# The new $\mu \mathrm{E} 4$ separator 

T. Prokscha, LMU, PSI

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The deflection angles $\phi_{E}$ and $\phi_{B}$ by the electric field $E$ and the magnetic field $B$ of an idealized separator are given by (see Sec. 1 and [1])

$$
\begin{align*}
\phi_{E} & \approx \frac{e \cdot l \cdot E}{p \cdot v}=\eta_{E} \cdot \frac{E}{\beta p}  \tag{1}\\
\phi_{B} & \approx \frac{e \cdot l \cdot B}{p}=\eta_{B} \cdot \frac{B}{p} \tag{2}
\end{align*}
$$

Here, $e$ is the electric charge, $l$ the effective length of the field ( $\sim 0.8 \mathrm{~m}$ for SEP61 in $\mu \mathrm{E} 4$ ), $v$ is the velocity of the particle, $p$ the momentum, and $\beta \equiv v / c$. The electric field $E=V / d$ is defined by the voltage difference $V$ between the electrodes and the gap $d=18 \mathrm{~cm}$. The parameters $\eta_{E, B}$ have for $l^{E}=0.803 \mathrm{~m}$ and $l^{B}=0.824 \mathrm{~m}$ values of

$$
\begin{align*}
& \eta_{E}=0.803 \frac{\mathrm{mrad}(\mathrm{MeV} / \mathrm{c})}{\mathrm{kV} / \mathrm{m}}  \tag{3}\\
& \eta_{B}=24.70 \frac{\mathrm{mrad}(\mathrm{MeV} / \mathrm{c})}{\mathrm{G}} \tag{4}
\end{align*}
$$

The total deflection angle $\phi_{t o t}$ of a particle is then simply given by

$$
\begin{equation*}
\phi_{t o t}=\phi_{B}-\phi_{E} . \tag{5}
\end{equation*}
$$

The maximum fields of the new $\mu \mathrm{E} 4$ separator will be $22.22 \mathrm{kV} / \mathrm{cm}$ ( 400 kV voltage difference between the electrodes, $\pm 200 \mathrm{kV}$ at the electrodes) and about 415 G .
The following table gives a short overview on deflecting angles for muons and positrons at $28 \mathrm{MeV} / \mathrm{c}$ and $40 \mathrm{MeV} / \mathrm{c}$ ( $\Delta \phi$ is the angle of muon spin precession):

| $\mathrm{p}[\mathrm{MeV} / \mathrm{c}]$ | particle | $\beta$ | $\mathrm{E}[\mathrm{kV} / \mathrm{m}]$ | $\mathrm{B}[\mathrm{G}]$ | $\phi_{B}[\mathrm{mr}]$ | $\phi_{E}[\mathrm{mr}]$ | $\phi_{\text {tot }}[\mathrm{mr}]$ | $\Delta \phi\left[{ }^{\circ}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 28 |  |  |  |  |  |  |  |  |
| 28 | $e$ | 1 | 1666 | 210 | 185 | 185 | 0 | 10.6 |
| 28 | $\mu$ | 0.2562 | 2220 | 210 | 185 | 47 | 138 |  |
| 28 | $e$ | 1 | 2220 | 281 | 248 | 248 | 0 | 14.2 |
| 40 | $\mu$ | 0.354 | 2220 | 204 | 126 | 126 | 0 | 7.2 |
| 40 | $e$ | 1 | 2220 | 204 | 126 | 44 | 82 |  |



Figure 1: Illustration of deflection angles in homogeneous a) electric and b), magnetic fields of length $l_{\text {eff }}$.

## 1 Deflection angles of electric and magnetic fields

In this section we present the derivation of deflection angles in electric and magnetic fields. Figure 1 illustrates the deflection of charged particles in fields of effective lengths $l_{\text {eff }}$. For the electric field the particle moves on a parabola, and the deflection $s$ in the electric field is given by

$$
\begin{align*}
s & =\frac{1}{2} a t^{2} \\
& =\frac{1}{2} \frac{e \cdot E}{m} \cdot \frac{l_{e f f}^{2}}{v^{2}} . \tag{6}
\end{align*}
$$

The slope of the particles trajectory is then given by

$$
\begin{align*}
\tan \phi_{E} & =\frac{d s}{d l}=2 \cdot \frac{1}{2} \frac{e \cdot E}{m} \cdot \frac{l_{e f f}}{v^{2}} \\
& =e \cdot l_{e f f} \cdot \frac{E}{p \cdot v}  \tag{7}\\
& \Longrightarrow\left(\phi_{E} \ll 1\right) \\
\phi_{E} & \cong \frac{e \cdot l_{e f f}}{c} \cdot \frac{E}{\beta \cdot p}=\eta_{E} \cdot \frac{E}{\beta \cdot p} . \tag{8}
\end{align*}
$$

For the deflection in the magnetic field one obtains from Fig. 1b):

$$
\begin{align*}
\sin \phi_{B} & =\frac{l_{e f f}}{r_{B}} \\
& =e \cdot l_{e f f} \cdot \frac{B}{p}  \tag{9}\\
& \Longrightarrow\left(\phi_{B} \ll 1\right) \\
\phi_{B} & \cong e \cdot l_{e f f} \cdot \frac{B}{p}=\eta_{B} \cdot \frac{B}{p} . \tag{10}
\end{align*}
$$

## References

[1] R. Frosch, Beam Optics with Electrostatic Separators, TM-11-95-01, PSI (1995).

