec{R}_l} e^{i(\vec{q} + \vec{K}) \cdot \vec{R}_v} e^{i\varphi}$$

$\vec{Q}_K \cdot \vec{p}_0 = 0$
 $k \neq 0$

Additional pure magnetic

$$W_4 \propto \sum_{vv'} P_v(q) P_{v'}(q) e^{-i \vec{q} \cdot \vec{R}_v} e^{i \vec{q} \cdot \vec{R}_{v'}} \left[\vec{M}_v \times \vec{M}_{v'} \right] \cdot \vec{p}_0$$

Scattering of non-polarized neutrons

$$R = a \pm f_m$$

Scattering amplitude Nuclear polarization Magnetic

$$\frac{d\sigma}{d\Omega} = R^2 = a^2 + f^2 \pm 2af_m$$

0

(17) $I_{\vec{k}=\vec{\tau}+\vec{K}} = \frac{(p_0 \langle \mu \rangle)^2}{4} \left| \sum_{\nu} f_{\nu}(\vec{k}) e^{\pm i \phi_{\nu}} e^{i \vec{\tau} \cdot \vec{r}_{\nu}} \right|^2 [1 + (\hat{\vec{k}} \cdot \hat{\vec{\zeta}})^2]$. Thus the spiral intensity varies with the factor $1 + \cos^2 \psi$, where ψ is the angle between the reflecting plane and the planes of magnetic moments.

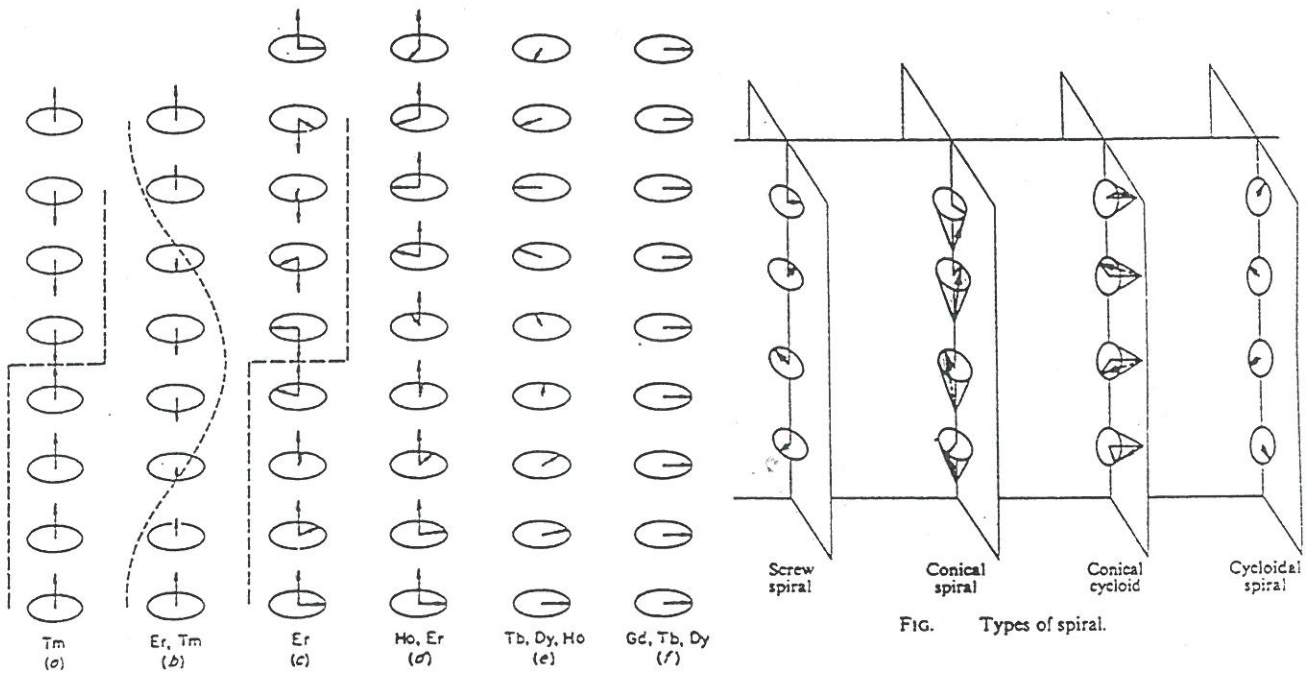


Fig. Magnetic structures of heavy lanthanide metals [after Koehler (1972)].

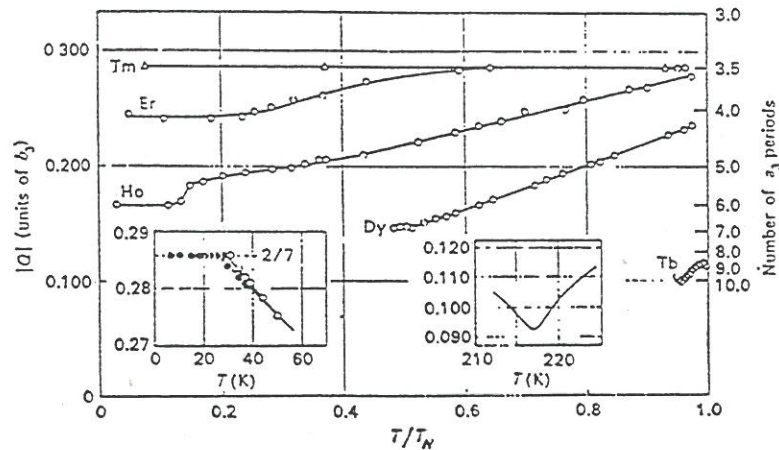


Fig. Variation of q_m with reduced temperature for the heavy rare earths. Inserts show details of data for Tm (Brun et al., 1970) and Tb (Dietrich and Als-Nielsen, 1967) on left and right hand sides respectively [after Koehler (1972)].

Ternary intermetallic compounds e.g.

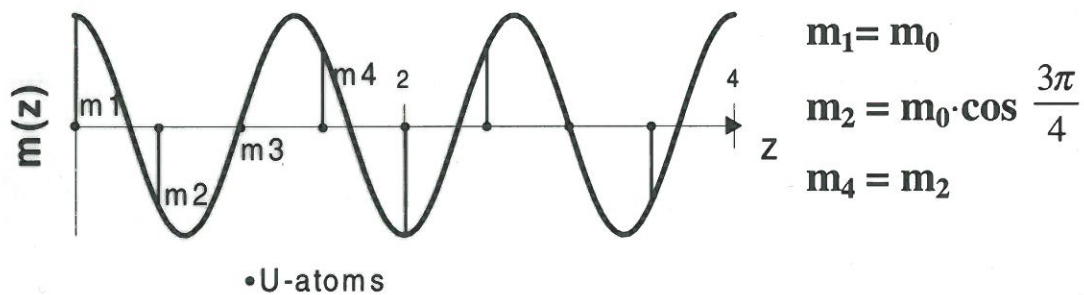
UNi₂Si₂ *H.Lin, et al PRB43, 12323, (1991)*

UPd₂Ge₂ *G.Andre, et al '96 SSC97, 923, (1996)*

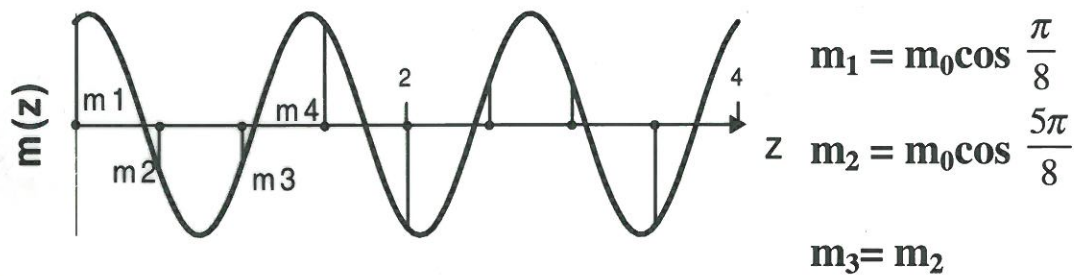
LSDW $m_U \parallel c$

$$m_U = m_0 \cos(2\pi kz + \varphi) \quad k=3/4$$

1. $\varphi = 0$



2. $\varphi = -\frac{\pi}{8}$



More sophisticated examples *J.Rodriguez-Carvajal,*

CsMnF₄, TbMn₆Ge₆

two - $\vec{k}$: TbGe₃

Modulated structures

- ◆ Spiral structures

$$\mathbf{m}(\mathbf{r}) = m_0 [\zeta \cos(2\pi \mathbf{k} \mathbf{r} + \varphi) + \eta \overset{\text{sin}}{\cos}(2\pi \mathbf{k} \mathbf{r} + \varphi)],$$

- ◆ Spin density wave (SDW)

$$\mathbf{m}(\mathbf{r}) = \mathbf{m}_0 \cos(2\pi \mathbf{k} \mathbf{r} + \varphi)$$

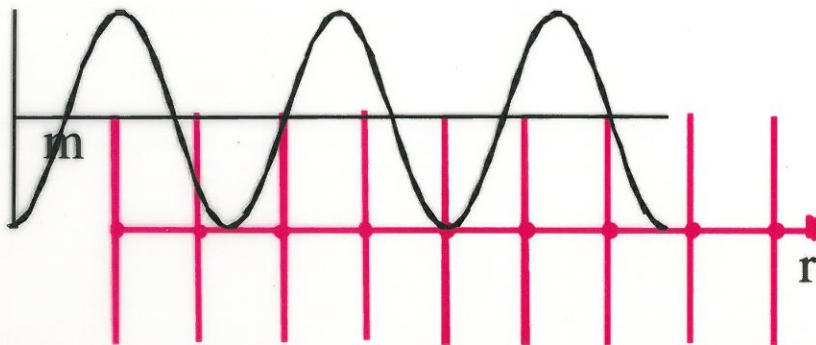
e.g. transverse SDW $\mathbf{m}_0 \perp \mathbf{k}$,
or longitudinal SDW $\mathbf{m}_0 \parallel \mathbf{k}$

- Incommensurate $k \neq (m/n)$:

φ has no sense

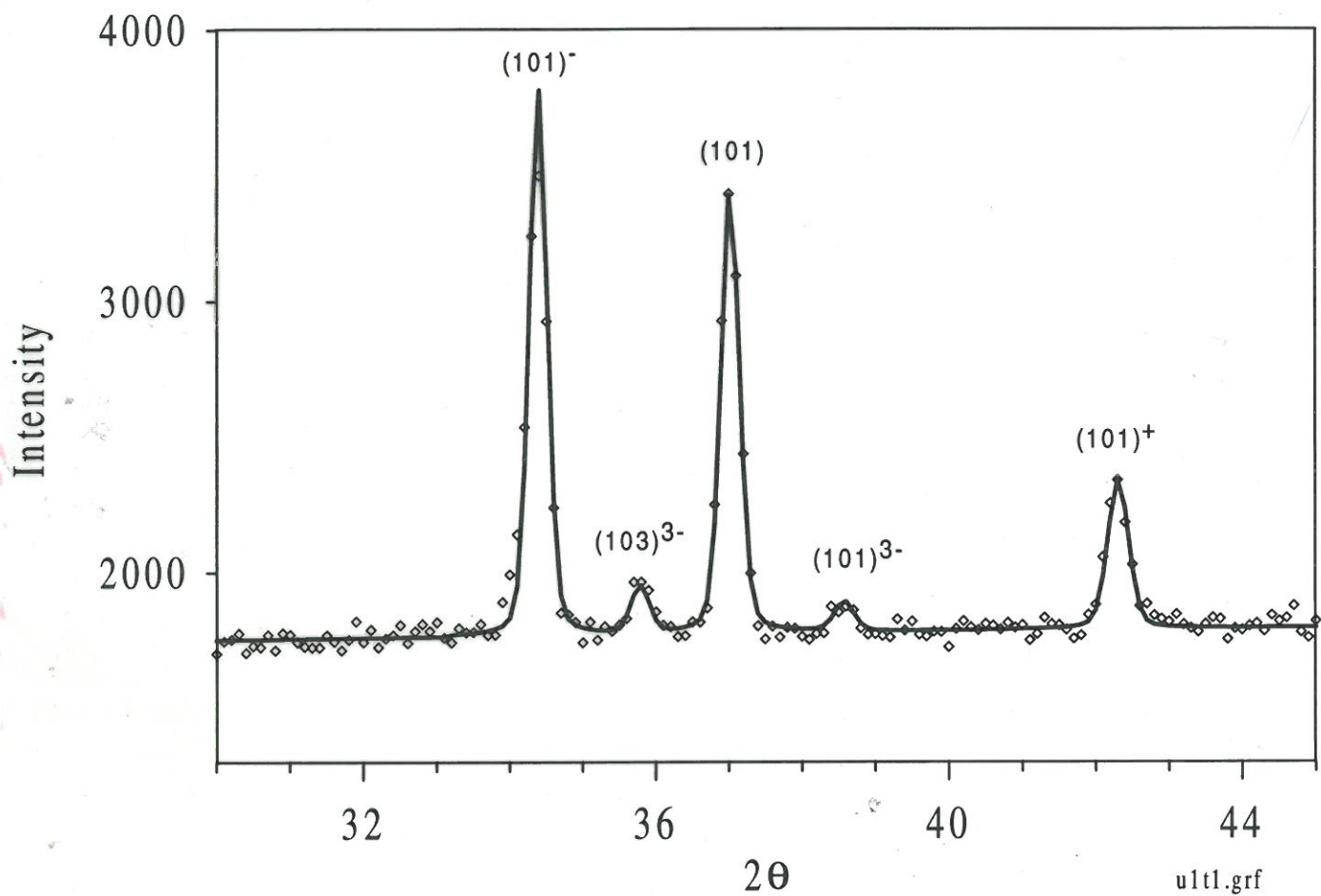
- Commensurate SDW with rational $k = (m/n)$:

φ affects the true magnetic moments on the atoms!



$\text{U}(\text{Pd}_{0.99}\text{Fe}_{0.01})_2\text{Ge}_2$, $T=1.5\text{K}$

LLB, G4.1, $\lambda=2.43\text{ \AA}$

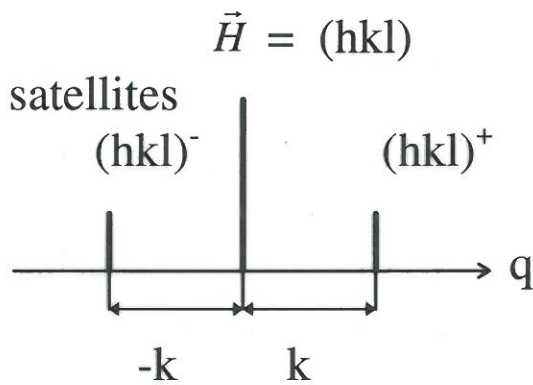


$$\vec{m}(\vec{R}) = 1/2 \sum_{(\vec{k}, -\vec{k})} e^{-(i\vec{k}\vec{R} + \varphi_k)} \vec{m}_{\vec{k}}$$

$$= \sum_{\vec{k}} \cos(\vec{k}\vec{R} + \varphi_k) \cdot \vec{m}_{\vec{k}}$$

$$\frac{dr_0}{d\Omega} \sim \left[m_k^2 - \left(\frac{\vec{q}}{q} \vec{m}_k \right)^2 \right] \cdot \delta(\vec{H} + \vec{k} - \vec{q}) + \dots \delta(\vec{H} - \vec{k} - \vec{q})$$

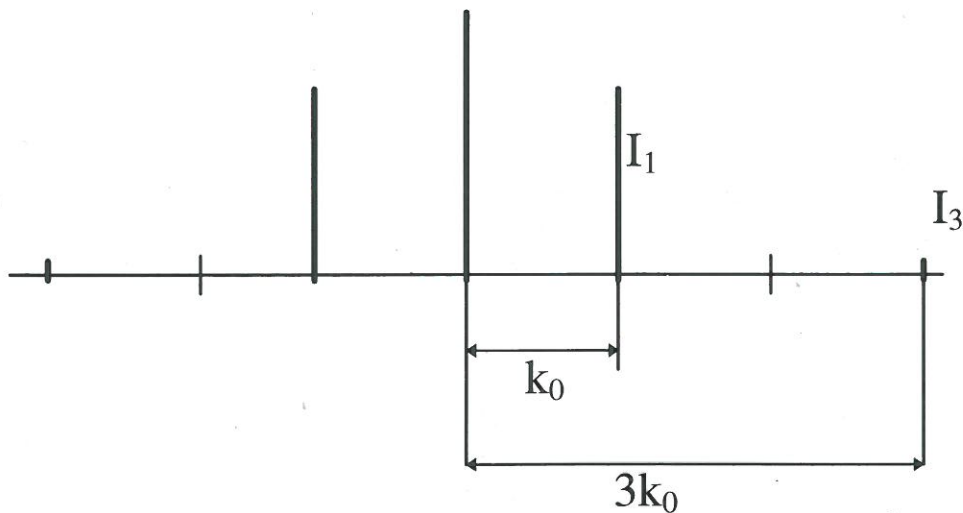
1. cos - modulation

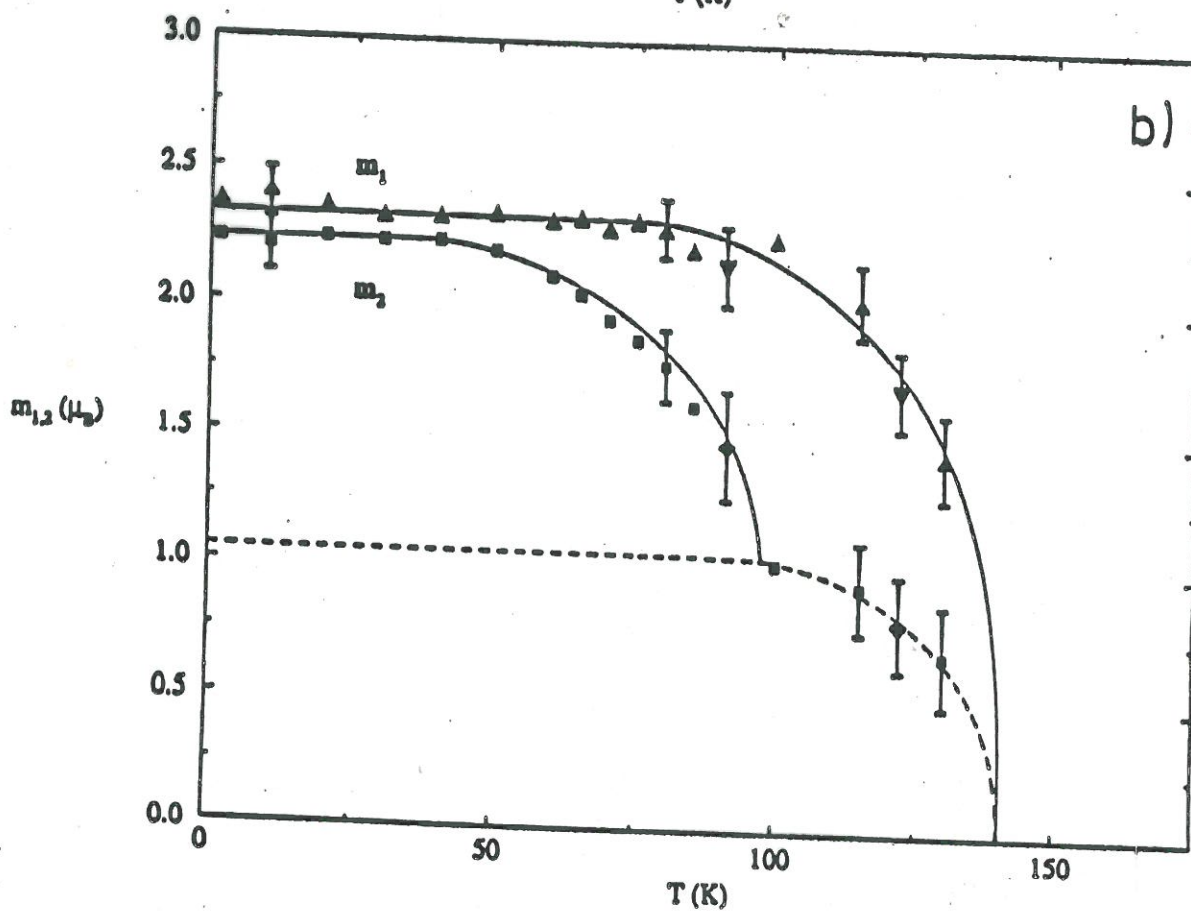
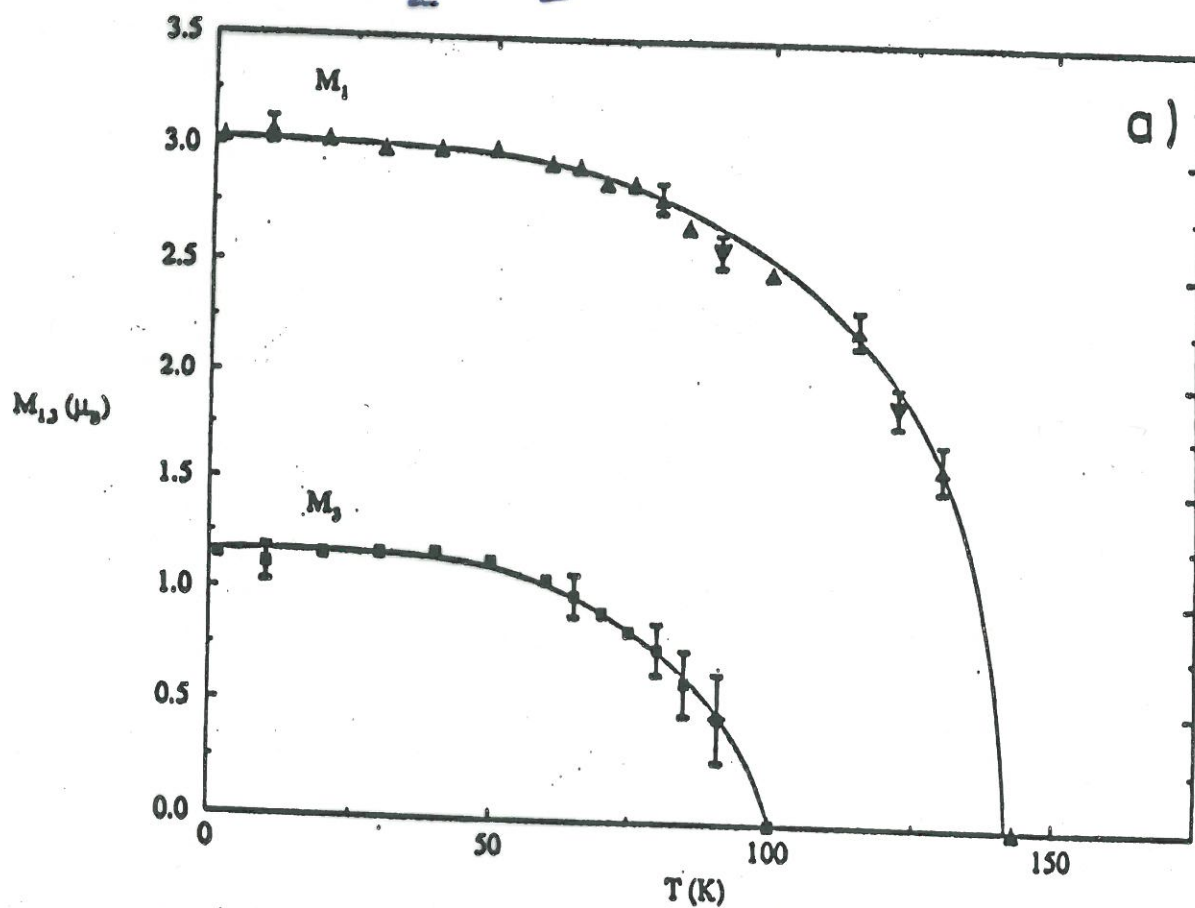


2.

$$= \frac{4}{\pi} \left[\cos k_0 x - \frac{1}{3} \cos 3k_0 x + \frac{1}{5} \dots \right]$$

k_0^{-1}





G. André, F. Bourée, et al, SSC 97, 923, (1996)

μ^+SR

cyclotron

≥ 500 MeV

protons

PROD. TGT.

ν_μ

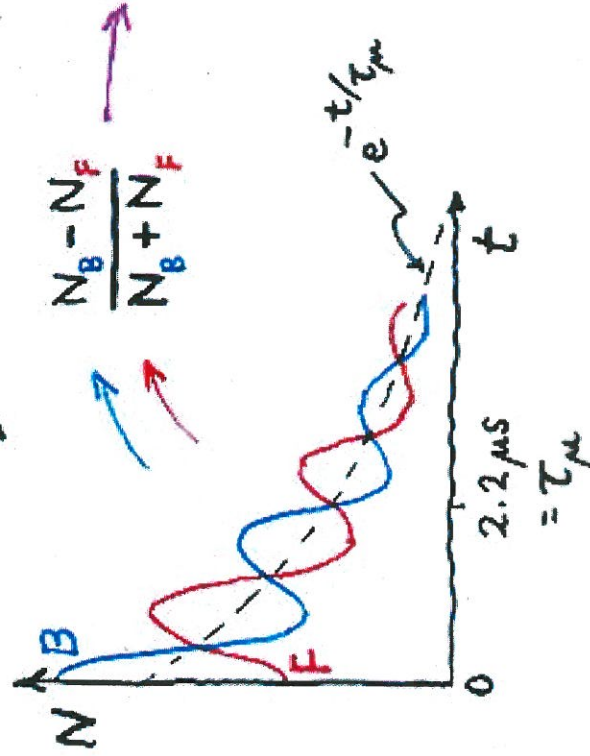
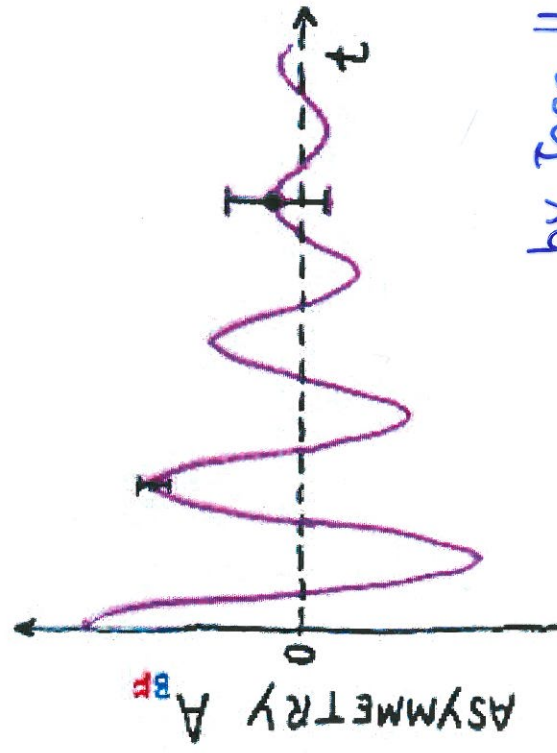
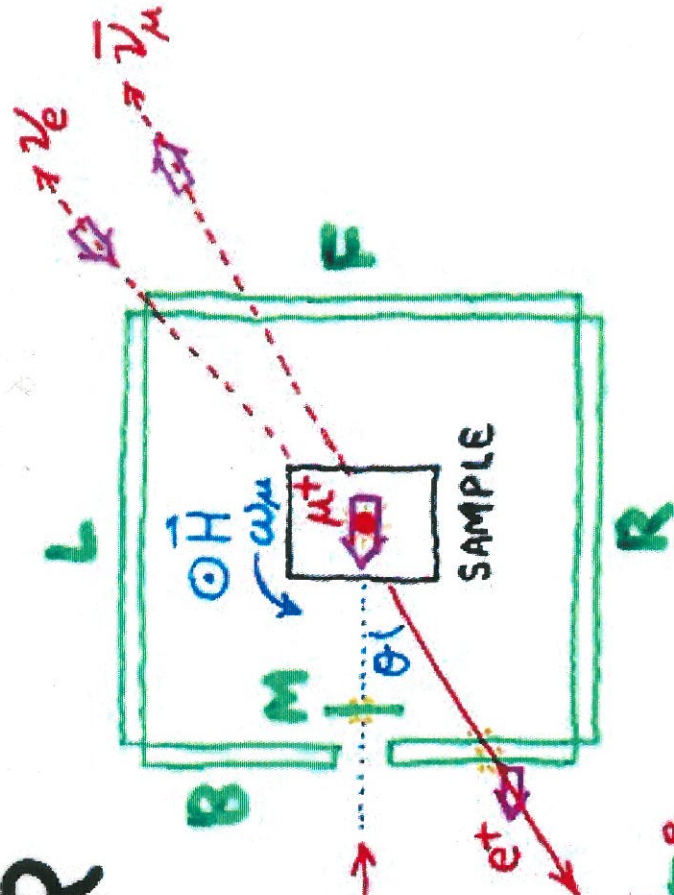
MUON

BEAMLINE

"CLOCK"

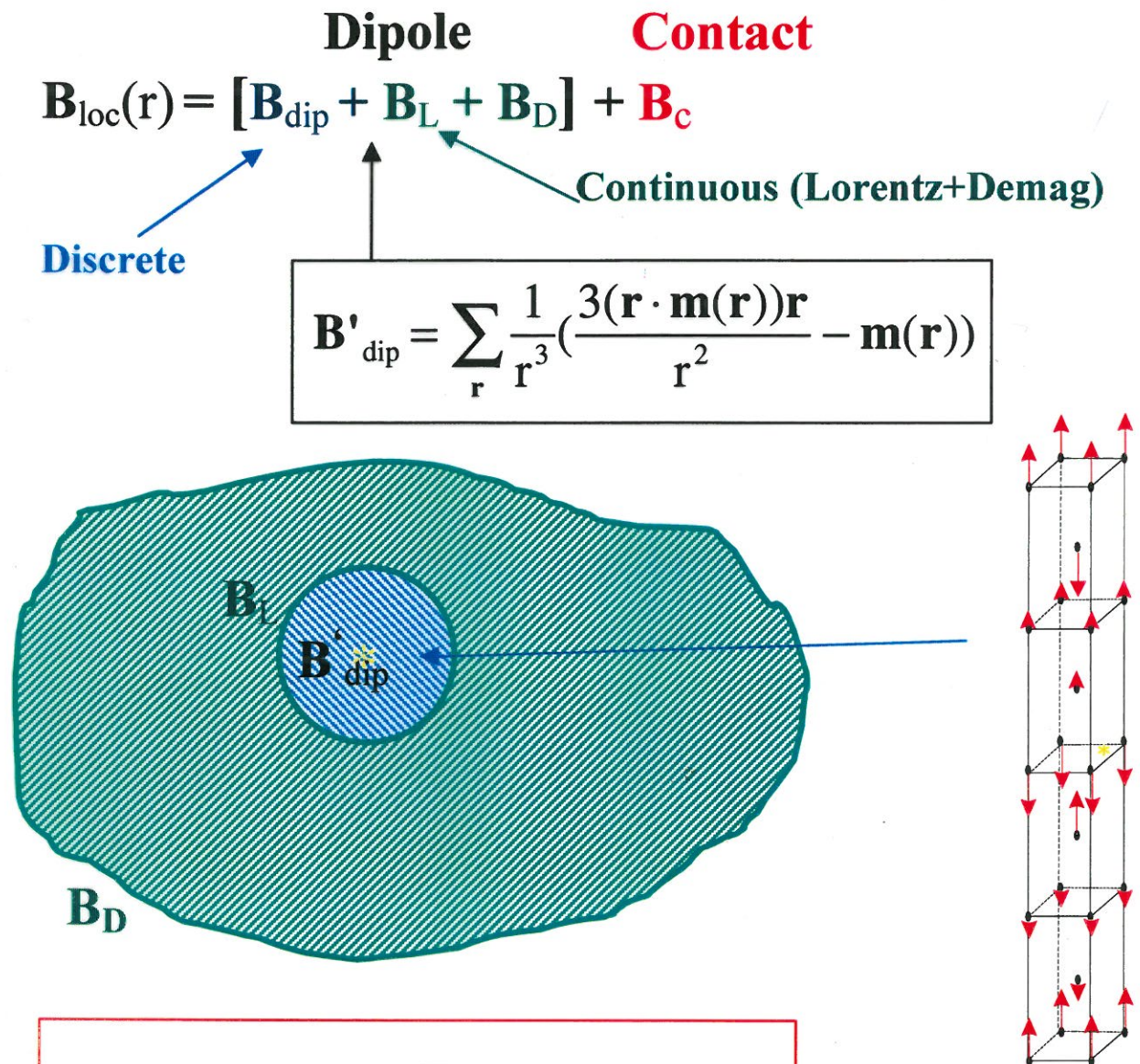
$\mu \rightarrow$ START

$e^- \rightarrow$ STOP



by Jess H. Brewer

Local magnetic field in an (anti)ferromagnet



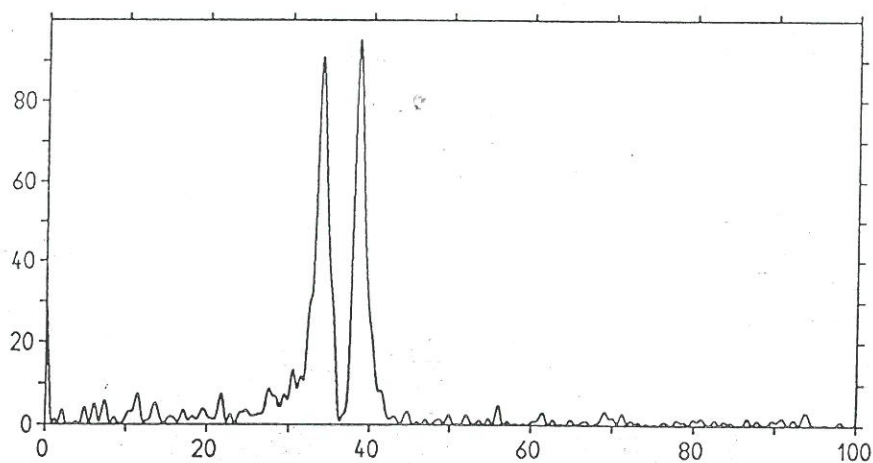
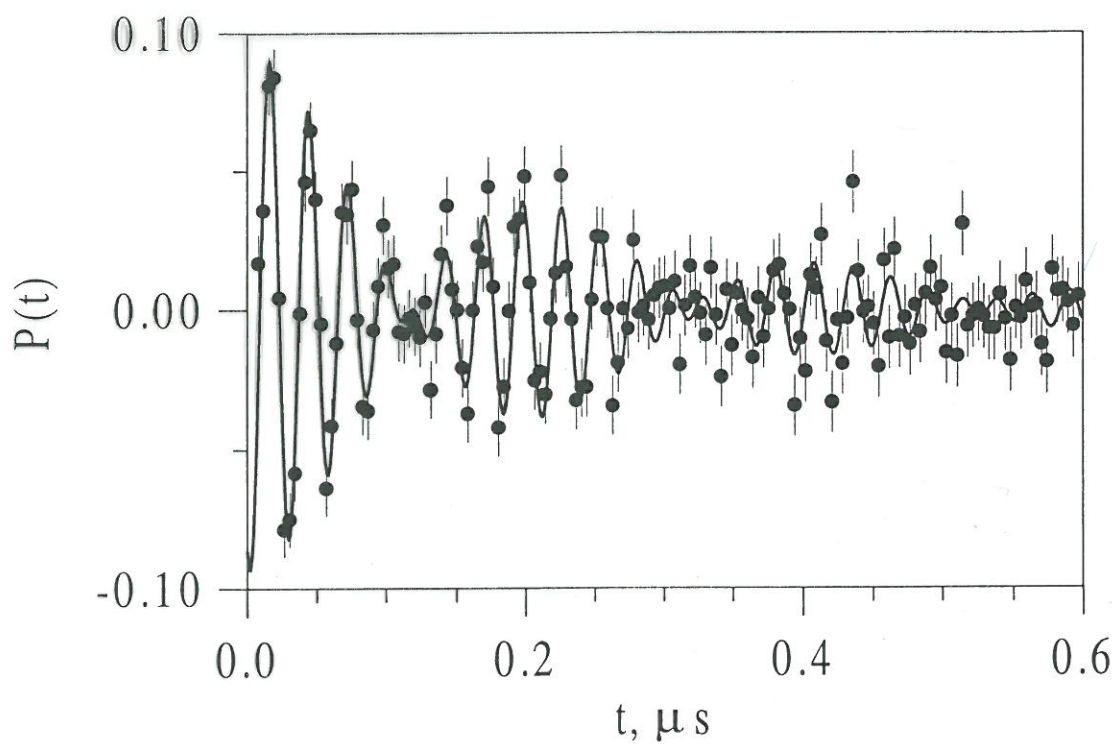
$$\mathbf{B}_c \sim J_{\text{sf}}(0)/E_F n_s \sum_{\mathbf{r}} F(2k_F \mathbf{r}) \mathbf{m}(\mathbf{r})$$

$\mathbf{m}(\mathbf{r})$: local magnetic moments

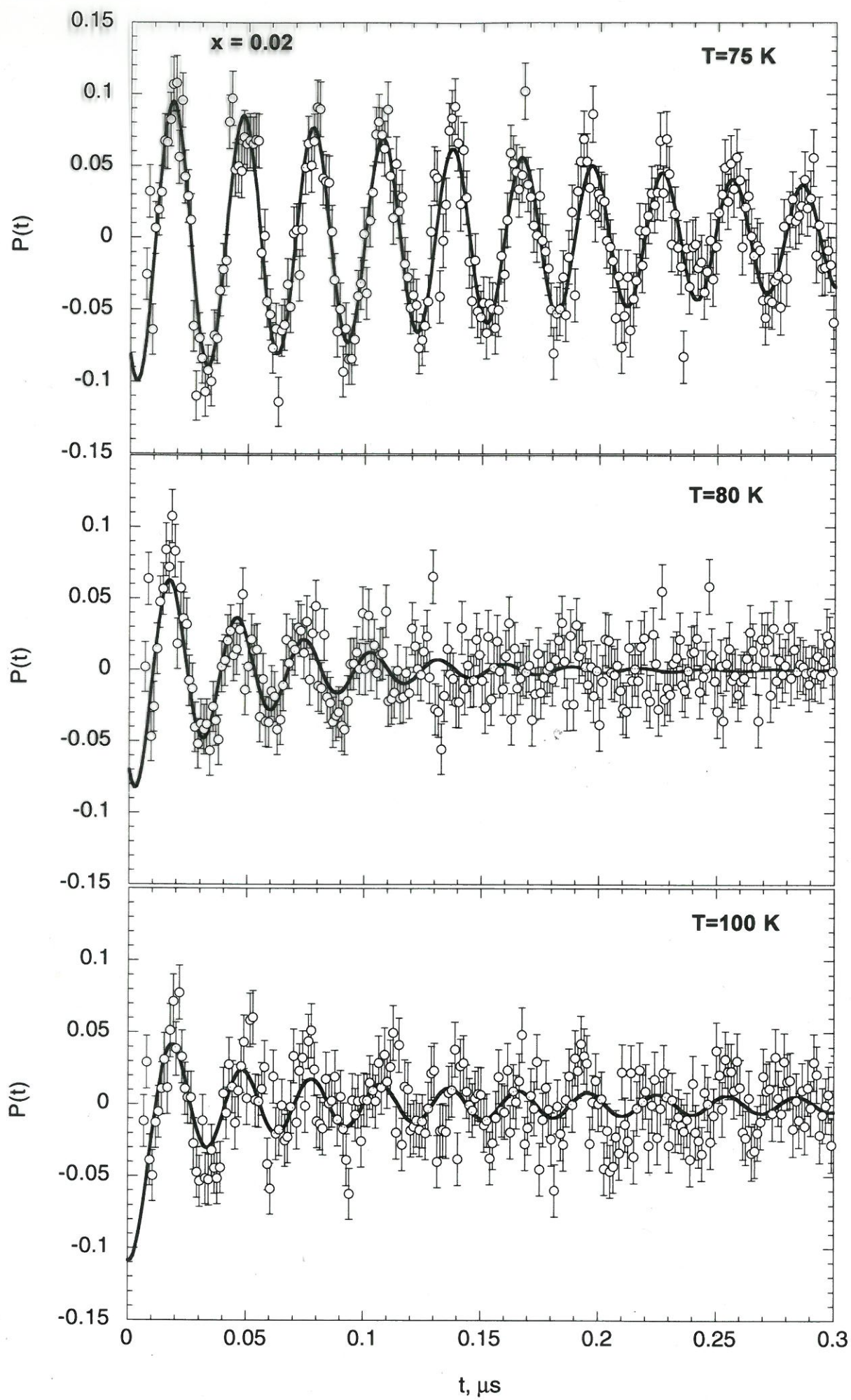
n_s electron spin density

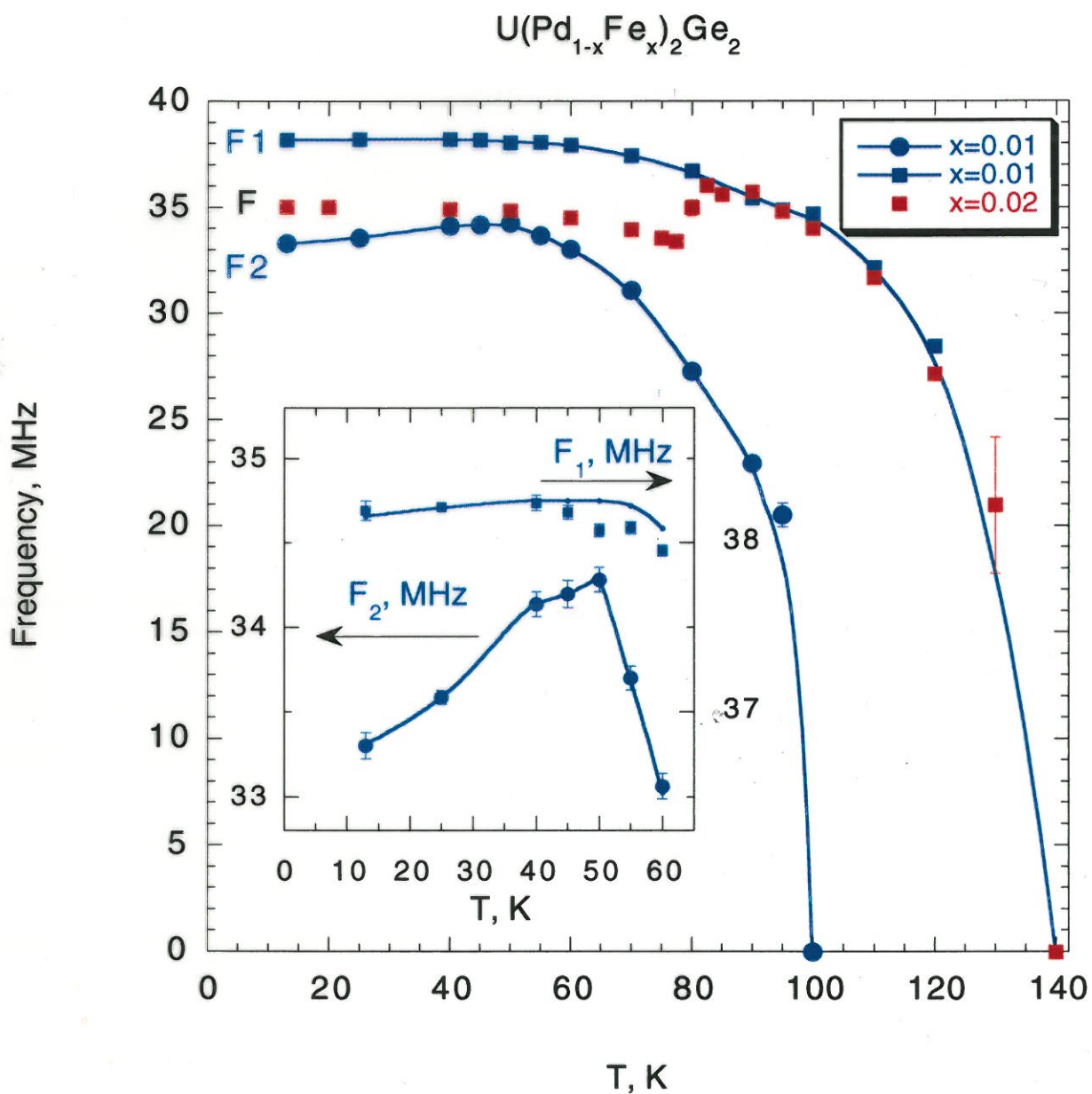
$J_{\text{sf}}(0)$ is the s-f exchange integral

$$F(x) = (1/x^4)(x \cos x - \sin x) \text{ /Rudderman-Kittel/}$$



Time dependence of the muon spin polarization in the $U(Pd_{0.99}Fe_{0.01})_2Ge_2$ in zero external field at temperature $T=40K$ and its Fourier transform



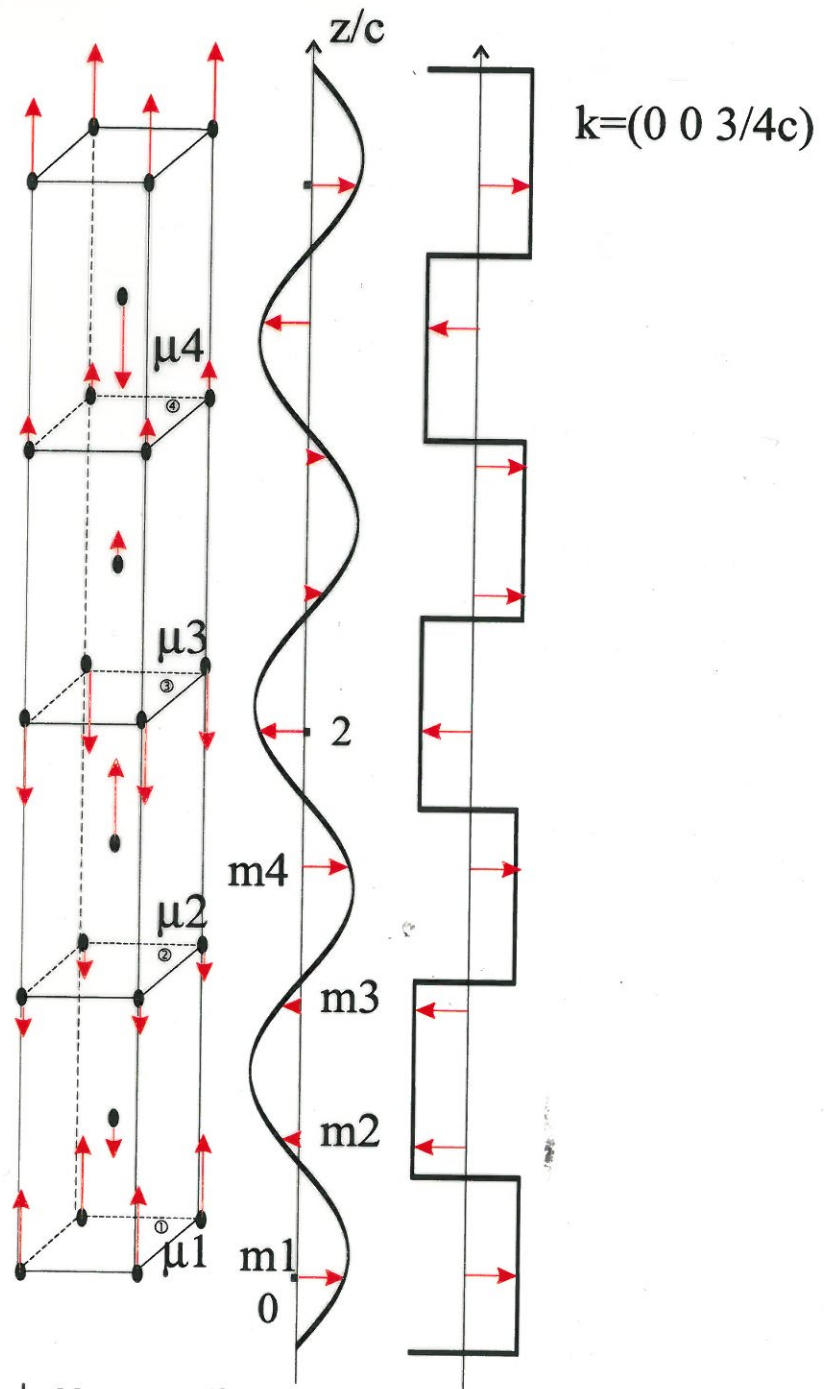


Muon spin precession frequencies F :

$$F_1 = \alpha_1 m_1 + \alpha_2 m_2 \quad F_3 = F_1$$

$$F_2 = \alpha_1' m_3 + \alpha_2' m_4 \quad F_4 = F_2$$

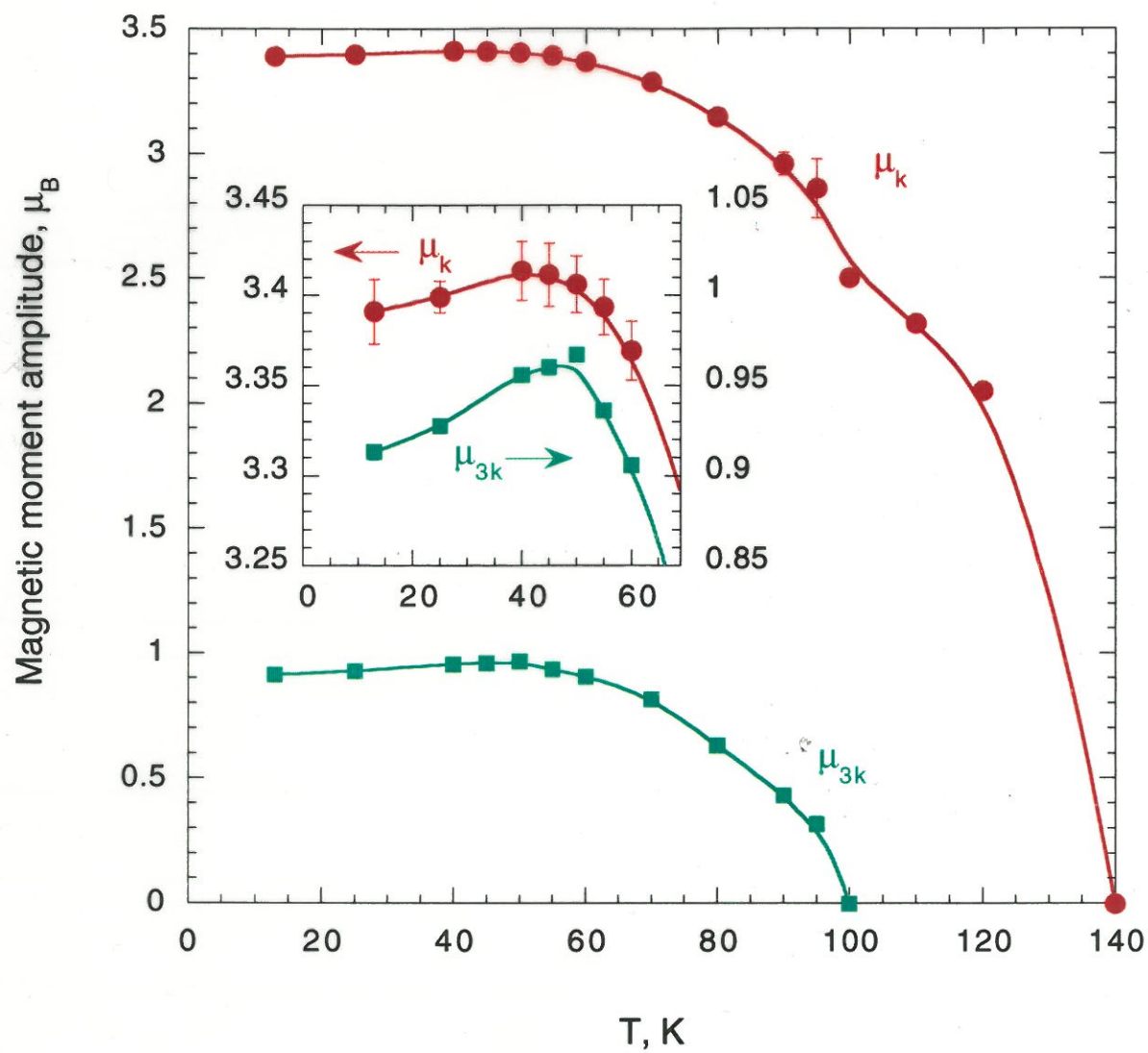
$$\alpha_1 \gg \alpha_2, \alpha_1' \approx \alpha_1$$



$$m_1 = m_{\square} + \mu_0 \cos \varphi,$$

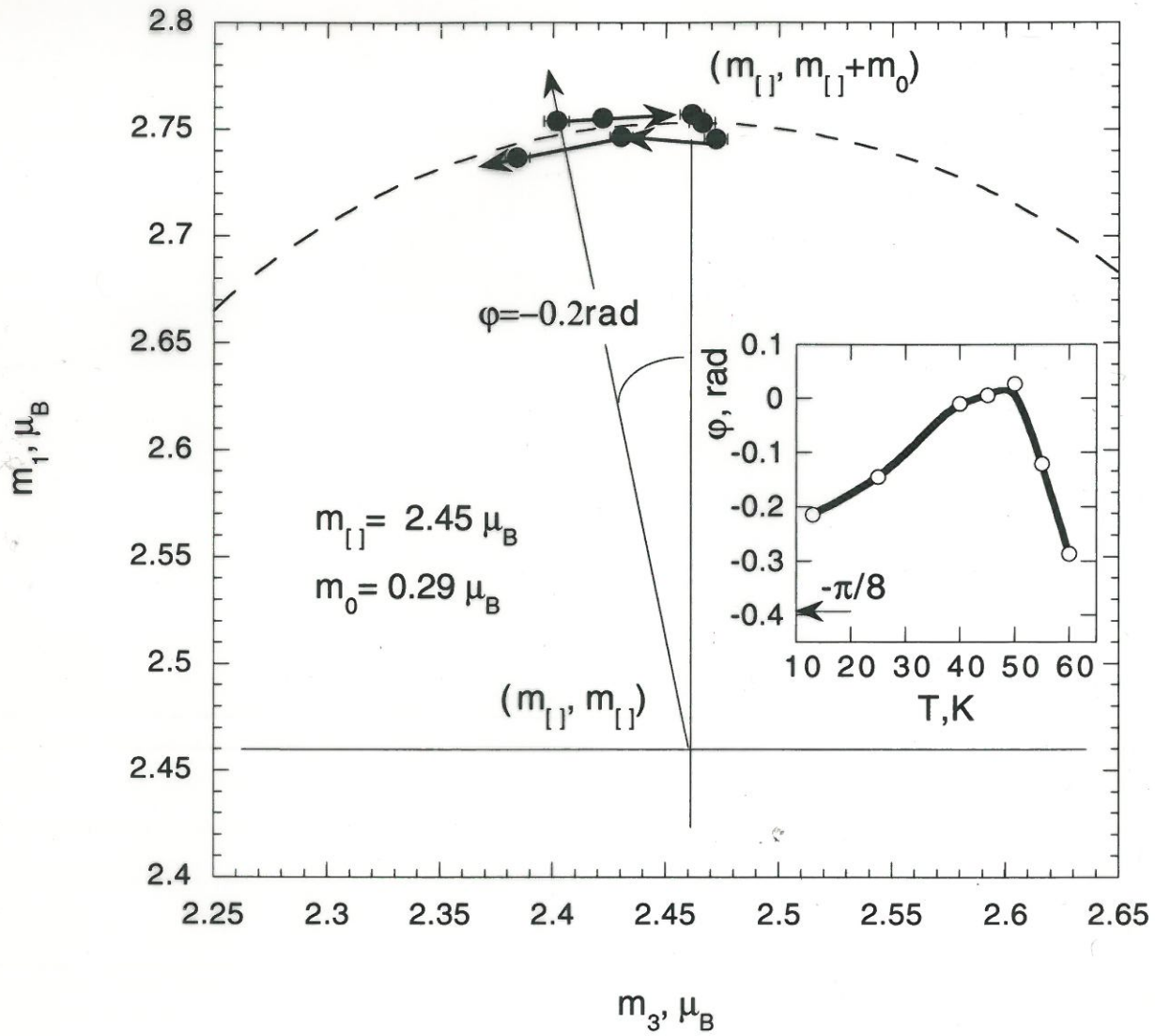
$$m_3 = m_{\square} - \mu_0 \sin \varphi,$$

μ_0 and $m_{\square}$ are the amplitudes of square and sine modulation.



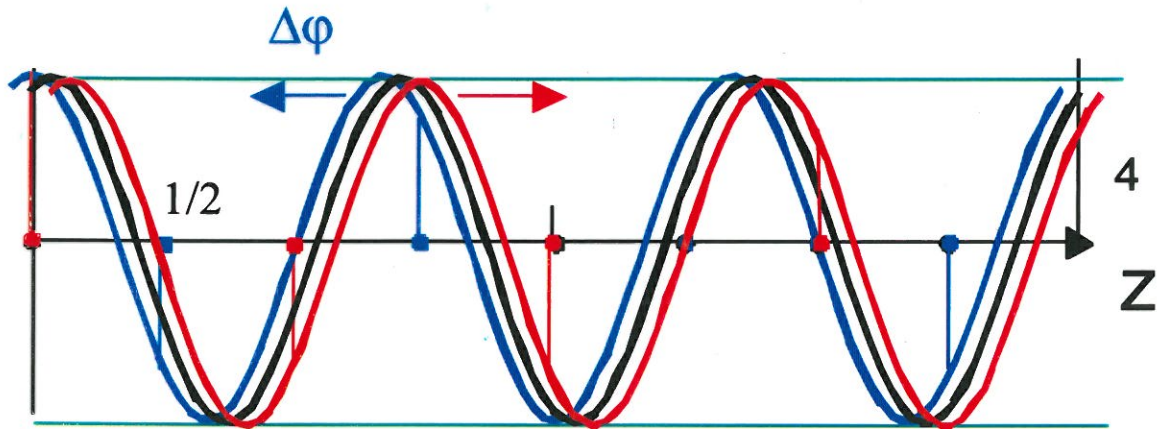
$$\mu_k = \mu_0 + 4/\pi m_{\square}$$

$$\mu_{3k} = 4/3\pi m_{\square}$$



$$m_1(m_3) = m_{\square} + (\mu_0^2 - (m_3 - m_{\square})^2)^{1/2}$$

The initial phase shift of modulation.



$$\varphi_1 = \Delta\phi - \pi/8 \quad (0 \ 0 \ 0)$$

$$\varphi_2 = -\Delta\phi - \pi/8 \quad (+1/2 \ +1/2 \ +1/2)$$

$$\varphi_1 = -\varphi_2 - \pi/4$$

Conclusion

In the LSDW phase of $\text{U}(\text{Pd}_{1-x}\text{Fe}_x)_2\text{Ge}_2$ ($x=0.01$) the peculiar temperature behavior of the uranium magnetic moments determined by μSR is connected to the variation of the initial phase φ of the LSDW modulation by $\Delta\varphi \approx 10^\circ$ – a parameter inaccessible in neutron diffraction experiment.

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U.Zimmermann *Villigen*

A.Gribanov *Moscow State University*
 Chemical department

Muon site	$B_{\text{dip}} / 1 \mu_B$, (G)	μ_U, μ_B
0 1/2 1/8	1016	2.55
0 0 1/2	2113	1.22
1/2 1/2 0	2113	1.22
1/4 1/4 1/4	0	-
0 0 1/4	901	2.87
1/2 1/2 5/16	2067	1.25
1/2 1/4 0	2485	1.04
1/2 0 1/16	2212	1.17
1/8 1/2 3/32	1607	1.61
1/4 1/4 5/32	1099	2.35
1/8 1/8 7/32	884	2.93
Experimental B_μ	2588(3)	

Table 1. The dipole field B_{dip} calculated for possible muon sites in $\text{U}(\text{Pd}_{0.98}\text{Fe}_{0.02})_2\text{Ge}_2$ lattice ($I4/mmm$, unit cell parameters $a = b = 4.18 \text{ \AA}$, $c = 10.21 \text{ \AA}$) for the simple antiferromagnetic structure with $\mathbf{k} = [0 \ 0 \ 1]$ for the uranium magnetic moment $\mu_U = 1 \mu_B$ along the c-axis and the experimentally measured B_μ at $T=15 \text{ K}$. The column μ_U shows the uranium magnetic moment expected in dipolar approximation from the experimentally measured B_μ .

