Higgs boson mass: the quest for precise predictions in SUSY models

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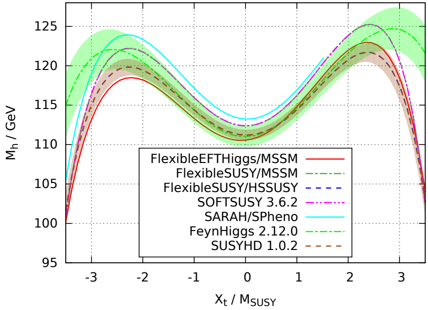
based on: [Athron, Park, Steudtner, DS, Voigt '16] + Kwasnitza

$$M_h^{\mathrm{Exp}} = 125.09 \pm 0.24\mathrm{GeV}$$



Outline

1. Fixed-order, EFT-type, and new combined approach
2. Numerical results
3. Uncertainty estimates
4. Application to non-minimal models, further improvements

Hallmark of SUSY: M_h predictable!

Standard Model: $m_h^2 = \lambda v^2$

MSSM: $\lambda \leftrightarrow$ gauge couplings

$HHHH \leftrightarrow HH\tilde{H}\tilde{H}$

$$m_h^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 - \sqrt{(m_A^2+m_Z^2)^2 - 4m_Z^2 m_A^2 c_{2\beta}^2}\right]$$
$$= v^2 \frac{1}{4}\left(g_Y^2 + g_2^2\right)\cos^2 2\beta + \mathcal{O}\left(\frac{1}{m_A^2}\right)$$

Extremely large loop corrections

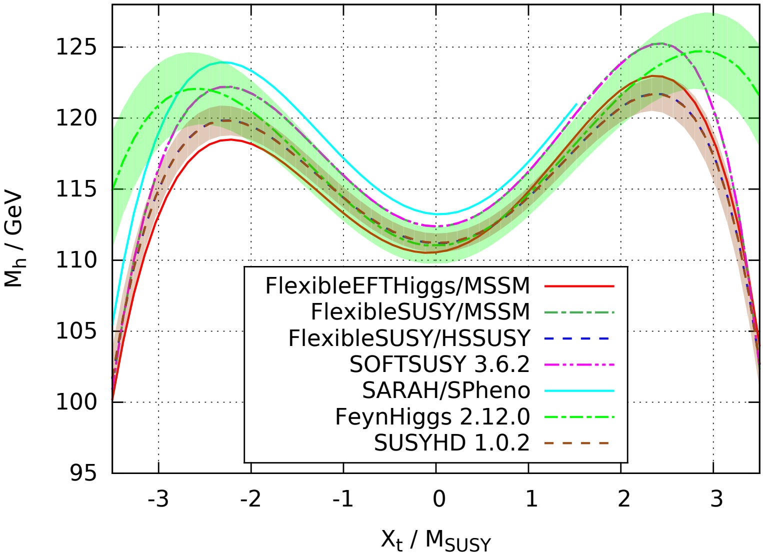
$$\Sigma_h^{\text{leading}} \propto v^2 y_t^4 \left(L \; , \; X_t^2 \; , \; X_t^4 \right) , \quad \text{where } L \equiv \ln \frac{M_{\text{SUSY}}}{M_{\text{weak}}}$$

Compare: RGE for λ has additive term, allows to predict the large log from simple EFT-arguments

$$\beta_\lambda^{\text{SM}} = -12 \kappa_L y_t^4 + \dots$$

In contrast, the X_t-terms originate from finite loop corrections

Plots illustrate qualitative behaviour $\propto X_t^2, X_t^4$



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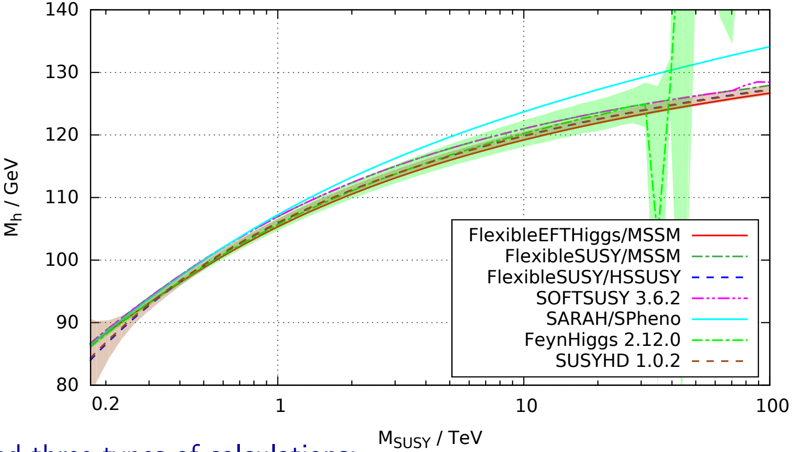
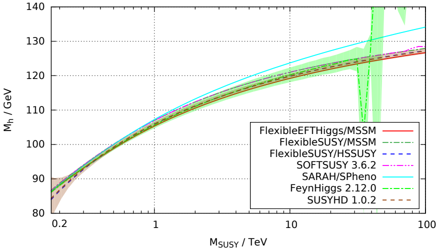
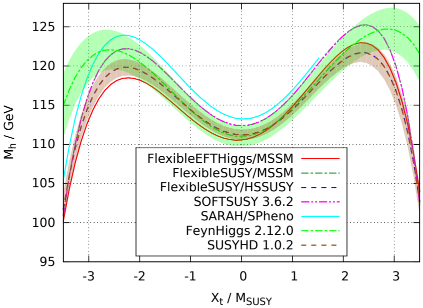


...and three types of calculations:

- Fixed order (Softsusy [Allanach], SPheno [Porod, Staub], FlexibleSUSY), 2-loop (gaugeless limit)
- EFT-type (SUSYHD [Vega,Villadoro], HSSUSY), 2-loop matching/3-loop running
- Combined (FeynHiggs [Hahn et al], FlexibleEFTHiggs)





Aims and questions

- How/why do the calculations differ?
- What is the theory uncertainty?
- Present improved method FlexibleEFTHiggs

Outline

1. Fixed-order, EFT-type, and new combined approach

Overview of M_h calculations

Two basic approaches

$$\text{standard P.T.} = \text{tree} + \mathcal{O}(\alpha) + \mathcal{O}(\alpha^2) + \mathcal{O}(\alpha^3) + \dots$$
$$\text{resummed logs} = \text{tree} + \mathcal{O}(\alpha^n L^n) + \mathcal{O}(\alpha^n L^{n-1}) + \dots$$

Resummation via EFT+RGE $\rightsquigarrow$ neglects terms $\mathcal{O}(1/M_{\text{SUSY}})$

[systematic improvement by orders of $M_{\text{weak}}/M_{\text{SUSY}}$ possible with higher-dimensional operators in EFT]

Combined approaches:

resummed logs + full M_{SUSY}-dependence at fixed order

Overview of Higgs mass calculators

Status of Flexible* and other Higgs mass calculations:

MSSM FS-versions of:		non-MSSM, generated	
FS-original (≈ Softsusy)	fixed 2L	FS-original	fixed 1L
Spheno	"		
SUSYHD,HSSUSY	EFT, 2L matching+3L running		
FEFTHiggs	EFT, 1L matching+3L running	FEFTHiggs	

⇒ many verifications; can study differences in detail. Note: ≥2-loop only in gaugeless limit!

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More Higgs mass calculations in MSSM:

- MSSM, fixed order: H3m, FeynHiggs, Softsusy, Spheno,. . .
- MSSM, EFT: SUSYHD, HSSUSY
- MSSM, combined: FeynHiggs, FlexibleEFTHiggs

Fixed order $\overline{\text{DR}}$ calculation in detail

1. Find $\overline{\text{DR}}$ parameters $\tilde{g}_i$, $\tilde{y}_t$, $\tilde{m}_Z$, ... at the SUSY scale. E.g. $\tilde{y}_t$ from

$$m_t^{\text{FS,SPh}} = M_t + \Sigma_t^{(1)}\left(\left\{\begin{array}{c} M_{t\text{FS}} \\ \tilde{m}_{t\text{SPh}} \end{array}\right\}\right)$$

(FS≈Softsusy)

2. Calculate the Higgs pole mass from the $\overline{\text{DR}}$ parameters.

$$0 = \det\left[p^2\delta_{ij} - (m_\phi^2)_{ij} + \tilde{\Sigma}_{\phi,ij}(p^2)\right]$$

($\tilde{\Sigma}$ includes tadpole term)

Example: leading logs in fixed-order calculations

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$$M_h^2 = m_h^2 + \tilde{v}^2\tilde{y}_t^4\left(12L\kappa_L + 192\tilde{g}_3^2L^2\kappa_L^2\right)$$

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1. Find $\overline{\text{DR}}$ parameters $\tilde{g}_i$, $\tilde{y}_t$, $\tilde{m}_Z$, …at the SUSY scale. E.g. $\tilde{y}_t$ from

   (in terms of low-energy SM parameters $\hat{y}_t$, $\hat{g}_3$:)

$$\tilde{y}_t^{\text{FS,SPh}} = \hat{y}_t \left(1 - 8\hat{g}_3^2 L\kappa_L + \begin{Bmatrix} \frac{976}{9} \text{FS} \\ \frac{1040}{9} \text{SPh} \end{Bmatrix} \hat{g}_3^4 L^2 \kappa_L^2\right) + \ldots$$

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$$M_h^2 = m_h^2 + \hat{v}^2 \hat{y}_t^4 \left[12L\kappa_L - 192\hat{g}_3^2 L^2 \kappa_L^2 + 4\hat{g}_3^4 L^3 \kappa_L^3 \begin{Bmatrix} 736_{\text{EFT}} \\ \frac{736}{3}\text{FS} \\ \frac{992}{3}\text{SPh} \end{Bmatrix} + \cdots\right]$$

Hence: not fixed order w.r.t. low-energy parameters, induced 3-loop terms different and wrong!

EFT-type calculation in detail

- assume SUSY masses at $M_{\text{SUSY}} \sim Q_{\text{match}} \gg M_{\text{weak}} \sim Q$
- assume SM = correct low-energy EFT below Q_{match}
- Then: setup for correct terms of order $\alpha^n L^n$, $\alpha^n L^{n-1}$

1. at $\mu = Q_{\text{match}}$: integrate out SUSY, match to SM — need 1-loop δp_i

$$p_i^{\text{SM}}(\mu) = p_i^{\text{SUSY}}(\mu) + \delta p_i$$

2. between $Q_{\text{match}} > \mu > Q$: run in SM — need 2-loop $\beta_{p_i}^{\text{SM}}$

$$\frac{\mathrm{d}p_i^{\text{SM}}(\mu)}{\mathrm{d}\ln\mu} = \beta_{p_i}^{\text{SM}}(\mu)$$

3. at $\mu = Q$: compute Higgs mass in SM, match to M_t, α_s^{Exp} ... — need 1-loop SM $\hat{\Sigma}_h$

$$M_h^2 = \lambda^{\text{SM}}(M_{\text{weak}})v^2 + \hat{\Sigma}_h$$

Example: leading logs in EFT-type calculations

1. at $\mu = Q_{\mathsf{match}}$: integrate out SUSY, match to SM

2. between $Q_{\mathsf{match}} > \mu > Q$: run in SM

3. at $\mu = Q$: compute Higgs mass in SM, match to M_t, α_s^{Exp} ...

Example: leading logs in EFT-type calculations

1. at $\mu = Q_{\mathsf{match}}$: integrate out SUSY, match to SM

$$\lambda \equiv \lambda^{\mathsf{SM}}(Q_{\mathsf{match}}) = \frac{m_h^{\mathsf{MSSM}}}{v^2}$$

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3. at $\mu = Q$: compute Higgs mass in SM, match to M_t, α_s^{Exp} ...

Example: leading logs in EFT-type calculations

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$$\hat{\lambda} = \lambda + \hat{y}_t^4 \left[12 L \kappa_L - 192 \hat{g}_3^2 L^2 \kappa_L^2 + 4 \hat{g}_3^4 L^3 \kappa_L^3 (736)\right] + \ldots$$

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$$M_h^2 = \hat{\lambda} \hat{v}^2$$

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EFT-type calculation in detail: matching at SUSY scale

"pure EFT" (SUSYHD, HSSUSY):

require $\Gamma^{\text{full}}(p=0) = \Gamma^{\text{EFT}}(p=0)$ in limit $M_{\text{SUSY}} \to \infty$

$$\lambda = \frac{1}{4}\left(g_Y^2 + g_2^2\right)\cos^2 2\beta + \Delta\lambda^{(1)} + \Delta\lambda^{(2)}$$

Pro: clean expansion in well-defined orders

Con: neglects $1/M_{\text{SUSY}}$-terms already at tree-level!

Possible improvements: EFT+non-renormalizable operators

FeynHiggs: combined approach, add resummed logs onto fixed-order calculation without double counting

EFT-type calculation in detail: matching at SUSY scale

FlexibleEFTHiggs: pole mass matching:

require $M_h^{\text{pole,full}} = M_h^{\text{pole,EFT}}$

$$\lambda = \frac{1}{v^2}\left[(M_h^{\text{MSSM}})^2 + \tilde{\Sigma}_h^{\text{SM}}((M_h^{\text{SM}})^2)\right]$$

$y_t, m_Z, \ldots$ similar

Pro: exact at tree-level and 1-loop (2-loop can/will be included)

Pro: easier to automate for non-minimal SUSY

Con: can contain superfluous 2-loop terms, e.g. $\propto \Delta y_t^4 * L$

Further details and discussion

"pure EFT"

$$\lambda = \frac{1}{4}\left(g_Y^2 + g_2^2\right)\cos^2 2\beta + \frac{1}{(4\pi)^2}\left[3(y_t^{\text{SM}})^4\left(\ln\frac{m_{Q_3}^2}{Q^2}\right) + \frac{6(y_t^{\text{SM}})^4 X_t^2 \ln\frac{m_{Q_3}^2}{m_{U_3}^2}}{m_{Q_3}^2 - m_{U_3}^2}\right] + \ldots$$

pole mass matching

$$\lambda = \frac{1}{v^2}\left[(m_h^{\text{MSSM}})^2 - \tilde{\Sigma}_h^{\text{MSSM}}((M_h^{\text{MSSM}})^2, y_t^{\text{MSSM}}, \ldots) + \tilde{\Sigma}_h^{\text{SM}}((M_h^{\text{SM}})^2, y_t^{\text{SM}}, \ldots)\right]$$

- Equivalence at one-loop up to $\mathcal{O}(M_{\text{weak}}^2/M_{\text{SUSY}}^2)$ [Athron,Park,Steudtner,DS,Voigt]
- Equivalence at two-loop [Kwasnitza, Voigt]
- "superfluous" 2-loop terms in one-loop matching

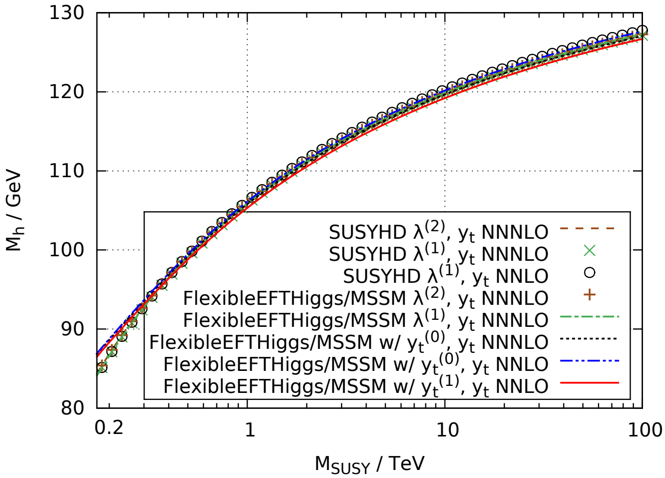
- form of terms: ln Q^2-dependent/independent! How to estimate missing higher-order terms?

Outline

2. Numerical results
   - Verify expected similarities, differences between calculations

Verification: step-by-step comparison to SUSYHD

$X_t = 0$, so $\Delta\lambda^{(2)} \approx 0$



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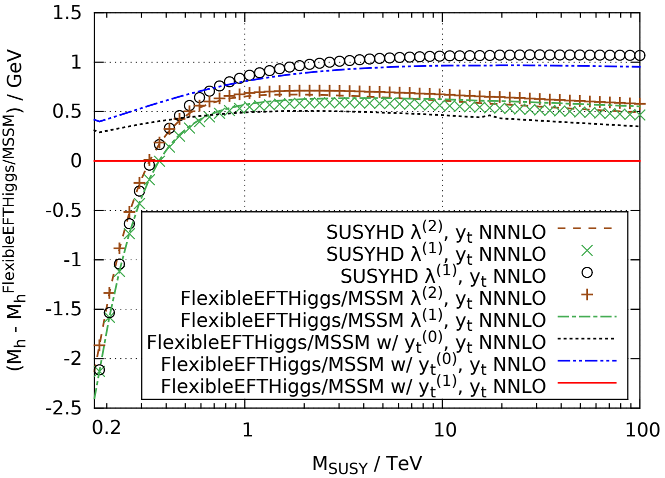
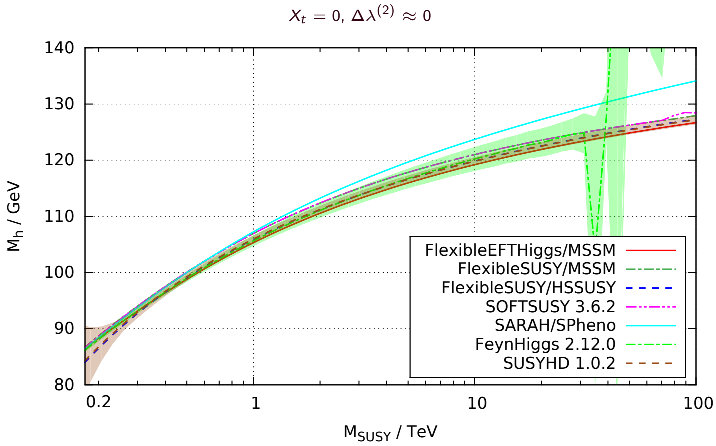


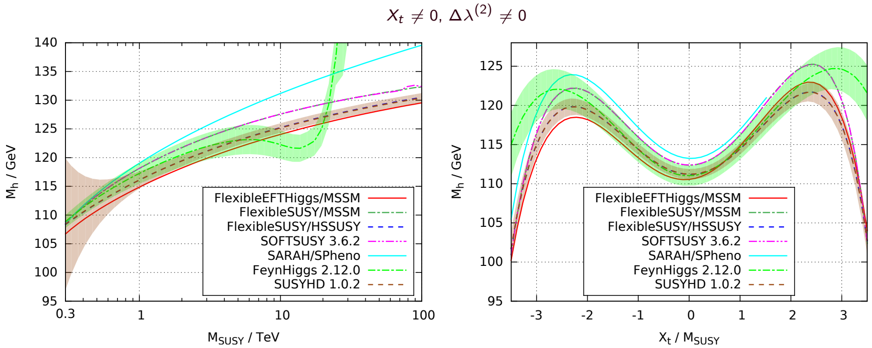
- agreement with replica of SUSYHD, two-loop matching here unimportant (common M_{SUSY}, $X_t = 0$)
- FlexibleEFTHiggs-like matching: drastic change at low M_{SUSY} $\rightsquigarrow$ uncertainty of SUSYHD
- changes of y_t at matching/low scales $\rightsquigarrow$ higher-order effect, numerically sizeable

Comparison fixed-order and EFT results



- FEFTHiggs agrees with pure EFT for large masses
- and agrees with fixed-order calculations for masses ⇝ "interpolates"
- fixed-order calculations differ strongly at high M_{SUSY} ⇝ theory uncertainty

Comparison fixed-order and EFT results



- non-log shift between FEFTHiggs and SUSYHD from missing 2-loop matching
- fixed-order calculations differ strongly at high M_{SUSY} $\rightsquigarrow$ theory uncertainty

Outline

3. Uncertainty estimates
   - Find comprehensive estimates for fixed-order and EFT

Uncertainties of fixed-order calculations

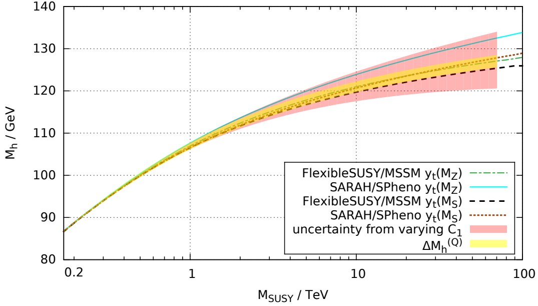
Source:

- missing ≥ 3 loop terms ($\propto L^3, L^2, L, 1$)

Estimates:

1. Estimate using known MSSM higher-order results
2. Estimate from generating motivated higher-order terms
   1. top-mass definition (sensitive to L^3)
   2. renormalization scale (sensitive to $\leq L^2$)

Illustrate fixed-order uncertainty estimates

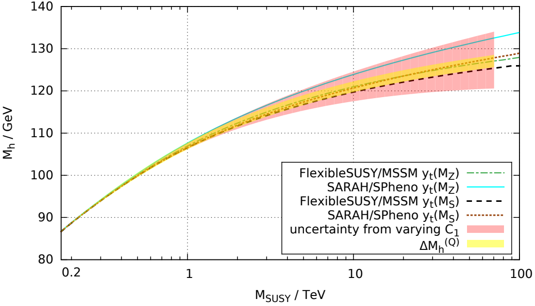


Estimate using known MSSM higher-order results: change

$$m_t^{FS,SPh} = M_t + \Sigma_t^{(1)}(\ldots) \pm \Delta m_t^{(2),\text{known}}$$

induces leading 3-loop change in M_h

Illustrate fixed-order uncertainty estimates

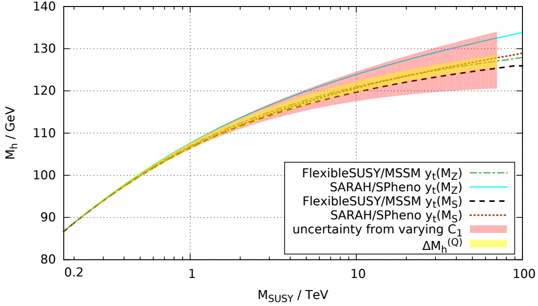


Estimate from generating motivated higher-order terms:

$$m_t^{FS,SPh} = M_t + \Sigma_t^{(1)}\left(\left\{\begin{array}{c} M_{t\,\text{FS}} \\ m_{t\,\text{SPh}} \end{array}\right\}\right) \qquad \text{at} \left\{\begin{array}{c} M_{\text{SUSY}} \text{ or} \\ M_{\text{weak}} \end{array}\right\}$$

four options also induce leading 3-loop changes in M_h

Illustrate fixed-order uncertainty estimates



renormalization scale Q varied by factor 2

induces change in M_h of

$\mathcal{O}$(3-loop $\times$ L^2 $\times$ ln(2)) and $\mathcal{O}$(2-loop, non-gaugeless $\times$ L $\times$ ln(2))

scale variation by itself not sufficient!

Uncertainties of fixed-order calculations

Source:

- missing ≥ 3 loop terms ($\propto L^3, L^2, L, 1$)

Estimates:

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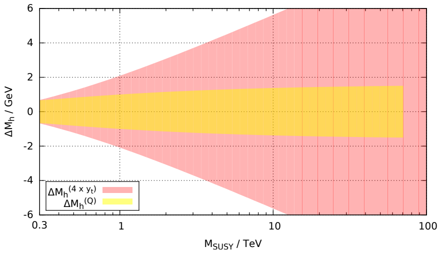
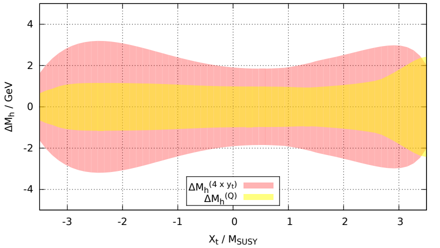
   2. renormalization scale (sensitive to $\leq L^2$)

Comments: method can be applied to non-minimal models. Could be an underestimate of the uncertainty: missing estimates for 3LL terms governed by not y_t, non-divergent 3NLL terms.

Summary of fixed-order uncertainty estimates





Uncertainties of new EFT calculation

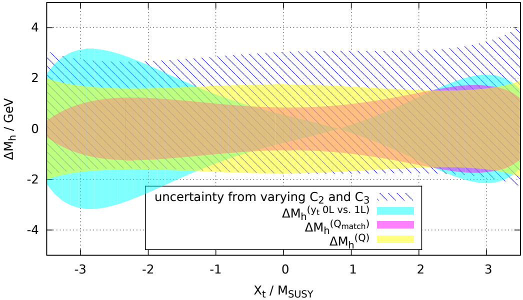
Sources:

- high-scale uncertainty: missing ≥ 2 loop matching corrections
- low-scale uncertainty: missing ≥ 2 loop terms in low-scale Higgs pole mass calculation
- EFT-uncertainty (in SUSYHD/HSSUSY): from missing $1/M_{\text{SUSY}}$-suppressed terms

Estimate high-scale uncertainty:

1. Estimate using known MSSM higher-order results
2. Estimate from generating motivated higher-order terms
   1. top-mass matching either at tree-level or 1-loop ($\Delta y_t^{1L} \sim X_t, m_{\text{gluino}}$)
   2. matching scale (sensitive to divergent terms, not to X_t)

Illustrate FlexibleEFTHiggs uncertainty estimates



Estimate using known MSSM higher-order results: change

$$\lambda \to \lambda \pm \Delta\lambda^{(2),\text{known}}$$

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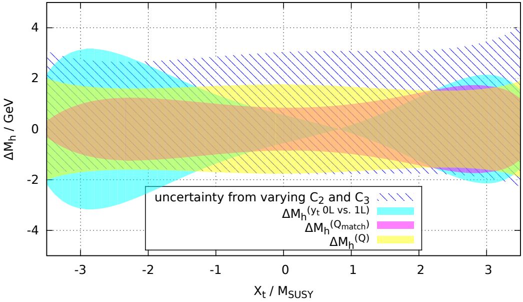
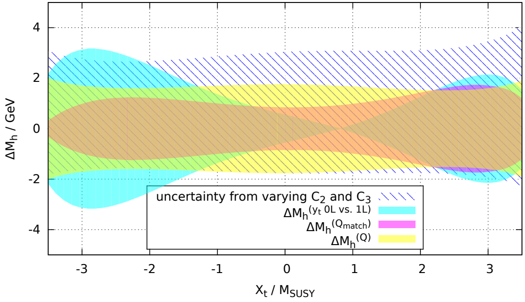


Estimate from generating motivated higher-order terms

- y_t-matching either at tree-level or 1-loop
- matching-scale variation

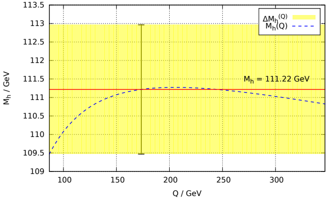
induce 2-loop changes in λ-matching

Illustrate FlexibleEFTHiggs uncertainty estimates

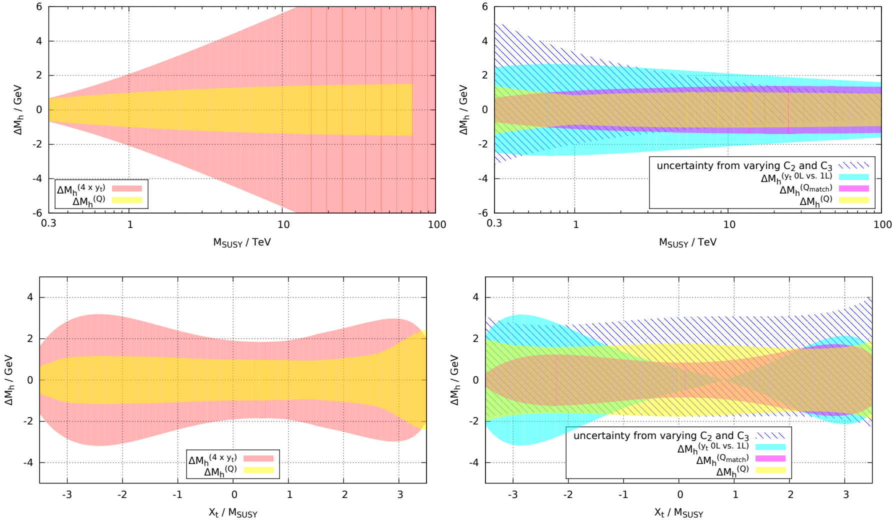


Low-scale uncertainty: missing SM higher orders

However: non-linear behaviour, different definitions possible



Summary of fixed-order and FlexibleEFTHiggs uncertainty estimates



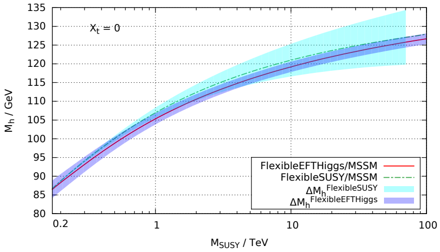
Combined uncertainty estimates

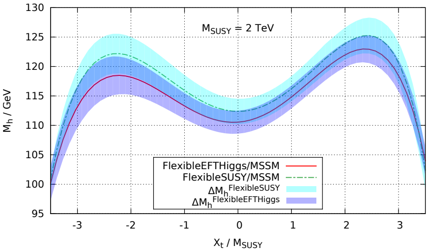
fixed-order: combine quadratically

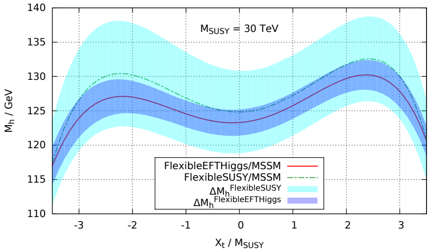
$$\Delta M_h^{(4\times y_t)}, \Delta M_h^{(Q)} \tag{1}$$

EFT: combine quadratically

$$\max\left[\Delta M_h^{(y_t \text{ 0L vs. 1L})}, \Delta M_h^{(Q_{\text{match}})}\right], \Delta M_h^{(Q)} \tag{2}$$





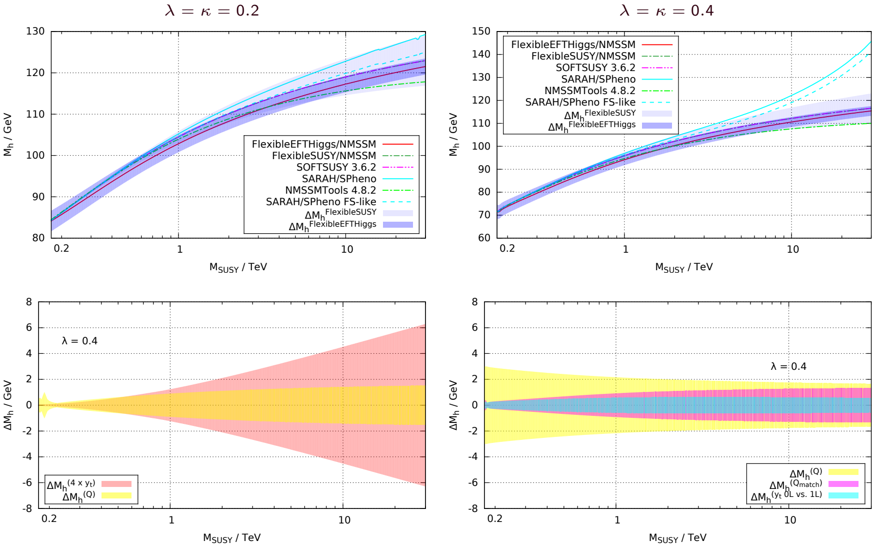


- FlexibleEFTHiggs more precise than fixed order above $\sim$ 2 TeV
- reliable at low and high M_{SUSY}
- uncertainty estimates are conservative and overlap consistently

Outline

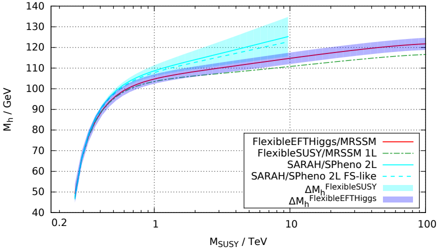
4. Application to non-minimal models, further improvements

NMSSM



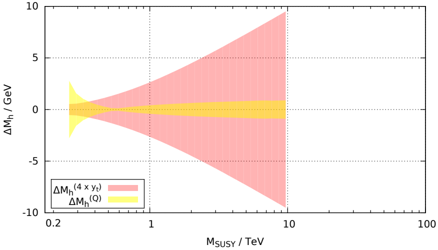
- Only SPheno has complete 2-loop $\rightsquigarrow$ IR catastrophe, result unreliable for high M_{SUSY}

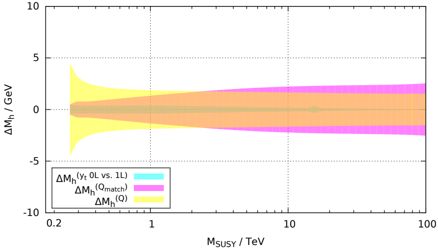
MRSSM



Point	SPheno 1L	SPheno 2L	SPheno 1L, (5)	SPheno 2L, (5)	FlexibleSUSY 1L	FlexibleEFT-Higgs 1L
BM1′	120.4	125.6 ± 1.3	120.0	125.1 ± 1.3	120.6	122.1 ± 1.7
BM2′	120.8	126.0 ± 1.1	120.4	125.6 ± 1.1	120.2	121.7 ± 1.8
BM3′	121.0	125.7 ± 1.3	120.5	125.2 ± 1.3	120.4	121.9 ± 1.9

points from Diessner, Kalinowski, Kotlarski, DS

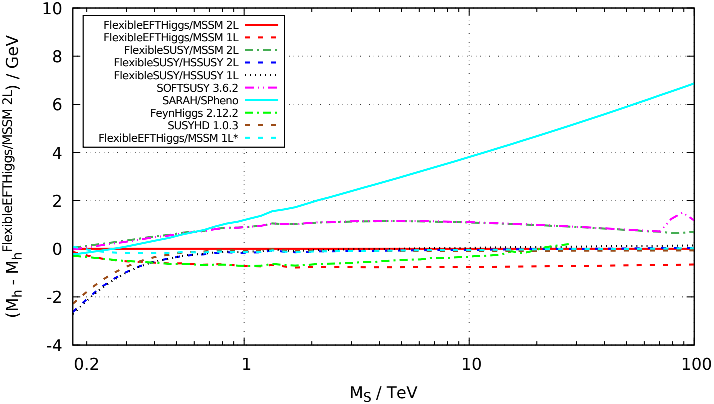




- Complementary to/more precise than Sarah/SPheno (FO 2-loop), but uncertainty probably still underestimated

Improvements

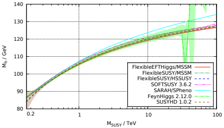
1. 1-loop matching → 2-loop matching
2. remove superfluous 2-loop terms induced in 1-loop matching



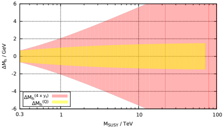
Alexander Voigt + Thomas Kwasnitza, preliminary

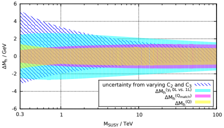
Conclusions

- New approach combines FO + EFT
  - understand differences, pros and cons

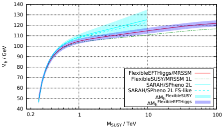


- Comprehensive uncertainty estimates
  - for fixed-order and EFT
  - y_t-change and scale variation





- Applicable to non-minimal models
  - NMSSM, MRSSM, ...
  - complementary to Sarah/SPheno



1. Find all $\overline{\text{DR}}$ parameters at the SUSY scale.

$$\alpha_s^{\overline{\text{DR}},\text{SUSY}}(M_Z) = \frac{\alpha_s^{\overline{\text{MS}},\text{SM(5)}}(M_Z)}{1 - \Delta\alpha_s^{\text{SM}}(M_Z) - \Delta\alpha_s^{\text{SUSY}}(M_Z)}, \tag{3}$$

$$m_Z^{\overline{\text{DR}},\text{SUSY}}(M_Z) = \sqrt{M_Z^2 + \text{Re}\,\Pi_{ZZ}^{T,\text{SUSY}}(M_Z^2)}, \tag{4}$$

$$\begin{aligned} m_t^{\overline{\text{DR}},\text{SOFTSUSY}} &= M_t + \text{Re}\left[\widetilde{\Sigma}_t^{(1),S}(M_t)\right] + M_t\,\text{Re}\left[\widetilde{\Sigma}_t^{(1),L}(M_t) + \widetilde{\Sigma}_t^{(1),R}(M_t)\right] \\ &+ M_t\left[\widetilde{\Sigma}_t^{(1),\text{qcd}}(m_t^{\overline{\text{DR}}}) + \left(\widetilde{\Sigma}_t^{(1),\text{qcd}}(m_t^{\overline{\text{DR}}})\right)^2 + \widetilde{\Sigma}_t^{(2),\text{qcd}}(m_t^{\overline{\text{DR}}})\right], \end{aligned} \tag{5}$$

$$\begin{aligned} m_t^{\overline{\text{DR}},\text{SPHENO}} &= M_t + \text{Re}\left[\widetilde{\Sigma}_t^{(1),S}(m_t^{\overline{\text{DR}}})\right] + m_t^{\overline{\text{DR}}}\,\text{Re}\left[\widetilde{\Sigma}_t^{(1),L}(m_t^{\overline{\text{DR}}}) + \widetilde{\Sigma}_t^{(1),R}(m_t^{\overline{\text{DR}}})\right] \\ &+ m_t^{\overline{\text{DR}}}\left[\widetilde{\Sigma}_t^{(1),\text{qcd}}(m_t^{\overline{\text{DR}}}) + \widetilde{\Sigma}_t^{(2),\text{qcd}}(m_t^{\overline{\text{DR}}})\right]. \end{aligned} \tag{6}$$

2. Calculate the Higgs pole mass from the $\overline{\text{DR}}$ parameters.

$$0 = \det\left[p^2\delta_{ij} - (m_\phi^2)_{ij} + \text{Re}\,\Sigma_{\phi,ij}(p^2) - \frac{t_{\phi,i}}{v_i}\right], \tag{7}$$

Example: leading logs in fixed-order calculations

$$M_h^2 = m_h^2 + \tilde{v}^2 \tilde{y}_t^4 \left(L 12 \kappa_L + L^2 (192 \kappa_L^2 \tilde{g}_3^2 - 108 \kappa_L^2 \tilde{y}_t^2) \right) \tag{8}$$

Yukawa running coupling expressed in terms of low-energy SM Yukawa $\sim M_t$:

$$\tilde{y}_t^{\text{FlexibleSUSY}} = \hat{y}_t + L\kappa_L \left(\frac{9\hat{y}_t^3}{2} - 8\hat{g}_3^2 \hat{y}_t \right) + L^2 \kappa_L^2 \left(\frac{976 \hat{g}_3^4 \hat{y}_t}{9} - 96 \hat{g}_3^2 \hat{y}_t^3 + \frac{63 \hat{y}_t^5}{2} \right) + \ldots, \tag{9}$$

$$\tilde{y}_t^{\text{SPheno}} = \hat{y}_t + L\kappa_L \left(\frac{9\hat{y}_t^3}{2} - 8\hat{g}_3^2 \hat{y}_t \right) + L^2 \kappa_L^2 \left(\frac{1040 \hat{g}_3^4 \hat{y}_t}{9} - 88 \hat{g}_3^2 \hat{y}_t^3 + \frac{135 \hat{y}_t^5}{4} \right) + \ldots, \tag{10}$$

EFT-procedure to obtain all leading logs correctly: integrate SM RGEs to obtain $\hat{\lambda}$ as a function of $\lambda(t)$ and $\hat{y}_t$; then compute M_h:

$$\hat{\lambda} = \lambda + 12 L \kappa_L \hat{y}_t^4 - 12 L^2 \kappa_L^2 \left(16 \hat{g}_3^2 \hat{y}_t^4 - 3 \hat{y}_t^6 \right) + 4 L^3 \kappa_L^3 \left(240 \hat{g}_3^2 \hat{y}_t^6 - 736 \hat{g}_3^4 \hat{y}_t^4 + 99 \hat{y}_t^8 \right) + \ldots \tag{11}$$

Hence:

$$\begin{aligned} (M_h^2)^X &= m_h^2 + \hat{v}^2 \hat{y}_t^4 \left[12 L \kappa_L - 12 L^2 \kappa_L^2 \left(16 \hat{g}_3^2 - 3 \hat{y}_t^2 \right) + 4 L^3 \kappa_L^3 \Delta_{\text{3LLL}}^X + \cdots \right], \\ \Delta_{\text{3LLL}}^X &= \begin{cases} 736 \hat{g}_3^4 - 240 \hat{g}_3^2 \hat{y}_t^2 - 99 \hat{y}_t^4 & (X = \text{EFT}), \\ \frac{736}{3} \hat{g}_3^4 + 144 \hat{g}_3^2 \hat{y}_t^2 - \frac{351}{2} \hat{y}_t^4 & (X = \text{FlexibleSUSY/SOFTSUSY}), \\ \frac{992}{3} \hat{g}_3^4 + 240 \hat{g}_3^2 \hat{y}_t^2 - \frac{297}{2} \hat{y}_t^4 & (X = \text{SPheno}), \end{cases} \end{aligned} \tag{12}$$

Example: leading logs in EFT-type calculations

$$M_h^2 = m_h^2 + \tilde{v}^2\tilde{y}_t^4\left(L12\kappa_L + L^2(192\kappa_L^2\tilde{g}_3^2 - 108\kappa_L^2\tilde{y}_t^2)\right) \quad (13)$$

Yukawa running coupling expressed in terms of low-energy SM Yukawa $\sim M_t$:

$$\tilde{y}_t^{\text{FlexibleSUSY}} = \hat{y}_t + L\kappa_L\left(\frac{9\hat{y}_t^3}{2} - 8\hat{g}_3^2\hat{y}_t\right) + L^2\kappa_L^2\left(\frac{976\hat{g}_3^4\hat{y}_t}{9} - 96\hat{g}_3^2\hat{y}_t^3 + \frac{63\hat{y}_t^5}{2}\right) + \ldots, \quad (14)$$

$$\tilde{y}_t^{\text{SPheno}} = \hat{y}_t + L\kappa_L\left(\frac{9\hat{y}_t^3}{2} - 8\hat{g}_3^2\hat{y}_t\right) + L^2\kappa_L^2\left(\frac{1040\hat{g}_3^4\hat{y}_t}{9} - 88\hat{g}_3^2\hat{y}_t^3 + \frac{135\hat{y}_t^5}{4}\right) + \ldots, \quad (15)$$

EFT-procedure to obtain all leading logs correctly: integrate SM RGEs to obtain $\hat{\lambda}$ as a function of $\lambda(t)$ and $\hat{y}_t$; then compute M_h:

$$\hat{\lambda} = \lambda + 12L\kappa_L\hat{y}_t^4 - 12L^2\kappa_L^2\left(16\hat{g}_3^2\hat{y}_t^4 - 3\hat{y}_t^6\right) + 4L^3\kappa_L^3\left(240\hat{g}_3^2\hat{y}_t^6 - 736\hat{g}_3^4\hat{y}_t^4 + 99\hat{y}_t^8\right) + \ldots \quad (16)$$

Hence:

$$\begin{aligned}(M_h^2)^X &= m_h^2 + \hat{v}^2\hat{y}_t^4\left[12L\kappa_L - 12L^2\kappa_L^2\left(16\hat{g}_3^2 - 3\hat{y}_t^2\right) + 4L^3\kappa_L^3\Delta_{3\text{LLL}}^X + \cdots\right],\\ \Delta_{3\text{LLL}}^X &= \begin{cases} 736\hat{g}_3^4 - 240\hat{g}_3^2\hat{y}_t^2 - 99\hat{y}_t^4 & (X = \text{EFT}),\\ \frac{736}{3}\hat{g}_3^4 + 144\hat{g}_3^2\hat{y}_t^2 - \frac{351}{2}\hat{y}_t^4 & (X = \text{FlexibleSUSY/SOFTSUSY}),\\ \frac{992}{3}\hat{g}_3^4 + 240\hat{g}_3^2\hat{y}_t^2 - \frac{297}{2}\hat{y}_t^4 & (X = \text{SPheno}),\end{cases}\end{aligned} \quad (17)$$

MRSSM parameter point

$$
\begin{aligned}
&m_S^2 = m_T^2 = m_O^2 = m_{R_d}^2 = m_{R_u}^2 = 10 M_{\text{SUSY}}^2, \\
&M_B^D = M_W^D = M_g^D = M_{\text{SUSY}}, \\
&\mu_u = \mu_d = 1\,\text{TeV}, \tan\beta = 5, \\
&\Lambda_u = \Lambda_d = -0.5, \lambda_u = \lambda_d = -0.01.
\end{aligned} \tag{18}
$$