Pumped-storage hydropower optimization: Effects of several reservoirs and of ancillary services

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Outline

Models with expected water constraints

- single-period: ancillary service, pumping
- multi-period: discontinuous df, continuous time, two reservoirs

More detail model on a scenario tree

- occupation times of price (‘price-duration curves’)
- numerical result

Ancillary services (control reserve)

Solar- and wind infeeds lower profitable price-peaks. Alternative:

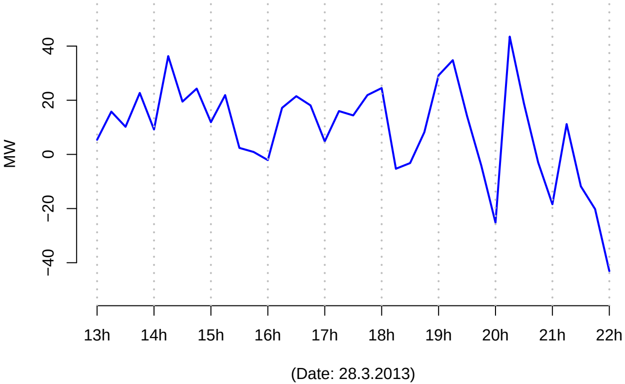
Flexible production capacity is sold to transmission system operator (TSO) to balance unpredictable deviations between supply and demand

Types of ancillary services

- Primary: fully available 30 sec. after imbalance is detected
- **Secondary:** fully available after 15 min.
- Tertiary: starts after 15 min.

$\pm$400 MW of secondary control reserved by TSO in Switzerland

A time series of requested secondary control



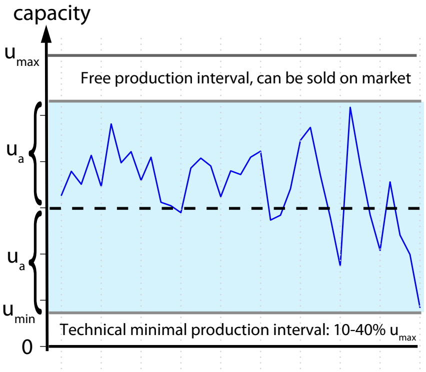
Data: SwissGrid

A major reason for imbalance is the hourly step-wise production schedule in Central Europe.

$\rightarrow$ Heuristically: $\sum$ up-regulation $\approx \sum$ down-regulation

(summed over a week or month)

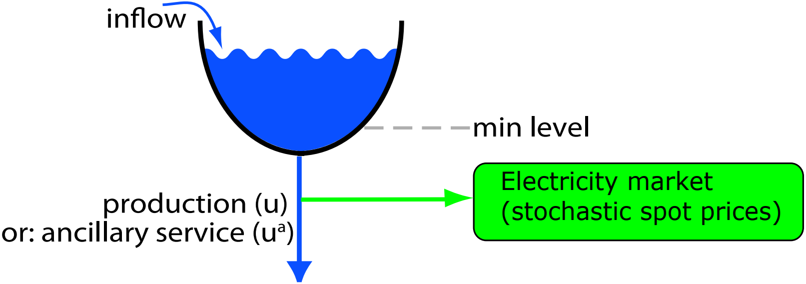
How is secondary control provided?



- Producer (with capacity $u_{\max}$) provides power $\pm u_a$ (MW) over a week or a month. Producer sells $u_{\min} + u_a$ at the market.
- TSO pays producer for providing the service
- TSO pays producer for up-regulation energy amount
- Producer pays TSO for down-regulation energy amount

Because payment rules change constantly, all service- and (expected) regulation-payments summed into p_a (EUR)

Single reservoir



Optimal control problem: How to dispatch?
Objective: Maximize profit.

Optimal rule [Massé, 1946]: Produce when the marginal utility of production is higher than the marginal utility of expected remaining water; else wait.

Single-period (steady-state) hydropower plant model

- Constraint on water-level in expectation
- $S \in L^1_+$ electricity spot price (EUR/MWh), continuous df
- $u\colon \mathbb{R}_+ \to \mathbb{R}_+$ ctrl-funct, $u(S)$: turbined water (MWh); $u_{\min} = 0$
- Max. capacity, initial, minimal water level: $u_{\max} > l_0 - l_{\min} > 0$
- Case without ancillary service

$$\max_{u(\cdot)} \mathbb{E}\big[S\, u(S)\big]$$

$$\text{s.t.} \begin{cases} l_0 - \mathbb{E}\big[u(S)\big] \geq l_{\min} \\ 0 \leq u(S) \leq u_{\max} \end{cases}$$

Optimal solution (bang-bang type):

$\hat{u}(S) = u_{\max} 1_{\{S \geq \hat{q}\}}$, $\quad \hat{q}$ given by $\mathbb{P}[S \geq \hat{q}] = (l_0 - l_{\min})/u_{\max}$.

Remark. Equivalent to financial coherent risk measures:

Optimal objective value $= -(l_0 - l_{\min})\mathrm{AVaR}_{\frac{l_0 - l_{\min}}{u_{\max}}}[-S]$

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Single-period (steady-state) model with ancillary service

- Interval $[-u_a, u_a]$ of reserved power; $u_{\min} = 0$
- up-regulation − down-regulation ≈ 0 (over a long period)
- $p_a \in \mathbb{R}$ represents service- and expected regulation-payments

$$\max_{u(\cdot),u_a} \mathbb{E}\big[S\,u(S) + S\,u_a + p_a u_a\big]$$

$$\text{s.t.} \begin{cases} l_0 - \mathbb{E}\big[u^+(S)\big] - u_a \geq l_{\min} \\ u(S) \geq 0, \quad u_a \geq 0 \\ u(S) + 2u_a \leq u_{\max} \end{cases}$$

Optimal solution in case $u_{\max} = 2(l_0 - l_{\min})$:

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where $\hat{q}$ is an optimal multiplier of the water constraint

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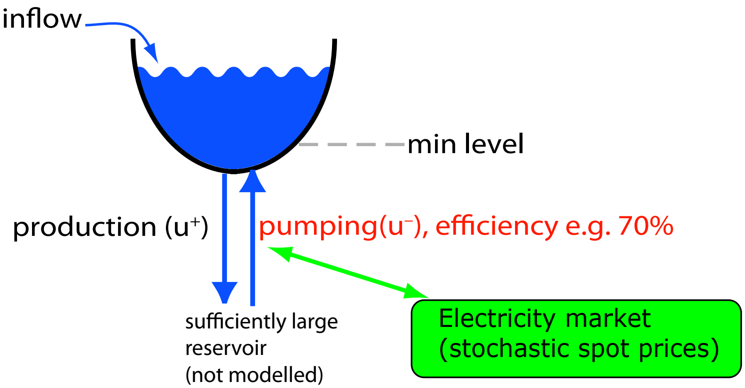
Single reservoir with pumping



Single-period (steady-state) pumped-storage plant model

- Constraint on water-level in expectation
- $S \in L^1_+$ electricity spot price (EUR/MWh), continuous df
- $u^{\pm}\colon \mathbb{R}_+ \to \mathbb{R}_+$ ctrl-funct, $u^{\pm}(S)$: turbined/pumped water (MWh)
- Max. capacity, initial, minimal water level: $u^+_{\max} > l_0 - l_{\min} > 0$
- $c \in (0,1)$ efficiency of pumping

$$\max_{u^{\pm}} \mathbb{E}\left[Su^+(S) - \frac{1}{c}Su^-(S)\right]$$

$$\text{s.t.} \begin{cases} l_0 - \mathbb{E}\left[u^+(S) - u^-(S)\right] \geq l_{\min} \\ 0 \leq u^{\pm}(S) \leq u^{\pm}_{\max} \end{cases}$$

Optimal solution:

$$\hat{u}^+(S) = u^+_{\max} 1_{\{S \geq \hat{q}\}}, \quad \hat{u}^-(S) = u^-_{\max} 1_{\{S \leq c\hat{q}\}}, \quad \hat{q} \text{ given by}$$

$$u^+_{\max}\mathbb{P}[S \geq \hat{q}] - u^-_{\max}\mathbb{P}[S \leq c\hat{q}] = l_0 - l_{\min}$$

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Single-period pumped-storage with discontinuous df of S

i.e., the distribution function of S may have discontinuities in

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Special case $u^-_{\max} = 0$:

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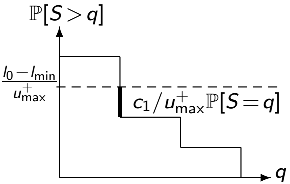


Multi-period pumped-storage plant in discrete time

- $(S_t)_{t=0,\dots,T}$, with $S_t \in L^1_+$, S_t continuous df, $\forall t$
- $U_t^\pm := u_t^\pm(S_0, S_1, \dots, S_t) \colon \mathbb{R}_+^{T+1} \to \mathbb{R}_+$ (non-anticipative)

$$\max_{(U_t^\pm)} \sum_{t=0}^{T} \mathbb{E}\Big[S_t U_t^+ - \frac{1}{c} S_t U_t^-\Big]$$

$$\text{s.t.} \begin{cases} l_0 + \displaystyle\sum_{t=0}^{t'} \Big(\mathbb{E}\Big[U_t^- - U_t^+\Big] + w_t\Big) \geq l_{\min} \\ 0 \leq U_t^\pm \leq u_{\max}^\pm, \quad t, t' = 0, \dots, T \end{cases}$$

Optimal solution:

$$\hat{U}_t^+ = u_{\max}^+ 1_{\left\{S_t \geq \sum_{s=t}^T q_s\right\}}, \quad \hat{U}_t^- = u_{\max}^- 1_{\left\{S_t \leq c \sum_{s=t}^T q_s\right\}}, \quad t = 0, \dots, T$$

Multi-period pumped-storage plant in continuous time

- $(S_t)_{0\le t\le T}$, $S_t \in L^1_+$, S_t continuous df, $\forall t$
- $U_t^\pm$ is a $\sigma((S_{t'})_{0\le t'\le t})$-measurable random variable

$$\max_{(U_t^\pm)} \int_{\Omega\times[0,T]} S_t U_t^+ - \frac{1}{c} S_t U_t^- \,(\mathrm{d}\mathbb{P}\times \mathrm{d}t)$$

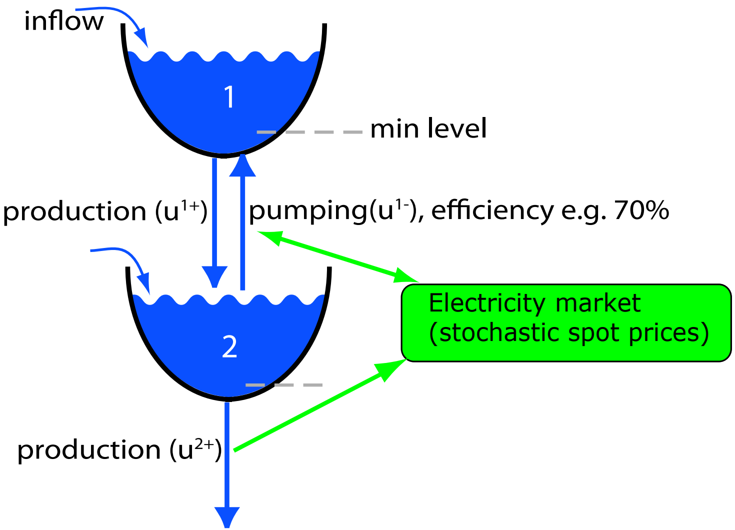
$$\text{s.t.} \begin{cases} l_0 + \displaystyle\int_{\Omega\times[0,t']} U_t^- - U_t^+ + w_t \,(\mathrm{d}\mathbb{P}\times \mathrm{d}t) \ge l_{\min} \\ 0 \le U_s^\pm \le u_{\max}^\pm, \quad s, t' \in [0, T] \end{cases}$$

Optimal solution:

$$\hat{U}_t^+ = u_{\max}^+ 1_{\left\{S_t \ge \int_t^T q_s \,\mathrm{d}s\right\}}, \quad \hat{U}_t^- = u_{\max}^- 1_{\left\{S_t \le c\int_t^T q_s \,\mathrm{d}s\right\}}, \text{ with}$$

$$\begin{cases} \displaystyle\int_0^T v_s q_s \,\mathrm{d}s = 0, \quad v_s, q_s \ge 0, \\ v_s = \displaystyle\int_0^s \left(u_{\max}^- \mathbb{P}\left[S_t \le c\int_t^T q_{s'} \,\mathrm{d}s'\right] - u_{\max}^+ \mathbb{P}\left[S_t \ge \int_t^T q_{s'} \,\mathrm{d}s'\right] + w_t + l_0 - l_{\min} \right) \mathrm{d}t \end{cases}$$

2 reservoirs



Multi-period pumped-storage: discrete time, 2 reservoirs

- $(S_t)_{t=0,\dots,T}$, with $S_t \in L^1_+$, S_t continuous df, $\forall t$
- $U_t^{1\pm} := u_t^{(1\pm)}(S_0, S_1, \dots, S_t)$
- $U_t^{2+} := u_t^{(2+)}(S_0, S_1, \dots, S_t)$
- same electricity-per-water ratio in reservoir 1 and 2

$$\max_{(U_t^{2\pm}),(U_t^{1+})} \sum_{t=0}^{T} \mathbb{E}\left[S_t U_t^{1+} - \frac{1}{c} S_t U_t^{1-} + S_t U^{2+}\right]$$

$$\text{s.t.} \begin{cases} l_0^1 + \sum_{t=0}^{t'} \left(\mathbb{E}\left[U_t^{1-} - U_t^{1+}\right] + w_t^1 \right) \geq l_{\min}^1, \\ l_0^2 + \sum_{t=0}^{t'} \left(\mathbb{E}\left[-U_t^{1-} + U_t^{1+} - U_t^{2+}\right] + w_t^2 \right) \geq l_{\min}^2, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad t' = 0, \dots, T \\ 0 \leq U_t^{1\pm} \leq u_{\max}^{1\pm}, \\ 0 \leq U_t^{2+} \leq u_{\max}^{2+}, \qquad t = 0, \dots, T \end{cases}$$

Optimal solution for 2 reservoirs

Let q_t^1, q_t^2 be optimal Lagrange multipliers of reservoir 1 and 2

$$\hat{U}_t^{1+} = u_{\max}^{1+} 1_{\left\{S_t \geq \sum_{s=t}^T q_s^1 - \sum_{s=t}^T q_s^2\right\}},$$
$$\hat{U}_t^{1-} = u_{\max}^{1-} 1_{\left\{S_t \leq c(\sum_{s=t}^T q_s^1 - \sum_{s=t}^T q_s^2)\right\}},$$
$$\hat{U}_t^{2+} = u_{\max}^{2+} 1_{\left\{S_t \geq \sum_{s=t}^T q_s^2\right\}}, \qquad t = 0, \dots, T$$

Water in lower reservoir 2 is more valuable:

$$\sum_{s=t}^T q_s^1 \leq \sum_{s=t}^T q_s^2 \implies \begin{cases} \hat{U}_t^{1+} = & u_{\max}^{1+} \\ \hat{U}_t^{1-} = & 0 \\ \hat{U}_t^{2+} > & 0 \ \exists \omega \end{cases}$$

water in reservoir 1 is reduced

Water in upper reservoir 1 is more valuable:

$$\sum_{s=t}^T q_s^1 > \sum_{s=t}^T q_s^2 > 0 \implies \begin{cases} \hat{U}_t^{1+} > & 0 \ \exists \omega \\ \hat{U}_t^{1-} > & 0 \ \exists \omega \\ \hat{U}_t^{2+} > & 0 \ \exists \omega \end{cases}$$

balanced operation

General mean-risk model (suitable for scenario tree)

- Monthly time steps for water-level and profit: $t = 0, 1, \dots, T$
- Electricity price: $(S_{t+\frac{h}{H}})_{h=1,\dots,H}$, changes hourly
- $u_t^{\pm}(S_{t+\frac{h}{H}}, \dots) \in \mathbb{R}_+$: production/pumping control variables
- Use of occupation times: $\sum_h u_t^{\pm}(S_{t+\frac{h}{H}}, \dots) \to \sum_i u_t^{\pm}(\bar{s}_i, \dots)$

$$\max_{u^{\pm}_{\min} \le u^{\pm} \le u^{\pm}_{\max}} \mathbb{E}[X_T]$$

$$\text{s.t.} \begin{cases} r[X_0, \dots, X_T] \geq \rho_{\min} \\ (L_t, P_t) \in \mathcal{X}_t, \quad t = 0, \dots, T \end{cases}$$

- L_t: water level (feasible in every scenario, with monthly stochastic inflow)
- P_t: cumulative profit-and-loss
- X_t: production value $:= P_t + \text{weight} \times \text{expected usable water}$
- $r[X_0, \dots, X_T] \in \mathbb{R}$: risk-adjusted value

Hourly time scale replaced by occupation times

Idea: Replace integration over time by integration over levels

Continuous time. State equations of profit-and-loss and water-level over a month, formally with suitable $f : \mathbb{R} \to \mathbb{R}$:

$$\int_t^{t+1} f(S_{t'})\, dt' = \int_0^{\infty} f(s)\, dF_{t+1}(s), \quad F_{t+1}(s) := \int_t^{t+1} 1_{\{S_t \le s\}} dt',$$

Stieltjes integral w.r.t. occupation time $F_{t+1}(s)$ at level s.

Discrete time. Discrete price levels $s_0 < s_1 < \cdots < s_N$:

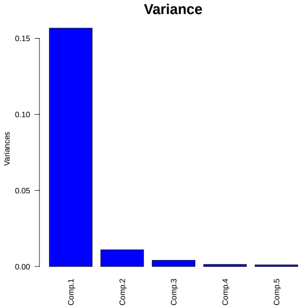
$$\sum_{h=1}^{H} f\left(S_{t+\frac{h}{H}}\right) \approx H \sum_{i=1}^{N} f(\bar{s}_i)\Big(F_{t+1}(s_i) - F_{t+1}(s_{i-1})\Big), \quad \bar{s}_i \in (s_{i-1}, s_i),$$

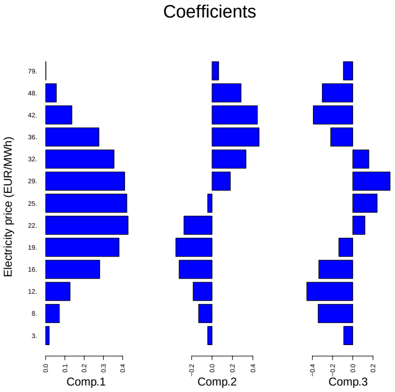
$$F_{t+1}(s) := \frac{1}{H} \sum_{h=1}^{H} 1_{\{S_{t+\frac{h}{H}} \le s\}} \left(= \begin{array}{l}\text{Fraction of hours where}\\ \text{price} \le s\end{array} \right).$$

Dimensional reduction of occupation times

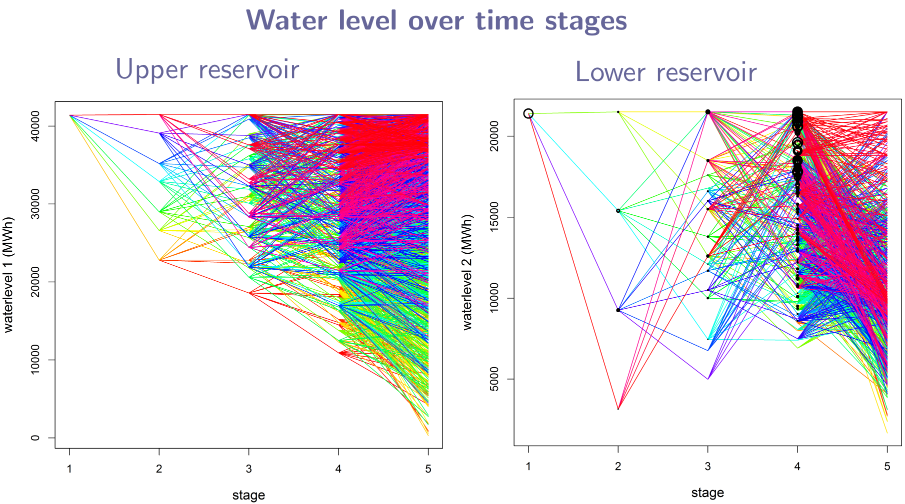
Goal: Dimensional reduction of stochastic vector of occupation times, $(F_t(s_0), \dots, F_t(s_N))$, N large, by Principal Component Analysis (PCA).

PCA of monthly occupation times from hourly electricity prices (EEX market, 2003-2005):





Ancillary service ($p_a = 60\,\mathrm{EUR/MW}$)

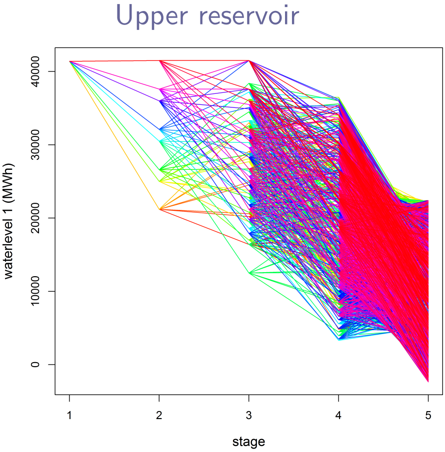


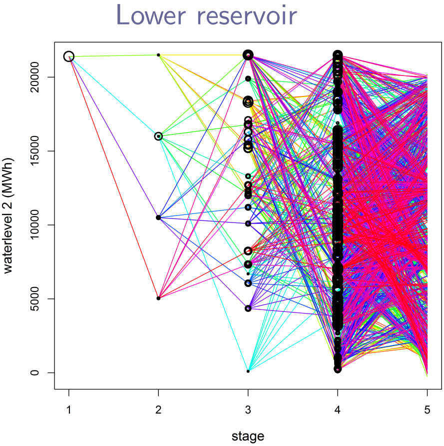
(diameter of circle in node = amount of ancillary service)

Ancillary service (lower reservoir) mainly during first and last stage

Ancillary service ($p_a = 195\,\mathrm{EUR/MW}$)

Water level over time stages





(diameter of circle in node = amount of ancillary service)

Upper reservoir is emptied for ancillary service in lower reservoir

Conclusion

For models with water levels in expectation:

- exact solutions for ancillary services or several reservoirs
- extendable to discontinuous df of price and to continuous time

For models on a scenario tree:

- Occupation time of electricity price ('price-duration curve', dual-scale modeling) allows to incorporate ancillary services
- Further ingredients: (i) principal component analysis of occupation times, (ii) time-consistent risk constraint

Outlook

- Exact solution for ancillary service in general
- Refine expected water constraint by using testfunctions H: $\mathbb{E}[\dots] \to \mathbb{E}[H \dots]$

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