

Quantum gravity phenomenology

Testing *CPT*- and Lorentz invariance

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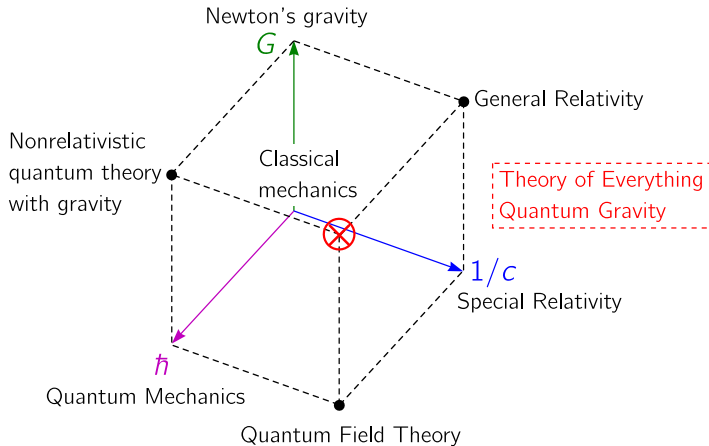
Paul Scherrer Institut, Villigen, Switzerland
January 21, 2020

Outline

- 1 Motivation
- 2 Modified dispersion relations
- 3 Standard-Model Extension
- 4 Experiments

The image features a dark background with a vibrant blue abstract graphic. On the left, a large, glowing blue sphere is partially visible. Below it, a complex, organic blue structure resembling a stylized figure or a network of interconnected nodes and lines extends across the bottom. The text "Lorentz symmetry" is written in a white, elegant cursive font across the upper middle, and "violation" is written in the same font below it, partially overlapping the blue graphic.

Lorentz symmetry
violation

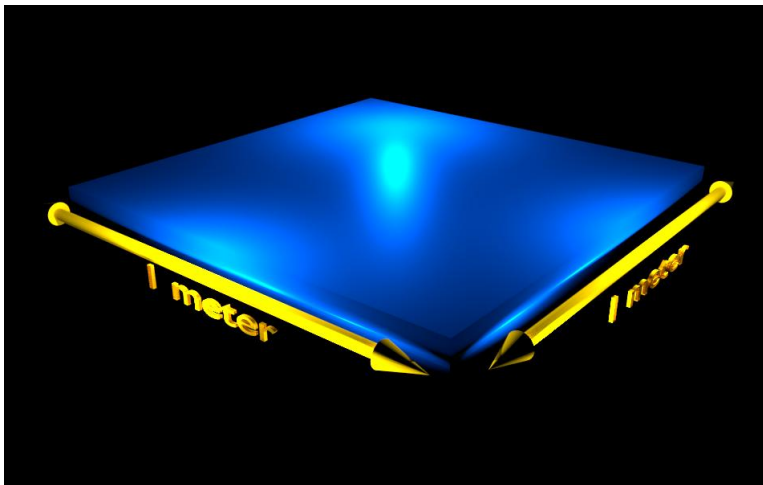


based on M. Bronshtein, A. Zelmanov

(cf. L.B. Okun, in *Multiple facets of quantization and supersymmetry*, M. Olshanetsky and A. Vainshtein, eds., hep-ph/0112339)

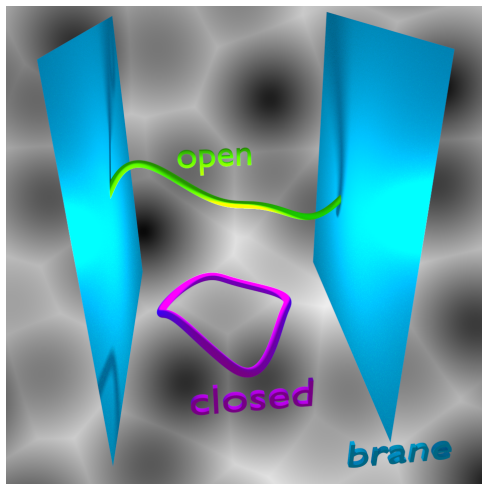
- ▶ Only prototype fundamental theories
- ▶ Main difficulty: no experimental signals
⇒ Better to look for possible experimental signals first!

The nature of spacetime



J.A. Wheeler, *Annals Phys.* **2**, 604 (1957); S.W. Hawking, *Nucl. Phys. B* **144**, 349 (1978); F.R. Klinkhamer and C. Rupp, *Phys. Rev. D* **70**, 045020 (2004), [hep-th/0312032](#); S. Bernadotte and F.R. Klinkhamer, *Phys. Rev. D* **75**, 024028 (2007), [hep-ph/0610216](#); MS, F. Sorba, and S. Thamyahpillai, *Phys. Rev. D* **88**, 125011 (2013), [arXiv:1211.0084 \[hep-th\]](#); S. Hossenfelder, *Adv. High Energy Phys.* **2014**, 950672 (2014), [arXiv:1401.0276 \[hep-ph\]](#); ...

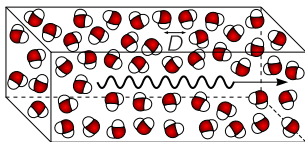
Lorentz violation from string theory



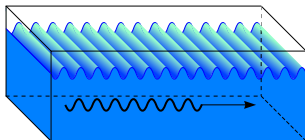
V.A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); V.A. Kostelecký and R. Potting, Nucl. Phys. B **359**, 545 (1991);

V.A. Kostelecký and R. Potting, Phys. Rev. D **51**, 3923 (1995), hep-ph/9501341

Low-energy effective theory



High-energy particle



Low-energy particle

- ▶ Particle propagating through water
- ▶ Wavelength $\sim D$: complicated physics
- ▶ Wavelength $\gg D$: effective theory (optics in a medium)
- ▶ Expect modifications of

$$E(\vec{p}) = \sqrt{(c\vec{p})^2 + (mc^2)^2}$$

$$\omega(\vec{k}) = c|\vec{k}|$$

- ▶ Medium defines a Lorentz-violating background

Modified dispersion laws

- ▶ Modified dispersion laws:

$$\omega^2 \left[1 \pm \frac{\hbar\omega}{E_{\text{Pl}}} \right] = c^2 \vec{k}^2, \quad E^2 = (c\vec{p})^2 + (mc^2)^2 \pm \frac{c|\vec{p}|^3}{M_{\text{Pl}}}.$$

G. Amelino-Camelia, J. R. Ellis, N.E. Mavromatos, D.V. Nanopoulos, and S. Sarkar, *Nature* **393**, 763 (1998), [astro-ph/9712103](#);

R. Aloisio, P. Blasi, P.L. Ghia, and A.F. Grillo, *Phys. Rev. D* **62**, 053010 (2000), [astro-ph/0001258](#)

- ▶ Problems:

- ▶ Very special models
- ▶ Description of kinematics, but not of *dynamics*
- ▶ Theoretical issues hidden (unitarity?, microcausality?)
- ▶ No observable physical effects(?)

- ▶ Framework based on an action: Standard-Model Extension

D. Colladay and V.A. Kostelecký, *Phys. Rev. D* **58**, 116002 (1998), [hep-ph/9809521](#); V.A. Kostelecký, *Phys. Rev. D* **69**, 105009

(2004), [hep-th/0312310](#)

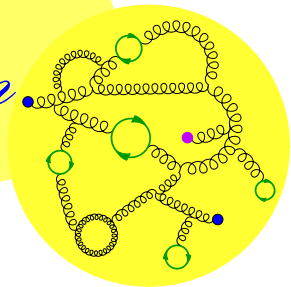


D. Colladay



V. A. Kostelecký

*Standard-Model
Extension*



Lorentz-violating background field

$$\mathcal{L}_\gamma = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}(k_F)^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \dots$$

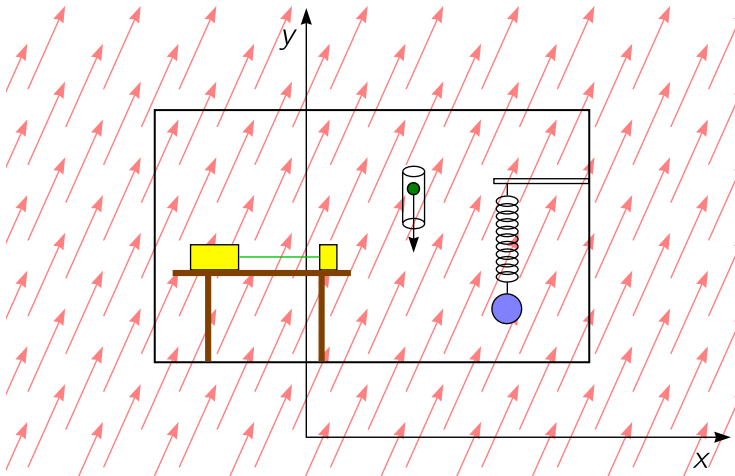
D. Colladay and V.A. Kostelecký, Phys. Rev. D **58**, 116002 (1998), hep-ph/9809521



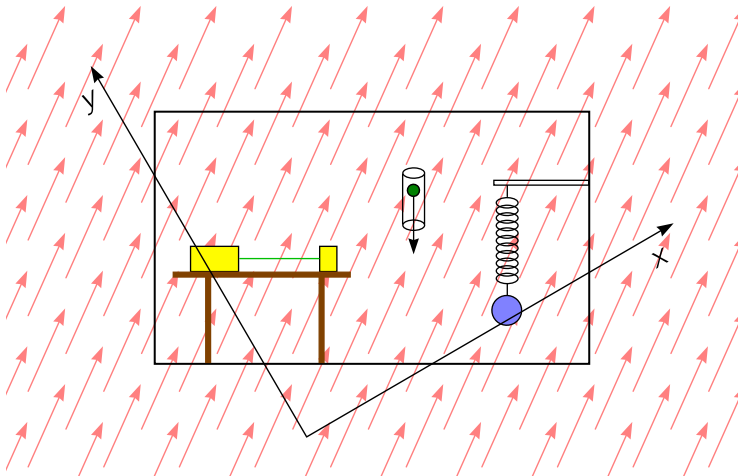
Behavior of background field:

- ▶ Tensor under **observer (coordinate) Lorentz transformations**
- ▶ Tensor under active Lorentz transformations (inverse of observer Lorentz transformations)
- ▶ Fixed under **particle Lorentz transformations**
- ▶ Low-energy effective description

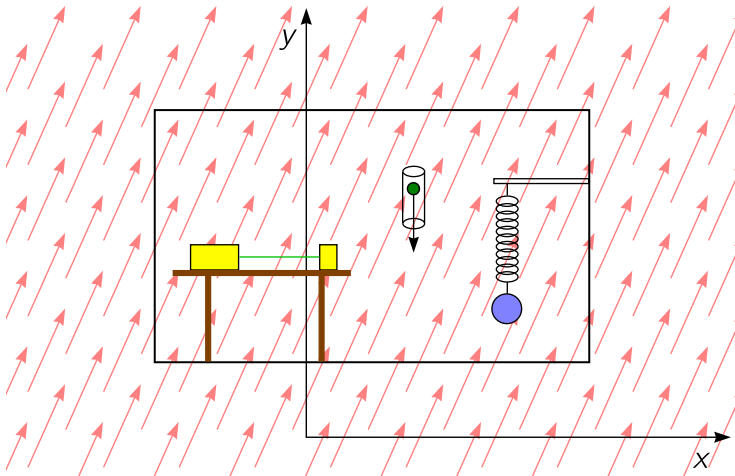
Observer Lorentz transformation



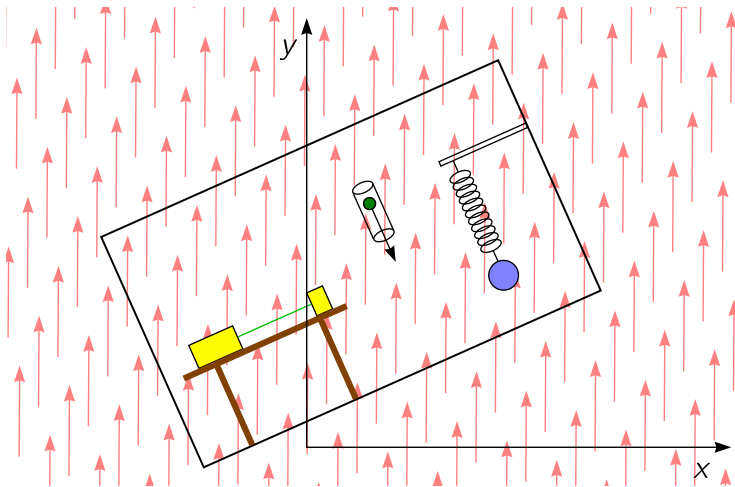
Observer Lorentz transformation



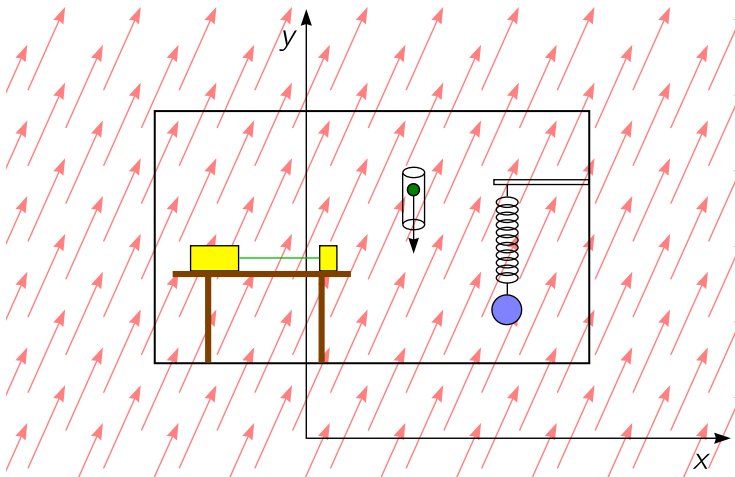
Active Lorentz transformation



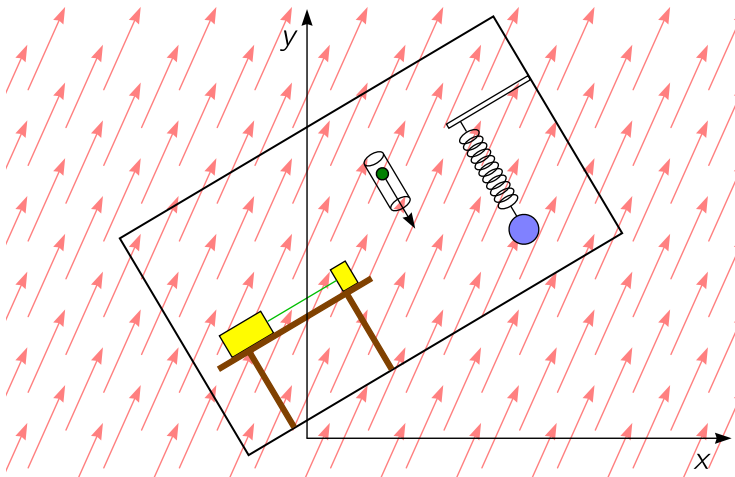
Active Lorentz transformation



Particle Lorentz transformation

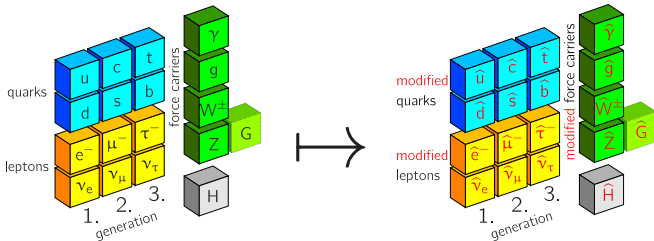


Particle Lorentz transformation



Standard-Model Extension (SME)

- ▶ SME describes possible Lorentz violation for Standard-Model particles + gravity



- ▶ All Lorentz-violating terms well organized
- ▶ *CPT*-violating contributions contained

O.W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002), hep-ph/0201258

- ▶ Besides **modified energy-momentum relations**:
 - ▶ **Modified Dirac equation** for electrons, quarks,
 - ▶ **Modified Maxwell's equations** for photons, etc.

Minimal Standard-Model Extension (SME)

- ▶ Low-energy effective photon sector

$$\mathcal{L}_\gamma = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}(k_F)^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \dots$$

- ▶ Fermion sector (minimally coupled)

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi} \left(\frac{i}{2}\gamma^\mu \overleftrightarrow{D}_\mu - m \right) \psi - a_\mu \bar{\psi} \gamma^\mu \psi - b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi + c_{\mu\nu} \bar{\psi} \gamma^\mu \partial^\nu \psi + \dots$$

- ▶ Embed it into Standard Model

$$\mathcal{L}_{\text{quark}} = \frac{i}{2}\bar{Q}_A \gamma^\mu \overleftrightarrow{D}_\mu Q_A + \dots - (a_Q)_{\mu,AB} \bar{Q}_A \gamma^\mu Q_B - (a_U)_{\mu,AB} \bar{U}_A \gamma^\mu U_B + \dots$$

$$\mathcal{L}_{\text{lepton}} = \frac{i}{2}\bar{L}_A \gamma^\mu \overleftrightarrow{D}_\mu L_A + \dots - (a_L)_{\mu,AB} \bar{L}_A \gamma^\mu L_B - (a_R)_{\mu,AB} \bar{R}_A \gamma^\mu R_B + \dots$$

$$\mathcal{L}_{\text{gluon}} = -\frac{1}{2}\text{Tr}(G^{\mu\nu}G_{\mu\nu}) - \frac{1}{2}(k_G)^{\mu\nu\rho\sigma}\text{Tr}(G_{\mu\nu}G_{\rho\sigma}) + \dots$$

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \phi)^\dagger D_\mu \phi + V(\phi) + \frac{1}{2}(k_{\phi\phi})^{\mu\nu}(D_\mu \phi)^\dagger D_\nu \phi - \frac{1}{2}(k_{\phi W})^{\mu\nu} \phi^\dagger W_{\mu\nu} \phi + \dots$$

$$\mathcal{L}_{\text{Yuk}} = -(G_U)_{AB} \bar{Q}_A \phi^c U_B + \dots - \frac{i}{4}(H_U)_{\mu\nu,AB} \bar{Q}_A \phi^c [\gamma^\mu, \gamma^\nu] U_B + \dots$$

CPT violation

- ▶ Charge conjugation C

$$C|q, \vec{p}, s_z\rangle = |-q, \vec{p}, s_z\rangle$$

- ▶ Parity transformation P ($\vec{r} \mapsto -\vec{r}$)

$$P|q, \vec{p}, s_z\rangle = |q, -\vec{p}, s_z\rangle$$

- ▶ Time reversal transformation T ($t \mapsto -t$)

$$T|q, \vec{p}, s_z\rangle = |q, -\vec{p}, -s_z\rangle$$

- ▶ CPT relates particle dynamics with antiparticle dynamics

$$CPT|q, \vec{p}, s_z\rangle = |-q, \vec{p}, -s_z\rangle$$

- ▶ All laws of nature are CPT -invariant

- ▶ Locality, causality, Hermiticity
- ▶ Energy bounded from below
- ▶ Lorentz invariance

- ▶ CPT violation implies Lorentz violation

O.W. Greenberg, *Phys. Rev. Lett.* **89**, 231602 (2002),
[hep-ph/0201258](https://arxiv.org/abs/hep-ph/0201258)

- ▶ Odd number of Lorentz indices: CPT violation



J. Schwinger



W. Pauli



G. Lüders



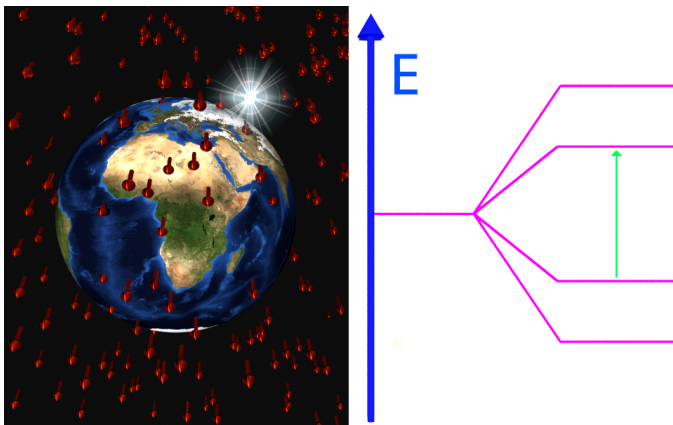
J.S. Bell

$$\mathcal{L}_{\text{Dirac}} \supset -a_{\mu} \bar{\psi} \gamma^{\mu} \psi + c_{\mu\nu} \bar{\psi} \gamma^{\mu} \partial^{\nu} \psi + \dots$$



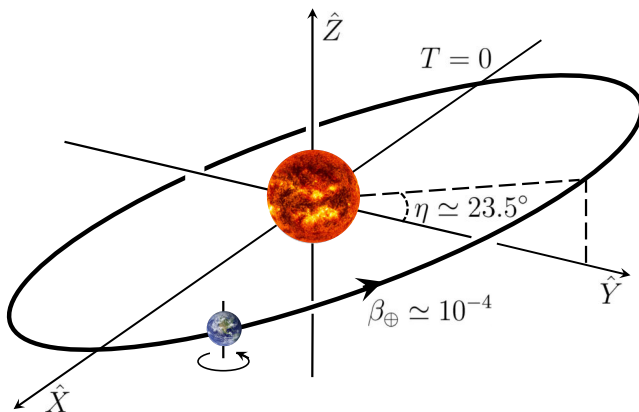
Experiments

Sidereal variations



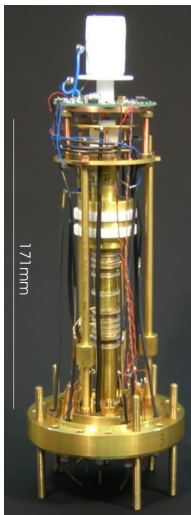
- ▶ Rotating laboratory frame on Earth
- ▶ Typical signal of Lorentz violation: Sidereal variations of atomic energy levels
- ▶ Problem: Noninertial coordinate frame!

Standard Sun-centered inertial reference frame

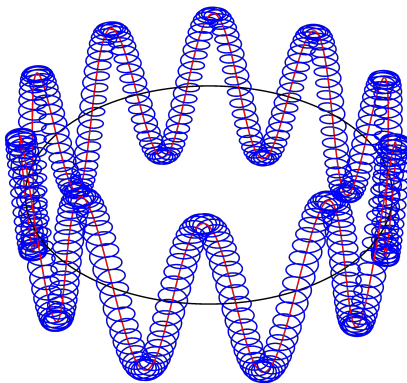


- ▶ Constraints compiled in V.A. Kostelecký and N. Russell, Data Tables for Lorentz and *CPT* Violation, Rev. Mod. Phys. **83**, 11 (2011) arXiv:0801.0287 [hep-ph]
- ▶ Start in 2008 (12 pages), presently version 13 (122 pages)

Penning traps: Classical treatment



MPI Heidelberg

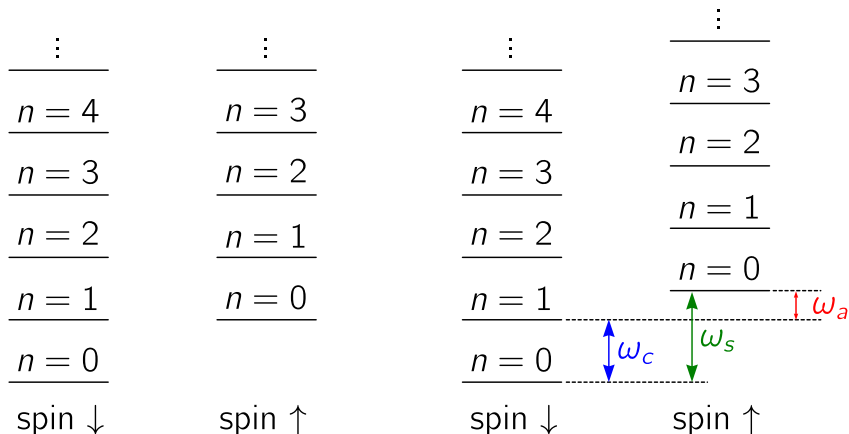


Classical trajectory of particle

L.S. Brown and G. Gabrielse, *Rev. Mod. Phys.* **58**, 233 (1986)

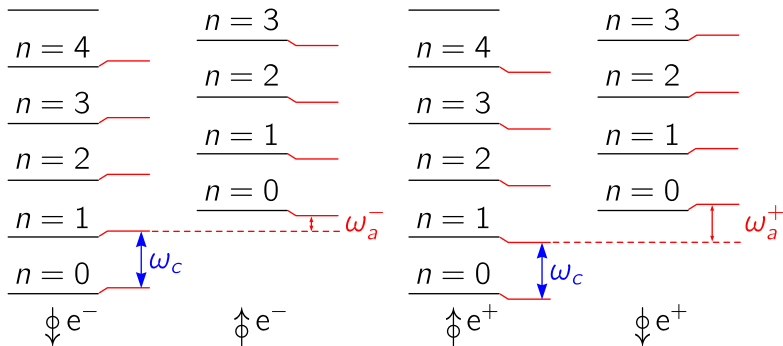
Penning traps: Quantum treatment

Solve Dirac equation for electron in uniform \vec{B} field (Landau problem)



Modified energy levels

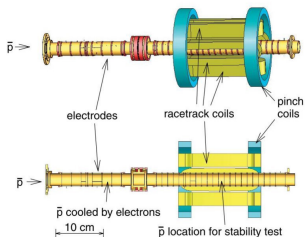
- Consider b_3^e : $\mathcal{L} = -b_\mu \bar{\psi} \gamma^5 \gamma^\mu \psi$



- Compare ω_a^- to ω_a^+ (also test of *CPT* invariance)
- Constrain Lorentz violation from the absence of a signal:

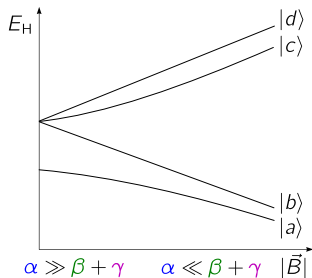
$$|\tilde{b}_X^e| < 1 \times 10^{-25} \text{ GeV}, \quad |\tilde{b}_X^{*,P}| < 9.7 \times 10^{-25} \text{ GeV}$$

Tests with anti-hydrogen



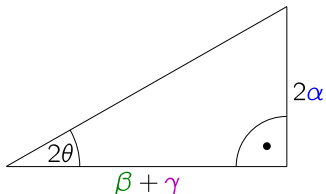
Penning trap + Ioffe-Pritchard trap

G. Gabrielse *et al.*, *Phys. Rev. Lett.* **98**, 113002 (2007)



L-S coupling Paschen-Back effect

$$H_s = \alpha \vec{\sigma}_e \cdot \vec{\sigma}_p + \beta \sigma_{e,3} - \gamma \sigma_{p,3}, \quad |s_e, s_p\rangle$$



$$|d\rangle = |\uparrow, \uparrow\rangle$$

$$|c\rangle = \sin \theta |\downarrow, \uparrow\rangle + \cos \theta |\uparrow, \downarrow\rangle$$

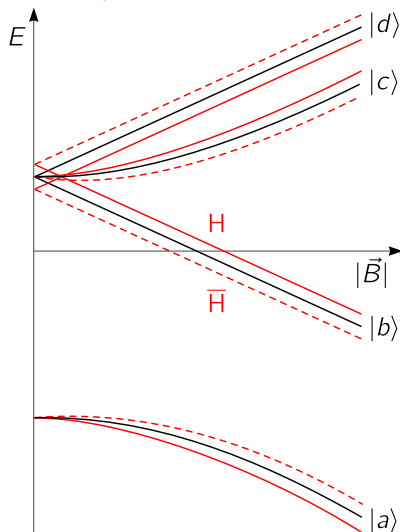
$$|b\rangle = |\downarrow, \downarrow\rangle$$

$$|a\rangle = \cos \theta |\downarrow, \uparrow\rangle - \sin \theta |\uparrow, \downarrow\rangle$$

For anti-hydrogen: Same \vec{B} field with spins reversed (CPT test)

Energy shifts by Lorentz violation

Consider b_3^P : $\mathcal{L} = -b_{\mu} \bar{\psi} \gamma^5 \gamma^{\mu} \psi$



- For hydrogen (red):

$$\Delta E_d = -b_3^P + \dots$$

$$\Delta E_c = b_3^P \cos 2\theta + \dots$$

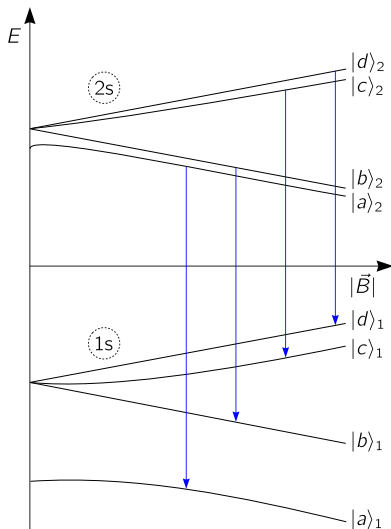
$$\Delta E_b = b_3^P + \dots$$

$$\Delta E_a = -b_3^P \cos 2\theta + \dots$$

- For anti-hydrogen (red, dashed):

$$b_3^P \mapsto -b_3^P$$

1s-2s transitions



- ▶ $|d\rangle_2 \mapsto |d\rangle_1$: no Lorentz-violating signal at leading order
- ▶ $|c\rangle_2 \mapsto |c\rangle_1$: has unsuppressed signal, as mixing angles θ_n differ
 - ▶ Optimum for $|\vec{B}| \approx 0.01 \text{ T}$
 - ▶ Advantage over $|d\rangle_2 \mapsto |d\rangle_1$: $\approx 10^5$
- ▶ Problems:
 - ▶ Dependence on magnetic field
 - ▶ Zeeman splitting (broadening)
- ▶ Nevertheless, searches performed for

$$c_{(TX)}^e = (3.1 \pm 1.9) \times 10^{-11}$$

$$0.92c_{(TY)}^e + 0.40c_{(TZ)}^e = (2.6 \pm 5.3) \times 10^{-11}$$

A. Matveev et al., Phys. Rev. Lett. 110, 230801 (2013)

Constraints from hyperfine splitting

- ▶ Consider hyperfine transition between $|c\rangle$ and $|d\rangle$
 - ▶ Independent of magnetic field
 - ▶ No Zeeman splitting
- ▶ Unsuppressed signal for Lorentz violation

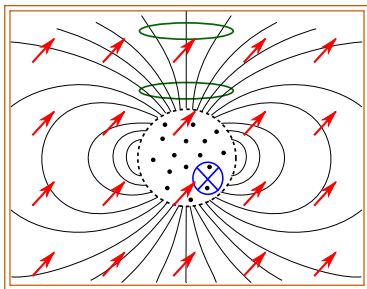
$$\delta\omega|_{LV,H} = -2b_3^P, \quad \delta\omega|_{LV,\bar{H}} = 2b_3^P$$

- ▶ With frequency resolution of 1 mHz: sensitivity for b_3^P at 10^{-27} GeV

He/Xe magnetometer

Nonrelativistic limit of b coefficients in neutron sector:

$$\mathcal{L} = -b_{\mu} \bar{\psi} \gamma^5 \gamma^{\mu} \psi, \quad V = -\vec{b}^n \cdot \vec{\sigma}$$

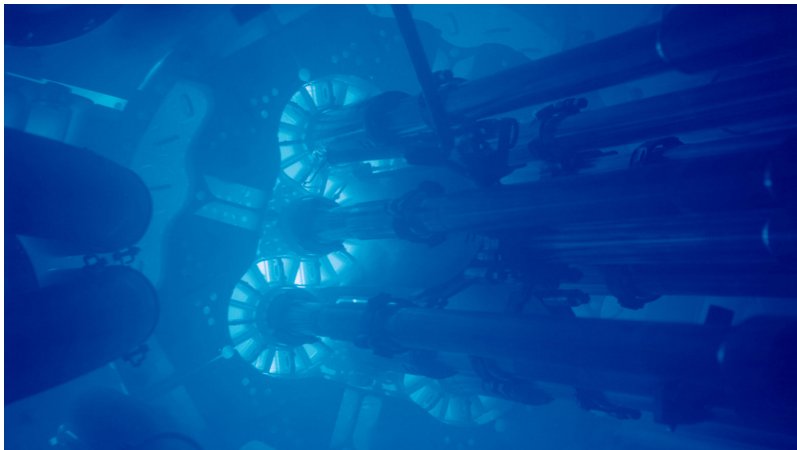


- ▶ Magnetically shielded room
- ▶ Homogeneous magnetic field
 $B_0 \approx 400 \text{ nT}$
- ▶ Polarize ^3He and ^{129}Xe nuclear spins in spherical glass shell
- ▶ Spin precession produces oscillation of magnetic flux: measurement by SQUIDS
- ▶ Look for sidereal variation of weighted phase difference

$$\Delta\Phi \equiv \Phi_{\text{He}} - \frac{\gamma_{\text{He}}}{\gamma_{\text{Xe}}} \Phi_{\text{Xe}} = \Delta\Phi_{\text{known}}(t) + \Delta\Phi_{\text{LV}}(t)$$

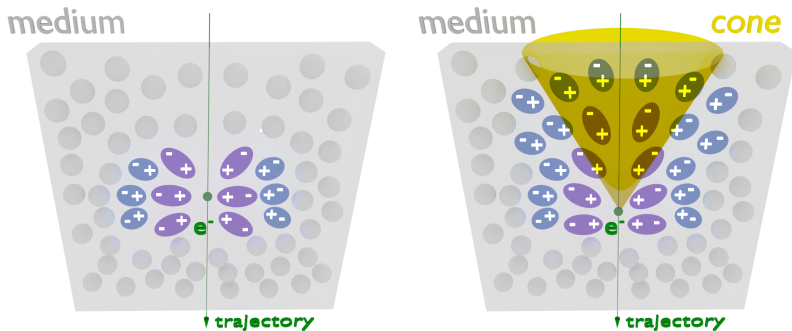
$$\tilde{b}_{\perp}^n < 1.3 \times 10^{-33} \text{ GeV}$$

Cherenkov radiation in media



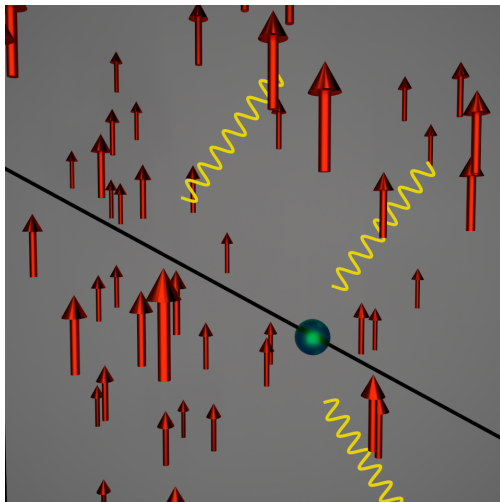
Idaho National Laboratory's Advanced Test Reactor core
(Argonne National Laboratory; Wikipedia)

Cherenkov radiation in media

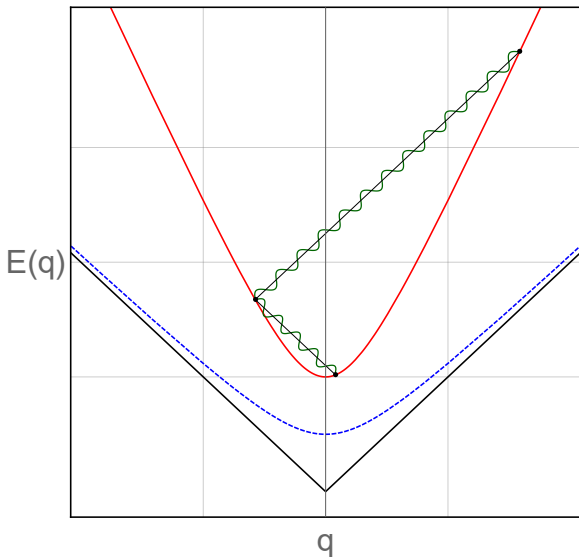


- ▶ Experimental discovery: Cherenkov, Vavilov (1934)
- ▶ Theoretical explanation: Frank, Tamm (1937)
- ▶ Book reference: J.V. Jelley, *Čerenkov Radiation and Its Applications* (Pergamon Press, London, 1958)

Cherenkov-type process in vacuo



Modified fermions: spin-degenerate case



Threshold analysis

- ▶ Consider operator $\mathcal{L} = c_{\mu\nu} \bar{\psi} \gamma^\mu i \partial^\nu \psi$
- ▶ Vacuum Cherenkov threshold energy

$$q_{\dot{c}}^{\text{th}} = \frac{1}{2} \sqrt{\frac{3}{2}} \frac{m_\psi}{\sqrt{-\dot{c}}}$$

- ▶ Simplified analysis based on Pierre-Auger event of $E = 212 \text{ EeV}$

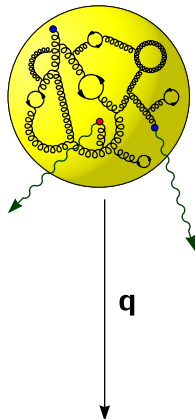
J. Abraham *et al.* [Pierre Auger Collaboration], *Astropart. Phys.* 27, 155 (2007)

- ▶ u and d quarks of proton carrying 10% of E

$$\dot{c}^u > -3 \times 10^{-23}$$

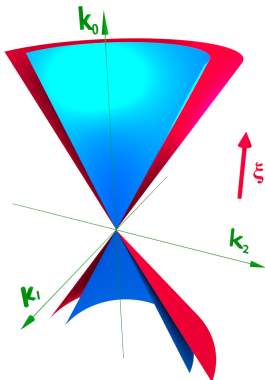
MS, *Phys. Rev. D* 96, 095026 (2017), arXiv:1702.03171 [hep-ph]

- ▶ To be refined by using parton model



Consistency of Standard-Model Extension

Many sectors tested for ...



- ▶ Microcausality
 - ▶ $[A_\mu(x), A_\nu(y)] = 0$ (photons)
 - ▶ $\{\psi(x), \bar{\psi}(y)\} = 0$ (fermions)
 outside of *modified* light cones
- ▶ Unitarity (reflection positivity, validity of optical theorem)
- ▶ Renormalizability (operators of dimensions 3, 4)
- ▶ Consistent cross sections

⇒ Quantum Field Theory very robust with respect to presence of background fields!

V.A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001) [hep-th/0012060]; F.R. Klinkhamer and MS, Nucl. Phys. B **848**, 90 (2011) arXiv:1011.4258 [hep-th]; MS, Phys. Rev. D **86**, 065038 (2012), arXiv:1111.4182 [hep-th]; ...

For higher-dimensional operators: MS, Phys. Rev. D **89**, 105019 (2014), arXiv:1312.4916 [hep-th]; MS, Phys. Rev. D **90**, 085025 (2014), arXiv:1403.6766 [hep-th]; R. Casana, M.M. Ferreira, Jr., A.R. Gomes, and P.R.D. Pinheiro, Phys. Rev. D **80**, 125040 (2009), arXiv:0909.0544 [hep-th]; C.M. Reyes, Phys. Rev. D **87**, 125028 (2013), arXiv:1307.5340 [hep-th]; ...

Outlook

- ▶ (Minimal) Lorentz violation tightly constrained by experiment

Matter			
	Electron	Proton	Neutron
\tilde{b}_X	10^{-31} GeV	10^{-33} GeV	10^{-33} GeV
\tilde{c}_{TX}	10^{-18} GeV	10^{-20} GeV	10^{-17} GeV
\tilde{d}_{XY}	10^{-26} GeV	—	10^{-27} GeV
Photons			
$k_{(V)00}^{(3)}$		10^{-43} GeV	
$(\tilde{\kappa}_{e-})^{XY}$		10^{-22}	
$(\tilde{\kappa}_{o+})^{XY}$		10^{-14}	
Neutrinos			
	electron-muon	electron-tau	mu-tau
$\text{Re}(a_L)^X$	10^{-20} GeV	10^{-19} GeV	10^{-23} GeV
$\text{Re}(c_L)^{XY}$	10^{-21}	10^{-17}	10^{-23}

V.A. Kostelecký and N. Russell, *Rev. Mod. Phys.* **83**, 11 (2011), [arXiv:0801.0287 \[hep-ph\]](https://arxiv.org/abs/0801.0287) and references therein

- ▶ Space for higher-dimensional operators
- ▶ Feedback on fundamental theories of quantum gravity

(Financial) support

Thanks to ...

- ▶ A. Crivellin for the invitation
- ▶ Brazilian research agencies:



- ▶ Federal University of Maranhão, Local particle physics group



- ▶ Some slides are based on material by N. Russell presented at the IUCSS Summer School on the Lorentz- and *CPT*-violating SME, Bloomington (IN), USA, 2012.

For Further Reading



V.A. Kostelecký and S. Samuel,
“Spontaneous breaking of Lorentz symmetry in string theory,”
Phys. Rev. D **39**, 683 (1989).



D. Colladay and V.A. Kostelecký,
“Lorentz-violating extension of the standard model,”
Phys. Rev. D **58**, 116002 (1998),
[hep-ph/9809521](#).



V.A. Kostelecký,
“Gravity, Lorentz violation, and the standard model,”
Phys. Rev. D **69**, 105009 (2004),
[hep-th/0312310](#).



V.A. Kostelecký and N. Russell,
“Data Tables for Lorentz and *CPT* Violation,”
Rev. Mod. Phys. **83**, 11 (2011),
[arXiv:0801.0287 \[hep-ph\]](#).