

European Commission

Composite Indicator Analysis and Optimization (CIAO) Tool

Codebook for practitioners, v.2

v.2: Revision January 2021

(v.1: November 2018)

David Lindén^{1,2} Dr. Marco Cinelli^{2,3,*} Dr. Matteo Spada^{4,2} Dr. William Becker⁵ Dr. Peter Burgherr ^{4,2}

¹Department of Sustainable Development, Environmental Science and Engineering, KTH Royal Institute of Technology, Stockholm, Sweden

²Future Resilient Systems (FRS), Swiss Federal Institute of Technology (ETH) Zürich, Singapore-ETH Centre (SEC), Singapore

³Institute of Computing Science, Poznań University of Technology, Poznań, Poland

⁴Technology Assessment Group, Laboratory for Energy Systems Analysis, Paul Scherrer Institute (PSI), Villigen PSI, Switzerland

⁵European Commission, Joint Research Centre (JRC), Ispra VA, Italy

*Corresponding author. Email: <u>marco.cinelli@put.poznan.pl</u>

Cite as: Lindén, D., Cinelli, M., Spada, M., Becker, W., Burgherr, P. 2021. Composite Indicator Analysis and Optimization (CIAO) Tool, v.2. http://dx.doi.org/10.13140/RG.2.2.14408.75520

Program description

The Composite Indicator Analysis and Optimization (CIAO) tool is an expansion of the automated menu-version of the Matlab[®] toolbox presented by Becker (2017) for the advanced assessment of Composite Indicators (CIs). It was developed in connection to assessing the implicit weights of a novel index – the Electricity Supply Resilience Index (ESRI), developed within the Future Resilient Systems (FRS) program, at the Singapore-ETH Centre (SEC); see Gasser et al. (2017) and Lindén (2018) for further details. The scripts for the CIAO tool are written in MathWorks's (http://www.mathworks.com) commercial software language, Matlab[®] 2017 version¹, and contain five main steps: (1) a correlation analysis, (2) a weighting and aggregation of the indicators, (3) an estimation of the influence of each indicator (Si), (4) a decomposition of such influence, and (5) an optimization of the indicators' weights.

Thus, the CIAO tool allows users to:

- 1. Perform a detailed examination of the linear and nonlinear relationships among (i) the set of indicators and (ii) between the indicators and the Composite Indicator (CI).
- 2. Assess how the indicators are "balanced" within the CI, i.e. to what extent the influence of each indicator matches its assigned weight.
- 3. Tune the weights of the indicators so that the influence of each one matches pre-defined importance values.

Details on the methodology are presented in Becker et al. (2017) and Lindén (2018).

Please NOTE that two additional commercial toolboxes are required to run the full analysis: The Statistics and Machine Learning Toolbox², and the Optimization Toolbox³. Also, this package uses one freely available script from the MATLAB Central/File Exchange: *Random Vectors with Fixed Sum*⁴. For this analysis, it is used for generating random starting points for the weight-optimization.

This codebook presents the structure of the package to run the CIAO tool and provides details on each script.

Revision January 2021 (v.2):

Adaptations have been made to the CIAO code and the codebook to include the application of the modules to perform the analysis of information transfer. This includes the addition of a second objective for the weight optimization, i.e., maximize information transfer. The user can now choose between two objectives: (i) *Adjust* the contribution of each indicator in relation to its assigned weight, or (ii) *Maximize* the information transferred from the set of indicators to the CI.

¹ Please NOTE that the current CIAO code was developed with Matlab[®] 2017 version. It is thus recommended not to run the CIAO code with another version of Matlab[®] as this might result in loss of functionality due to possible changes/updates of functions and underlying math libraries (implemented by MathWorks between Matlab[®] version updates), which the authors cannot take responsibility for.

² URL: <u>https://se.mathworks.com/products/statistics.html</u>

³ URL: <u>https://se.mathworks.com/products/optimization.html</u>

⁴ Author: Roger Stafford, 2006 - URL: <u>https://se.mathworks.com/matlabcentral/fileexchange/9700-random-vectors-with-fixed-sum</u>

Scripts

A flowchart of the CIAO tool is shown in Fig. 1. The scripts run in the tool are described in the following sections.

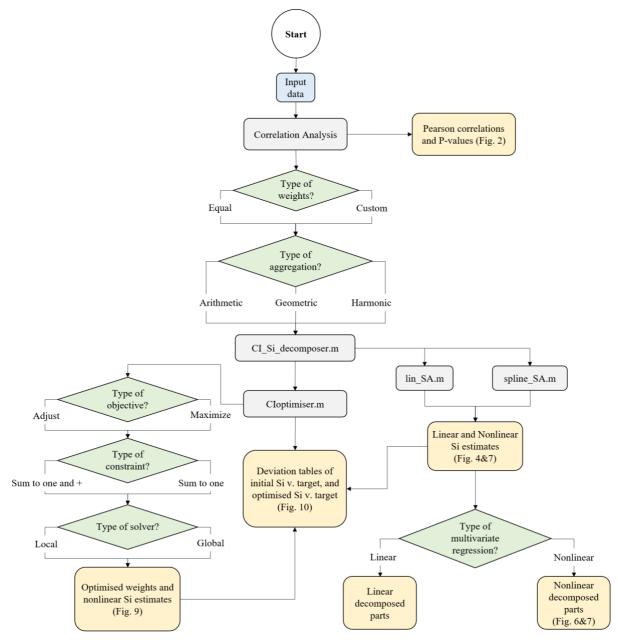


Fig. 1. Flow of the menu-version for implicit weights analysis and optimization in CIs; color coded accordingly: input (blue), outputs (orange), functions (grey), menu choices (green).

MENU_version.m

MENU_version.m is the main script of this package. By running this script, a user-friendly GUI menu is initiated. Although most sub-function settings are automated, the GUI allows the user to "click" through the analysis with a certain amount of freedom to choose different settings (see the green menu choices in Fig. 1). For an explanation on what the script needs to run, see "Input" section at page 9. Default setting loads *test_data.m*, which is a 58-by-4 matrix (see Input section). Then, the main script automatically generates a correlation matrix (with Pearson correlation coefficients) and its corresponding p-values for the matrix of indicators (Fig. 2).

17 18	%%	Check corr	elation an	d P-values	1		
19 -	[R,	P] = corro	oef(X);				
20 -	fpr	intf('Corr	elation ma	trix >>\n')		
21 -	cor	r=array2ta	ble(round(R,4), 'RowN	lames', rownamez,	'VariableNames',	varnamez);
22 -		p(corr)					
23 -	fpr	intf('P-va	lues >>\n')			
24 -	, pva	l=array2ta	ble(round(P,4), 'RowN	lames', rownamez,	'VariableNames',	varnamez);
25 -	dis	p(pval)				-	
6	1.1.4					0	
Comma	nd W	Indow					
Corre	elati	on matrix	>>				
		X_1	X_2	Х_З	X_4		
:	x_1	1	0.6891	0.7286	0.4145		
:	X_2	0.6891	1	0.8224	0.6249		
:	Х_З	0.7286	0.8224	1	0.5997		
2	X_4	0.4145	0.6249	0.5997	1		

Fig. 2. Output from the correlation analysis, which is obtained automatically by running the script. Note that default setting is to name variables X_i , where i=1,2,...,n.

0.0012

0

0

1

Subsequently, the user is provided with the choice of what weights to apply to the indicators (either equal or custom) and also asked to choose an aggregation function (either arithmetic "ArAv", geometric "GeAv", or harmonic average "HarAv"); see Fig. 3. NOTE: The showcased results are obtained using the default data (*test_data.m*), aggregated with arithmetic average and equal weights.



Fig. 3. The GUI menu for the selection of weights (left) and the selection of aggregation function (right).

CI_Si_decomposer.m

P-values >

fx

X_1

1

0

0

0.0012

X 2

0

1

0

0

ХЗ

0

0

1

CI_Si_decomposer.m is a function called by the main script *MENU_version.m*. It uses the X and y data to provide the first order sensitivity indices (i.e. influence of each indicator, Si), plus their decomposed values into correlated and uncorrelated contributions. The function runs accordingly:

function outSi = CI_Si_decomposer(X, y, plott)

The input parameters are the matrix of indicators (X) and the vector (y), which is the aggregated CI. Additionally, the choice of plotting the regression models (plott=1) or not (plott=0) can be made; plott=1 gives scatter plots with splines and linear regression fits (Fig. 4).

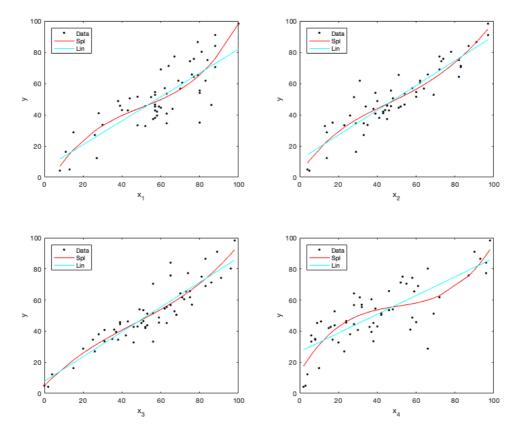


Fig. 4. Output from the regression analysis. Plots showing linear (cyan) and the nonlinear splines (red) regression models.

By running this function, three steps are performed:

First, the function calls two sub-functions for the estimation of the first order sensitivity indices (Si):

function $[R2, par] = lin_SA(y, X, plott)$

this calculates R^2 (R2), i.e. the linear estimates of Si, for the X and y data. The linear regression parameters (e.g. the predicted outcome variables) are stored in a structured array (*par*) with two fields, including: the predicted outcome values of the regression model (*par.yhats*), and X-values sorted in ascending order (*par.xsrts*). Also,

function [S_E, S_V, par] = spline_SA(y, X, plott, p, knots)

this uses the X and y data to calculate the Si, using nonlinear (splines) regression. The $CI_Si_decomposer.m$ uses a cubic order of the spline (p=3) and a row vector of knot numbers to try (knots = [1 2 3 4 5]) as standard, but this can be altered if desired. The outputs are a column vector of first order sensitivity indices estimated via E(y|Xi) (S_E) or V(y|Xi) (S_V). The splines regression parameters are stored in a structured array (*par*) with eight fields, including: order of spline (*par.p*), trial knots (*par.trialknots*), the predicted outcome and variance values of the regression model (*par.yhats & par.varhats*), number of knots (*par.knotN*), derivatives (*par.derivs & par.derivs_xx*), and X-values sorted in ascending order (*par.xsrts*).

Second, a multivariate regression of Xi on X-i (representing the matrix X without the ith column, i.e. indicator) is carried out, using either a standard linear model, according to the method of Xu and Gertner (2008), or MATLAB's inbuilt Gaussian process regression model

 $(fitrgp)^5$ for a nonlinear model. The user is free to choose between these two models, as shown in Fig. 5.

	MENU
Select depe	endence modelling for decomposition
Linear]
Nonlinear	

Fig. 5. The GUI for the selection of model to perform the multivariate regression.

The results from the multivariate regression are then used to obtain a residual (i.e. what is "left" after correlation is removed). Subsequently, a nonlinear regression (splines) of y on the residuals of previous regression is performed to obtain the uncorrelated part (SU). Finally, the correlated part (SC) is obtained simply by subtracting the uncorrelated from the Si, i.e. SC=Si-SU. These are the decomposed values, which are then plotted in a bar chart (Fig. 6).

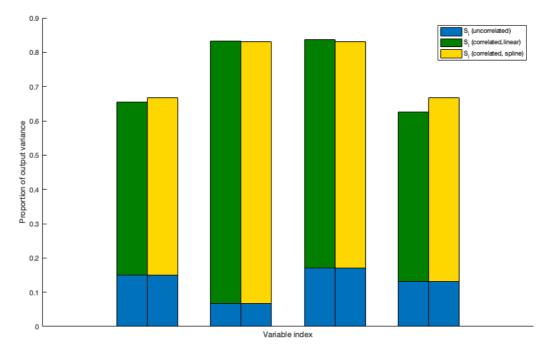


Fig. 6. The decomposed Si values; full bars represent the Si values estimated in the previous regression analysis. The uncorrelated parts (blue), estimated with nonlinear multivariate regression, and the resulting correlated parts, from either the linear (green) and nonlinear (yellow) regression results.

After these steps have been completed, the output of the *CI_Si_decomposer.m* is, beside the two plots, a structure array (*outSi*) with 3 fields containing: (1) linear regression results (*outSi.lin*), (2) spline regression results (*outSi.spline*), and (3) a summarized table of both estimates and the decomposed values (*outSi.results*), shown in Fig. 7.

⁵ NOTE: for this nonlinear dependence modelling, the Statistics and Machine Learning toolbox is required. If this is not detected, linear dependence modelling will be used.

🗞 Estimation and Decomposition of influence
so Estimation and Decomposition of influence
decomp_choice=menu('Select dependence modelling for decomposition','Linear','Nonlinear');
if decomp_choice==1
<pre>outSi=CI_Si_decomposer(X,y,1,decomp_choice);</pre>
<pre>fprintf('Si (lin and nonlin) and decomposed parts (linear) >>\n')</pre>
<pre>disp(outSi.results) % display results-table with all results</pre>
elseif decomp_choice==2
<pre>outSi=CI_Si_decomposer(X,y,1,decomp_choice);</pre>
<pre>fprintf('Si (lin and nonlin) and decomposed parts (nonlinear) >>\n')</pre>
<pre>disp(outSi.results) % display results-table with all results</pre>
end and a second s
•

Si (lin an	d nonlin) S_L	and decompo S_Sp	sed parts (r SU_L	onlinear) >> SU_Sp	SC_L	SC_Sp
X_1	0.65455	0.66823	0.15051	0.15051	0.50404	0.51772
X_2	0.8324	0.83133	0.067245	0.067245	0.76515	0.76408
X_3	0.83636	0.83085	0.17052	0.17052	0.66584	0.66034
X_4	0.62556	0.66625	0.13171	0.13171	0.49385	0.53455

Fig. 7. Output from the estimation and decomposition of influence. These are the values used for the bar chart in Fig. 6 above. S_L and S_Sp are the full bar values, i.e. the influence (Si), estimated with linear or nonlinear (splines) regression. The SU and SC are the uncorrelated and correlated parts, respectively.

Cloptimiser.m

Cloptimiser.m is a self-contained function which optimizes CI weights, using nonlinear regression (penalized splines). It requires three choices (Fig. 8) prior to initiating the optimization algorithm⁶.

e o Menu	e o Menu	O MENU
Select weight optimization objective	Select weight constraint for opt	timization Select solver
Adjust	Sum to one	Local
Maximize	Sum to one AND be positive	Global (Multistart)

Fig. 8. The GUI for the choice of objective (left), optimization constraint (middle) and solver (right).

First, the objective of the optimization must be specified. There are two choices: (i) *Adjust* the contribution of each indicator in relation to its assigned weight, or (ii) *Maximize* the information transferred from the set of indicators to the CI. Second, a constraint on the weights must be chosen. The weights are either only constrained to sum to one (meaning they can take on negative values), OR they must sum to one but also be positive. Third, the choice is between two different types of solver, either a local or a global one. Although both are based on the same search method, the Nelder-Mead simplex, they differ in two aspects: (a) staring point(s) and (b) number of runs. The local solver uses the input weights as starting point, running the search algorithm once. Conversely, the global solver (MultiStart⁷) offers an option to specify the quantity of randomly generated starting points, re-running the search method for each one. This might be desired if the local solver fails to converge to an optimal solution⁸. However, for most cases, the local solver is sufficient and is thus recommended. After these choices are made, the function runs accordingly:

⁶ NOTE: for constrained optimization, Optimization Toolbox is required. If this is not detected, will automatically revert to unconstrained.

⁷ For a more detailed overview on this solver, see <u>https://se.mathworks.com/help/gads/multistart.html</u>

⁸ For more info on this topic, see <u>https://se.mathworks.com/help/optim/ug/local-vs-global-optima.html</u>

function outOpt = Cloptimiser(X, Sd, objective, constr, Cleq)

this function uses the X data to find the optimal set of weights according to a predefined "target" (*Sd*), determined by the objective (*objective*) and a constraint defined by (*constr*); *constr=1* represents the sum-to-one constraint, and *constr=2* represents the sum-to-one constraint together with a positive value constraint. Cleq is the aggregation form (e.g. 'ArAv' for arithmetic) of the CI and it is set automatically by the *MENU_version.m* script according to which function was chosen for aggregation at the beginning (Fig. 9). The output of this function is a structured array (*outOpt*) with six fields containing, for example: the optimized weights (*outOpt.wopt*), Si at optimized weights (*outOpt.Sopt*) and some other solver-specific parameters.

81 82	%% Optimization
33 -	w choice=menu('Select weight constraint for optimization', 'Sum to one', 'Sum to one AND be positive');
34 -	if aggregation==1
35 -	<pre>outOpt=CIoptimiser(X,w,w_choice,'ArAv');</pre>
36 -	elseif aggregation==2
37 -	<pre>outOpt=CIoptimiser(X,w,w_choice,'GeAv');</pre>
38 -	elseif aggregation==3
39 -	<pre>outOpt=CIoptimiser(X,w,w_choice,'HarAv');</pre>
	o o
Loca	minimum found that satisfies the constraints.
feas	ization completed because the objective function is non-decreasing in <u>ble directions</u> , to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> .
feas: and	<u>ble directions</u> , to within the default value of the <u>optimality tolerance</u> ,
feas: and o < <u>sto</u>	<u>ble directions</u> , to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> .
feas: and o < <u>stop</u> Optim	<u>ble directions</u> , to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> . ping criteria details>
feas: and o <stop< td=""><td><u>ble directions</u>, to within the default value of the <u>optimality tolerance</u>, onstraints are satisfied to within the default value of the <u>constraint tolerance</u>. <u>ping criteria details</u>> ised weights >></td></stop<>	<u>ble directions</u> , to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> . <u>ping criteria details</u> > ised weights >>
feas: and o <stop< th=""><th><u>ble directions</u>, to within the default value of the <u>optimality tolerance</u>, onstraints are satisfied to within the default value of the <u>constraint tolerance</u>. <u>ping criteria details</u>> ised weights >> .4443 .0782 .1002</th></stop<>	<u>ble directions</u> , to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> . <u>ping criteria details</u> > ised weights >> .4443 .0782 .1002
feas: and o <stop< td=""><td>ble directions, to within the default value of the <u>optimality tolerance</u>, onstraints are satisfied to within the default value of the <u>constraint tolerance</u>. ping criteria details> ised weights >> .4443 .0782</td></stop<>	ble directions, to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> . ping criteria details> ised weights >> .4443 .0782
feas: and o <stop< td=""><td>ble directions, to within the default value of the <u>optimality tolerance</u>, onstraints are satisfied to within the default value of the <u>constraint tolerance</u>. ping criteria details> ised weights >> .4443 .0782 .1002 .3773</td></stop<>	ble directions, to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> . ping criteria details> ised weights >> .4443 .0782 .1002 .3773
feas: and o <stop Optin (((Si ar</stop 	<u>ble directions</u> , to within the default value of the <u>optimality tolerance</u> , onstraints are satisfied to within the default value of the <u>constraint tolerance</u> . <u>ping criteria details</u> > ised weights >> .4443 .0782 .1002
feas: and c <stop Optim ((() Si at</stop 	<pre>ble directions, to within the default value of the optimality tolerance, onstraints are satisfied to within the default value of the constraint tolerance. ping criteria details> ised weights >> .4443 .0782 .1002 .3773 optimised weights >></pre>
feas: and c <stop Optin () () () () () () () () () (</stop 	<pre>ble directions, to within the default value of the optimality tolerance, onstraints are satisfied to within the default value of the constraint tolerance. ping criteria details> ised weights >> .4443 .0782 .1002 .3773 optimised weights >> .7207</pre>

Fig. 9. Output from weight-optimization, using the sum to one AND positive constraint. As seen from the stopping criterion, a local solver is used. Judging by the results, it successfully finds an optimal set of weights that achieves an equal Si.

- After all the steps are carried out, the main script *MENU_version.m* automatically compiles two summarizing tables (Fig. 10), which showcase the discrepancy between initial Si values and the target values (weights), and optimized Si values and the target values (weights). For this purpose, both sets of Si values are first normalized to render them comparable to the weights (summing to one). Note that this is only performed if "Adjust" objective is selected.

Normalised	Si at initi Sis_norm	al weights >> Sis_target	Sis_dev	Dev_	_ratio
X_1	0.22299	0.25	-0.02701	'-1	1%'
X_2	0.27742	0.25	0.02742	' 13	1%'
х_з	0.27726	0.25	0.02726	' 13	1%'
x_4	0.22233	0.25	-0.02767	'-1:	1%'
Normalised	Si at optim	ised weights >	>		
	Sisopt_norm	Sis_target	Sisopt_	dev	Dev_rati
		0.05	0		'0%'
X_1	0.25	0.25			
X_1 X_2	0.25 0.25	0.25	ø		'0%'
					'0%' '0%'

Fig. 10. Summarizing tables of the results, comparing both the Si values at initial weights (above) and Si values at optimized weights (below) with reference to the target values (i.e. weights), in this case equal.

Input

A matrix (X), with *N*-alternatives (rows) by *k*-variables (columns). Default setting loads *test_data.m*, which is a 58-by-4 matrix (58 alternatives, 4 indicators) containing data that is normalized to a 0-100 scale. To add your own dataset from, for example, an excel spreadsheet, simply use MATLAB's inbuilt import tool⁹. Follow the checklist below to confirm that data is imported correctly (format, name, etc.).

Checklist:

- 1) First, create/prepare an excel-file (*.xlsx*) with the input data. NOTE: The data must be normalized¹⁰ prior to importing it. Example of a correct dataset can be found <u>here</u>.
- 2) When importing the data, set the "output type" to be a numeric matrix (Fig. 11).

I	MPORT	VIEW					
	Range	: A1:D58 🔻	Output Type	.	Replace	▼ unimportable cells with ▼ NaN	-+ 🖌 🖌
Variab	le Names Row	r. 1 🗘	Table				Import Selection
	SELECTI	ON	Column			UNIMPORTABLE CELLS	IMPORT
ſt	est_data.xls	x x	Numeric				
	А	В	Cell Arra				
		testdata		,			
v	arName1 V	/arName2 Vai	rName3 Var	Name4			
N	umber 🔻 N	umber v Num	nber v Num	ber 🔻			
1	100	97	98	98			
2	88	97	89	90			
3	79	91	83	93			
4	88	87	65	96			
5	81	78	96	66			
6	84	82	81	53			
7	67	72	74	96			
8	77	74	65	87			
9	75	73	91	58			
10	64	83	86	52			
11	88	83	56	55			
12	60	72	83	61			
13	77	82	70	28			
14	76	66	75	46			
15	79	51	73	59			
16	83	62	71	31			

Fig. 11. Screenshot representing the choice of output types. Set to Numeric Matrix.

- 3) Be sure to assign your imported matrix to the global variable "X", as this is fundamental for running the scripts. This is easily done by right-clicking on the matrix name in the workspace and selecting "Rename" (Fig. 12).
- 4) Save your imported X-matrix as a .mat file in the same folder as the scripts (Fig. 12).

⁹ For more info on the import tool in MATLAB see for example, <u>https://se.mathworks.com/help/matlab/ref/importtool-app.html</u>

¹⁰ Examples of appropriate normalization methods, (a) Min-Max (b) Target (c) Ranking. Avoid negative values. If standardization is used, it requires that all values are brought to a positive scale. This can be done by, for example, finding the largest negative value in the data set and adding its absolute form to all values (e.g. X + abs(min(X))). This ensures that the minimum value is zero. For more details on normalization methods, see OECD (2008).

orkspace		\odot	Workspace	(
ame ≜	Value		Name 🔺 Value	
testdata	58×4	t double		doub
Oper	Selection	企業D	Open Selection	
Save	As	ዮ₩S	Save As	ት ጋር መ
Сору		жc	Сору	æ
Dupl	cate	жD	Duplicate	жI
Delet		\boxtimes	Delete	D
Rena	me		Rename	
Edit \	/alue		Edit Value	
plot(estdata)		plot(X)	
	testdata)		area(X)	
	estdata)		bar(X)	
	estdata)		pie(X)	
	gram(testd	lata)	histogram(X)	
			contour(X)	
	our(testdat	a)		
surf(estdata)		surf(X)	
mesh	(testdata)		mesh(X)	
Plot (Catalog		Plot Catalog	

Fig. 12. Screenshots representing the renaming of the imported matrix (left). Then, when renamed/assigned to the global variable X (right), select Save As... and save as a *.mat* file.

5) In the *MENU_version.m* script, simply replace the "load *test_data*" (Fig. 13) with the name that your data is saved as.

```
%% Load input data
load test_data % load X matrix containing indicator data
```

Fig. 13. The line in the script *MENU_version.m* that loads the X matrix for analysis.

6) Now, everything should be set for running the analysis on your data.

Output

Pearson correlations, linear and nonlinear regression estimates (Si), decomposed values (SU and SC), optimized Si and weights (wopt), and discrepancy tables.

Acknowledgments

The research was conducted at the Future Resilient Systems (FRS) at the Singapore-ETH Centre (SEC), which was established collaboratively between ETH Zürich and Singapore's National Research Foundation (FI 370074011) under its Campus for Research Excellence And Technological Enterprise (CREATE) program. This study has also been supported by the Technology Assessment Group of the Laboratory for Energy Systems Analysis at the Paul Scherrer Institute (PSI) in Switzerland. Marco Cinelli acknowledges that this project has received funding from the European Union's Horizon 2020 research and innovation program under the Marie Skłodowska-Curie grant agreement No 743553.

References

- Becker, W. 2017. Matlab toolbox for analysis and adjustment of weights in composite indicators. European Commission, Joint Research Centre, <u>http://dx.doi.org/10.13140/RG.2.2.27035.46883.</u>
- Becker, W., M. Saisana, P. Paruolo, and I. Vandecasteele. 2017. Weights and importance in composite indicators: Closing the gap. Ecological Indicators **80**:12-22.
- Gasser, P., P. Lustenberger, T. Sun, W. Kim, M. Spada, P. Burgherr, S. Hirschberg, and B. Stojadinović. 2017. Security of electricity supply indicators in a resilience context. Pages 1015-1022 European Safety and Reliability Conference. 2017 Taylor & Francis Group, London, ISBN 978-1-138-62937-0.
- Lindén, D. 2018. Exploration of implicit weights in composite indicators: the case of resilience assessment of countries' electricity supply. KTH Royal Institute of Technology, KTH DIVA.
- OECD. 2008. Handbook on constructing composite indicators: Methodology and user guide. OECD publishing, Paris.
- Xu, C., and G. Z. Gertner. 2008. Uncertainty and sensitivity analysis for models with correlated parameters. Reliability Engineering & System Safety **93**:1563-1573.